Recursion in 20”

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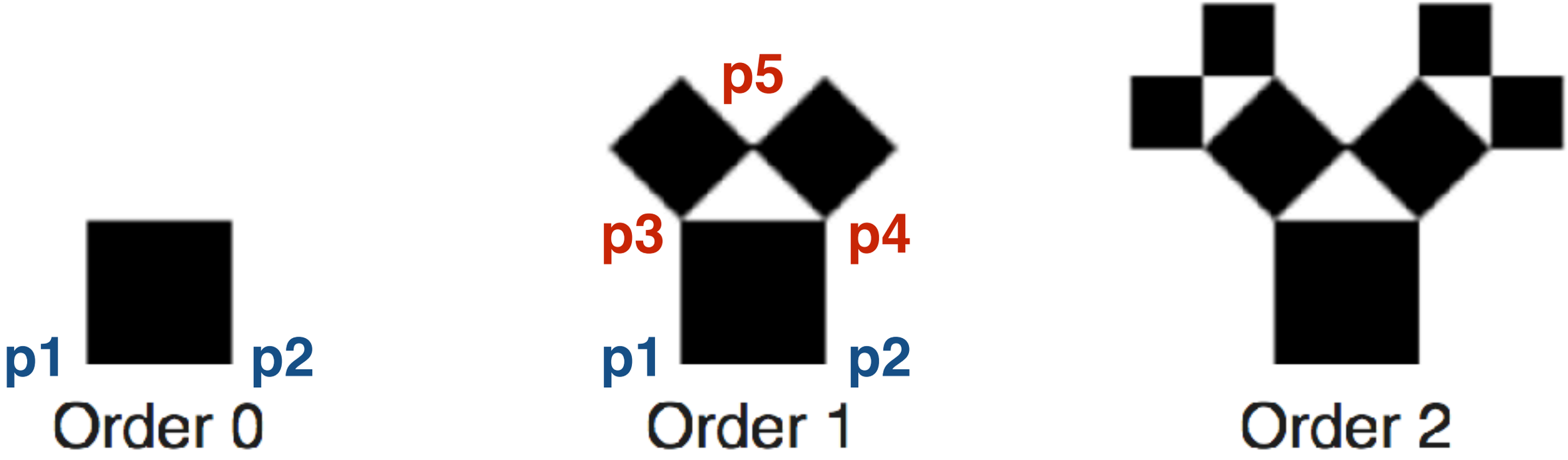
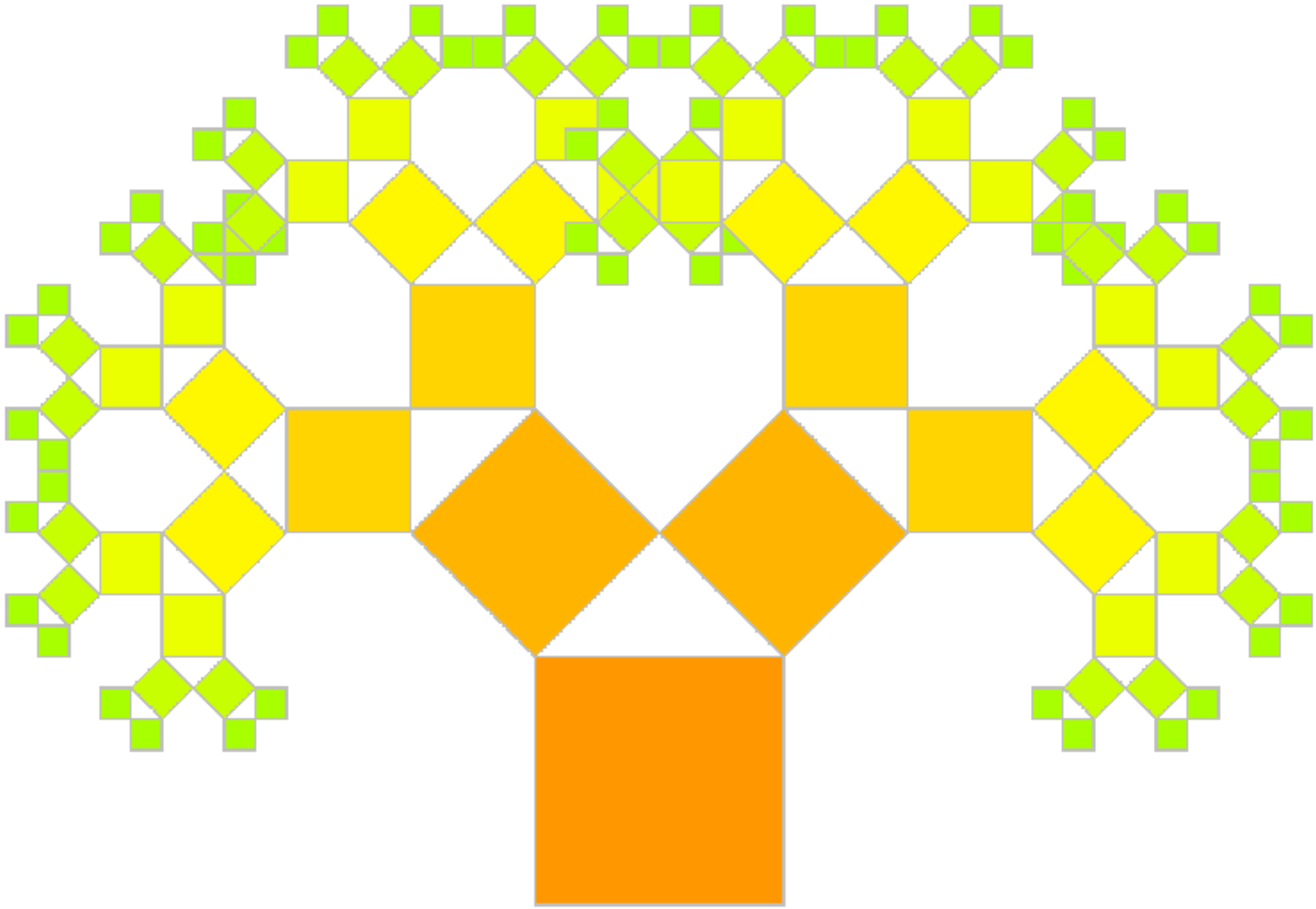
https://github.com/mircealungu/open_lectures/recursion

Pythagoras' Tree



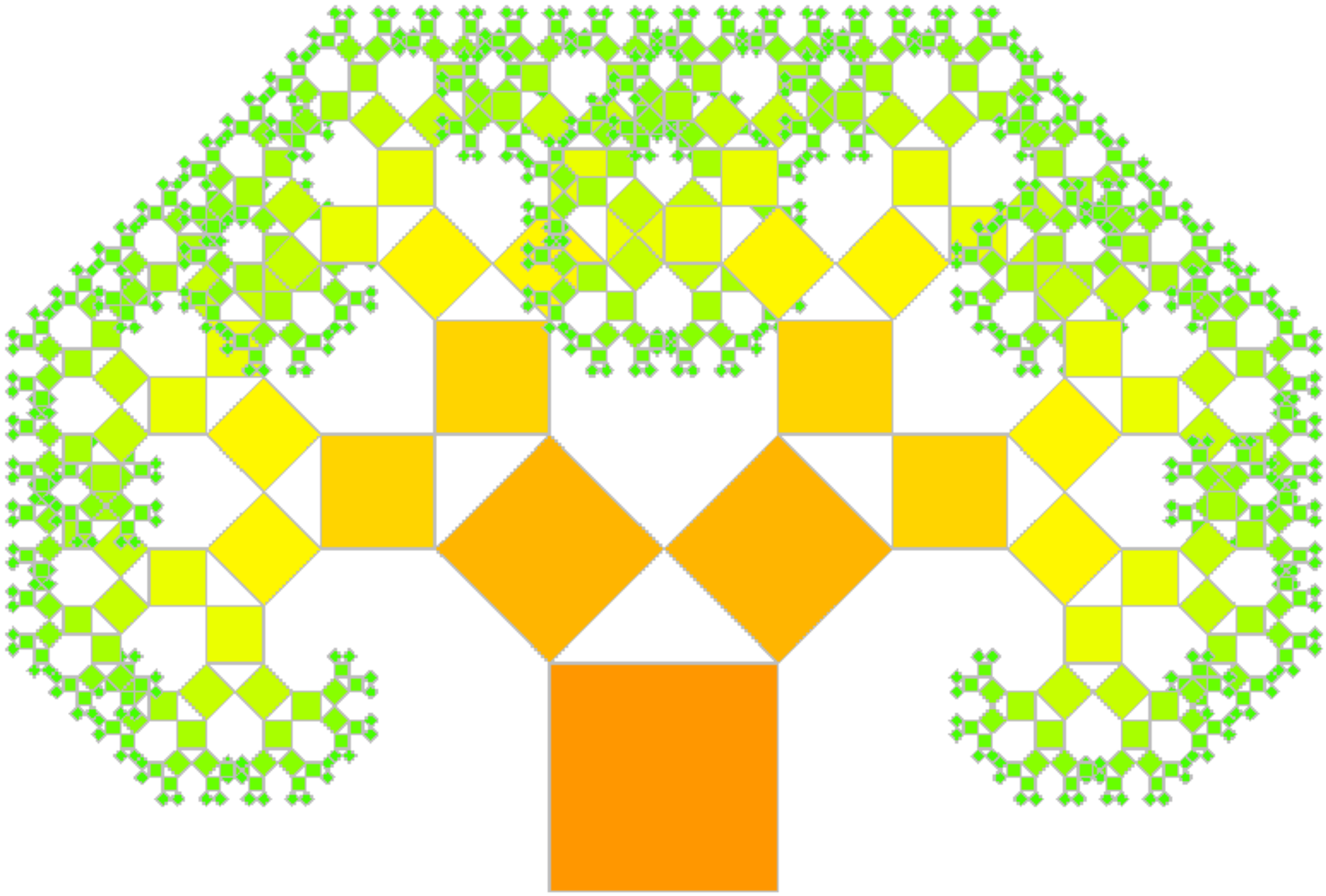
Code on GitHub

Order 7



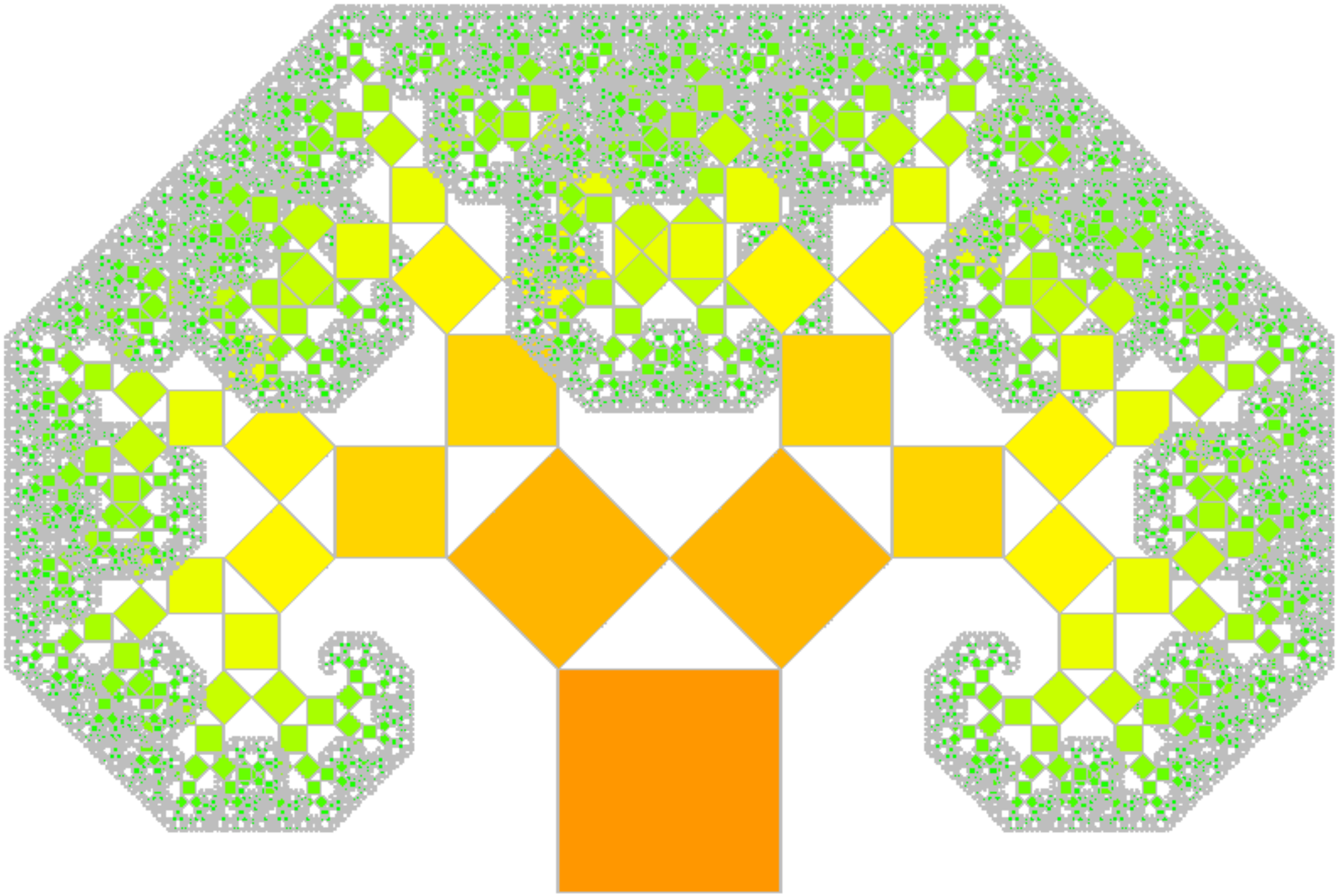
Code on GitHub

Order 10



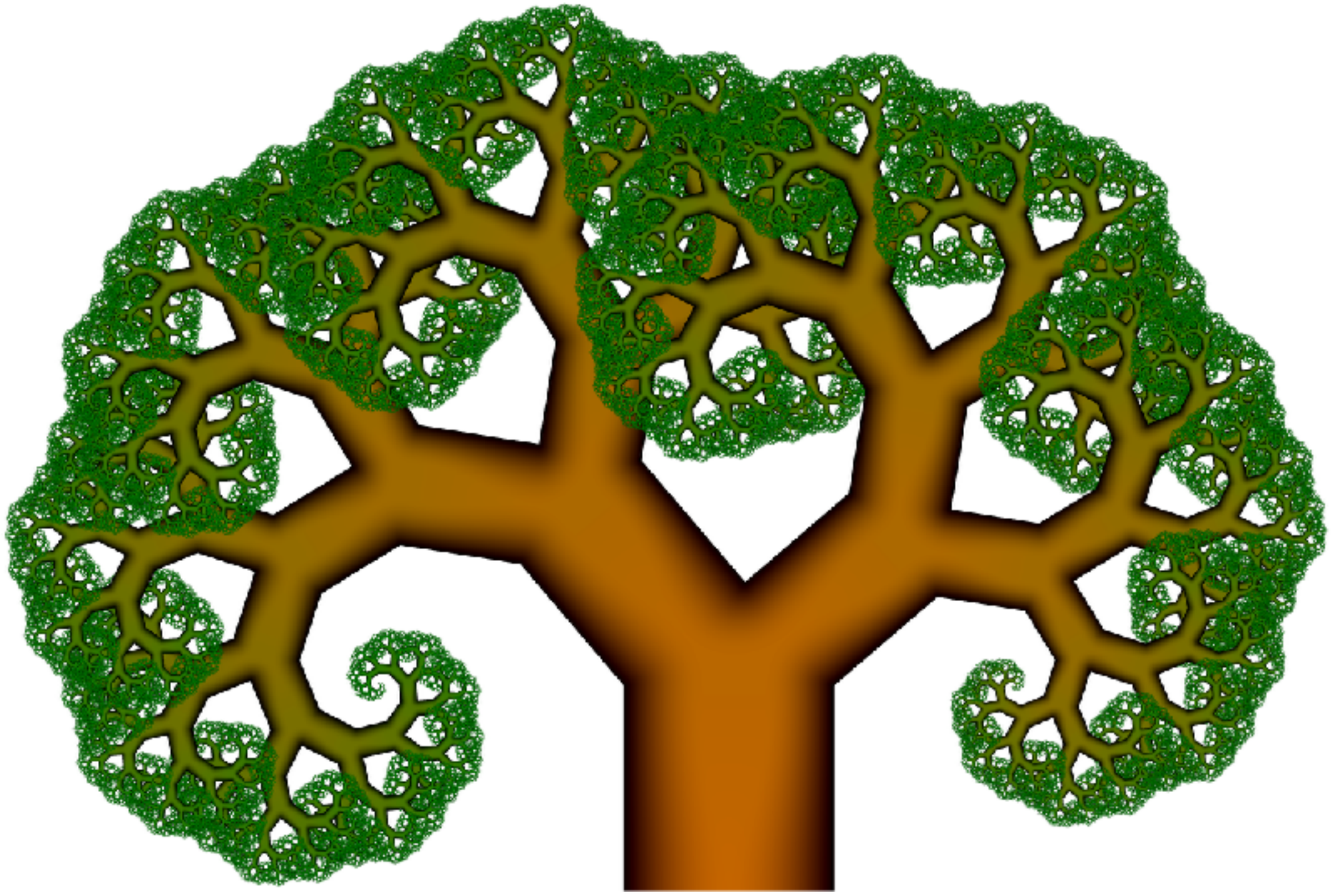
Code on GitHub

Order 15



Code on GitHub

Order ??



Warning: not on GitHub anymore!!!

By Atze van der Ploeg Atzecsse - Own work,
Attribution, https://commons.wikimedia.org/w/index.php?curid=28183362

```
private void drawTree(Graphics2D g, Point2D p1, Point2D p2,
                      int depth) {

    if (depth == depthLimit)
        return;

    Point2D p3 = new Point2D.Double();
    Point2D p4 = new Point2D.Double();
    Point2D p5 = new Point2D.Double();

    computeNewPoints(p1, p2, p3, p4, p5);

    drawSquare(g, p1, p2, p3, p4, depth);

    drawTree(g, p4, p5, depth + 1);
    drawTree(g, p5, p3, depth + 1);
}
```

Code on GitHub

```
private void drawTree(Graphics2D g, Point2D p1, Point2D p2,
                      int depth) {

    if (depth == depthLimit)
        return;

    Point2D p3 = new Point2D.Double();
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    computeNewPoints(p1, p2, p3, p4, p5);

    drawSquare(g, p1, p2, p3, p4, depth);

    drawTree(g, p4, p5, depth + 1);
    drawTree(g, p5, p3, depth + 1);
}
```

base case

when reached causes recursion to end

recursive calls

Code on GitHub

The power of recursion lies in the possibility of defining

- **an infinite set of objects by a finite statement**

- **an infinite number of computations by a finite recursive program**

Algorithms + Data Structures = Programs,
Wirth, Niklaus (1976)

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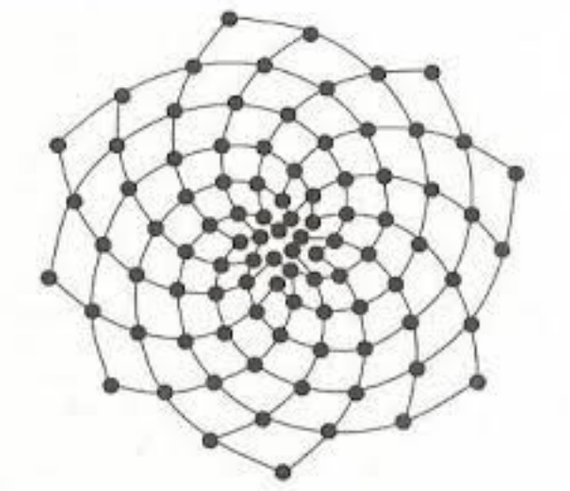
$$F_{n-2} = F_n - F_{n-1}$$

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610



http://www.sciencemag.org/news/2016/05/sunflowers-show-complex-fibonacci-sequences

Fibonacci in Java

```
public static long fibonacci(final int n)
{
    return (n < 2) ? n : fibonacci(n - 1) + fibonacci(n - 2);
}
```

base case

recursive calls

Exponential Growth

```
                    F(n)
                   /    \
              F(n-1)     F(n-2)
              /    \       /      \
         F(n-2) F(n-3) F(n-3)   F(n-4)
        /     \
    F(n-3) F(n-4)
```

Dynamic Programming - method for solving a complex problem by breaking it down into a collection of simpler subproblems, solving each of those subproblems just once, and storing their solutions

Demo — scalability of naïve Fibonacci

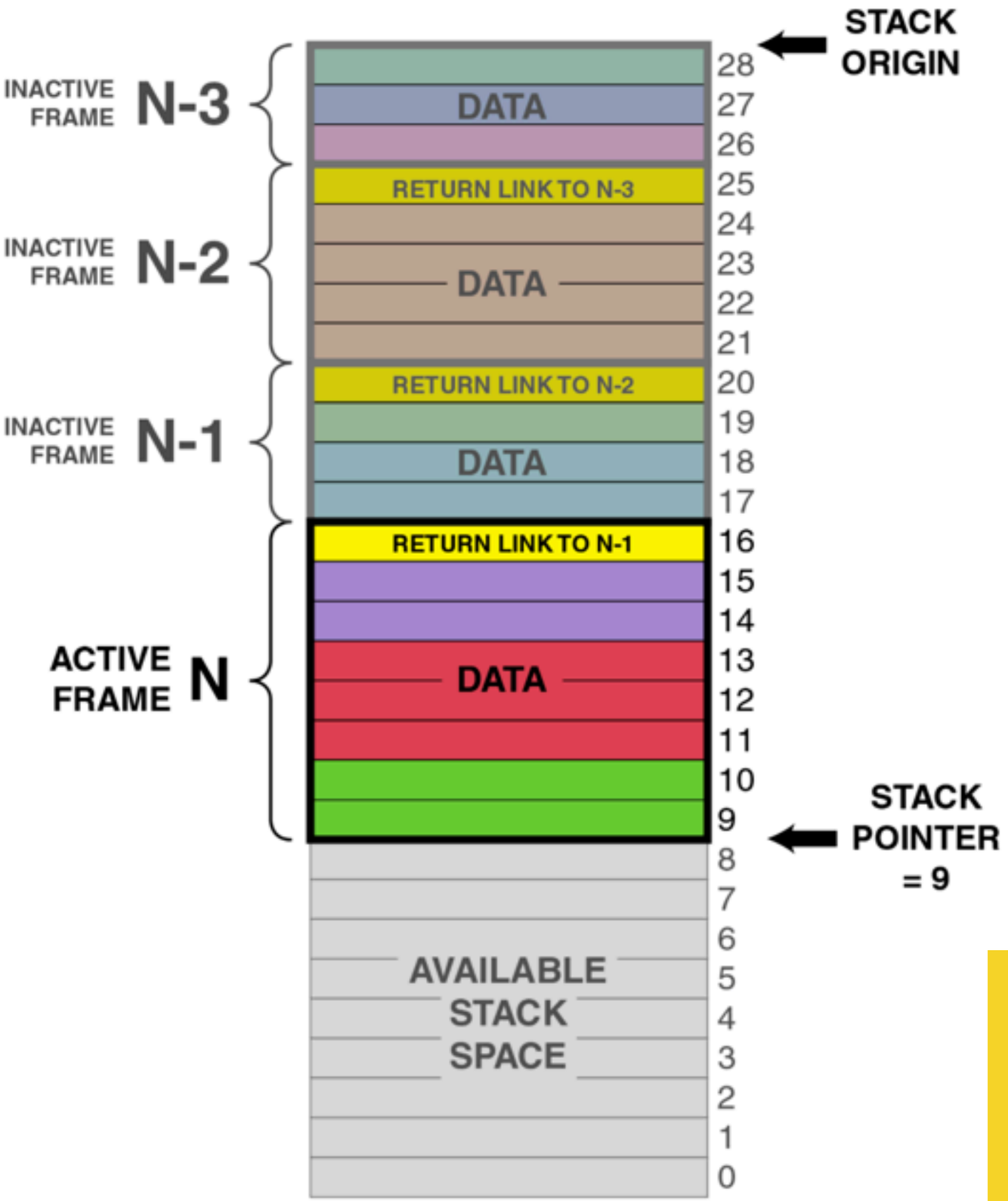
Accelerated Version

```
public static long fasterFibonacci(final int n)
{
    if ( n <= 1 )
        return n;

    return fibonacciAcc(0, 1, n);
}
```

```
private static long fibonacciAcc(long prev, long curr, int n)
{
    if (n == 1)
        return curr;

    return fibonacciAcc(curr, prev + curr, n - 1);
}
```



The Call Stack

language implementation structure which allows interrupting and resuming a function

Peril: stackoverflow

```java
public static long fasterFibonacci(final int n)
{
    if ( n == 1 )
        return n;

    return fibonacciAcc(0, 1, n);
}

public static void main(String[] args) {
    for ( int counter = 0; counter <= 50; counter++ )
        System.out.printf( "Fibo(%d) = %d\n", counter, fasterFibonacci(counter)
}
```

Fibonacci

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_25.jdk/Contents/Home/bin/java ...
Exception in thread "main" java.lang.StackOverflowError
    at Fibonacci.fibonacciAcc(Fibonacci.java:11)
    at Fibonacci.fibonacciAcc(Fibonacci.java:11)
    at Fibonacci.fibonacciAcc(Fibonacci.java:11)
```

Demo - Testing the depth of Java's Stack

Tail Recursion

```
private static long fibonacciAcc(long prev, long curr, int n)
{
    if (n == 1)
        return curr;

    return fibonacciAcc(curr, prev + curr, n - 1);
}
```

can be optimized with *tail call optimization*
easy to transform in iteration

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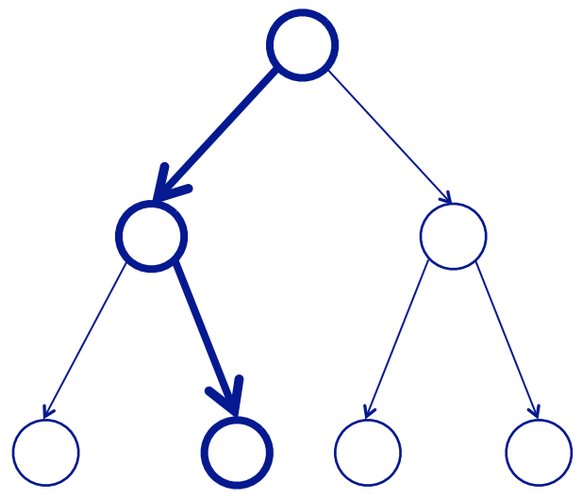
Modeling Binary Trees

```
class Node<T> {
    T value;
    Node<T> left;
    Node<T> right;

    Node(T value) {
        this.value = value;
    }

    void visit() {
        System.out.print(this.value + " ");
    }
}
```



Traversing Trees

recursive algos for recursive structures

```
static void traverse(Node<?> node, ORDER order) {
    if (node == null) {
        return;
    }
    switch (order) {
        case PREORDER:
            node.visit();
            traverse(node.left, order);
            traverse(node.right, order);
            break;
```

base case

recursive calls

“structural recursion”

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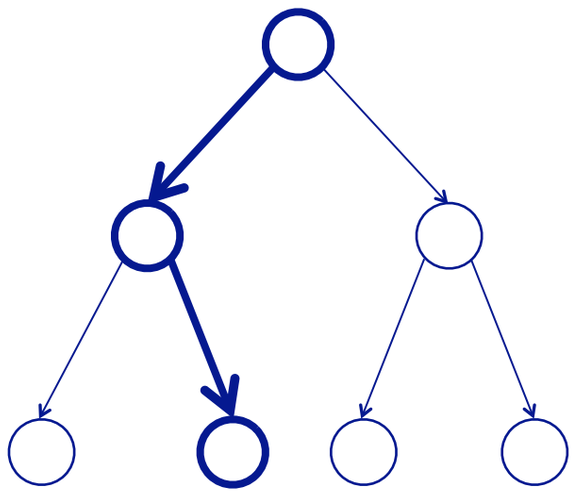
Binary search

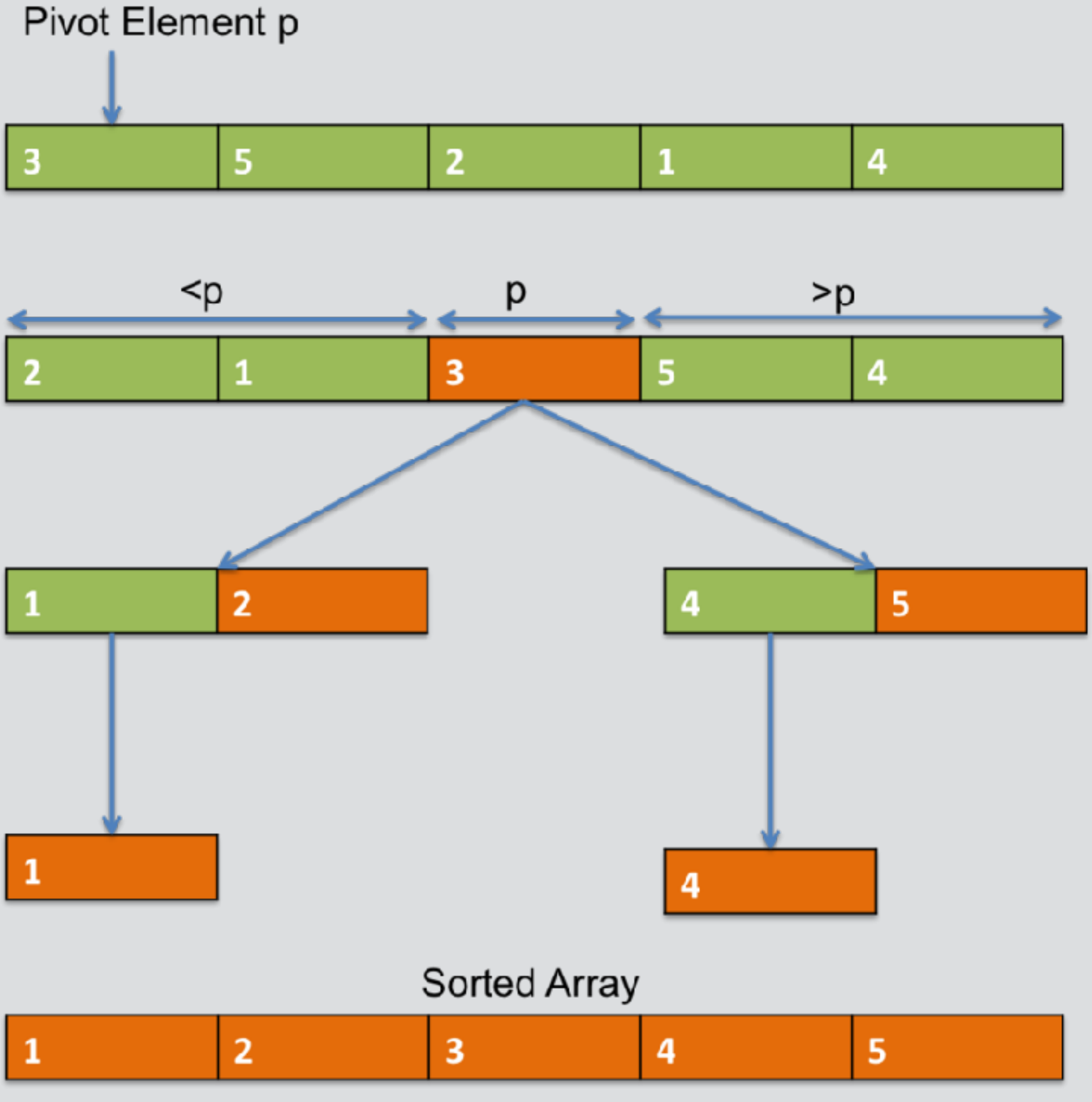


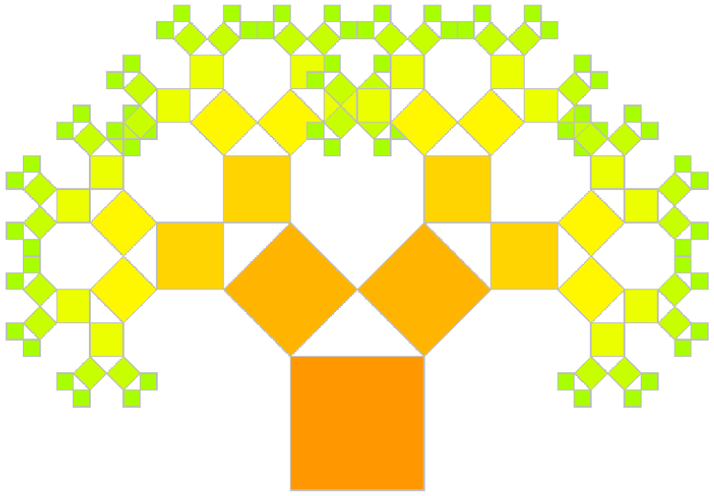
```java
/*
     a - array with integer elements
     x - element we want to know whether present
     left - leftmost position where element could be
     right - rightmost position where element could be
 */

private int binarySearch(int[ ] a, int x, int left, int right) {

    if (left > right) return -1;

    int mid = (left + right)/2;

    if (a[mid] == x) return mid;

    else if (a[mid] < x)
        return binarySearch(a, x, mid+1, right);

    else
        return binarySearch(a, x, left, mid-1);
}
```

base case

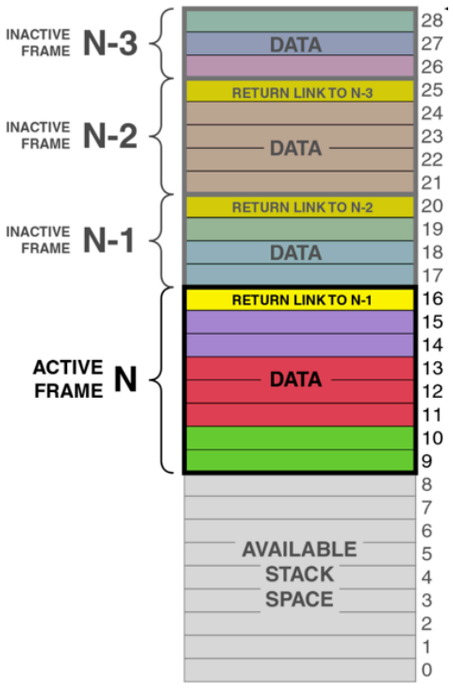
recursive calls

Quicksort





```
                F(n)
               /    \
          F(n-1)    F(n-2)
          /   \      /      \
     F(n-2) F(n-3) F(n-3)  F(n-4)
    /    \
F(n-3) F(n-4)
```



Code: https://github.com/mircealungu/open_lectures/recursion

Further Reading

- Dynamic Programming (CodeChef)
- Tail Call Optimization on JVM (DrDobbs)
- Replacing Recursion with Iteration (ThoughtWorks)

Credits

- Cover Art: http://qcc-art.deviantart.com/art/Tony-Monahan-Turtles-All-the-Way-Down-590318917

Backup Materials

Replacing Recursion with Iteration

```
Stack<Object> stack;
stack.push(first_object);
while( !stack.isEmpty() ) {
    // Do something
    my_object = stack.pop();

  // Push other objects on the stack.

}
```

```
void quicksort(int *array, int left, int right)
{
    int stack[1024];
    int i=0;

    stack[i++] = left;
    stack[i++] = right;

    while (i > 0)
    {
        right = stack[--i];
        left = stack[--i];

        if (left >= right)
            continue;

        int index = partition(array, left, right);
        stack[i++] = left;
        stack[i++] = index - 1;
        stack[i++] = index + 1;
        stack[i++] = right;
    }
}
```

iterative quicksort

Composite Design Pattern