```
> with(Student[LinearAlgebra]) : with(LinearAlgebra)
```

(1)

```
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,
  BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
  ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
  CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy,
  CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal,
  DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
  Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm,
  FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
  GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
  HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix,
  IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary,
  JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition,
  LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential,
  MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
  MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
  NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot,
  PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank,
  RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
  RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
  SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
  SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,
  UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm,
  VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]
```

```
> A := Matrix([[0, -2, 0], [1, -2, 0], [0, 0, -2]])
```

$$A := \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \tag{2}$$

```
> Determinant(A)
```

$$-4 \tag{3}$$

```
> A^(-1)
```

$$\begin{bmatrix} -1 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \tag{4}$$

```
> CharacteristicPolynomial(A, r)
```

(5)

$$r^3 + 4\,r^2 + 6\,r + 4 \tag{5}$$

> $lam := Eigenvalues(A)$

$$lam := \begin{bmatrix} -2 \\ -1 - \mathrm{I} \\ -1 + \mathrm{I} \end{bmatrix} \tag{6}$$

> $lam, P := Eigenvectors(A)$

$$lam, P := \begin{bmatrix} -2 \\ -1 + \mathrm{I} \\ -1 - \mathrm{I} \end{bmatrix}, \begin{bmatrix} 0 & 1 + \mathrm{I} & 1 - \mathrm{I} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{7}$$

> $lam[1]$

$$-2 \tag{8}$$

> $Column(P, 1)$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

> $A \cdot Column(P, 1)$

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{10}$$

> $Column(P, 1) \cdot lam[1]$

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{11}$$

> $A \cdot Column(P, 2)$

$$\begin{bmatrix} -2 \\ -1 + \mathrm{I} \\ 0 \end{bmatrix} \tag{12}$$

> $Column(P, 2) \cdot lam[2]$

$$\begin{bmatrix} -2 \\ -1 + \mathrm{I} \\ 0 \end{bmatrix} \tag{13}$$

> $A \cdot Column(P, 3)$

$$\begin{bmatrix} -2 \\ -1 - \mathrm{I} \\ 0 \end{bmatrix} \tag{14}$$

> $Column(P, 3)\cdot lam[3]$

$$\begin{bmatrix} -2 \\ -1-I \\ 0 \end{bmatrix} \quad (15)$$

> $J := DiagonalMatrix(lam)$

$$J := \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1+I & 0 \\ 0 & 0 & -1-I \end{bmatrix} \quad (16)$$

> P

$$\begin{bmatrix} 0 & 1+I & 1-I \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (17)$$

> $P\cdot J\cdot P\hat{}(-1)$

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (18)$$

> $MatrixExponential(A)$

$$\begin{bmatrix} e^{-1}(\sin(1)+\cos(1)) & -2\,e^{-1}\sin(1) & 0 \\ e^{-1}\sin(1) & e^{-1}(\cos(1)-\sin(1)) & 0 \\ 0 & 0 & e^{-2} \end{bmatrix} \quad (19)$$

> $MatrixExponential(J)$

$$\begin{bmatrix} e^{-2} & 0 & 0 \\ 0 & e^{-1}\cos(1)+I\,e^{-1}\sin(1) & 0 \\ 0 & 0 & e^{-1}\cos(1)-I\,e^{-1}\sin(1) \end{bmatrix} \quad (20)$$

> $Map(limit, MatrixExponential(t\cdot A), t=\text{infinity})$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (21)$$

> $P := Matrix([[1, 0, 3, 14], [5, 6, 7, 28], [9, 10, 11, 142], [0, 0, 0, 1]])$

$$P := \begin{bmatrix} 1 & 0 & 3 & 14 \\ 5 & 6 & 7 & 28 \\ 9 & 10 & 11 & 142 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)$$

> $Determinant(P)$

$$-16 \quad (23)$$

```
> J := DiagonalMatrix([2, 2, -1, 0])
```

$$J := \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

```
> A := P . J . P^(-1)
```

$$A := \begin{bmatrix} -\frac{1}{4} & -\frac{45}{8} & \frac{27}{8} & -\frac{1273}{4} \\ -\frac{21}{4} & -\frac{89}{8} & \frac{63}{8} & -\frac{2933}{4} \\ -\frac{33}{4} & -\frac{165}{8} & \frac{115}{8} & -\frac{5393}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

```
> A := -8*A
```

$$A := \begin{bmatrix} 2 & 45 & -27 & 2546 \\ 42 & 89 & -63 & 5866 \\ 66 & 165 & -115 & 10786 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

```
> solve(1 - x^2 = 0, x)
```

$$-1, 1 \tag{27}$$

```
> eq := diff(x(t), t) = 1 - x(t)^2
```

$$eq := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = 1 - x(t)^2 \tag{28}$$

```
> ic := x(0) = -2
```

$$ic := x(0) = -2 \tag{29}$$

```
> sol := dsolve({eq, ic}, x(t))
```

$$sol := x(t) = \coth\left(-\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \tag{30}$$

```
> sol := rhs(sol)
```

$$sol := \coth\left(-\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \tag{31}$$

```
> fl := convert(convert(sol, exp), exp)
```

$$fl := \frac{\mathrm{e}^{2t} + 3}{\mathrm{e}^{2t} - 3} \tag{32}$$

```
> ic := x(0) = 0
```

$$ic := x(0) = 0 \tag{33}$$

```
> sol := dsolve({eq, ic}, x(t))
```

$$sol := x(t) = \tanh(t) \tag{34}$$

```
> sol := rhs(sol)
```

$$sol := \tanh(t) \tag{35}$$

```
> f2 := convert(sol, exp)
```

$$f2 := \frac{e^{t} - e^{-t}}{e^{t} + e^{-t}} \tag{36}$$

```
> ic := x(0) = -1
```

$$ic := x(0) = -1 \tag{37}$$

```
> sol := dsolve({eq, ic}, x(t))
```

$$sol := x(t) = -1 \tag{38}$$

```
> ic := x(0) = 1
```

$$ic := x(0) = 1 \tag{39}$$

```
> sol := dsolve({eq, ic}, x(t))
```

$$sol := x(t) = 1 \tag{40}$$

```
> ic := x(0) = 2
```

$$ic := x(0) = 2 \tag{41}$$

```
> sol := dsolve({eq, ic}, x(t))
```

$$sol := x(t) = \coth\left(\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \tag{42}$$

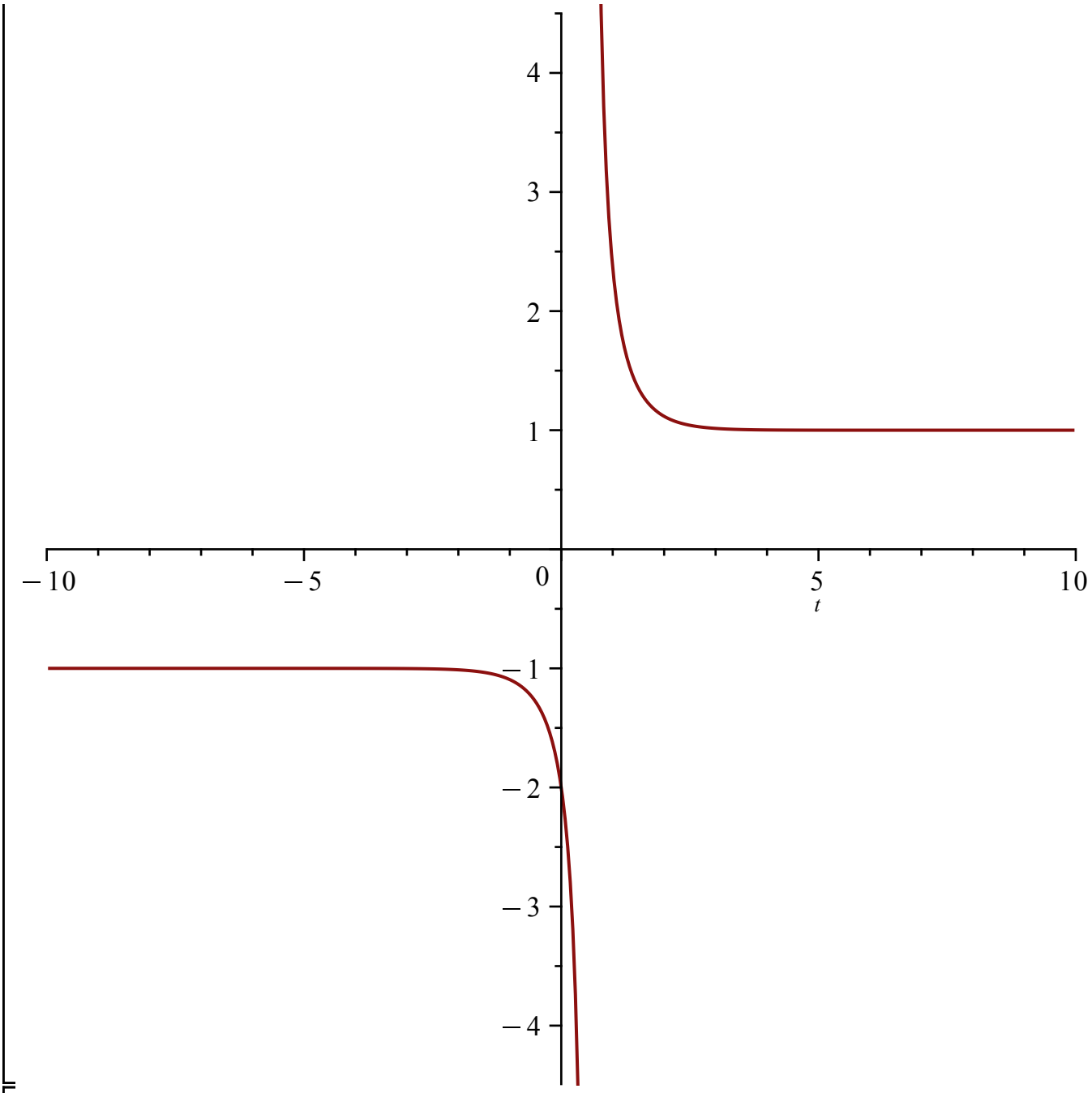
```
> sol := rhs(sol)
```

$$sol := \coth\left(\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \tag{43}$$

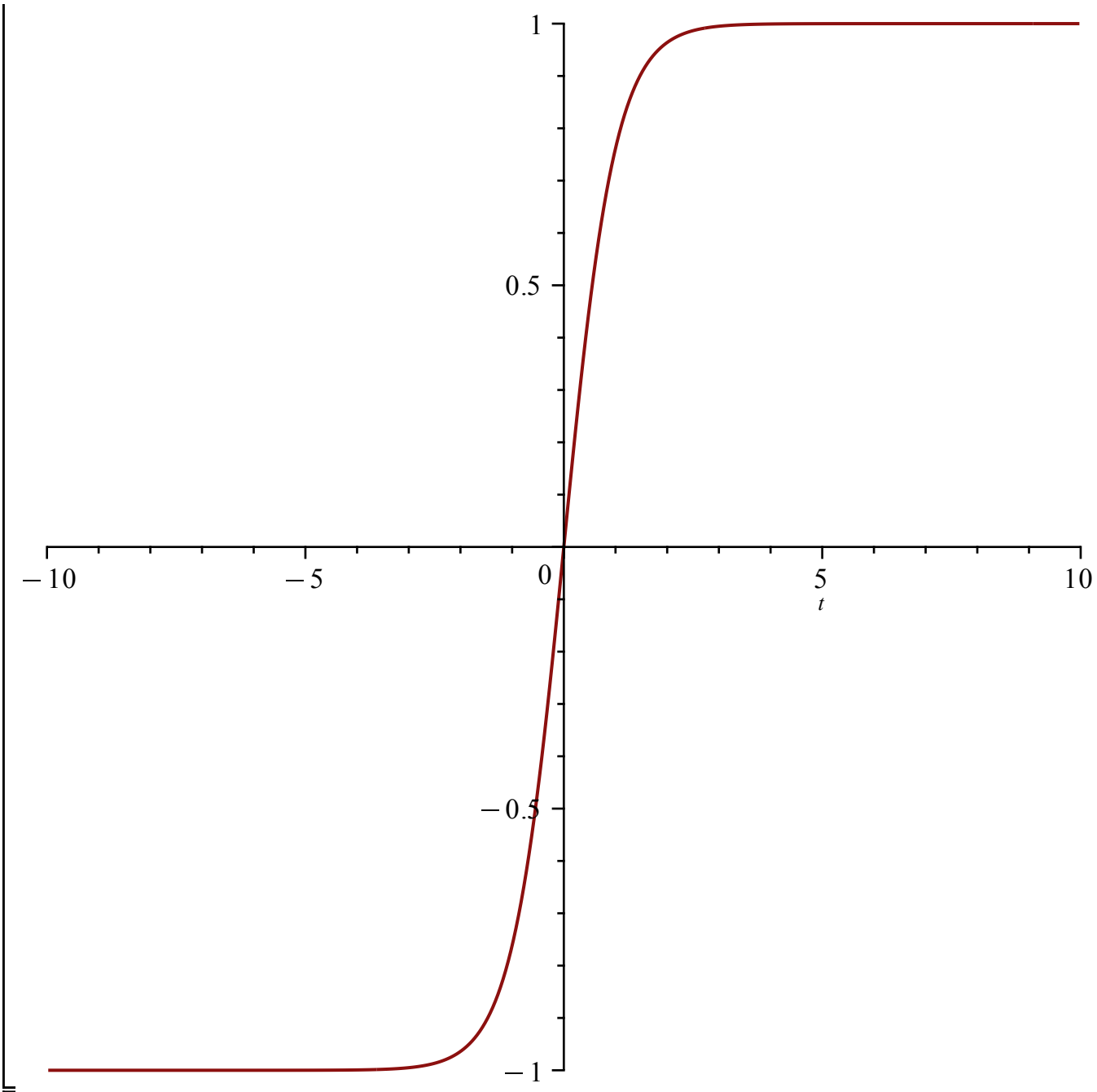
```
> f3 := convert(convert(sol, exp), exp)
```

$$f3 := \frac{3\,e^{2t} + 1}{3\,e^{2t} - 1} \tag{44}$$

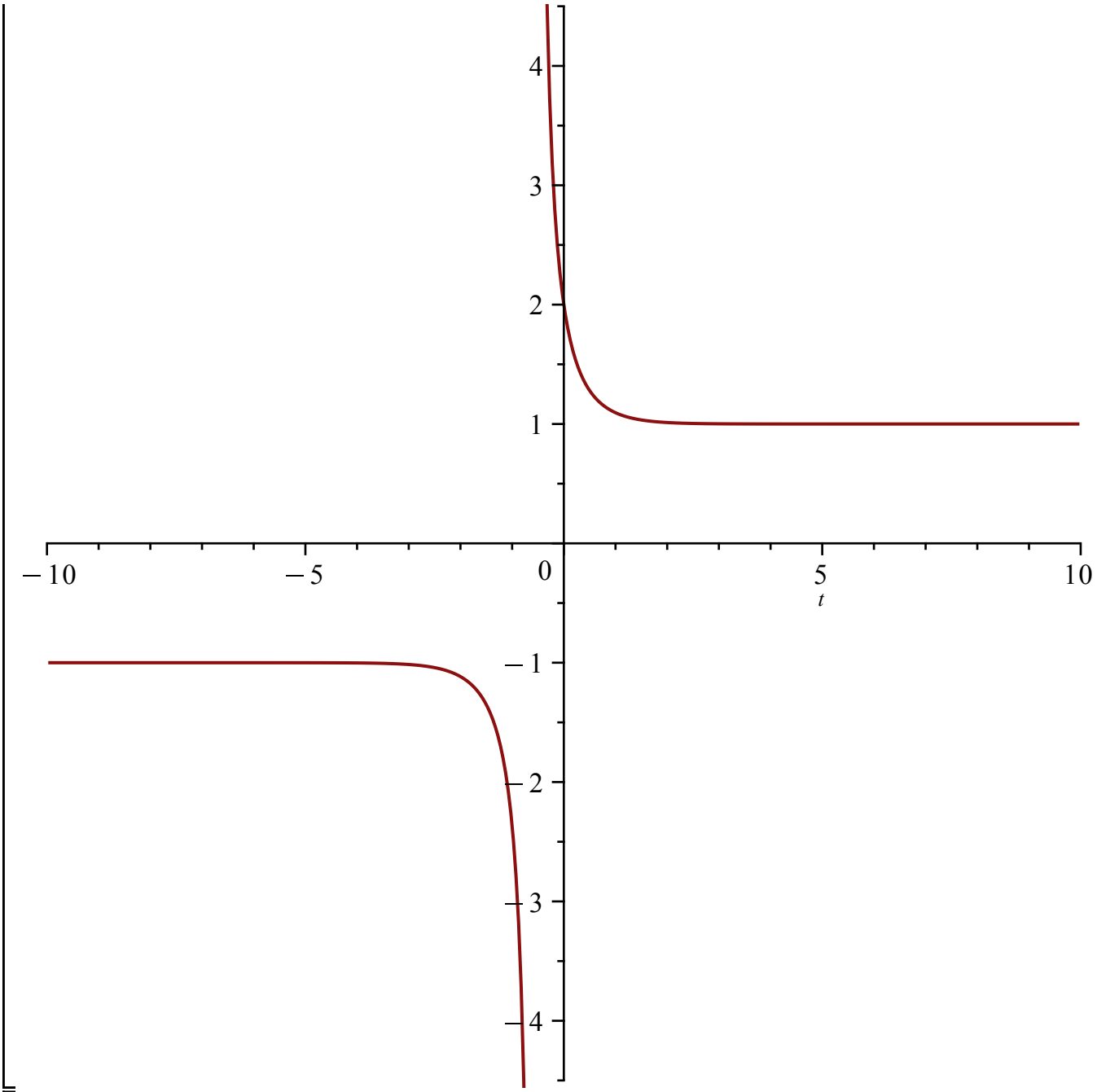
```
> plot(f1)
```




```
> plot( f2 )
```




```
> plot( f3 )
```



> $limit(f1, t = -\text{infinity})$

-1 **(45)**

> $limit(f2, t = \text{infinity})$

1 **(46)**

> $limit(f3, t = \text{infinity})$

1 **(47)**

>