Seminar 13

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C - linear code

H - parity check matrix

$v \in C \Leftrightarrow H \cdot [v]_E = 0$

(code vector) $v \in \mathbb{Z}_2^n$

$H \cdot [v]_E$ the syndrome associated to the vector we will call the coset leader (the most likely error for the vector in this case)

2. Using the parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

and the syndromes and coset leaders

Syndrome	000	001	010	011
Coset leader	000000	001000	010000	000010

Syndrome	100	101	110	111
Coset leader	100000	000110	000100	000001

decode the following words: 101110, 011000, 001011, 111111, 110011.

a) $H \cdot \begin{pmatrix}1\\0\\1\\1\\1\\0\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix}1\\0\\1\\1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow e = 000000 \Rightarrow$ The message is 110

101110 + 000000 = 101110

b) $H \cdot \begin{pmatrix}0\\1\\1\\0\\0\\0\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix}0\\1\\1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix} \Rightarrow e = 000010 \Rightarrow$ The message is 010

$$\begin{array}{r} 000010 \\ \underline{011000} \\ 011010 \end{array}$$

c) $H \cdot \begin{pmatrix}0\\0\\1\\0\\1\\1\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\\0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} \Rightarrow e = 000110 \Rightarrow$ The message is 101

$$\begin{array}{r} 000110 \\ \underline{001011} \\ 001101 \end{array}$$

d) $H \cdot \begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} \Rightarrow e = 000110 \Rightarrow$ The message is 001

$$\begin{array}{r} 000110 \\ \underline{111111} \\ 111001 \end{array}$$

e) $H \cdot \begin{pmatrix}1\\1\\0\\0\\1\\1\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix}1\\1\\0\\0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \Rightarrow e = 010000 \Rightarrow$ The message is 011

5. Construct a table of coset leaders and syndromes for the (7,4)-code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

S	000	001	010	011	100	101	110	111
CL	0000000	0010000	0100000	0000001	1000000	0000010	0000100	0001000

$v \in CL$, $H \cdot [v]_E = [S]_E$

Add columns in order to obtain 1s

8. Construct a table of coset leaders and syndromes for the (7,3)-code generated by $p = 1 + X^2 + X^3 + X^4 \in \mathbb{Z}_2[X]$.

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

S	0000	0001	0010	0011	0100	0101	0110	0111
CL	0000000	0001000	0010000	0011000	0100000	0000110	0110000	0000001

	1000	1001	1010	1011	1100	1101	1110	1111
	1000000	0000011	0001100	0000100	1100000	1101000	0000010	0001010

Decode 1101011, 0011110

$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix}1\\1\\0\\1\\0\\1\\1\end{pmatrix} = (0\ 1\ 0\ 1) \Rightarrow e = 0000110 \Rightarrow$ The message is 101

$$\begin{array}{r} 0000110 \\ \underline{1101011} \\ 1101101 \end{array}$$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\\1\\1\\1\\0\end{pmatrix} = (0\ 1\ 1\ 0) \Rightarrow e = 0110000 \Rightarrow$ The message is 110

$$\begin{array}{r} 0110000 \\ \underline{0011110} \\ 0101110 \end{array}$$

3. A (7,4)-code is defined by the equations $u_1 = u_4 + u_5 + u_7$, $u_2 = u_4 + u_6 + u_7$, $u_3 = u_4 + u_5 + u_6$, where u_4, u_5, u_6, u_7 are the message digits and u_1, u_2, u_3 are the check digits. Write its generator matrix and parity check matrix. Decode the received words 0000111 and 0001111.

$$H \cdot \begin{pmatrix}u_1\\u_2\\u_3\\u_4\\u_5\\u_6\\u_7\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \qquad \begin{cases} u_1 = u_4 + u_5 + u_7 \Leftrightarrow u_1 + u_4 + u_5 + u_7 = 0 \\ u_2 = u_4 + u_6 + u_7 \Rightarrow u_2 + u_4 + u_6 + u_7 = 0 \\ u_3 = u_4 + u_5 + u_6 \Rightarrow u_3 + u_4 + u_5 + u_6 = 0 \end{cases} \Rightarrow$$

$$\left.\begin{array}{l} H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \\ H = (I_3 | P) \end{array}\right\} \Rightarrow P = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \Rightarrow G = \frac{P}{I_4} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

S	000	001	010	011	100	101	111	110
CL	0000000	0001000	0010000	0000010	1000000	0000100	0001000	1100000

$H \cdot \begin{pmatrix}0\\0\\0\\0\\1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow$ The message is 0111

$H \cdot \begin{pmatrix}0\\0\\0\\1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow$ The coset leader is 0001000 $\Rightarrow$ the message is 1000

$$\begin{array}{r} 0001000\,+ \\ \underline{0001111} \\ 0000111 \end{array}$$