SAUUUUUUUUUUU

Thursday, May 29, 2025 9:21 PM

$(0,1)$, $(1,-3)$, $(2,7)$

x_i	0	1	2
y_i	1	-3	7

$m=3$

$y(x) = ax+b$

Least Squares Approximation

$$a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$
$$a\sum_{i=1}^{n} x_i + nb = \sum_{i=1}^{n} y_i$$

$$a\sum_{i=1}^{3} x_i + mb = \sum_{i=1}^{3} y_i$$

$= a(0+1+2) + 3\cdot b = 3a+3b = 3(a+b)$

$\sum_{i=1}^{3} y_i = 1-3+7 = 5$

$3(a+b) = 5 \Rightarrow a+b = \frac{5}{3}$

Least Squares Approximation

$$a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

$a(0^2+1^2+2^2) + b(0+1+2) = (0\cdot 1 + 1\cdot(-3) + 2\cdot 7)$

$\Rightarrow 5a + 3b = 11$

$$\begin{cases} a+b = \frac{5}{3} \\ 5a+3b = 11 \end{cases} \Rightarrow$$

$a = \frac{5}{3} - b$ $\Rightarrow$ $\frac{25}{3} - 2b = 11$

$\frac{25}{3} - 5b + 3b = 11$

$25 - 6b = 33 \Rightarrow -6b = 33-25 \Rightarrow b = -\frac{8}{6} = -\frac{4}{3}$

$a = \frac{5}{3} + \frac{8}{6} = \frac{18}{6} = 3$

$$\boxed{p(x) = 3x - \frac{4}{3}}$$

$$a\sum_{i=1}^{n} x_i^4 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^2 y_i,$$
$$a\sum_{i=1}^{n} x_i^3 + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i,$$
$$a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i + nc = \sum_{i=1}^{n} y_i,$$

x_i	0	1	2
y_i	1	-3	7

$p(x) = ax^2+bx+c$

$$\begin{cases} 5a+3b+3c = 5 \\ 9a+5b+3c = 11 \\ 17a+9b+5c = 25 \end{cases} \Rightarrow \begin{cases} 4a+2b = 6 \\ \cdot \end{cases} \Rightarrow 4a = 6-2b$$

$\Rightarrow 2a = 3-b$

$a = \frac{3-b}{2}$

$5\frac{(3-b)}{2} + 3b + 3c = 5 \quad | \cdot 2$

$5(3-b) + 6b + 6c = 10$

$15 - 5b + 6b + 6c = 10$

$15 + b + 6c = 10$

$b = -5 - 6c$

$a = \frac{3+5+6c}{2}$

$5\left(\frac{3+5+6c}{2}\right) + 3(-5-6c) + 3c = 5$

$\frac{15+25+30}{2} - 15 - 18c + 3c = 5$

$\cdots - 15 - 15c = 5$

$b = -5 - 6c$

$\frac{15 + 25 + 30}{2} - 15 - 18c + 9c$

$\frac{70}{2} - 15 - 15c = 5 \Rightarrow 35 - 15 - 15c = 5$

$20 - 15c = 5 \Rightarrow c = 1$

$c = 1$

$a = \frac{8 \pm 6}{2} = 7$

$b = -5 - 6 = -11$

$f(x) = 7x^2 - 11x + 1$

$x_1^0 = 0 \quad x_2^0 = 0$

$k = 0, 1, \ldots$

3) $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$; $x^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; use gauss seidel (do 3 iterations)

$2x_1 + x_2 = -1$

$x_1 + 2x_2 = -2$

$x_1 = \frac{-1 - x_2}{2}$

$x_2 = \frac{-2 - x_1}{2}$

$k = 0$ Iteratia 1

$$\begin{cases} x_1^1 = \frac{-1 - x_2^0}{2} = \frac{-1 - 0}{2} = -\frac{1}{2} \\ x_2^1 = \frac{-2 - x_1^1}{2} = \frac{-2 + \frac{1}{2}}{2} = -\frac{3}{4} \end{cases}$$

$k = 1$

$$\begin{cases} x_1^2 = \frac{-1 - x_2^1}{2} = \frac{-1 + \frac{3}{4}}{2} = -\frac{1}{8} \\ x_2^2 = \frac{-2 - x_1^2}{2} = \frac{-2 + \frac{1}{8}}{2} = -\frac{15}{16} \end{cases}$$

$k = 2$

$$\begin{cases} x_1^3 = \frac{-1 + \frac{15}{16}}{2} = \frac{-1}{32} \\ x_2^3 = \frac{-2 - x_1^3}{2} = \frac{-2 + \frac{1}{32}}{2} = -\frac{63}{64} \end{cases}$$

$x_1^0 = 0, \quad x_2^0 = 0$

$k = 0$ It 1

$$\begin{cases} x_1^1 = \frac{-1 - x_2^0}{2} = \frac{-1 - 0}{2} = -\frac{1}{2} \\ x_2^1 = \frac{-2 - x_1^0}{2} = -1 \end{cases}$$

$k = 2$ It 2

$$\begin{cases} x_1^3 = \frac{-1 - x_2^2}{2} \\ x_2^3 = \frac{-2 - x_1^2}{2} \end{cases}$$

$k = 1$ It 2

$$\begin{cases} x_1^2 = \frac{-1 - x_2^1}{2} = 0 \\ x_2^2 = \frac{-2 + \frac{1}{2}}{2} = -\frac{3}{4} \end{cases}$$