

1 Algorithm Example

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1: counter ← Sketch.total
2: for h = 1 to H do
3:   i ← Sketch[h].hash(k1)
4:   if Sketch[h][i].total ≤ counter then
5:     index ← h
6:     hash ← i
7:     counter ← Sketch[h][i].total
8:   end if
9: end for
10: result ← Sketch[index][hash][k2]

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2 Theory Example

Lemma 1 Let $h_1 \dots h_n : \{1 \dots k\} \rightarrow \{1 \dots v\}$, H be a set of sets H_i where $h_i(x) \in H_i$, $S = \{h_i^{-1}(x) | x \in \bigcup H\}$.

$$\forall H_i \in H, |H_i| = \sum_j^{j \in S} \begin{cases} 0 & \text{if } h_i(j) \notin H_i \\ 1 & \text{otherwise} \end{cases}.$$

Corollary 1 As a consequence of Lemma 1,

$$\forall H_i \in H, \sum_j^{j \in H_i} j = \sum_j^{j \in S} \begin{cases} 0 & \text{if } h(j) \notin H_i \\ h(j) & \text{otherwise} \end{cases}.$$

Therefore, if we were to remove an arbitrary element x from S , then the sum would become:

$$\forall H_i \in H, \sum_j^{j \in H_i} j = \sum_j^{j \in S - \{x\}} \begin{cases} 0 & \text{if } h(j) \notin H_i \\ h(j) & \text{otherwise} \end{cases} + h_i(x).$$

Finally, this allows us to conclude that in the case of the sum of all elements in $S - \{x\}$ which have images in H_i is equal to $\sum_j^{j \in H_i} j - h_i(x)$.

3 Tikz Example

