

# Dynamic Event-triggered Control and Estimation: A Survey

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**Abstract:** The efficient utilization of computation and communication resources became a critical design issue in a wide range of networked systems due to the finite computation and processing capabilities of system components (e.g., sensor, controller) and shared network bandwidth. Event-triggered mechanisms (ETMs) are regarded as a major paradigm shift in resource-constrained applications compared to the classical time-triggered mechanisms, which allows a trade-off to be achieved between desired control/estimation performance and improved resource efficiency. In recent years, dynamic event-triggered mechanisms (DETs) are emerging as a promising enabler to fulfill more resource-efficient and flexible design requirements. This paper provides a comprehensive review of the latest developments in dynamic event-triggered control and estimation for networked systems. Firstly, a unified event-triggered control and estimation framework is established, which empowers several fundamental issues associated with the construction and implementation of the desired ETM and controller/estimator to be systematically investigated. Secondly, the motivations of DETMs and their main features and benefits are outlined. Then, two typical classes of DETMs based on auxiliary dynamic variables (ADVs) and dynamic threshold parameters (DTPs) are elaborated. In addition, the main techniques of constructing ADVs and DTPs are classified, and their corresponding analysis and design methods are discussed. Furthermore, three application examples are provided to evaluate different ETMs and verify how and under what conditions DETMs are superior to their static and periodic counterparts. Finally, several challenging issues are envisioned to direct the future research.

**Keywords:** Networked systems, dynamic event-triggered control, dynamic event-triggered estimation, dynamic event-triggered mechanisms, vehicle active suspension system, water distribution and supply system.

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## 1 Introduction

Advances in communication and computer techniques are leading to a new paradigm of control and estimation for modern networked systems. In the past, the plant was controlled and monitored via networks configured with sufficient communication resources. Accordingly, desired control and state estimation actions are implemented in a time-triggered fashion (namely, at predetermined and periodic instants of time) since this allows system performance analysis and design procedures to be readily performed by using the celebrated sampled-data system theory<sup>[1–4]</sup>. However, it is well acknowledged that these time-triggered control and estimation approaches often lead to over-utilization of available computation and communication resources, given that embedded system components are constantly battery-operated<sup>[5]</sup> and data communication is often energy-costly<sup>[6]</sup>. Nowadays, control

lers and state estimators are deployed spatially and remotely over some wireless and digital network mediums, where control and estimation tasks persistently share precious resources with other neighboring tasks<sup>[7]</sup>. Under this new paradigm, the constrained resource issue, posed by finite bandwidth and scarce computation and processing capabilities of battery-powered system components, should be adequately taken into account during controller and estimator design. It is thus of theoretical and practical significance to address resource-efficient control and estimation problems for networked systems.

In order to efficiently utilize limited computation and communication resources, data sampling and transmission actions or control and estimate updates should be kept to the minimum required to preserve desired control and estimation performance, which motivates an event-triggered mechanism (ETM)<sup>[1]</sup>. It is noteworthy that event-triggered ideas have been prevalently adopted to account for a wide array of capabilities, such as data sampling, transmission, communication, scheduling, and controller/estimator update. Understanding that event-

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<sup>1</sup>We use the terms “event-triggered mechanism” and “event trigger” interchangeably throughout the paper whenever without causing confusion.

based controller and estimator take the triggered and released data packets as their inputs, we employ, hereafter, the term ETM to refer to either a sampling event or a transmission event of the data of interest (e.g., system state or measurement output), which depends on the triggering strategies in terms of event-triggered sampling (ETS) and event-triggered transmission (ETT). Specifically, an ETM decides when or how often data samplings and/or transmissions should be performed based on some well-defined events rather than at fixed points of time. From this perspective, ETMs can be regarded as an addition of certain intelligence to conventional sampling and/or transmission decisions in a time-triggered mechanism (TTM). Hence, ETMs are capable of significantly reducing the number of data samplings and/or transmissions compared with TTMs, while retaining satisfactory system performance. On the other hand, it is noted that reducing unnecessary data transmissions contributes to a relief of network traffic or congestion, which in turn alleviates network-induced phenomena (e.g., transmission delays, packet dropouts) that inevitably affect desired control/estimation performance. In this sense, ETMs may also be beneficial for meeting a fundamental quality-of-service requirement during networked controller/estimator design.

An ETM is composed of two essential components: an error function  $g(e(\tilde{t}))$  that evaluates the data of interest

between the last triggering instant  $t_m$  and the present time  $\tilde{t}$  with the error being denoted as  $e(\tilde{t}) = z(t_m) - z(\tilde{t})$ ; and a threshold function  $\underline{\sigma}f(Z(\tilde{t}))$  that is used to quantitatively characterize the change/variation of the data amplitude, where  $g(\cdot)$  and  $f(\cdot)$  are class  $\mathcal{K}$  functions, and the other notations are given in Table 1. Then, the traditional static event-triggered mechanism (SETM) decides when to sample and/or transmit the data  $z(\tilde{t})$  at every instant of time  $\tilde{t}$  according to the following decision rule

$$t_{m+1} = \inf\{\tilde{t} > t_m \mid g(e(\tilde{t})) > \underline{\sigma}f(Z(\tilde{t}))\}. \quad (1)$$

By properly designing the triggering condition  $g(e(\tilde{t})) > \underline{\sigma}f(Z(\tilde{t}))$ , it is obvious that the triggering actions (or the instants  $t_{m+1}$ ) can be only invoked when the data  $z(\tilde{t})$  is truly needed to be sampled and/or transmitted for ensuring stability and performance requirements, which thus leads to a noticeable reduction of occupancy of the available computation and communication resources. The following two facts are noted from the above SETM: 1) The threshold function  $\underline{\sigma}f(Z(\tilde{t}))$  directly accounts for the triggering frequency of events. Specifically, the larger the threshold, the less the number and thus the frequency of the events. The existing literature has considered a wide variety of threshold functions to cater to different system models and problem formulations, e.g.,

Table 1 Mathematical notations used in ETMs

Notation	Meaning
$h > 0$	The constant sampling period
$\{kh \mid k \in \mathbf{N}\}$	The monotonically increasing time sequence of sampling instants on sensor/sampler
$\{t_m \mid m \in \mathbf{N}; t_0 = 0\}$	The monotonically increasing time sequence of event triggering/releasing instants, where $t_m \in \mathbf{R}$ in ETS, $t_m \in \{kh\}$ in ETT, and $t_m \in \mathbf{N}$ in the discrete-time case
$\{s_m \mid s_m = t_m + mh, m = 0, 1, \dots, t_{m+1} - t_m - 1\}$	The monotonically increasing time sequence of sampling instants between any two consecutive triggering instants in ETT
$T_m = t_{m+1} - t_m$	The $m$ -th IET between two consecutive events
$z(t)$	The data of interest, i.e., the system state $x(t)$ or the system measurement output $y(t)$
$\tilde{z}(t) \triangleq z(t_m)$	The last triggered/released data, i.e., $\tilde{x}(t) \triangleq x(t_m)$ or $\tilde{y}(t) \triangleq y(t_m)$ at time $t_m$
$\tilde{t}$	The present instant of time, i.e., $\tilde{t} \triangleq t \in \mathbf{R}$ in the ETS case or $\tilde{t} \triangleq kh \in \mathbf{R}$ in the ETT case or $\tilde{t} \triangleq t \in \mathbf{N}$ in the discrete-time case
$z(\tilde{t})$	The data at the present instant of time $t$ or $kh$ , i.e., $z(\tilde{t}) \triangleq x(t)$ or $z(\tilde{t}) \triangleq y(t)$ in ETS or discrete-time case, and $z(\tilde{t}) \triangleq x(kh)$ or $z(\tilde{t}) \triangleq y(kh)$ in ETT
$Z(\tilde{t})$	The last triggered data $\tilde{z}(t)$ or the present data $z(\tilde{t})$
$e(\tilde{t}) = \tilde{z}(t) - z(\tilde{t})$	The triggering error between the last triggered data and the present data
$\Phi > 0$	The weighting matrix in the relevant triggering condition to be designed
$\underline{\sigma} \geq 0$	The static threshold parameter in the relevant triggering condition
$\sigma(\tilde{t}) \geq 0$	The DTP in the relevant triggering condition
$\lambda(\tilde{t}) \geq 0$	The ADV (or internal dynamic variable) in the relevant triggering condition

Section 2.4 for some typical threshold functions. 2) There is a fundamental trade-off between desired control/estimation performance and expected resource efficiency. In other words, along with decreased resource occupancy, it is not uncommon that the overall control/estimation performance is degraded to some extent because less data from the plant is transmitted for controller/estimator design and implementation.

Based on the same parameter selection as in SETM (1), a question naturally arises: *How can one further reduce the number of events without sacrificing too much the desired control/estimation performance?* A solution to this question is to enlarge the threshold function  $\underline{\sigma}f(Z(\tilde{t}))$  by adding a non-negative (or strictly positive) auxiliary dynamic variable (ADV) to the right-hand side of the triggering condition such that only the more significantly changed data can cross the newly defined threshold, which motivates a dynamic event-triggered mechanism (DETM)<sup>[8–10]</sup>. For a simple illustration, a comparative example of triggering the signal  $x(t) = \sin(\pi t)e^{-0.3t}$ ,  $t \in \mathbf{R}$  under TTM, SETM and DETM, respectively, is provided in Fig. 1. It can be seen that introducing a positive ADV  $\lambda(t)$  results in significantly sporadic events compared with the SETM. Apparently, such a DETM incorporates additional dynamics into deciding when to sample and/or re-

lease the data at every sampling instant of time. In contrast, an SETM represents a single decision-maker via merely the predefined threshold function. The promise of further decreasing the frequency of samplings on sensor devices and/or data packet transmissions over some shared communication medium serves as the primary motivation that stimulates recent developments of this class of ADV-based DETMs in networked systems.

Although the advantages of the ADV-based DETMs are well motivated, these DETMs are implemented in a decisive manner to reduce the frequency of sampling and/or transmission actions as much as possible. However, in many practical situations, one needs ways of implementing dynamic triggering ideas in a more flexible and versatile manner. For example, network conditions intrinsically vary over time, and network bandwidth may be only busy during some specific peak periods but idle during other periods. In this sense, an intelligent DETM should trigger events more often when actual network bandwidth is idle but less frequently when bandwidth is busy. On the other hand, from a convergence perspective, more data packets are expected to be released either at an early stage of system evolution or when a system undergoes external disturbances to seek fast transient response and quick settling, while fewer data packets can

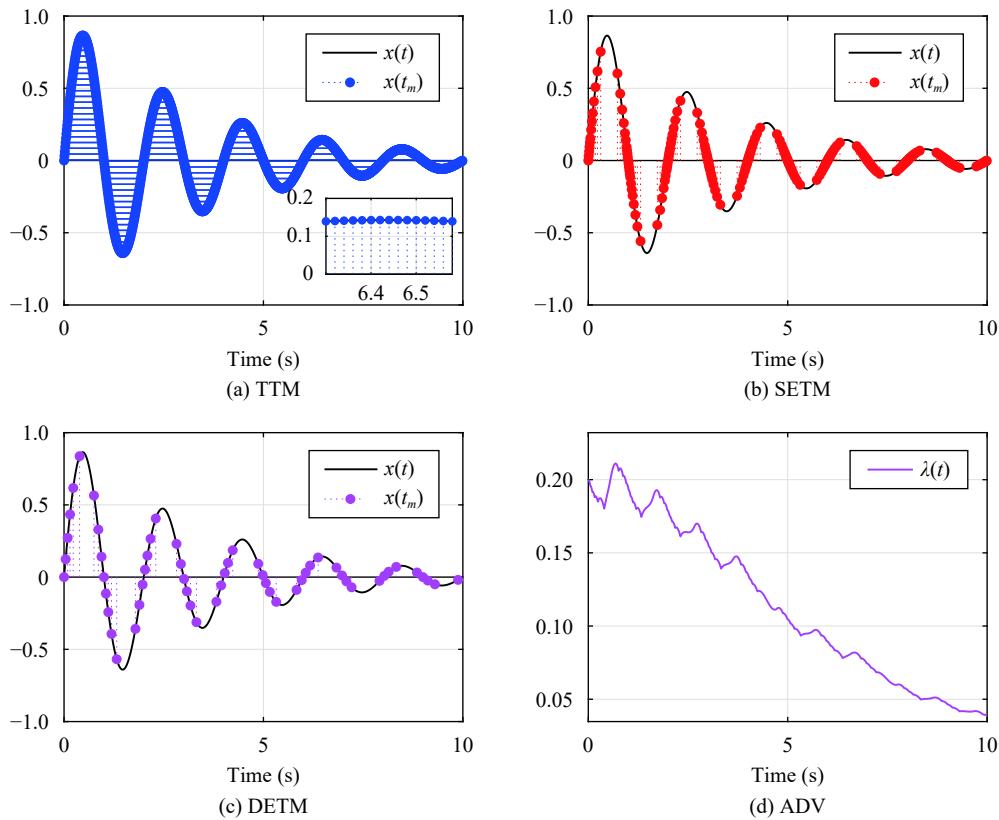


Fig. 1 Illustration of periodic triggering and event triggering of the signal of interest  $x(t) = \sin(\pi t)e^{-0.3t}$ ,  $t \in \mathbf{R}$ : (a) TTM (or periodic sampling) with  $t_m = mh$ ,  $m \in \mathbf{N}$ ;  $h = 0.01$ s; (b) SETM of the form  $t_{m+1} = \inf\{t > t_m | |x(t_m) - x(t)| > 0.2|x(t_m)|\}$ ; (c) ADV-based DETM of the form  $t_{m+1} = \inf\{t > t_m | |x(t_m) - x(t)| > 0.2|x(t_m)| + 0.5\lambda(t)\}$ ; (d) ADV  $\lambda(t) > 0$  evolving along  $\dot{\lambda}(t) = -0.1\lambda(t) - (|x(t_m) - x(t)| - 0.2|x(t_m)|)$ ,  $\lambda(0) = 0.2$

be triggered when a system is approaching its steady-state, and no disturbance is acting on it to relieve network traffic. This also necessitates a DETM to be dynamic and adaptive to respond to different system stability and performance requirements. Letting the fixed threshold parameter  $\underline{\sigma}$  in SETM (1) be dynamically or adaptively adjusted, namely in the form of  $\sigma(\bar{t})$ , constitutes one of the possible solutions, which leads to the so-called dynamic threshold parameter based (DTP-based) DETM<sup>[10]</sup>. In a nutshell, DETMs based on either ADVs or DTPs introduce extra dynamics and further design freedom to event-triggered systems, which can thus be regarded as a promising alternative to the traditional SETMs.

The rationale behind an ETM is to selectively execute sampling and/or transmission actions in order to efficiently accomplish various control and estimation tasks. Hence, ETMs intrinsically trade real-time control and estimation performance for resource efficiency since the designed event-based controllers and estimators are merely executed intermittently. Furthermore, the majority of existing DETMs make sampling and/or transmission actions work in a more sporadic fashion to confront a severe shortage of computation and communication resources. It is therefore imaginable that most existing dynamic event-triggered control and estimation approaches may sacrifice more real-time control and estimation performance in exchange for significantly decreased resource utilization. Still, understanding that no single best DETM can meet all design objectives and application requirements, there is a clear need to present ways of evaluating different DETMs that successfully achieve the same control/estimation task in order to understand which one is advantageous and at what cost. It is also noted that some existing DETM strategies in the literature have certain limitations that hamper their implementation in practice. An insightful examination of existing DETMs for various event-triggered control and estimation problems is also needed.

Albeit the control and estimation theory of sampled-data systems and networked control systems has been well developed, there is a lack of mature theory for event-triggered systems, not mentioning dynamic triggering. As dynamic triggering ideas gain increasing popularity and application in the field of event-triggered control and estimation, any progress made in DETMs will benefit this emerging field as well as the widespread areas of systems and control, detection, and optimization. The overall aim of this survey is to emphasize the motivations of DETMs and the wide technical context of dynamic event-triggered control and estimation, and further promote the dynamic triggering ideas to other related tasks that can be implemented in a resource-efficient and intelligent manner. Specifically, this survey presents a comprehensive review of dynamic triggering techniques, their main

features and benefits, and the relevant analysis and design techniques for event-triggered control and estimation of resource-constrained networked systems. A general event-triggered control/estimation framework is firstly presented, which enables several key issues during construction, design, and implementation of desired event trigger and controller/estimator to be comprehensively examined. An emphasis is then placed on two typical classes of DETMs based on ADVs and DTPs. The main techniques of constructing ADVs and DTPs and the corresponding analysis and design methods are elaborated. Furthermore, several application examples are presented to demonstrate how and when DETMs outperform the classical SETMs and TTM. It is shown through fair comparisons that DETMs are more flexible and intelligent to strike a trade-off between desired control/estimation performance and satisfactory resource efficiency, which is useful for identifying scenarios where dynamic triggering offers potential benefits.

Notice that there are several reviews of the advances in ETMs in the published literature on different design objectives and networked systems, such as [11–14] on static event-triggered control of networked control systems, [15] on static sampled-data-based event-triggered control and filtering of networked systems, [16, 17] on static event-triggered control and filtering/estimation of networked systems, [18] on static event-triggered distributed estimation of wireless sensor network-based monitoring systems, [19, 20] that focus on static event-triggered consensus of multi-agent systems, and [10] on dynamic event-triggered distributed coordination control of multi-agent systems. Meanwhile, a book is edited in [6] to cover the latest developments of static event-based control and signal processing for a variety of networked systems. A bibliometric analysis of the published results on event-based control in the last twenty years is conducted in [21], which identifies the most relevant articles, authors, institutions, and journals. This survey, however, is dedicated to DETMs and reviewing the related studies that are not covered in the surveys and book mentioned above, paying special attention to those published in recent seven years. Moreover, this paper presents a comprehensive compilation of state-of-the-art dynamic triggering techniques, which will serve as a direct reference for interested readers in the field of event-triggered control and estimation. A snapshot of the structure of this survey is shown in Fig. 2.

## 2 A general event-triggered control and estimation framework

### 2.1 A plant

For simplicity of exposition, the dynamics of the plant are modeled by the following state-space equations:

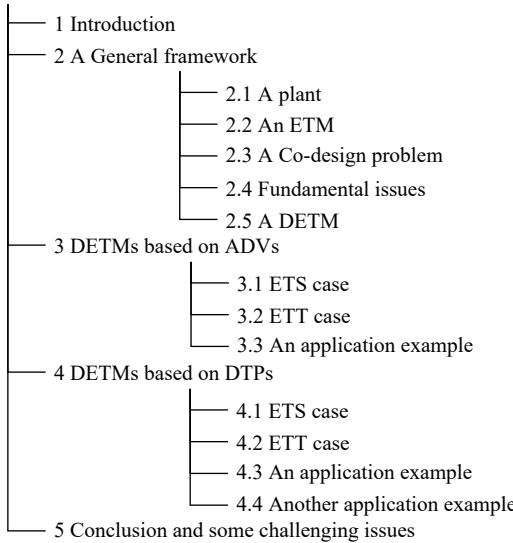


Fig. 2 Skeleton structure of this survey

$$\begin{cases} x^\Delta(t) = Ax(t) + Bu(t) + Ew(t) \\ y(t) = Cx(t) + Dv(t) \\ p(t) = Fx(t) + Gu(t) \end{cases} \quad (2)$$

where  $t$  denotes the time variable with  $t \in \mathbf{R} = \{t|t \geq 0\}$  in the continuous-time case and  $t \in \mathbf{N} = \{t|t = 0, 1, 2, \dots\}$  in the discrete-time case;  $x(t) \in \mathbf{R}^{n_x}$  denotes the state vector;  $x^\Delta(t)$  denotes the differential operator  $\dot{x}(t)$  in the continuous-time case and the one step ahead operator  $x(t+1)$  in the discrete-time case, respectively;  $u(t) \in \mathbf{R}^{n_u}$  denotes the desired control input;  $w(t) \in \mathbf{R}^{n_w}$  represents the general unknown input (e.g., process noise, external disturbance);  $y(t) \in \mathbf{R}^{n_y}$  stands for the system measurement output recorded by an on-board sensor device;  $v(t) \in \mathbf{R}^{n_v}$  denotes the unknown measurement noise;  $p(t) \in \mathbf{R}^{n_p}$  stands for the controlled system output;  $x(0)$  represents an initial state;  $A, B, C, D, E, F$  and  $G$  are matrices of appropriate dimensions.

## 2.2 An event-triggered mechanism

We first recall the traditional state-feedback controller and state estimator (or observer) that employ continuous or periodic system information. We then present a general form of the event-based state-feedback controller and state estimator and discuss several key issues that need to be carefully addressed. For concision and convenient development, the mathematical notations used in ETMs are clarified in Table 1.

The traditional state-feedback controller for system (2) takes the following form:

$$u(t) = Kx(t) \quad (3)$$

and the state estimator (or observer) for system (2) is described as

$$\begin{cases} \hat{x}^\Delta(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)) \\ \hat{p}(t) = F\hat{x}(t) + Gu(t) \end{cases} \quad (4)$$

where  $\hat{x}(t) \in \mathbf{R}^{n_x}$  denotes the state estimate of the system state  $x(t)$  at time  $t$ ;  $\hat{x}^\Delta(t)$  denotes the differential operator  $\dot{\hat{x}}(t)$  in the continuous-time case, and the one step ahead operator  $\hat{x}(t+1)$  in the discrete-time case, respectively;  $\hat{p}(t) \in \mathbf{R}^{n_p}$  denotes the output of the estimator;  $K$  and  $L$  represent the controller and estimator gain matrices to be designed. Obviously, the system state  $x(t)$  (or measurement output  $y(t)$ ) needs to be continually supplied to the controller (3) (or the estimator (4)) at all times  $t \in \mathbf{R}$  in the continuous-time case or  $t \in \mathbf{N}$  in the discrete-time case. Such a requirement is inapplicable or even invalid in a networked communication setting where only digitized and sampled data packets are permitted to be intermittently transmitted between networked and distributed system components.

A common form of an event-triggered control and scheduling policy  $\pi_m^x$  for system (2) is given by

$$\pi_m^x : \begin{cases} u(t) = Kx(t_m), & t \in [t_m, t_{m+1}) \\ t_{m+1} = \inf\{\tilde{t} > t_m | g(e_x(\tilde{t})) > \underline{g}f(X(\tilde{t}))\} \end{cases} \quad (5)$$

and similarly, an event-triggered estimation and scheduling policy  $\pi_m^y$  for system (2) is described as

$$\pi_m^y : \begin{cases} \hat{x}^\Delta(t) = A\hat{x}(t) + Bu(t) + L(y(t_m) - C\hat{x}(t)), \\ t \in [t_m, t_{m+1}) \\ \hat{p}(t) = F\hat{x}(t) + Gu(t) \\ t_{m+1} = \inf\{\tilde{t} > t_m | g(e_y(\tilde{t})) > \underline{g}f(Y(\tilde{t}))\} \end{cases} \quad (6)$$

where  $g(e_x(\tilde{t}))$  (or  $g(e_y(\tilde{t}))$ ) denotes a function of the triggering error  $e_x(\tilde{t})$  (or  $e_y(\tilde{t})$ ) and  $X(\tilde{t})$  (or  $Y(\tilde{t})$ ) denotes either the last triggered or present state (or output).

It can be seen from the above control (or estimation) and scheduling policy  $\pi_m^x$  (or  $\pi_m^y$ ) that the data  $x(t)$  (or  $y(t)$ ) will be sampled and/or transmitted only when the relevant triggering condition is satisfied but not over the releasing interval  $[t_m, t_{m+1})$  in the continuous-time case or in the set of releasing instants  $\{t_m, t_m + 1, \dots, t_{m+1} - 1\}$  in the discrete-time case. In this sense, it is expected that the number of data samplings and/or transmissions over a communication network can be significantly reduced under (5) and (6), which further leads to less resource consumption.

## 2.3 An event-triggered control/estimation and scheduling co-design problem

The co-design problem to be pursued for system (2) is stated as follows: For system (2), the objective is to design a suitable event-triggered control (or estimation) and scheduling policy  $\pi_m^x$  of the form (5) (or  $\pi_m^y$  in the

form of (6)) such that the equilibrium point of the resulting closed-loop system (or estimation error system) is uniformly ultimately bounded, namely, if there exists a compact set  $\mathcal{S} \subset \mathbf{R}^{n_x}$ , then for all  $x(0) \in \mathcal{S}$ , there exists a bound  $\delta \geq 0$  and a time  $t_s(\delta, x(0))$  such that  $\|x(t)\| \leq \delta$  (or  $\|\psi(t)\| = \|x(t) - \hat{x}(t)\| \leq \delta$ ) for all  $t \geq t_s$ .

We note the following two points for the problem above.

1) Stability, performance and optimization: Due to the presence of the generally unknown inputs  $w(t)$  and  $v(t)$  in system (2), the bound  $\delta$  needs to be suitably regulated to an acceptable level in a practical scenario, which leads to bounded/practical solutions of the resulting closed-loop system (or estimation error system). Generally, one may distinguish different concepts of stability for the closed-loop dynamics (or estimation error dynamics), or convergence of the established control/estimation algorithms, depending on various noise assumptions and different control (or estimation) objectives, such as asymptotic stability<sup>[22,23]</sup>, exponential stability<sup>[24–26]</sup>, mean-square stability<sup>[27–29]</sup>, finite-time and fixed-time stability<sup>[30–34]</sup>.

Apart from stability, performance evaluation and optimization are also of great importance. In order to measure the response quality of the closed-loop (or estimation error) dynamics, several performance and optimization indices can be suitably explored to evaluate the closed-loop (or estimation error) system responses, such as bounded error covariance<sup>[35, 36]</sup>,  $H_\infty$  performance<sup>[22, 26, 37]</sup>, mixed  $H_2/H_\infty$  performance<sup>[38, 39]</sup>, and set-valued performance<sup>[40–42]</sup>. Nevertheless, formalizing a suitable performance index and a notion of optimality generally depends on the type of disturbance and noise as well as the concerned system model.

2) Continuous-time and discrete-time system cases: In the continuous-time system case, the event triggering instants  $\{t_m\}$  in the policy  $\pi_m^x$  (or  $\pi_m^y$ ) can be determined by the following two triggering strategies.

**ETS.** The continuous-time system state  $x(t)$  (or measurement output  $y(t)$ ) is sampled and transmitted over some communication networks at the same time. Under such an ETS: i) The actions of sampling and transmitting occur simultaneously. ii) Each triggering instant satisfies  $t_m \in \mathbf{R}$  and the set  $\{t_m\} \subset [0, \infty)$  holds. iii) Some extra hardware/device may be required to continuously monitor the system state  $x(t)$  or measurement output  $y(t)$  in order to judge the triggering condition  $g(e_x(\tilde{t})) \geq \underline{\sigma} f(X(\tilde{t}))$  (or  $g(e_y(\tilde{t})) \geq \underline{\sigma} f(Y(\tilde{t}))$ ) and decide whether an event should be released at any time  $\tilde{t}$ . Undoubtedly, this may increase system expenditure and difficulty of practical implementation of the trigger.

**ETT.** The continuous-time system state  $x(t)$  (or output  $y(t)$ ) is firstly sampled at discretized and equidistant instants of time  $\{kh | k \in \mathbf{N}\}$ . The sampled data  $x(kh)$  (or  $y(kh)$ ) with its time-stamp  $k$  are then encapsulated into a single data packet  $(k, x(kh))$  (or  $(k, y(kh))$ ) for possible

transmission. The decision of when to transmit the sampled data packet is made by the ETT-based event trigger at every sampling instant  $kh$ . It is thus clear that: i) The actions of sampling and transmitting take place at discrete but different instants. ii) The event triggering instants belong to the set of sampling instants, i.e.,  $t_m \in \{kh\}$ . iii) There is no need to persistently monitor the continuous data  $x(t)$  (or  $y(t)$ ). In the literature, there are some alternative terminologies for the ETT-based mechanism, such as sampled-data-based ETM<sup>[13, 15, 19, 43, 44]</sup> or periodic ETM<sup>[45–48]</sup>.

In the discrete-time system case, although the time variable  $t$  of system (2) and policies (5) and (6) takes values in the set of non-negative integers, i.e.,  $t \in \mathbf{N}$ , it is noted that the discrete-time system is often derived via approximating the original continuous-time system at discretized instants of time, e.g.,  $t = kh$  in a periodic sampling case. From this perspective, the ETM in the discrete-time system case is similar to the ETT scenario in the continuous-time system case. Hence, unless otherwise clarified, we do not explicitly distinguish the discrete-time and continuous-time cases in the subsequent discussions.

## 2.4 Fundamental issues

The triggers in (5) and (6) can be unified in a general form of the trigger in (1). It is noteworthy that a variety of triggering conditions have been explored in the existing literature for the static event trigger (1). For example, let the error function be

$$g(e(\tilde{t})) \triangleq \|\Phi^{\frac{1}{2}}e(\tilde{t})\|^2. \quad (7)$$

Some common form of the threshold functions  $\underline{\sigma}f(Z(\tilde{t}))$  can be given as follows.

Constant threshold<sup>[41, 49–51]</sup>:

$$\underline{\sigma}f(Z(\tilde{t})) \triangleq \underline{\sigma}, \underline{\sigma} \geq 0. \quad (8a)$$

Continuous-data-dependent threshold<sup>[52–55]</sup>:

$$\underline{\sigma}f(Z(\tilde{t})) \triangleq \underline{\sigma}\|\Phi^{\frac{1}{2}}z(t)\|^2, \underline{\sigma} \geq 0. \quad (8b)$$

Sampled-data-dependent threshold<sup>[43, 56, 57]</sup>:

$$\underline{\sigma}f(Z(\tilde{t})) \triangleq \underline{\sigma}\|\Phi^{\frac{1}{2}}z(kh)\|^2, \underline{\sigma} \geq 0. \quad (8c)$$

Triggered-data-dependent threshold<sup>[44, 58, 59]</sup>:

$$\underline{\sigma}f(Z(\tilde{t})) \triangleq \underline{\sigma}\|\Phi^{\frac{1}{2}}z(t_m)\|^2, \underline{\sigma} \geq 0. \quad (8d)$$

Time-dependent threshold<sup>[60–62]</sup>:

$$\underline{\sigma}f(Z(\tilde{t})) \triangleq \underline{\sigma} + \sigma_1 e^{-\sigma_2 t}, \underline{\sigma}, \sigma_1 \geq 0, \sigma_2 > 0. \quad (8e)$$

It can be also a combination of the above thresholds<sup>[63–66]</sup>.

To guarantee practical implementation of the event trigger (1) and well-posedness of the event-triggered control/estimation and scheduling co-design problem, the following three fundamental issues are required to be well addressed.

1) Minimal inter-event time (IET): Event triggers are often embedded in advanced sensing or transceiver devices. A critical design issue is thus to determine how fast the device should release the so-called events, or equivalently, how often the desired remote controller/estimator should be updated with the newly arrived data. Such an issue is commonly interpreted as Zeno-freeness or exclusion of Zeno behavior of the event-based sensor/controller/estimator, i.e., there must exist a strictly positive minimal IET  $T_{\min} > 0$  such that  $T_m \geq T_{\min}$ . Only in this way the event-based sensor/controller/estimator will not perform an infinite number of updates in a finite time period on digital platforms.

In the ETT case, it is clear that the IETs satisfy that  $T_m \geq h > 0$  for all  $m \in \mathbb{N}$ , and the sampling period  $h$  guarantees the strict positiveness of the minimal IET. However, it is generally difficult to prove the existence of a strictly positive lower bound of the IETs for ETS-based triggers. Therefore, ETTs may offer more simplicity and convenience than ETSs for real-time implementation of event-based controllers and estimators.

2) Continuous monitoring VS. periodic sampling: As mentioned in Section 2.3, the ETS-based triggers dictate the continuous system state  $x(t)$  or output  $y(t)$  to be available at all times  $t \in \mathbf{R}$ , which means that some dedicated hardware is demanded to meet such a continuous monitoring requirement. This requirement may increase the overheads of system monitoring and operation or may find its technical impossibility in a cyber-physical application scenario. Whereas, the ETT-based triggers operate only at sampling instants of time, e.g.,  $\{kh, k \in \mathbb{N}\}$ , which naturally excludes continuous monitoring and makes them better suited for practical implementation in standard time-sliced embedded hardware and software architectures. However, under such an ETT-based trigger, the sensor or trigger is configured to sample the system state  $x(t)$  or output  $y(t)$  after each fixed interval of time, no matter whether it is actually needed for preserving stability and performance. This may shorten sensor lifetime because one of the main causes for energy consumption of a real-world sensor devise arises from its persistent message listening and sampling<sup>[18]</sup>.

In the context of ETSs, there are two common techniques that can be adopted to eliminate the continuous monitoring of the system data at all times  $t \in \mathbf{R}$ : One is called time regularization<sup>[67–69]</sup>, where a positive time threshold  $T_w$  (acting as time regularization or waiting time between two contiguous events) is inserted during the verification of the triggering condition such that the next triggering instant  $t_{m+1}$  is always produced after at

least  $T_w$  units of time, the other is to introduce an additional positive constant threshold into the right-hand side of the triggering condition<sup>[63, 66]</sup> (e.g., in (8a) and (8e)) at the expense of a sacrifice in accurate stability (asymptotic convergence) in exchange for practical/bounded system stability (convergence to a neighborhood around zero).

3) Triggering condition constraint: Under the event trigger (1), it is clear that within each IET, there is no occurring event, namely,

$$g(e(\tilde{t})) \leq \underline{\sigma}f(Z(\tilde{t})), \quad \forall t \in [t_m, t_{m+1}).$$

Therefore, the above inequality constraint is required to be suitably accommodated in the desired analysis and design criteria so as to preserve the existence of an admissible event-based controller or estimator.

It is further noted that in the continuous-time case, the system behaviour under event-triggered control (or estimation) is inherently hybrid, which means that both continuous as well as discrete signals are incorporated in the resulting closed-loop dynamics (or estimation error dynamics). This also poses a challenge to the analysis and design procedures of the event-triggered controller/estimator.

## 2.5 A dynamic event-triggered mechanism

Among several ETMs for networked systems, the subsequent focus is put on DETMs because of the introduced extra dynamics and the potential design freedom that will be unfolded hereinafter. Before elaborating on dynamic event triggers, our discussion shall begin with the conventional static event triggers that have been widely studied in the literature.

Static event triggers can be arguably referred to as the triggers whose threshold functions  $\underline{\sigma}f(Z(\tilde{t}))$  are dependent on only the system information  $Z(\tilde{t})$  (e.g., state, output) and/or the time information (e.g.,  $e^{-\sigma_2 t}$ ) during the entire implementation of the ETM. For example, the event triggers equipping the thresholds (8a)–(8e) are typical SETMs.

Dynamic event triggers are classified as the triggers whose threshold functions include not only the system information  $Z(\tilde{t})$  (e.g., state, output) and/or the time information but also some auxiliary variables or dynamic parameters possessing their own dynamics. A versatile structure of such a DETM can be given as

$$t_{m+1} = \inf\{\tilde{t} > t_m \mid g(e(\tilde{t})) > \sigma(\tilde{t})f(Z(\tilde{t})) + \frac{1}{\epsilon}\lambda(\tilde{t})\} \quad (9)$$

with  $\epsilon > 0$  being a prescribed constant,  $\sigma(\tilde{t}) \geq 0$  denoting a DTP and  $\lambda(\tilde{t}) \geq 0$  denoting an ADV. Compared with SETM (1), it is clear that the introduction of DTP  $\sigma(\tilde{t})$  and/or ADV  $\lambda(\tilde{t})$  into (9) brings extra dynamics and

sometimes adaptiveness to event triggering decisions. In a particular case, by fixing  $\sigma(\tilde{t}) \equiv \underline{\sigma}$  and letting  $\epsilon \rightarrow +\infty$ , DETM (9) reduces to SETM (1). The DETM of the form (9) thus offers a comprehensive trade-off analysis between desired system performance and satisfactory resource efficiency than the traditional SETM (1).

Dynamic triggering ideas have long been pursued to deal with various event-triggered control and estimation problems in the literature. For example, an ADV-based DETM of the following form:

$$t_{m+1} = \inf\{t > t_m \mid g(\|e(t)\|) > \underline{\sigma}f(\|x(t)\|) + \frac{1}{\epsilon}\lambda(t)\} \quad (10)$$

is firstly developed in [8] to investigate the stability of the resulting continuous-time closed-loop nonlinear and linear control systems under guaranteed minimal IET, where the ADV  $\lambda(t)$  is given by

$$\dot{\lambda}(t) = -\alpha(\lambda(t)) + \underline{\sigma}f(\|x(t)\|) - g(\|e(t)\|) \quad (11)$$

with  $\alpha(\cdot)$  denoting a locally Lipschitz continuous  $\mathcal{K}_\infty$  function. The discrete-time case of DETM (10) under ADV (11) for stability analysis is studied in [9]. It is proved in [8] that ADV  $\lambda(t)$  in the form of (11) is non-negative for any  $t \in \mathbf{R}$ . This implies that in comparison to the following SETM:

$$t_{m+1} = \inf\{t > t_m \mid g(\|e(t)\|) > \underline{\sigma}f(\|x(t)\|)\} \quad (12)$$

it will be much more stringent for the triggering error function  $g(\|e(t)\|)$  of DETM (10) to exceed the new threshold  $\underline{\sigma}f(\|x(t)\|) + \frac{1}{\epsilon}\lambda(t)$  because  $\frac{1}{\epsilon}\lambda(t) \geq 0$  for any  $t \in \mathbf{R}$ . In [68], a DETM in the form of

$$t_{m+1} = \inf\{t > t_m + T_w \mid \lambda(t) < 0\} \quad (13)$$

is presented to reduce communication cost while guaranteeing desired stability and performance criteria despite the presence of packet losses, where the ADV evolves according to

$$\begin{cases} \dot{\lambda}(t) = \phi(o(t), \lambda(t)), \lambda(t) \geq 0 \\ \lambda(t^+) = \lambda_0(o(t)), \lambda(t) = 0 \text{ & release successful} \\ \lambda(t^+) = \lambda(t), \lambda(t) = 0 \text{ & release failed} \end{cases} \quad (14)$$

with  $o(t)$  denoting all the information locally available at the trigger. On account of the time regularization, the DETM (13) subject to (14) generates the next event always after at least  $T_w$  time units even in the presence of disturbance, which thus guarantees Zeno-freeness and preserves robustness of the event trigger. Similar DETMs of (13) subject to (14) are studied in [67, 69, 70] for different problem formulations.

Although it is generally difficult to theoretically prove that the DETM in the form of (10) outperforms SETM

(12) in terms of guaranteed resource efficiency, it is shown in [8, 68] through simulations that the generated IETs under the relevant DETMs are typically larger than those of their static counterparts (i.e., SETM (12) for [8] and SETM under the triggering condition  $\phi(o(t), 0) < 0$  for [68]). Furthermore, it is formally proved in [8] that for a given state  $x(t_m)$ , the next triggering time under DETM (10) will be larger than or equal to that under SETM (12), namely,  $t_{m+1}^{\text{DETM (10)}} \geq t_{m+1}^{\text{SETM (12)}}$ . In other words, the minimal IET for the DETM (10) cannot be smaller than that for the SETM (12), which serves as the essential profit motive of the ADV-based DETMs for networked systems. Since the minimum IET can be interpreted as how far or safe the event trigger is away from the Zeno behavior, it is generally believed that an ADV-based DETM produces more sporadic data samplings/transmissions than the relevant static analogue.

### 3 Dynamic event-triggered mechanisms based on auxiliary dynamic variables

In this section, depending on the triggering strategies, i.e., ETS or ETT, the existing DETMs based on the ADV technique are classified and discussed. Note that a focus is placed on only the construction of the triggering mechanism, specifically, the ADV, while the Zeno-freeness analysis is left out as one may either formally prove the existence of a strictly positive minimal IET or employ the time regularization technique or constant threshold techniques aforementioned.

#### 3.1 Event-triggered sampling case

Consider the following ADV- and ETS-based triggering mechanism:

$$t_{m+1} = \inf\{t > t_m \mid q^\sigma(t) > \frac{1}{\epsilon}\lambda(t)\} \quad (15)$$

where  $q^\sigma(t) \triangleq \|\Phi^{\frac{1}{2}}e(t)\|^2 - \underline{\sigma}\|\Phi^{\frac{1}{2}}Z(\tilde{t})\|^2$  with  $e(t) = z(t_m) - z(t)$  and  $\underline{\sigma} \geq 0$ ,  $\epsilon > 0$  being two given constants.

1) Continuous-time case: The ADV  $\lambda(t)$  in (15) can be defined as follows:

$$\dot{\lambda}(t) = -\mu\lambda(t) - \chi q^\sigma(t), \quad t \in [t_m, t_{m+1}] \quad (16)$$

with  $\mu \geq 1/\epsilon > 0$  and  $\chi > 0$  denoting two prescribed constants and  $\lambda(0) \geq 0$  being a given initial condition. Note that a salient feature of the DETM (15) is that the introduced ADV  $\lambda(t)$  of the form (16) is non-negative, i.e.,  $\lambda(t) \geq 0$  holds for all  $t \in \mathbf{R}$ . Indeed, it is clear that no event is triggered for  $t \in [t_m, t_{m+1})$  and thus the following condition that  $q^\sigma(t) \leq \frac{1}{\epsilon}\lambda(t)$  holds. Recalling (16), it can be derived that  $\dot{\lambda}(t) \geq -\mu\lambda(t) - \frac{\chi}{\epsilon}\lambda(t) =$

$-\left(\mu + \frac{\chi}{\epsilon}\right)\lambda(t) \triangleq -\tilde{\mu}\lambda(t)$  for  $t \in [t_m, t_{m+1}]$ . Then, multiplying its both sides by  $e^{\tilde{\mu}t}$  and noting that  $\frac{d}{dt}(e^{\tilde{\mu}t}\lambda(t)) = \tilde{\mu}e^{\tilde{\mu}t}\lambda(t) + e^{\tilde{\mu}t}\dot{\lambda}(t)$ , we further have that  $\frac{d}{dt}(e^{\tilde{\mu}t}\lambda(t)) \geq 0$ ,  $t \in [t_m, t_{m+1}]$ . Then integrating its both sides from  $t_m$  to  $t$  and noting  $\tilde{\mu} > 0$ , one obtains that  $\lambda(t) \geq \lambda(t_m)e^{-\tilde{\mu}(t-t_m)} \geq \lambda(t_{m-1})e^{-\tilde{\mu}(t-t_{m-1})} \geq \dots \geq \lambda(0)e^{-\tilde{\mu}t} \geq 0$  for  $t \in [t_m, t_{m+1}]$  and thus all  $t \in \mathbf{R} = \bigcup_{m=0}^{\infty} [t_m, t_{m+1}]$ .

During system performance analysis and control design, an additional Lyapunov function term  $V_\lambda = \lambda(t)$  may be introduced to deal with the inequality constraint posed by the triggering condition under the non-negative ADV (16). Then, it is easy to derive that  $\dot{V}_\lambda \leq \dot{V}_\lambda + \frac{1}{\epsilon}\lambda(t) - q^\sigma(t) = \left(\frac{1}{\epsilon} - \mu\right)\lambda(t) - (\chi + 1)q^\sigma(t) \leq -(\chi + 1)q^\sigma(t) = -(\chi + 1)\|\Phi^{\frac{1}{2}}e(t)\|^2 + (\chi + 1)\underline{\sigma}\|\Phi^{\frac{1}{2}}Z(t)\|^2$  since  $\left(\frac{1}{\epsilon} - \mu\right)\lambda(t) \leq 0$ .

The continuous ADV  $\lambda(t)$  in the form of (16) has been intensively investigated for various dynamic event-triggered multi-agent coordination control problems. For example, two dynamic event-triggered control laws are proposed in [71] to cope with the average consensus problem for a class of first-order continuous-time multi-agent systems under undirected and connected graphs. In [72], via the time regularization technique, a hybrid dynamic event trigger is devised to solve the consensus problem for general linear multi-agent systems with external disturbances. In [73], both the dynamic event-triggered leaderless and leader-follower consensus problems of general linear multi-agent systems are studied. In [74], the formation-containment control of general linear multi-agent systems is addressed, where the leader-to-leader and follower-to-follower communications are regulated by a dynamic event trigger. Recent advances in dynamic event-triggered distributed coordination control can be found in the survey [10]. For a linear time-invariant networked system, an ETS-based dynamic event-triggered controller is designed in [75] with an  $L_2$ -gain performance guarantee. In the context of event-triggered estimation, the continuous ADV  $\lambda(t)$  has also been widely investigated. To name a few, a DETM of the form (15) with ADV (16) and  $q^\sigma(t) \triangleq \|\Phi^{\frac{1}{2}}e(t)\|^2 - \underline{\sigma}$  is studied in [76] for a class of continuous-time polynomial nonlinear systems with external disturbance. In [77], the DETM (15) is employed to address the fault estimation and accommodation problem for continuous-time linear systems.

2) Discrete-time case: The ADV  $\lambda(t)$  in (15) can be described by

$$\lambda(t+1) = \mu\lambda(t) - \chi q^\sigma(t), \quad t \in \{t_m, t_m + 1, \dots, t_{m+1} - 1\} \quad (17)$$

with  $\epsilon > 0, \chi > 0, \mu \in (0, 1)$  and  $\mu \geq \chi/\epsilon$  denoting some prescribed constants and  $\lambda(0) \geq 0$  being a given initial

condition. Similarly, the ADV  $\lambda(t) \geq 0$  holds at all time steps  $t \in \mathbf{N}$ . Actually, it is readily seen from (15) that  $q^\sigma(t) \leq \frac{1}{\epsilon}\lambda(t)$  for any  $t \in \{t_m, t_m + 1, \dots, t_{m+1} - 1\}$ . Recalling (17), it can be inferred that  $\lambda(t+1) \geq \mu\lambda(t) - \frac{\chi}{\epsilon}\lambda(t) \triangleq \tilde{\mu}\lambda(t) \geq \dots \geq \tilde{\mu}^{t+1}\lambda(0) \geq 0$  by recalling that  $\tilde{\mu} \triangleq \mu - \chi/\epsilon \geq 0$  and  $\lambda(0) \geq 0$ .

Analogously, one may adopt the following Lypaunov function term  $V_\lambda = \lambda(t)$  to cope with the relevant triggering condition constraint. Then, one has that  $\Delta V_\lambda = \lambda(t+1) - \lambda(t) = -(1 - \mu)\lambda(t) - \chi q^\sigma(t) \leq -(\epsilon(1 - \mu) + \chi)q^\sigma(t) \triangleq -\tilde{\mu}q^\sigma(t) = -\tilde{\mu}\|\Phi^{\frac{1}{2}}e(t)\|^2 + \tilde{\mu}\underline{\sigma}\|\Phi^{\frac{1}{2}}Z(t)\|^2$ .

For a class of nonlinear discrete-time systems represented by polynomial fuzzy models, a decentralized version of the DETM (15) subject to (17) is adopted in [78] for solving an  $H_\infty$  state feedback control problem. In [79], an ADV-based event-triggered  $H_\infty$  dynamic output feedback control method is devised for a class of sensor saturated systems with external disturbances. In [80], a dynamic event-triggered estimation approach under ADV (17) and  $q^\sigma(t) \triangleq \|\Phi^{\frac{1}{2}}e(t)\|^2 - \underline{\sigma}$  is presented for a class of discrete-time singularly perturbed systems with distributed time-delays. In a multi-sensor network setting, a novel dynamic event-triggered distributed set-membership estimation approach is developed in [40] for a class of discrete-time linear time-varying systems subject to unknown-but-bounded process and measurement noises. A cluster of ellipsoidal sets centered at the computed state estimates are derived for the distributed and networked sensors such that the plant's true state always resides in each sensor's bounding ellipsoid at each time step regardless of the process and measurement noises. In [81], the recursive distributed filtering problem under the DETM (15) of ADV (17) and a constant threshold is studied for a class of discrete nonlinear time-varying systems over Gilbert-Elliott channels.

### 3.2 Event-triggered transmission case

Consider the ADV-based and ETT-based triggering mechanism of the following form:

$$t_{m+1} = \inf\{s_m > t_m \mid q^\sigma(s_m) > \frac{1}{\epsilon}\lambda(s_m)\} \quad (18)$$

where  $q^\sigma(s_m) \triangleq \|\Phi^{\frac{1}{2}}e(s_m)\|^2 - \underline{\sigma}\|\Phi^{\frac{1}{2}}Z(\tilde{t})\|^2$  with  $e(s_m) = z(t_m) - z(s_m)$  and the ADV  $\lambda(s_m)$  being defined as

$$\dot{\lambda}(t) = -\mu\lambda(t) - \chi q^\sigma(s_m), \quad t \in [s_m, s_{m+1}] \quad (19)$$

with  $\mu > 0$  and  $\chi > 0$  denoting two prescribed constants and satisfying that  $\epsilon\mu < \chi$ , and  $\lambda(0) \geq 0$  being a given initial condition.

For any  $t \in [s_m, s_{m+1}]$ , solving the differential equation (19) yields that  $\lambda(t) = e^{-\mu(t-s_m)}\lambda(s_m) + \frac{1}{\mu}(1 - e^{-\mu(t-s_m)})$ .

$(-\chi q^\sigma(s_m)) \geq \left( \frac{(\epsilon\mu + \chi)e^{-\mu(t-s_m)} - \chi}{\epsilon\mu} \right) \lambda(s_m)$ . Notice that for all  $t \in [s_0, s_1]$ , one has that  $\lambda(t) \geq \left( \frac{(\epsilon\mu + \chi)e^{-\mu(t-s_0)} - \chi}{\epsilon\mu} \right) \lambda(s_0) \geq \left( \frac{(\epsilon\mu + \chi)e^{-\mu h} - \chi}{\epsilon\mu} \right) \lambda(s_0)$ . Letting  $\epsilon \geq \chi(e^{\mu h} - 1)/\mu$ , it is clear that  $\lambda(t) \geq 0$  for all  $t \in [s_0, s_1]$ . Since  $\lambda(t)$  is a continuous function with respect to the time variable  $t$ , one then has  $\lambda(s_1) \geq 0$ . Using the mathematical induction, one can conclude that  $\lambda(t) \geq 0$  holds for all  $t \in \mathbf{R}$  if  $\epsilon \geq \chi(e^{\mu h} - 1)/\mu$  (and thus  $\mu h < \ln(2)$ ).

During analysis and design, one may consider an additional Lyapunov function term<sup>[82, 83]</sup>  $V_\lambda = \lambda(t)|_{t=s_m}$ . Then, it is derived that  $\dot{V}_\lambda|_{t=s_m} = -\mu\lambda(s_m) - \chi q^\sigma(s_m) \leq (\mu\epsilon - \chi)q^\sigma(s_m) \triangleq -\tilde{\mu}q^\sigma(s_m) = -\tilde{\mu}\|\Phi^{\frac{1}{2}}e(s_m)\|^2 + \tilde{\mu}\sigma\|\Phi^{\frac{1}{2}}Z(\tilde{t})\|^2$  given that  $\tilde{\mu} > 0$ . For example, to deal with the limited bandwidth allocation, the above dynamic event trigger is adopted in [82] for primary-redundancy communication channels, where both event-triggered primary and redundancy state feedback control laws are designed to guarantee the exponential stability of the resulting closed-loop switched delay system. In [83], the observer-based dynamic event-triggered control problem under power-constrained denial-of-service attacks is considered. A dynamic event trigger based on sampled state estimates is employed to economize the limited bandwidth.

Alternatively, DETM (18) can be modified as

$$t_{m+1} = \inf\{s_m > t_m \mid q^\sigma(s_m) > \frac{1}{\epsilon}\lambda(t)\} \quad (20)$$

which involves a continuous ADV  $\lambda(t)$  under  $\mu \geq \frac{1}{\epsilon}$ . A benefit of such a triggering mechanism is that the ADV  $\lambda(t)$  can be readily shown to be non-negative for all  $t \in \mathbf{R}$ . More specifically, (19) implies that  $\dot{\lambda}(t) = -\mu\lambda(t) - \chi q^\sigma(s_m) \geq -(\mu + \chi/\epsilon)\lambda(t) \triangleq -\tilde{\mu}\lambda(t)$  for all  $t \in [s_m, s_{m+1}]$ , which further means that  $\lambda(t) \geq \lambda(s_m)e^{-\tilde{\mu}(t-s_m)} \geq \lambda(s_{m-1})e^{-\tilde{\mu}(t-s_{m-1})} \geq \dots \geq \lambda(0)e^{-\tilde{\mu}t} \geq 0$ . Then, employing  $V_\lambda = \lambda(t)$ , it can be derived that  $\dot{V}_\lambda \leq \dot{V}_\lambda - q^\sigma(s_m) + \frac{1}{\epsilon}\lambda(t) = \left(\frac{1}{\epsilon} - \mu\right)\lambda(t) - (\chi + 1)q^\sigma(s_m) \leq -(\chi + 1)\|\Phi^{\frac{1}{2}}e(s_m)\|^2 + (\chi + 1)\sigma\|\Phi^{\frac{1}{2}}z(t_m)\|^2$  given that  $\left(\frac{1}{\epsilon} - \mu\right)\lambda(t) \leq 0$ . However, one drawback of the DETM (20) is that it demands the continuous ADV  $\lambda(t)$ , which seems to break a promise of eliminating the continuous monitoring of any signal under ETT in (20). Similar DETMs with hybrid signals can be also found in [84–87], where the triggering conditions therein are checked based on both periodically sampled data and continuously evolving threshold parameter  $\sigma(t)$ . As a result, these DETMs still need to be implemented and performed continuously.

As discussed in Section 2.4, most ETT-based DETMs leverage a periodic sampling paradigm, which may cause excessive energy consumption on persistent message

listening and sampling. To further promote resource-efficient control and estimation applications, it is desirable to develop refined ETT-based dynamic event triggers based on aperiodic/nonuniform sampling instants  $\{s_m \mid m \in \mathbf{N}, s_m \in \mathbf{R}, 0 < s \leq s_{m+1} - s_m \leq \bar{s}\}$ . In this case, the triggers employ only sporadically sampled data  $z(s_m)$  to decide whether or not the sampled data packet should be released at time  $s_m$ . For example, for a class of stochastic linear systems, two ADV-based dynamic event-triggered control strategies are proposed in [88] to preserve the stability of the resulting closed-loop system under sporadic measurements and communication delays. The ADV  $\lambda(t)$  therein is regulated according to some impulsive differential equations.

As in [8], it can be formally proved that for a given signal  $z(t_m)$ , the next triggering time under DETM (15) or (18) or (20) will not be smaller than that under the relative SETM in either of the following form:

$$t_{m+1} = \inf\{t > t_m \mid q^\sigma(t) > 0\} \quad (21a)$$

$$t_{m+1} = \inf\{s_m > t_m \mid q^\sigma(s_m) > 0\}. \quad (21b)$$

In other words, the minimum IET of DETM (15) or (18) or (20) cannot be less than that of the relative SETM, namely,  $\min\{T_m^{(21a)}\} \leq \min\{T_m^{(15)}\}$  and  $\min\{T_m^{(21b)}\} \leq \min\{T_m^{(18) \text{ or } (20)}\}$ .

### 3.3 An application to dynamic event-triggered control of an in-vehicle networked active suspension system

In this section, for comparison purposes, we conduct some quantitative performance analysis between the ADV-based DETM and the relative SETM by investigating an event-triggered control problem of an in-vehicle networked active suspension system which has been widely studied in the literature<sup>[84, 89–92]</sup>.

The schematic diagram of the studied event-triggered active suspension control system is demonstrated in Fig. 3, where the quarter vehicle suspension system, consisting of an one-quarter vehicle chassis (represented by a sprung mass  $m_s$ ), a vehicle wheel assembly (represented by an unsprung mass  $m_u$ ), suspension spring ( $f_s = k_s(x_s(t) - x_u(t))$ ) and damper ( $f_d = c_s(\dot{x}_s(t) - \dot{x}_u(t))$ ), and an elastic pneumatic tire component (with the stiffness  $k_t$  and the constant damping coefficient  $c_t$ ), is controlled over a digital and shared controller area network. The dynamical model of the 2-DOF quarter vehicle suspension system can be described by the continuous-time state-space equation (2) with  $x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T = [x_s(t) - x_u(t), x_u(t) - x_r(t), \dot{x}_s(t), \dot{x}_u(t)]^T \in \mathbf{R}^4$  denoting the stacked state vector,  $w(t) = \dot{x}_r(t) \in \mathbf{R}$  denoting the road disturbance input vector,  $u(t) \in \mathbf{R}$  representing the desired actuator force, and

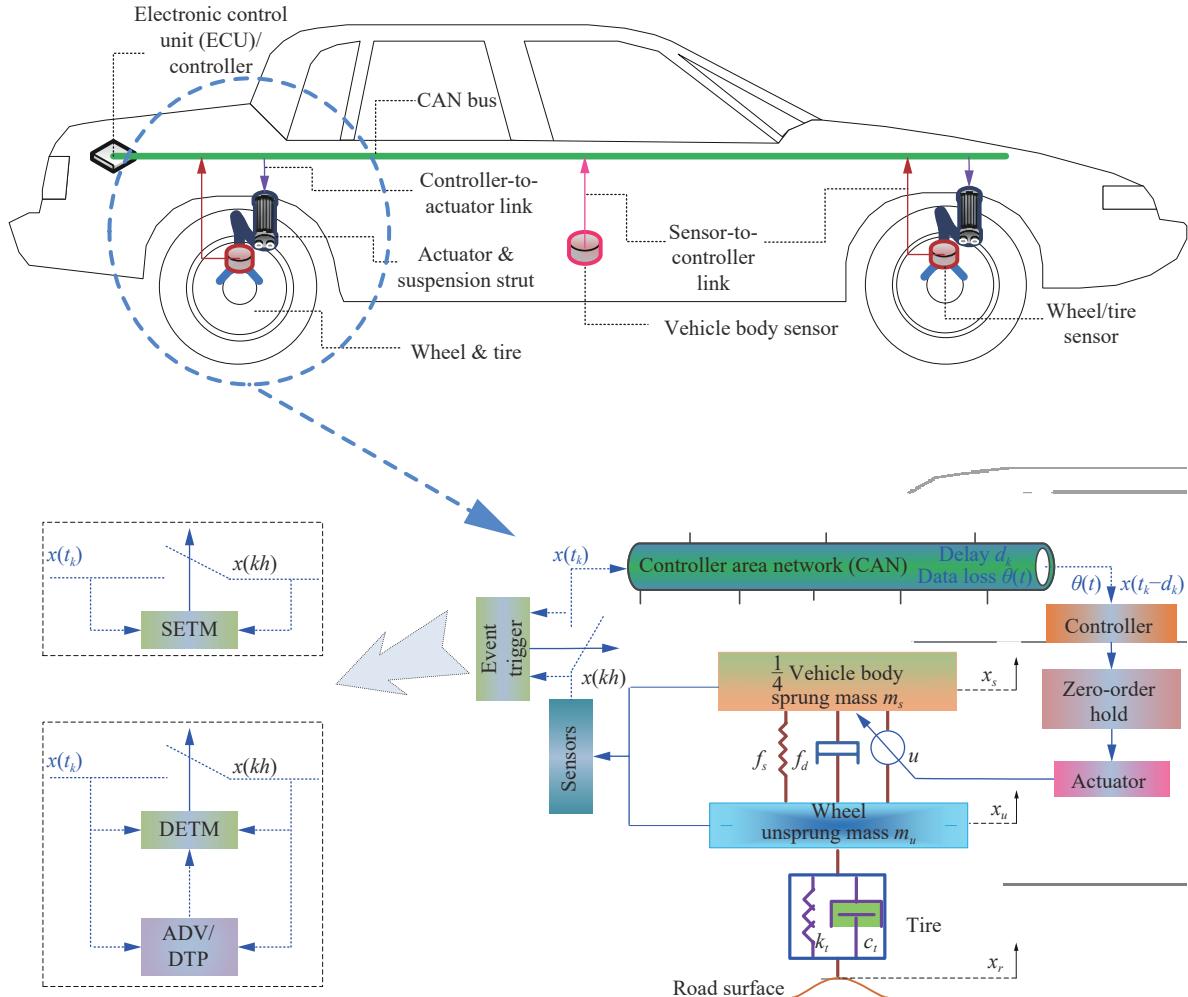


Fig. 3 An event-triggered active suspension control configuration over a controller area network<sup>[92]</sup>

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s + c_t}{m_u} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}, E = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}.$$

Note that the vehicle loads may not be precisely known in practice because of payload changes, which results in the uncertain and time-varying sprung and unsprung masses  $m_s(t)$  and  $m_u(t)$ . In this sense, the Takagi-Sugeno fuzzy modeling approach can be employed to approximate and represent the uncertain active suspension system in terms of a finite number of fuzzy rules. The interested readers are referred to [92] in more detail for the Takagi-Sugeno fuzzy active suspension system.

The main problem to be addressed is now stated as:

For the in-vehicle networked uncertain quarter vehicle active suspension system subject to the external road disturbance input  $w(t) \in \mathcal{L}_2[0, \infty)$ , the objective is to design an ETT-based and ADV-based event-based control and scheduling policy  $\pi_m^x$  of the following form:

$$\begin{cases} u(t) = K(t)\bar{x}_\theta(t) = K(t)\theta(t)x(t_m), \\ t \in [t_m + d_m, t_{m+1} + d_{m+1}] \\ t_{m+1} = \inf\{s_m > t_m \mid q^\sigma(s_m) > \frac{1}{\epsilon}\lambda(s_m)\} \end{cases} \quad (22)$$

where  $\theta(t) \in [\underline{\theta}, \bar{\theta}] \in [0, 1]$  characterizes the data loss ratio of the sensor transmission links;  $d_m \in [\underline{d}, \bar{d}]$  stands for the transmission delay at time step  $m$  induced by the network;  $q^\sigma(s_m) \triangleq \|\Phi^{\frac{1}{2}}(x(t_m) - x(s_m))\|^2 - \sigma\|\Phi^{\frac{1}{2}}x(t_m)\|^2$ , such that 1) (System stability) the resulting closed-loop system with  $w(t) \equiv 0$  is asymptotically stable; 2) (Ride comfort) under zero initial condition and nonzero  $w(t) \in \mathcal{L}_2[0, \infty)$ , the vehicle body acceleration  $p(t) = \ddot{x}_s(t) = \dot{x}_3(t) = Fx(t) + Gu(t)$  with  $F = \left[ -\frac{k_s}{m_s(t)}, 0, -\frac{c_s}{m_s(t)}, \frac{c_s}{m_s(t)} \right]$  and  $G = \frac{1}{m_s(t)}$  is minimized, namely,

$\sup_{\substack{w(t) \neq 0 \\ \gamma > 0}} \frac{\|p(t)\|_2}{\|w(t)\|_2} \leq \gamma$  for a prescribed performance level 3) (Road holding) the dynamic tire load  $k_t x_2(t) = k_t(x_u(t) - x_r(t))$  does not exceed the minimal static load  $(\underline{m}_s + \underline{m}_u)g$ , i.e.,  $r_1(t) = H_1 x(t) \leq 1$  with  $H_1 = [0, k_t / ((\underline{m}_s + \underline{m}_u)g), 0, 0]$ ; and 4) (Suspension stroke) the suspension deflection  $x_1(t) = x_s(t) - x_u(t)$  does not surpass the allowable limit  $x_{\max}$ , i.e.,  $|r_2(t)| = |H_2 x(t)| \leq 1$  with  $H_2 = [1/x_{\max}, 0, 0, 0]$ .

To solve the problem formulated above, Algorithm 1 is presented to achieve the co-design of the desired event-triggered controller and DETM co-design algorithm. A detailed derivation of the inequalities can be found in [92].

**Algorithm 1.** Dynamic event-triggered control and scheduling co-design

1) For suitable scalars,  $h, \gamma, \underline{d}, \bar{d}, \underline{\theta}, \bar{\theta}, \nu, \varpi, \eta, \underline{\sigma}, \mu, \epsilon, \chi$ , solve the following linear matrix inequalities:

$$\begin{bmatrix} \Xi_{ij} & \phi_{1i} & \phi_{2j}^T \\ \phi_{1i}^T & -\nu^{-2} \mathbf{I} & \mathbf{0} \\ \phi_{2j} & \mathbf{0} & -\nu^2 \mathbf{I} \end{bmatrix} < 0, \quad 1 \leq i \leq j \leq 4 \quad (23)$$

$$\begin{bmatrix} -P & \sqrt{\varpi} U^T H_q^T \\ \sqrt{\varpi} H_q U & -\mathbf{I} \end{bmatrix} < 0, \quad q = 1, 2 \quad (24)$$

to find the positive matrices  $P, Q, R_1, R_2, \tilde{\Phi}$  and matrices  $\tilde{U}, \tilde{K}_j$ , where  $\Xi_{ij} = 2e_1^T P e_2 + e_1^T Q e_1 - e_3^T Q e_3 + e_2^T (\underline{d}^2 R_1 + (\bar{d} - \underline{d} + h)^2 R_2) e_2 - (e_1 - e_3)^T R_1 (e_1 - e_3) - \gamma^2 e_6^T e_6 - \frac{\pi^2}{4} (e_3 - e_4)^T R_2 (e_3 - e_4) + 2(e_1^T + \eta e_2^T) (-U e_2 + A_i U e_1 + \bar{\theta}^o B_i \tilde{K}_j (e_4 + e_5) + E_i e_6) + \tilde{\mu} \underline{\sigma} (e_4 + e_5)^T \times \tilde{\Phi} (e_4 + e_5) - \tilde{\mu} e_5^T \tilde{\Phi} e_5 + 2e_1^T U^T F_i^T e_7 + 2e_4^T \bar{\theta}^o \tilde{K}_j^T G_i^T e_7 + 2e_5^T \bar{\theta}^o \tilde{K}_j^T G_i^T e_7 - e_7^T e_7, \phi_{1i} = [B_i^T, \eta B_i^T, \mathbf{0}, \mathbf{0}, \mathbf{0}, G_i^T]^T, \phi_{2j} = [\mathbf{0}, \mathbf{0}, \mathbf{0}, \bar{\theta}^o \tilde{K}_j, \bar{\theta}^o \tilde{K}_j, \mathbf{0}, \mathbf{0}], \tilde{\mu} = \chi - \mu \epsilon, \bar{\theta}^o = (\bar{\theta} + \underline{\theta})/2, \underline{\theta}^o = (\bar{\theta} - \underline{\theta})/2$ ; and  $e_i, \forall i = 1, 2, \dots, 7$ , denoting some block entry matrices, e.g.,  $e_6 = [\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{I}, \mathbf{0}]$ .

if (23) and (24) are infeasible, repeat Step 1) otherwise, determine a feasible solution  $(\tilde{K}_j, \Phi) = (\tilde{K}_j U^{-1}, U^{-T} \tilde{\Phi} U^{-1})$  and continue

2) Set the simulation time  $T_{sim}$ ,  $m = 0$ ,  $t_m = 0$  and  $\tilde{x} = x(0)$

- 3) **for**  $k = 0 : T_{sim}/h - 1$  **do**
- 4) Compute  $u(t)$  in (22) under the newly received data  $\tilde{x}$
- 5) Derive the system dynamics of  $x(t)$  under  $u(t)$
- 6) Obtain the present sampled data packet  $(k, x(kh))$  from the sensor/sampler at time step  $k$
- 7) Determine the ADV  $\lambda(t)$  in (19)
- 8) **if**  $q^{\underline{\sigma}}(k) > \frac{1}{\epsilon} \lambda(k)$  at time step  $k$  **then**
- 9) Release the sampled data packet  $(k, x(kh))$
- 10) Update  $m = m + 1$ ,  $t_{m+1} \leftarrow kh$  and  $\tilde{x} \leftarrow x(t_{m+1})$
- 11) **else** Keep  $\tilde{x}$  unchanged
- 12) **end if**
- 13) **end for**

In the subsequent simulation, we examine the resulting active suspension and control performance as well the resource efficiency under three different ETMs: 1) SETM in the form of (22) without  $\lambda(s_m)$  and under a fixed threshold parameter  $\underline{\sigma} = 0.08$ ; 2) DETMm in the form of (22) under the sampled-data version of  $\lambda(s_m)$ ; and 3) DETMt in the form of (22) under the continuous-time version of  $\lambda(t)$ . In the last two cases, we set  $\lambda(0) = 2, \chi = 1.2, \mu = 0.6, \epsilon = 1.8$ , which straightforwardly implies that  $\chi > \mu \epsilon$  in DETMm and  $\mu > \frac{1}{\epsilon}$  in DETMt. The vehicle dynamics parameters are the same as those in [92]. The other design parameters are set as  $h = 1\text{ms}$ ,  $\gamma = 40$ ,  $\varpi = 0.015$ ,  $\eta = 0.01$ ,  $\nu = 10$ . By applying the co-design algorithm above, it is found that the problem is feasible in all cases. The bump responses of the controlled vehicle suspension system under the bump road disturbance input,  $x_r(t) = \frac{h_b}{2} \left( 1 - \cos \left( \frac{2\pi v_0}{l_b} t \right) \right)$  for  $t \in \left[ 0, \frac{l_b}{v_0} \right]$  and  $x_r(t) = 0$  otherwise, are presented in Fig. 4, where  $h_b = 0.06\text{m}$  and  $l_b = 5\text{m}$  are the height and length of the bump, respectively, and  $v_0 = 45\text{km/h}$  is the vehicle forward velocity. One can observe that regardless of the simultaneous presence of the road disturbance  $x_r(t)$ , transmission delay ( $d_m = 2\text{ms}$ ) and data loss ( $\theta(t) = 0.7$ ), 1) the controlled vehicle body vertical acceleration is greatly suppressed compared with the open-loop case; and 2) the suspension deflection and the dynamic tyre load are all regulated below the specified limits. A further comparison between the resulting control performance and the anticipated resource efficiency under the three ETMs is provided in Fig. 5. It can be seen that 1) via inserting the ADV  $\lambda(s_m)$  or  $\lambda(t)$ , the DETMm and DETMt can significantly reduce the data transmissions than the SETM case due to the prolonged average IETs, and 2) although the greatly improved resource efficiency under DETMm and DETMt, the control performance is compromised to a certain degree compared with the SETM case, which can be clearly observed from the trajectory of  $\|x(t)\|$ .

It is noted from the above simulation results that: 1) In the context of event-triggered control or estimation, the existing literature testifies vastly via numerical verification that the DETM in the form of (9) is capable of releasing much fewer data packets over networks than the relative SETM of the form (1), thereby offering great potential for saving more resources. However, the price to be paid is that the resulting control or estimation performance of the concerned system is often compromised to some extent. This is reasonable as the central motivation of an ETM is to trade a certain level of system performance for improved resource efficiency, which is especially profitable when communication networks exhibit insufficient bandwidth or wireless sensor devices possess some finite battery for continual data sampling and

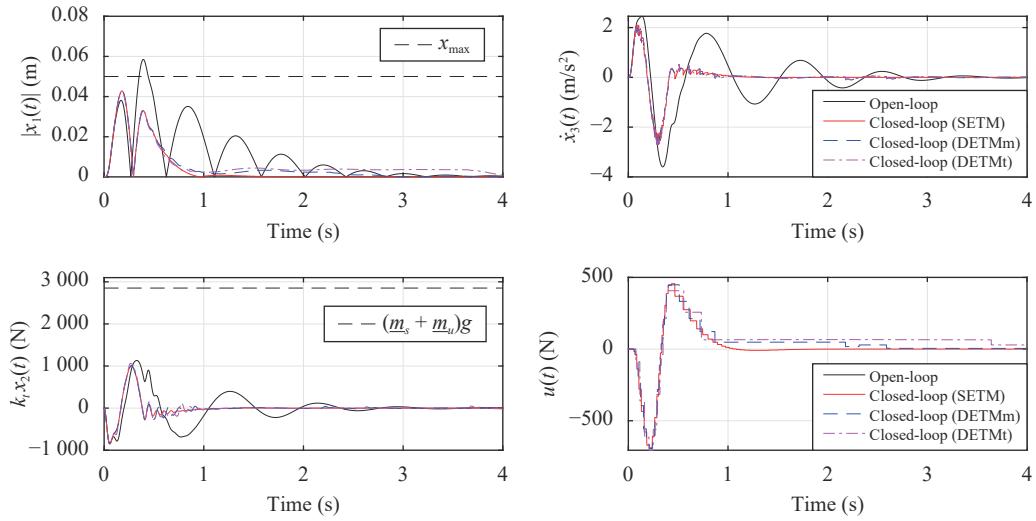


Fig. 4 Bump responses of the open-loop and the resulting closed-loop active suspension system in terms of the suspension stroke  $|x_1(t)|$ , vehicle body vertical acceleration  $\dot{x}_3(t)$ , dynamic tire load  $k_t x_2(t)$ , and actuator power  $u(t)$  under SETM, DETMm, and DETMt

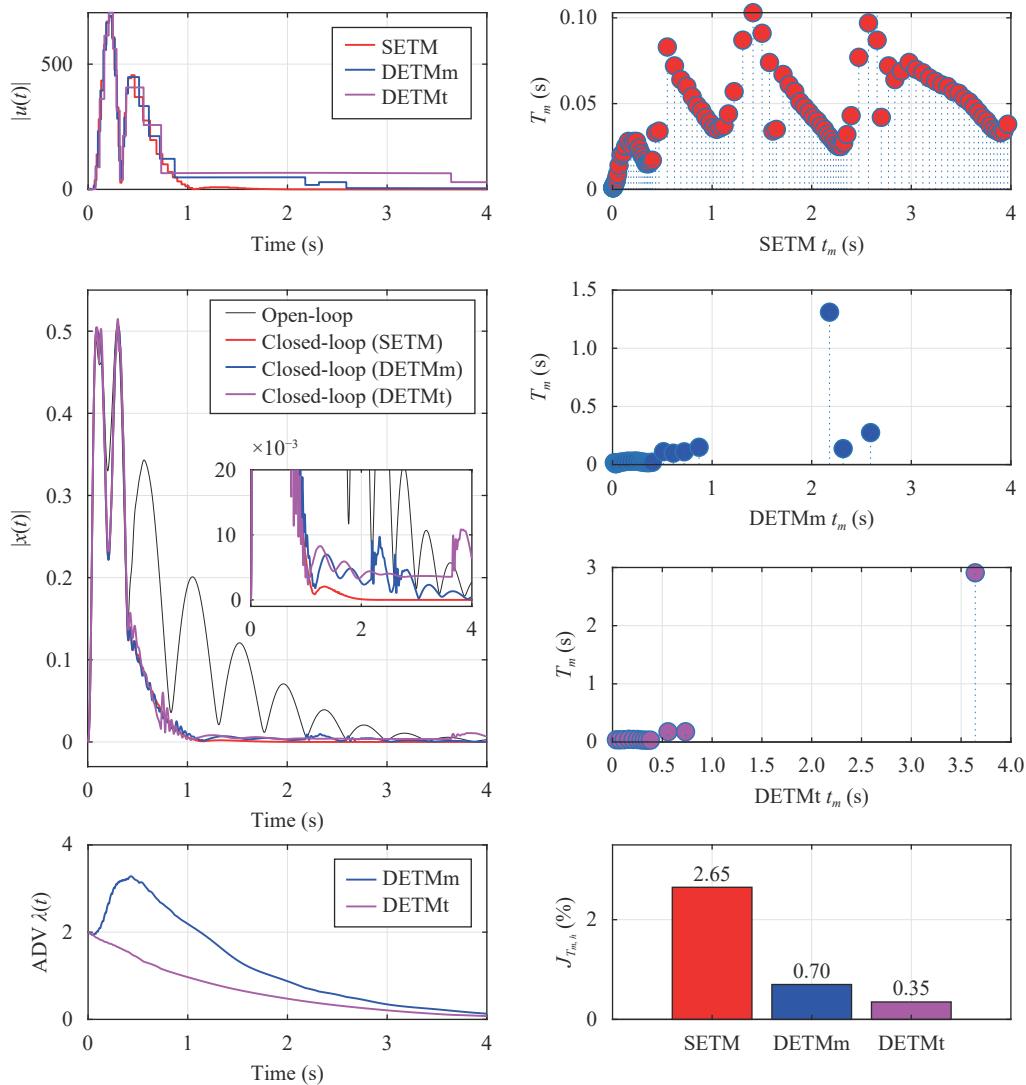


Fig. 5 Comparison of the control performance (in terms of control power  $|u(t)|$  and trajectory of  $\|x(t)\|$ ) and the resource efficiency (in terms of IET  $T_m$  and data packet transmission rate  $J_{Tm,h}$  in percentage) of the vehicle active suspension system between SETM, DETMm and DETMt

broadcasting. 2) There, however, exist certain application scenarios that desired control or estimation performance should not be sacrificed too much. For example, for a mobile target tracking application, the latest target state information (e.g., position and velocity) is the key to accomplish accurate and successful target state estimation. In this sense, when a dynamic event-triggered estimation objective is pursued, it is expected that the DETM should release sensor data packets as often as possible such that the desired tracking performance can be maintained at a satisfactory level or not disrupted.

A maneuvering target tracking example under different measurement transmission rates: Consider a maneuvering target equipped with a global positioning system for measuring its position. The objective is to design a state estimator based on only the noisy and intermittent position measurements such that the real-time target motion can be estimated as accurately as possible.

The target motion dynamics under a prescribed command acceleration<sup>[93, 94]</sup> can be described as the discrete-time state-space equations (2), where  $x(t) = [p_n(t), p_e(t), v_n(t), v_e(t)]^T$  with  $(p_n(t), p_e(t))$  being the northerly and easterly positions and  $(v_n(t), v_e(t))$  being the relative velocity components,  $A = [1, 0, h, 0; 0, 1, 0, h; 0, 0, 1, 0; 0, 0, 1]$ ,  $B = [0, 0, h \sin(\varphi), h \cos(\varphi)]^T$ ,  $E = [2, 1, 1, 1]^T$ ,  $C = [1, 0, 0, 0; 0, 1, 0, 0]$ ,  $D = [1, 2]^T$ ,  $F = [0.25, 0.25, 0.25, 0.25]$ ,  $G = 0$ , the sampling period  $h = 1.5\text{s}$ , the road orientation angle  $\varphi = 120\text{deg}$  (in the counterclockwise direction from due east), the commanded acceleration  $u(t) = 0.15\text{m/s}^2$  for  $t \leq 60$  and  $u(t) = -0.15\text{m/s}^2$  for  $t > 60$ , the disturbance  $w(t) = 0.3 \sin(0.8t)e^{-0.05t}$  and the measurement noise  $v(t) = 0.2 \cos(t)e^{-0.05t}$ . The target starts with the initial positions of  $(10\text{ m}, 10\text{ m})$  and velocities of  $(5\text{ m/s}, -3\text{ m/s})$ . Choosing

$$L = \begin{bmatrix} 1.709 & 8 & 0.499 & 7 & 0.511 & 9 & 0.293 & 9 \\ -0.117 & 6 & 0.963 & 7 & -0.050 & 2 & 0.197 & 1 \end{bmatrix}^T$$

for the estimator (4), it is easy to verify that the matrix  $A - LC$  is Schur stable. To demonstrate the significance of the real-time target motion information for preserving accurate state estimation performance, we examine three scheduling scenarios of sensor measurements  $y(t)$ : 1) 100% transmission rate; 2) 80.83% transmission rate; and 3) 27.50% transmission rate, where the last two cases are randomly determined. The estimation (tracking) performance of the estimator is provided in Fig. 6. It can be inferred that the desired estimation performance becomes increasingly deteriorated when the measurement transmission rate decreases. Apparently, the ADV-based DETMs should be carefully exploited in the communication scheduling scenarios that the accurate control/estimation performance is a major concern or the concerned dynamical system exhibits fast evolutions.

#### 4 Dynamic event-triggered mechanisms based on dynamic threshold parameters

The threshold parameter  $\underline{\sigma}$  plays a vital role in scheduling data samplings and/or transmissions under the SETM of the form (1). It is generally true that a larger  $\underline{\sigma}$  leads to a lower frequency of data packet transmissions<sup>[10, 44, 45]</sup>. Similarly, different values of  $\sigma(\tilde{t})$  in DETM (9) affect the ADV  $\lambda(\tilde{t})$  and thus result in varying threshold functions  $\sigma(\tilde{t})f(Z(\tilde{t})) + \frac{1}{\epsilon}\lambda(\tilde{t})$ . Therefore, the threshold parameter could be deemed as a scheduling parameter that corresponds to data sampling/transmission rate over networks.

It is undesirable to permanently fix the threshold parameter  $\underline{\sigma}$  during the implementation of the existing SETMs mainly due to the following two reasons.

1) The resilience requirement of an event trigger: Event triggers are often embedded in smart devices such as sensors as they are required to monitor, either continuously or periodically, the data of interest. These devices, in most cases, possess restricted energy resources, which means that event triggers may not be able to perform accurately at a fixed level of scheduling performance due to varying power allocations, finite chipset's processing capacity, inherent parameter shifts, and changes or runtime errors. This thus requests desired event triggers to possess a certain level of resilience to tolerate these uncertainties. On the other hand, the resilience of an event trigger may emanate from inaccurate event detection caused by either exogenous disturbance, measurement outlier, or even malicious attack/injection. For example, during normal operation of an ETS-based static event trigger with a fixed threshold of 0.1, whenever the triggering function that  $\|\Phi^{\frac{1}{2}}(z(t) - z(t_m))\|^2 > 0.1$  holds, the continuous data  $z(t)$  will be sampled and released. However, when there is a persistent exogenous disturbance or malicious data injection, causing the weighting error function always to be greater than 0.1, the notorious Zeno phenomenon inevitably occurs. In this sense, event triggers should be resilient enough to inaccurate event detection and remain functional. It becomes apparent that a time-varying or dynamically adjusted threshold parameter represents a possible way to circumvent this inaccurate event detection, and thus may contribute to such a resilience requirement.

2) The time-varying network traffic and bandwidth status: The majority of existing ETMs are designed to alleviate the utilization of limited computation and/or communication resources during the entire system operation and runtime. In other words, they are decisively engaged in preventing data samplings and/or transmissions over networks as long as the event trigger and the controller/estimator are implemented. It is true, in most network communication scenarios, that the computation

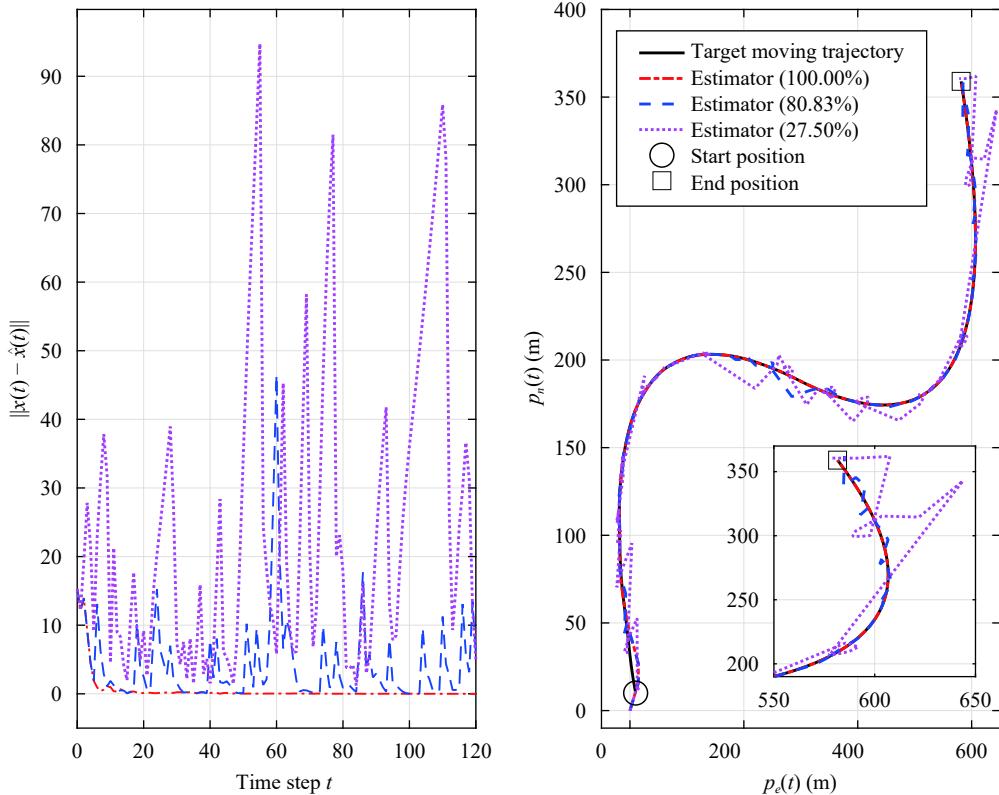


Fig. 6 Comparison of the estimation (tracking) performance (in terms of  $\|x(t) - \hat{x}(t)\|$ ) of the moving target tracking system under different data transmission rates

and/or communication resources are often shared in a multipurpose way or accessed by other neighboring processes and tasks, which thus requires the precious resources to be occupied as little as possible. Meanwhile, it is also stressed that network conditions essentially vary from time to time. Consequently, the bandwidth may be only busy during some specific peak periods but idle during others. If the traditional SETM and ADV-based DETM are employed in this case, some useful data packets (that are important for accomplishing real-time and accurate control/estimation tasks) might still be prevented from being transmitted even during off-peak periods. An intelligent DETM is thus expected to schedule data samplings and/or transmissions more frequently when actual network traffic is low, and real-time bandwidth status is idle to gain better control and estimation performance, while releasing data packets less often when network traffic is high, and the bandwidth is busy to yield better resource efficiency. This also motivates a DTP-based DETM.

It should be further noted that the existing SETMs and ADV-based DETMs are still possible to sustain a trade-off between desired system performance and satisfactory resource consumption via carefully prescribing the threshold and ADV parameters. Nevertheless, the ascertainment of a suitable threshold or ADV parameter for the desired SETM or DETM often relies on either certain design experience or extensive simulations and exper-

iments of the established event-triggered control/estimation algorithm. In contrast, the DTP-based DETMs, thanks to the dynamic nature of the threshold parameters, alleviate these requirements. Still, more importantly, the DTPs add a certain level of flexibility and intelligence to the desired event triggers via directed scheduling in such a way as to permit data transmissions under real-time network conditions, which will be elaborated in Sections 4.1 and 4.2.

In what follows, the existing DTP-based DETMs are classified and reviewed in detail.

#### 4.1 Event-triggered sampling case

Consider the following DTP- and ETS-based triggering mechanism:

$$t_{m+1} = \inf\{t > t_m \mid \|\Phi^{\frac{1}{2}} e(t)\|^2 - \sigma(t) \|\Phi^{\frac{1}{2}} Z(\tilde{t})\|^2 > 0\} \quad (25)$$

where  $e(t) = z(t_m) - z(t)$  and  $\sigma(t) \geq 0$  is a DTP to be suitably designed. Specifically, some common strategies to dynamically adjust the DTP are summarized as follows.

1) A monotonically nonincreasing continuous function  $\sigma(t)$  is in the following form:

$$\dot{\sigma}(t) = -\epsilon \sigma^2(t) \|\Phi^{\frac{1}{2}} e(t)\|^2 \quad (26)$$

with  $\epsilon > 0$  and  $0 < \sigma(0) \leq \underline{\sigma} < 1$  being a given initial

condition. Clearly, (26) implies that  $\dot{\sigma}(t) \leq 0$ , which straightforwardly shows the monotonically nonincreasing property of the DTP and further  $\sigma(t) \leq \sigma(0) \leq \underline{\sigma} < 1$  for all  $t \in \mathbf{R}$ . Next, one only needs to ensure that  $\sigma(t) > 0$ , namely, lower-bounded. From (26), one has that  $-\frac{\dot{\sigma}(t)}{\sigma^2(t)} = \epsilon \|\Phi^{\frac{1}{2}} e(t)\|^2$ , which means that  $\frac{d(\sigma^{-1}(t))}{dt} = \epsilon \|\Phi^{\frac{1}{2}} e(t)\|^2$ . Integrating both sides of the equation from 0 to  $t$ , we obtain that  $\int_0^t d\sigma^{-1}(s) = \int_0^t \epsilon \|\Phi^{\frac{1}{2}} e(s)\|^2 ds$ , which further yields that  $\sigma(t) = (\sigma^{-1}(0) + \int_0^t \epsilon \|\Phi^{\frac{1}{2}} e(s)\|^2 ds)^{-1} > 0$  by noting that  $\sigma(0) > 0$ ,  $\epsilon > 0$  and  $\Phi > 0$ . Therefore, the continuous function  $\sigma(t)$  is monotonically nonincreasing for all  $t \in \mathbf{R}$  and satisfies  $0 < \sigma(t) \leq \sigma(0) \leq \underline{\sigma} < 1$ . During the analysis and design procedures, one may consider the worst-case triggering scenario by including the upper bound  $\underline{\sigma}$  of  $\sigma(t)$  in the derived criteria or employ an additional Lyapunov function term  $V_\sigma = (\sigma^{-1}(t) - \kappa)^2/2\epsilon$  to deal with the triggering condition constraint under the DTP (26), where  $\kappa$  is a prescribed positive scalar. Then, a simple calculation yields that  $\dot{V}_\sigma = (\sigma^{-1}(t) - \kappa) \|\Phi^{\frac{1}{2}} e(t)\|^2 \leq -\kappa \|\Phi^{\frac{1}{2}} e(t)\|^2 + \|\Phi^{\frac{1}{2}} z(t_m)\|^2$ . For example, the above DETM is adopted to solve a distributed leader-follower consensus problem for a nonlinear multi-agent system subject to input saturation<sup>[95]</sup>.

2) A non-monotonic continuous function  $\sigma(t)$  is one of the following forms:

$$\underline{\sigma} \leq \sigma(t) \leq \bar{\sigma}, \quad 0 < \underline{\sigma} \leq \bar{\sigma} < 1 \quad (27a)$$

$$\dot{\sigma}(t) = \frac{1}{\sigma(t)} \left( \frac{1}{\sigma(t)} - \epsilon \right) \|\Phi^{\frac{1}{2}} e(t)\|^2 \quad (27b)$$

where  $0 < \underline{\sigma} \leq \sigma(0)$  and  $\epsilon > 0$  in (27b). It is noteworthy that the DTP  $\sigma(t)$  in (27b) does not exhibit a strictly monotonic property. In order to derive the desired analysis and design criteria under (27b), an additional Lyapunov function term  $V_\sigma = \frac{1}{2}\sigma^2(t)$  can be employed to deal with the triggering condition constraint. Furthermore, it should be noted that  $\sigma(t)$  in (27b) will eventually converge to some finite steady value if the system approaches its equilibrium point in the presence of vanishing disturbance/noise. On the other hand, the DTP  $\sigma(t)$  in (27a) allows to be generally unknown and time-varying with attainable upper and lower bounds. These bounds can be further exploited in the analysis and design criteria to guarantee robust control/estimation. For example, such a DTP (27a) is adopted to deal with a distributed event-triggered  $H_\infty$  consensus filtering problem for a class of discrete-time linear systems over sensor networks<sup>[96]</sup>. Under a polytope-like transformation regarding the DTP, a threshold-parameter-dependent approach is developed to determine both the desired distributed consensus filters and event triggers. In [97], a

distributed event-based communication mechanism under a time-varying threshold parameter is proposed to cope with the leader-following consensus problem for multi-agent systems with unknown but-bounded process and measurement noises.

The DTP in (27b) has yet been widely employed to address various event-triggered control and estimation problems under an ETT strategy; see, e.g., [86] on event-triggered stabilization of a class of networked Takagi-Sugeno fuzzy systems, [87] on decentralized event-triggered  $H_\infty$  filtering of a class of interconnected Takagi-Sugeno fuzzy systems, [84, 92] on event-triggered  $H_\infty$  control of vehicle active suspension control systems, and [85] on automatic steering control of autonomous ground vehicles. Nevertheless, as noted in Section 3.2, when the continuous DTP  $\sigma(t)$  is embedded in an ETT-based dynamic event trigger, the triggering law therein may still need to perform continuously rather than periodically, which represents a clear limitation of such a dynamic event trigger.

3) An adaptive continuous function  $\sigma(t)$  is one of the following forms:

$$\sigma(t) = \underline{\sigma} + \sigma_1 e^{-\sigma_2 \|\Phi^{\frac{1}{2}} z(t)\|^2}, \quad t \in \mathbf{R} \quad (28a)$$

$$\sigma(t) = \underline{\sigma} + \sigma_1 e^{-\sigma_2 \|\Phi^{\frac{1}{2}} z(t_m)\|^2}, \quad t \in [t_m, t_{m+1}] \quad (28b)$$

$$\sigma(t) = \underline{\sigma} + \sigma_1 e^{-\sigma_2 \|\Phi^{\frac{1}{2}} e(t)\|^2}, \quad t \in [t_m, t_{m+1}] \quad (28c)$$

where  $\sigma_2 \geq 0$  and  $\sigma_1 = \bar{\sigma} - \underline{\sigma}$ . It is clear that  $\sigma(t) \in [\underline{\sigma}, \bar{\sigma}]$  for all the DTPs in (28). Furthermore, a key feature of the above DTPs is that their values can be adaptively adjusted on the interval  $[\underline{\sigma}, \bar{\sigma}]$  based on the weighting data or error term. More specifically, when the system data  $z(t)$  (or  $z(t_m)$ ) or the triggering error  $e(t)$  suffers large fluctuation, a smaller value of  $\sigma(t)$  will be selected to verify the event trigger. Generally, a smaller threshold parameter leads to more data packets to be sampled and/or transmitted over networks with an aim to achieve faster convergence of the resulting closed-loop system or estimation error system. On the other hand, when the system data  $z(t)$  (or  $z(t_m)$ ) or the triggering error  $e(t)$  experiences little fluctuation, it may mean that the system is now approaching its equilibrium point without external disturbance/noise, and thus a larger threshold parameter  $\sigma(t)$  should be prescribed to reduce unnecessary data samplings and/or transmissions. From this perspective, the DTPs in (28) offer certain adaptiveness between maintaining desired system performance and resource efficiency.

It is shown in [48] that dynamic triggering with an adaptive threshold can be employed to better shape the resulting network traffic over some shared medium when

successive packet dropouts occur. Specifically, the proposed adaption technique for adjusting the threshold parameter depends on the prediction horizon and network congestion status. For example, when packet losses take place due to network congestion, the prediction horizon becomes shortened, and a larger threshold is thus selected to reduce the transmissions. Obviously, such a dynamic triggering mechanism requires an acknowledgement scheme that determines the last successful transmission and the number of consecutive dropouts. A similar acknowledgement-based event-triggered protocol is considered in [68] for dynamic event-triggered control systems subject to packet losses, however, under a different dynamic triggering mechanism and problem setup.

Adaptive techniques have been intensively studied in conventional control literature. It also seems natural to make the DTP and thus the event trigger dependent on some adaptive gain/parameter in such a way as to gain more adaptiveness during the scheduling and control co-design. For example, an adaptive event-triggered control method is presented in [98] for a class of single-input and single-output uncertain nonlinear systems, where the DTP and the controller gain are both adaptively adjusted via some adaptive weights. However, such an adaptive event-triggered control method may exhibit a potential limitation of practical implementation since it requires both the controller, normally remotely located, and the event-trigger, often locally embedded in an intelligent sensor device, to be synchronously orchestrated at all times. In [99], an adaptive event-triggered output-feedback control scheme is developed for a class of upper-triangular uncertain nonlinear systems. Whether or not the observer state should be transmitted is decided by an adaptive event trigger whose threshold parameter is adaptively adjusted via a dynamic observer gain.

A closer look at the DTPs (26) and (28) reveals that  $0 < \sigma(t)|_{(26)} \leq \underline{\sigma} \leq \sigma(t)|_{(28)} \leq \bar{\sigma}$ . This further indicates that for a given signal  $z(t_m)$ , the next triggering times for the event triggers (25) under DTP (26) and DTP (28) and the static threshold parameter  $\sigma(t) \equiv \underline{\sigma}$  satisfy  $t_{m+1}^{\text{DTP}(26)} \leq t_{m+1}^{\sigma(t) \equiv \underline{\sigma}} \leq t_{m+1}^{\text{DTP}(28)}$ . Therefore, the theoretical relationship between the minimum IETs can be expressed as  $\min\{T_m^{\text{DTP}(26)}\} \leq \min\{T_m^{\sigma(t) \equiv \underline{\sigma}}\} \leq \min\{T_m^{\text{DTP}(28)}\}$ .

## 4.2 Event-triggered transmission case

In this section, we review some existing DTP-based and ETT-based triggering mechanisms. It is noted that the exclusion of Zeno behavior follows naturally under ETT owe to the positive sampling period  $h$ .

Consider the following ETT-based triggering mechanism in the form of

$$t_{m+1} = \inf\{s_m > t_m \mid \|\Phi^{\frac{1}{2}}e(s_m)\|^2 - \sigma(s_m)\|\Phi^{\frac{1}{2}}Z(\tilde{t})\|^2 > 0\} \quad (29)$$

where  $e(s_m) = z(t_m) - z(s_m)$  and  $\sigma(s_m) \geq 0$  is a DTP to be designed. Some common strategies for constructing the DTP are elaborated below.

1) A monotonically nonincreasing discrete function  $\sigma(s_m)$  is of the following forms:

$$\sigma((k+1)h) = \frac{\sigma(kh)}{1 + \epsilon_1 \sigma(kh) \|\Phi^{\frac{1}{2}}e(kh)\|^2} \quad (30)$$

with  $\epsilon \geq 0$  and  $0 \leq \sigma(0) \leq \underline{\sigma} \in [0, 1)$  being a given initial condition. Using the mathematical induction, it can be easily shown that  $\sigma((k+1)h) \leq \sigma(kh) \leq \sigma(0) \leq \underline{\sigma} < 1$  because  $1 + \epsilon \sigma(kh) \|\Phi^{\frac{1}{2}}e(kh)\|^2 \geq 1$ . Therefore, the sequence  $\{\sigma_1(kh)\}$  is monotonically nonincreasing and lower-bounded for all  $k \in \mathbb{N}$  and satisfies that  $0 \leq \sigma_1(kh) \leq \sigma_1(0) \leq \underline{\sigma} < 1$ .

Due to the monotonic nonincreasing property of the DTP  $\sigma(t)$  in (27b) or  $\sigma(s_m)$  in (30), more and more data packets may be transmitted and released over networks before the system reaches its steady-state or when there remains external disturbance acting on the system. As such, the DETM equipped DTP  $\sigma(t)$  (27b) or DTP  $\sigma(s_m)$  (30) may be advantageous when the network bandwidth is identified as idle, or the system seeks fast convergence and high-performance requirement during its operation.

The DTP of the form (30) has been well explored in several different control problem formulations; see, e.g., [100] on distributed formation control of linear multi-agent systems, [95] on leader-follower consensus control of a class of nonlinear and input-saturated multi-agent systems, [89, 92] on vehicle active suspension control, and [101] on vehicle platooning control of a group of wirelessly connected automated vehicles.

2) A monotonically nondecreasing discrete function  $\sigma(s_m)$  is one of the following forms:

$$\sigma((k+1)h) = \frac{\sigma(kh) \|\Phi^{\frac{1}{2}}e(kh)\|^2 + \epsilon \bar{\sigma}}{\epsilon + \|\Phi^{\frac{1}{2}}e(kh)\|^2} \quad (31a)$$

$$\sigma((k+1)h) = \frac{\sigma(kh) + \epsilon \bar{\sigma} \|\Phi^{\frac{1}{2}}e(kh)\|^2}{1 + \epsilon \|\Phi^{\frac{1}{2}}e(kh)\|^2} \quad (31b)$$

with  $\epsilon \geq 0$  and  $0 < \underline{\sigma} \leq \sigma(0) \leq \bar{\sigma}$  being a given initial condition. Notice that the upper bound of  $\sigma(s_m)$  in (31) can be easily proved by the mathematical induction. As a matter of fact, it follows from (31a) and (31b) that  $\sigma((k+1)h) - \bar{\sigma} = \frac{(\sigma(kh) - \bar{\sigma}) \|\Phi^{\frac{1}{2}}e(kh)\|^2}{\epsilon + \|\Phi^{\frac{1}{2}}e(kh)\|^2} \leq 0$  and  $\sigma((k+1)h) - \bar{\sigma} = \frac{\sigma(kh) - \bar{\sigma}}{1 + \epsilon \|\Phi^{\frac{1}{2}}e(kh)\|^2} \leq 0$  by recalling that  $\sigma(kh) - \bar{\sigma} \leq 0$ .

$\bar{\sigma} \leq 0$ ,  $\epsilon \geq 0$  and  $\Phi > 0$ . On the other hand, the monotonic and nondecreasing property of  $\{\sigma(kh)\}$  in (31) can be straightforwardly verified by noting that  $\sigma((k+1)h) - \sigma(kh) = \frac{\epsilon(\bar{\sigma} - \sigma_2(kh))}{\epsilon_2 + \|\Phi^{\frac{1}{2}}e(kh)\|^2} \geq 0$  and  $\sigma((k+1)h) - \sigma(kh) = \frac{\sigma(kh) - \sigma_2(kh)}{\epsilon_2 + \|\Phi^{\frac{1}{2}}e(kh)\|^2} \geq 0$ .

$\sigma(kh) = \frac{\epsilon(\bar{\sigma} - \sigma(kh))}{1 + \epsilon\|\Phi^{\frac{1}{2}}e(kh)\|^2} \geq 0$ . Therefore, the sequence  $\{\sigma(kh)\}$  in (31) is monotonically nondecreasing and upper-bounded for all  $k \in \mathbf{N}$  and satisfies that  $0 \leq \underline{\sigma} \leq \sigma(0) \leq \sigma(kh) \leq \bar{\sigma}$ . As a result, the data samplings and/or transmissions are to be scheduled in a directed manner, i.e., less and less data packets are expected to be sampled and/or transmitted over networks to alleviate resource shortage. Clearly, such a DETM may be beneficial to account for heavy traffic load and busy bandwidth scenarios during system operation<sup>[102]</sup>.

An event-triggered filtering scheme based on a similar form of DTP (31a) is proposed in [103] to deal with a probabilistic-constrained filter design problem for a class of time-varying systems with stochastic nonlinearities and state constraints. In [104], an event-triggered predictive control scheme under a similar DTP in (31b) is studied to tackle the leader-follower consensus problem of discrete-time multi-agent systems with communication delays. In [102], a unified DETM incorporating both the DTP (30) and DTP (31a) is exploited to deal with the co-design of resource-efficient scheduling and platooning control for a convoy of automated vehicles. Based on a bandwidth acknowledgement parameter, it is shown that the vehicle-to-vehicle data transmissions can be dynamically regulated in accord with the bandwidth status.

3) An adaptive discrete function  $\sigma(s_m)$  is one of the following forms:

$$\sigma(kh) = \underline{\sigma} + \sigma_1 e^{-\sigma_2 \|\Phi^{\frac{1}{2}}z(kh)\|}, \quad k \in \mathbf{N} \quad (32a)$$

$$\sigma(kh) = \underline{\sigma} + \sigma_1 e^{-\sigma_2 \|\Phi^{\frac{1}{2}}e(kh)\|}, \quad k \in \mathbf{N} \quad (32b)$$

where  $\sigma_2 \geq 0$  and  $\sigma_1 = \bar{\sigma} - \underline{\sigma}$ . Similar to (28), one has that  $\sigma(kh) \in [\underline{\sigma}, \bar{\sigma}]$ .

An adaptive DTP in the form of (32a) (with  $z(kh) = x(s_m)$ ) is employed in [105] to initiate a memory-based dynamic event-triggered  $H_\infty$  control approach for a class of continuous-time linear systems with network-induced delays. It is shown that by incorporating a series of previously released data packets, the control performance can be greatly improved since the latest released dynamic information is well used. Such a memory-based event trigger is further explored in [106] to deal with  $H_\infty$  load frequency control for power systems over a bandwidth-constrained network subject to deception attacks. Recently, based on the adaptive DTP (32b), a decentralized co-design approach for dynamic event-triggered communication and active suspension control is developed in [107] for a class of in-vehicle networked in-wheel motor-driven electric vehicles subject to dynamic damping. It is demonstrated that the co-design approach is promising for guaranteeing both prescribed suspension performance and satisfactory resource efficiency.

Notice that a defining feature of the invert tangent function  $\arctan(\cdot)$  is that it is naturally bounded by  $\frac{\pi}{2}$ , i.e.,  $\arctan(\cdot) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ . This feature is fully explored in [108, 109] to construct an adaptive DTP-based triggering mechanism for addressing event-triggered control problems. Specifically, the DTP  $\sigma(s_m)$  in (29) is replaced by  $\sigma(t_m)$  which takes the following form:

$$\sigma(t_m) = \max\{\underline{\sigma}, \kappa\sigma(t_{m-1})\} \quad (32c)$$

where  $\sigma(0) = \underline{\sigma}$  and  $\kappa = 1 - \frac{2\epsilon}{\pi} \arctan\left(\frac{\|z(t_m)\| - \|z(t_{m-1})\|}{\|z(t_m)\|}\right)$  with  $\epsilon > 0$  being a prescribed constant. The following facts are noted: 1)  $\sigma(t_m) \geq \underline{\sigma}$  holds for all  $t_m$ ; 2) if  $\|z(t_m)\| = \|z(t_{m-1})\|$ ,  $\sigma(t_m) = \underline{\sigma}$ ; if  $\|z(t_m)\| > \|z(t_{m-1})\|$ , it is clear that  $\kappa \in (1 - \epsilon, 1)$  and  $\sigma(t_m) < \sigma(t_{m-1})$ ; and if  $\|z(t_m)\| < \|z(t_{m-1})\|$ , one has that  $\kappa \in (1, 1 + \epsilon)$  and thus  $\sigma(t_m) > \sigma(t_{m-1})$ ; and 3) the constant  $\epsilon$  affects the change rate of the DTP. The larger  $\epsilon$ , the larger the change rate of  $\sigma(t_m)$ . It is clear that the DTP  $\sigma(t_m)$  can adaptively adjust its value at each triggering instant based on the latest and past transmitted data to regulate the data transmission rate. For example, if  $\|z(t_m)\| < \|z(t_{m-1})\|$ , a larger DTP  $\sigma(t_m)$  is chosen, which further means that a lower transmission frequency of the sampled data packets is configured to gain improved resource efficiency.

It is also noted that the DTP  $\sigma(t_m)$  in (32c) is rarely lower-bounded as  $\underline{\sigma}$ . In many situations, it seems natural to also pose an upper bound  $\bar{\sigma}$  on the DTP. As a result, the adaptive DTP (32c) can be refined as

$$\sigma(t_m) = \min\{\max\{\underline{\sigma}, \kappa\sigma(t_{m-1})\}, \bar{\sigma}\}. \quad (32d)$$

Recalling the boundedness of the DTPs (30)–(32), it is easy to infer that for a given signal  $z(t_m)$ , the next triggering times for the event triggers (29) under DTPs (30)–(32), and the static threshold parameter  $\sigma(s_m) \equiv \underline{\sigma}$  satisfy that  $t_{m+1}^{\text{DTP}(30)} \leq t_{m+1}^{\sigma(s_m) \equiv \underline{\sigma}} \leq t_{m+1}^{\text{DTP}(31)}$  and  $t_{m+1}^{\text{DTP}(30)} \leq t_{m+1}^{\sigma(s_m) \equiv \underline{\sigma}} \leq t_{m+1}^{\text{DTP}(32)}$ . Analogously, one can establish the theoretical relationship between the minimum IETs as  $0 < h \leq \min\{T_m^{\text{DTP}(30)}\} \leq \min\{T_m^{\sigma(s_m) \equiv \underline{\sigma}}\} \leq \min\{T_m^{\text{DTP}(31)}\}$  and  $0 < h \leq \min\{T_m^{\text{DTP}(30)}\} \leq \min\{T_m^{\sigma(s_m) \equiv \underline{\sigma}}\} \leq \min\{T_m^{\text{DTP}(32)}\}$ .

### 4.3 An application to dynamic event-triggered control of a mass-spring-damper mechanical system

In this section, we employ a mass-spring-damper mechanical system as an example to evaluate the effectiveness and performance of different DTP-based and ETT-based dynamic event triggers presented in Section 4.2. Specifically, consider the following nonlinear mass-spring-damper mechanical system, as also shown in Fig. 7,

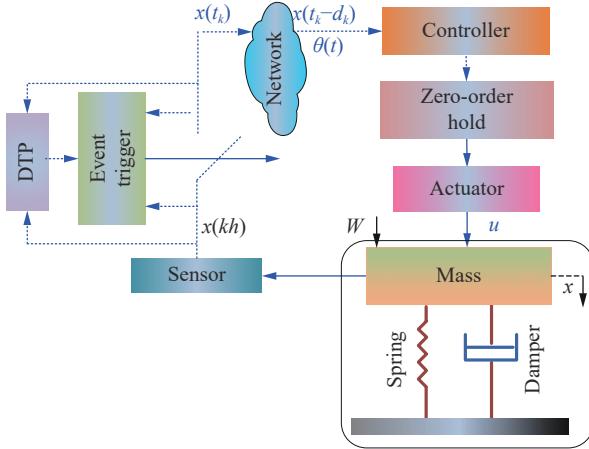


Fig. 7 A dynamic event-triggered control configuration for a mass-spring-damper mechanical system

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -0.01x_1(t) - 0.67x_1^3(t) + w(t) + u(t) \end{cases}$$

where  $x_1(t) \in [-1, 1]$  denotes the mass displacement;  $x_2(t)$  represents the mass velocity;  $w(t) = 2.5 \sin(\pi t)e^{-0.2t}$  stands for the external disturbance; and  $u(t)$  is the desired control force. Detailed parameter selections of the above system can be found in [110]. Choosing the fuzzy membership functions as  $\alpha_1(x_1(t)) = 1 - x_1^2(t)$  and  $\alpha_2(x_1(t)) = x_1^2(t)$ , the inferred Takagi-Sugeno fuzzy system can be given as [56]

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 \alpha_i(x_1(t)) \{A_i x(t) + B_i u(t) + E_i w(t)\} \\ p(t) = F x(t) = [1 \ 0] x(t) \end{cases}$$

with  $A_1 = \begin{bmatrix} 0 & 1 \\ -0.01 & 0 \end{bmatrix}$ ,  $A_2 = \begin{bmatrix} 0 & 1 \\ -0.68 & 0 \end{bmatrix}$ ,  $B_1 = B_2 = [0, 1]^T$ ,  $E_1 = E_2 = [0, 1]^T$ , and the initial condition  $x(0) = [1, 0]^T$ .

The objective is to design an ETT-based and DTP-based event-based control and scheduling policy  $\pi_m^x$  of the following form:

$$\begin{cases} u(t) = \sum_{j=1}^2 \alpha_j K_j \theta(t) x(t_m), \quad t \in [t_m + d_m, t_{m+1} + d_{m+1}] \\ t_{m+1} = \inf\{s_m > t_m \mid q^\sigma(s_m) > 0\} \end{cases}$$

where  $\theta(t) \in [\underline{\theta}, \bar{\theta}] \in [0, 1]$  characterizes the data loss ratio;  $d_m \in [\underline{d}, \bar{d}]$  corresponds to the transmission delay;  $q^\sigma(s_m) \triangleq \|\Phi^{\frac{1}{2}}(x(t_m) - x(s_m))\|^2 - \sigma(s_m)\|\Phi^{\frac{1}{2}}x(t_m)\|^2$  such that the resulting closed-loop system with  $w(t) \equiv 0$  is asymptotically stable; and under zero initial condition and nonzero  $w(t) \in \mathcal{L}_2[0, \infty)$ , the following performance index  $\sup_{w(t) \neq 0} \frac{\|p(t)\|_2}{\|w(t)\|_2} \leq \gamma$  holds for prescribed  $\gamma > 0$ .

For comparison purposes, we next examine the following five different ETMs. 1) SETM1: Under a constant small threshold parameter  $\sigma(s_m) = \underline{\sigma} \equiv 0.05$ ; 2) SETM2: Under a constant large threshold parameter  $\sigma(s_m) = \bar{\sigma} \equiv 0.75$ ; 3) DETM1: Under a monotonically nonincreasing DTP (30) with  $\underline{\sigma} = 0.05$  and  $\epsilon_1 = 1.5$ ; 4) DETM2: Under a monotonically nondecreasing DTP (31b) with  $\sigma = 0.05$ ,  $\bar{\sigma} = 0.75$ , and  $\epsilon_2 = 0.2$ ; and 5) DETM3: Under an adaptive DTP (32b) with  $\underline{\sigma} = 0.05$ ,  $\sigma_1 = 0.7$  and  $\sigma_2 = 50$ .

The co-design Algorithm 1 (without (24) and with  $\tilde{\mu} = 1$ ,  $d_m = 20\text{ms}$ ,  $h = 10\text{ms}$ ,  $\theta(t) = 0.8$ ,  $\gamma = 0.7$ ,  $\eta = 0.008$ ,  $\nu = 1$ ) is adopted here to obtain the comparative simulation results. Fig. 8 shows a comparison between the resulting control performance and the preserved resource efficiency under the five different ETMs. It can be observed that 1) a larger threshold parameter leads to a much lower number of data packet transmissions over the network. Not surprisingly, SETM2 contributes to the lowest data packet transmission rate among the other ETMs due to the largest value of  $\sigma(s_m) = \bar{\sigma} \equiv 0.75$ . In contrast, during its implementation, DETM1 employs the smallest threshold parameter owing to its monotonic nonincreasing property and upper bound of 0.05. It is thus reasonable that DETM1 results in the highest data transmission rate; and 2) although the significantly reduced data transmissions and thus improved resource efficiency under SETM2, the control performance is compromised greatly compared with the SETM1 case, which can be seen from the trajectory of  $\|x(t)\|$ . Similar conclusions can be drawn between DETM2 and DETM3 and the other ETMs.

#### 4.4 An application to dynamic event-triggered estimation of a water distribution and supply system

Supervisory control and data acquisition (SCADA) represents a system of software and hardware elements. It empowers real-time monitoring and control of geographically dispersed assets, such as electrical power grids, water, oil and gas pipelines, and sewage treatment plants, to be conducted reliably, timely, and remotely. SCADA is regarded as the backbone of modern critical infrastructure, including water distribution and supply systems. Among the many functions of SCADA systems, state estimation plays an essential role in achieving an effective monitoring task as it allows the unavailable full system state to be estimated/observed on the basis of partial and noisy sensor measurements.

In what follows, we outline a dynamic event-triggered estimation framework for a remote SCADA water distribution and supply system, as shown in Fig. 9. Some major components of the concerned system include: 1) two waste water treatment plants; two water storage reservoirs (R1 and R2); one water tank (T1); numerous water

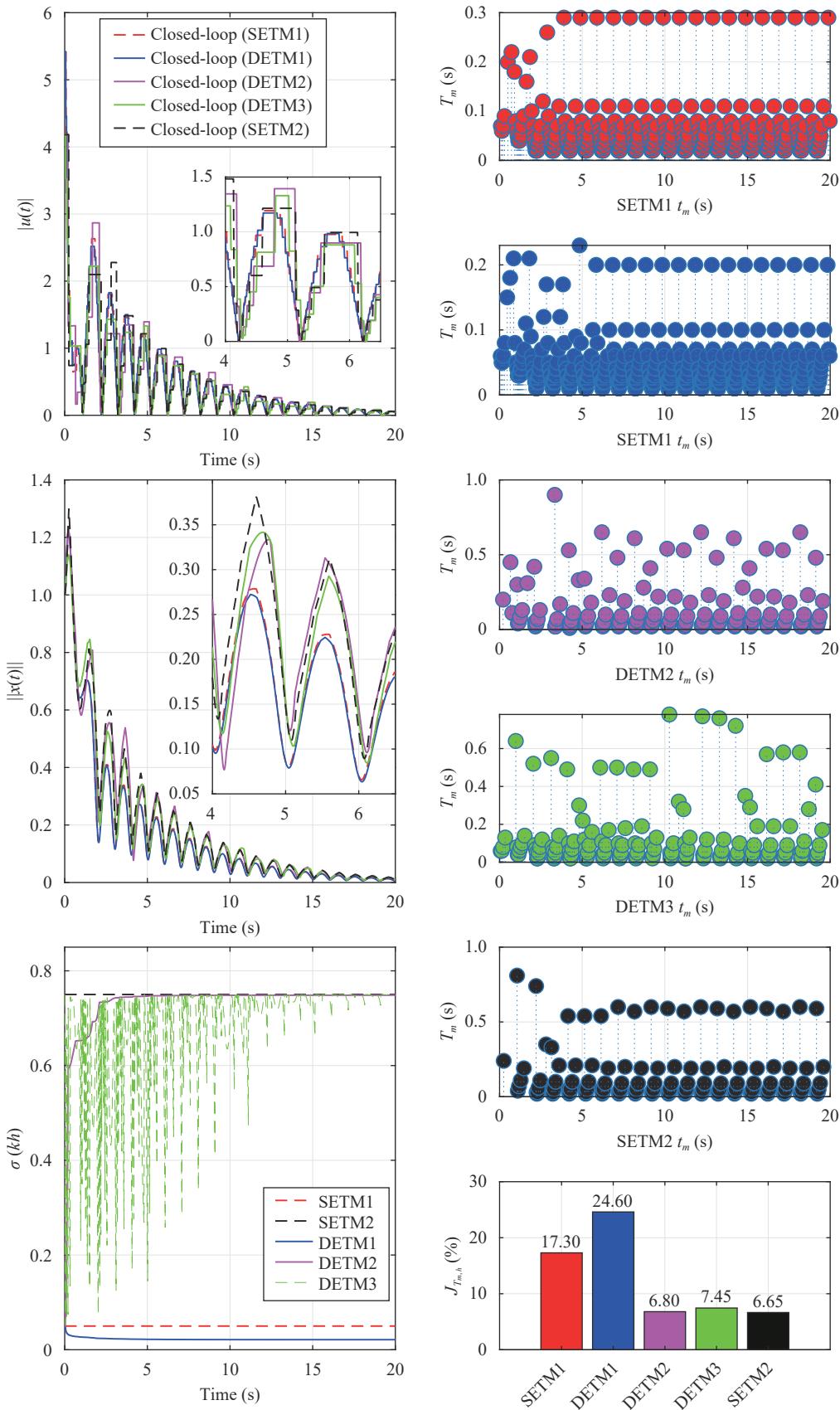


Fig. 8 Comparison of the control performance (in terms of control power  $|u(t)|$  and trajectory of  $\|x(t)\|$ ) and the resource efficiency (in terms of IET  $T_m$  and data packet transmission rate  $J_{T_m,h}$  in percentage) of the mass-spring-damper system between SETM1, SETM2, DETM1, DETM2, and DETM3

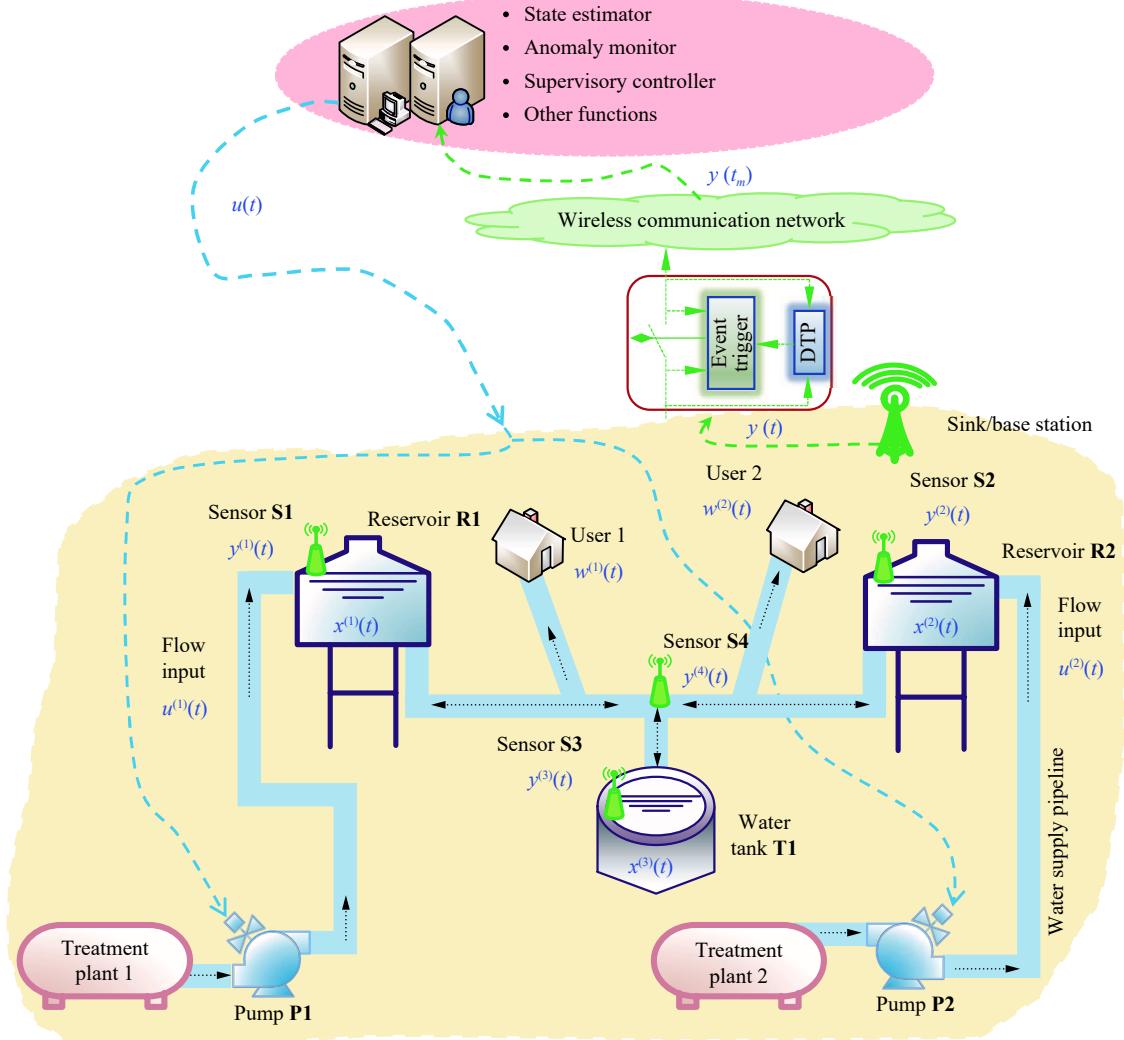


Fig. 9 A dynamic event-triggered estimation configuration for remote monitoring of a water distribution and supply system

supply pipelines and junctions; two pumps (P1 and P2) for regulating the flow rates of R1 and R2; four wireless sensors for measuring the water pressure heads of R1, R2, T1, and the main junction, respectively; two end-users; and some other hydraulic devices; 2) a remote SCADA control center for real-time monitoring and supervising control; and 3) a wireless communication network for enabling data transmissions from wireless sensors to remote control center. As in [10, 111–113], the discrete-time version of the state-space model (2) is employed to describe the SCADA system, where  $x(t) = [x_1(t), x_2(t), x_3(t)]^T$  denotes the system state that incorporates the pressure heads of R1, R2 and T1, respectively;  $u(t) = [u_1(t), u_2(t)]^T$  represents the control signals sent to local pumps for regulating the flow rates of R1 and R2;  $w(t) = [w_1(t), w_2(t)]^T$  stands for the water consumption by two end-users and represents the external disturbance input to the system;  $y(t) = [y_1(t), y_2(t), y_3(t), y_4(t)]^T$  denotes the sensor measurements;  $v(t)$  is the measurement noise affecting all sensor readings. It is assumed that the disturbance and noise are unknown but bounded and satisfy

$w(t) \in \mathcal{E}(\mathbf{0}, Q, 1)$  and  $v(t) \in \mathcal{E}(\mathbf{0}, R, 1)$ , where  $\mathcal{E}(c(t), \Theta, \Delta) \triangleq \{z(t) : (z(t) - c(t))^T \Theta (z(t) - c(t)) \leq \Delta\}$  represents an ellipsoid enclosing a real vector  $z(t) \in \mathbb{R}^{n_z}$  with a real vector  $c(t) \in \mathbb{R}^{n_z}$  being the center, a real-valued matrix  $\Theta = \Theta^T > 0$  being the shape matrix, and a positive scalar  $\Delta > 0$  being the radius of the ellipsoid, respectively.

The dynamic event-triggered estimation and scheduling co-design problem for the concerned SCADA water distribution and supply system is formulated as follows: For any unknown but bounded disturbance and noise inputs  $w(t) \in \mathcal{E}(\mathbf{0}, Q, 1)$  and  $v(t) \in \mathcal{E}(\mathbf{0}, R, 1)$ , the objective is to design a DTP-based event-triggered estimation and scheduling policy  $\pi_m^y$  in the form of (6) with the triggering law being specified by

$$\begin{cases} t_{m+1} = \inf\{t > t_m \mid \|y(t_m) - y(s_m)\|^2 > \sigma(s_m)\} \\ \sigma(s_m) = \min\{\max\{\underline{\sigma}, \kappa\sigma(s_m - 1)\}, \bar{\sigma}\} \\ \kappa = 1 - \frac{2\epsilon}{\pi} \arctan\left(\frac{\|y(s_m)\| - \|y(t_m)\|}{\|y(s_m)\|}\right) \end{cases}$$

such that the SCADA system's true state  $x(t+1) \in \mathcal{E}(\hat{x}(t+1), P, \Delta)$  always holds at every time step  $t$  regardless of the simultaneous disturbance  $w(t)$  and measurement noise  $v(t)$ , namely, there exists a bounding ellipsoidal set  $\mathcal{E}(\hat{x}(t+1), P, \Delta)$  for any  $t \in \mathbf{Z}$  to guarantee always the enclosing of the true state  $x(t+1)$ , where the ellipsoid center, represented by the desired state estimate  $\hat{x}(t+1)$ , the shape matrix  $P = P^T > 0$ , and the radius  $\Delta > 0$  are to be determined.

To solve the above problem, Algorithm 2, which outlines the main steps for the co-design of the DTP-based event-triggered estimation and scheduling policy  $\pi_m^y$  is provided.

**Algorithm 2.** Dynamic event-triggered estimation and scheduling co-design

1) For positive scalars  $\bar{\sigma}$  and  $\eta$ , solve the following linear matrix inequality:

$$\begin{bmatrix} -P & \Phi \\ \Phi^T & -\Lambda \end{bmatrix} \leq 0 \quad (33)$$

to find the positive matrix  $P$ , matrix  $\tilde{L}$ , and positive scalars  $\rho_1, \rho_2, \rho_3$  and  $\Delta$ , where  $\Phi = [\mathbf{0}, PA - \tilde{L}C, -\tilde{L}, PE, -\tilde{L}D]; \Lambda = \text{diag}\{(1 - \eta)\Delta - \rho_1 - \rho_2 - \rho_3\bar{\sigma}, \eta P, \rho_3 I_{n_y}, \rho_1 Q, \rho_2 R\}$ .

if (33) is infeasible, repeat Step 1) otherwise, determine a feasible solution  $L = P^{-1}\tilde{L}$  and continue

2) Set the simulation time  $T_{sim}$ ,  $m = 0$ ,  $t_m = 0$  and  $\tilde{y} = y(0)$

3) **for**  $t = 0 : T_{sim}$  **do**

4) Derive  $\hat{x}(t+1)$  in (6) under  $L$  and  $\tilde{y} = y(t_m)$

5) Compute the ellipsoidal state estimate set  $\{x(t+1) : x(t+1) = \hat{x}(t+1) + \Delta^{\frac{1}{2}}E\alpha, \alpha \in \mathbf{R}^{n_x}, \|\alpha\| \leq 1\}$  based on  $\hat{x}(t+1)$  and  $P^{-1} = EE^T$ .

6) Determine the DTP  $\sigma(t)$  at time step  $t$

7) **if**  $\|y(t_m) - y(t)\|^2 > \sigma(t)$  at time step  $t$  **then**

8) Release the sampled data packet  $(t, y(t))$

9) Update  $m = m + 1$ ,  $t_{m+1} \leftarrow t$  and  $\tilde{y} \leftarrow y(t_{m+1})$

10) **else** Keep  $\tilde{y}$  unchanged

11) **end if**

12) **end for**

It is noted from the above co-design algorithm that 1) the calculated state estimates  $\hat{x}(t+1) + \Delta^{\frac{1}{2}}E\alpha$  at any  $t+1$  in Step 5) form an ellipsoidal set in state-space rather than a single vector generated by some traditional estimation methods such as Kalman filtering and  $H_\infty$  estimation. In this sense, the resulting state estimate ellipsoid  $\mathcal{E}(\hat{x}(t+1), P, \Delta)$  guarantees to always contain all possible values of the true SCADA system state  $x(t+1)$  for any  $t \in \{0, 1, \dots, T_{sim}\}$  regardless of the unknown but bounded disturbance  $w(t)$  and measurement noise  $v(t)$ ; and 2) the derived criterion in terms of inequality (33) enables the upper bound  $\bar{\sigma}$  of the DTP and the shape matrix  $P$  and radius  $\Delta$  of the ellipsoid to be jointly designed. Therefore, the co-design algorithm empowers us

to perform a trade-off analysis between the desired estimation performance and the expected event-triggered scheduling performance in a unified framework.

In the following simulation, the system and measurement matrices are given as

$$A = \begin{bmatrix} 0.995 & 1 & 0.000 & 9 & 0.004 & 0 \\ 0.001 & 2 & 0.992 & 2 & 0.006 & 6 \\ -0.016 & 2 & 0.019 & 8 & 0.996 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.625 & 0 & 0 \\ 0 & 0.833 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} -0.229 & 3 & -0.054 & 0 \\ -0.095 & 9 & -0.365 & 7 \\ -1.295 & 0 & -1.187 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.366 & 9 & 0.115 & 1 & 0.518 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

During system operation and runtime, the reservoirs R1 and R2 are replenished with constant flow rates  $u(t) = [0.453 & 3, 0.553 & 9]^T \text{m}^3/\text{s}$ ; the first customer's water usage demand  $w^{(1)}(t)$  varies randomly between  $0.8 \sim 1.0 \text{ m}^3/\text{s}$  for  $t < 100$  and fluctuates randomly between  $1.3 \sim 1.5 \text{ m}^3/\text{s}$  for  $t \geq 100$ ; the second customer's demand  $w^{(2)}(t)$  varies randomly between  $0.9 \sim 1.1 \text{ m}^3/\text{s}$  for  $t < 120$  and fluctuates randomly between  $1.0 \sim 1.2 \text{ m}^3/\text{s}$  for  $t \geq 120$ ; and the four sensors experience some persistent random measurement noise  $v(t)$  that causes the changes of  $-0.3 \sim 0.3 \text{ m}$  on their readings. Furthermore, set  $Q = \frac{1}{3}I_{n_w}, R = \frac{1}{2}I_{n_v}, x(0) = [100, 80, 60]^T \text{m}, \hat{x}(0) = [101, 81, 59]^T \text{m}, \sigma(0) = \sigma = 0.1\bar{\sigma}, \epsilon = 200, \eta = 0.95$ . We next examine the resulting estimation performance and event-triggered scheduling performance of the SCADA monitoring system under four different DTPs  $\sigma(s_m)$ , i.e.,  $\bar{\sigma} = 1, \bar{\sigma} = 10, \bar{\sigma} = 20, \bar{\sigma} = 50$ , respectively.

Implementing the co-design Algorithm 2 straightforwardly implies the feasibility of the formulated co-design problem in different cases of DTP  $\sigma(s_m)$ . Fig. 10 presents the comparison results of the resulting estimation and scheduling performance under the four different DTPs. It is noted that in the context of ellipsoidal estimation, the conservatism of the resultant bounding ellipsoid lies in its tightness, i.e., the width between the upper and lower bounds centered at the state estimate  $\hat{x}(t)$ . Generally, the tighter the ellipsoid, the less conservative the ellipsoidal estimation method. It can be observed from Fig. 10 that 1) a smaller DTP generally results in more sensor measurement transmissions over the network (and thus sacri-

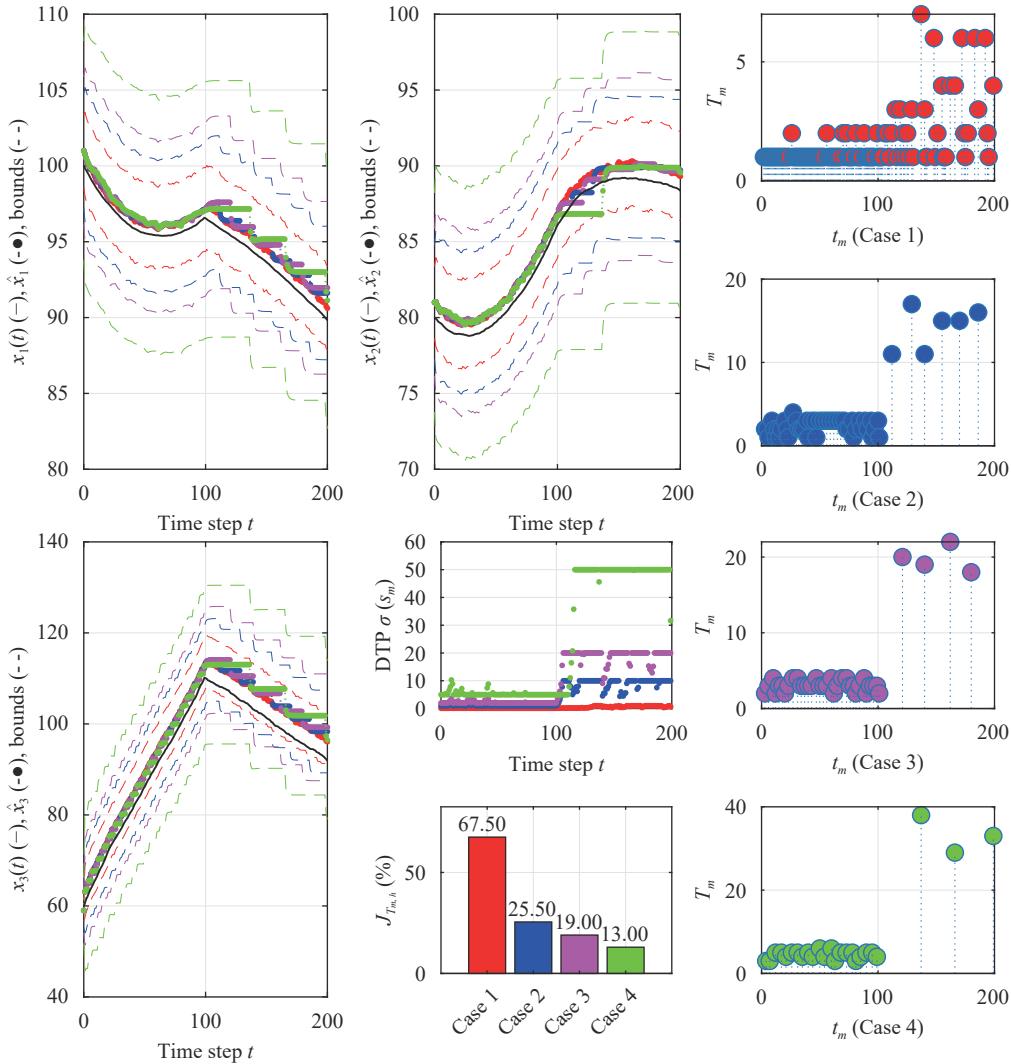


Fig. 10 Comparison of the estimation performance (in terms of trajectories of  $x(t), \hat{x}(t)$  and their bounds as well as the adaptive DTPs  $\sigma(s_m)$ ) and the resource efficiency (in terms of IET  $T_m$  and data packet transmission rate  $J_{T_{m,h}}$  in percentage) of the SCADA water distribution and supply system under different adaptive DTPs: Case 1 (red):  $\sigma(s_m) \in [\underline{\sigma}, \bar{\sigma}] = [0.1, 1]$ ; Case 2 (blue):  $\sigma(s_m) \in [\underline{\sigma}, \bar{\sigma}] = [1, 10]$ ; Case 3 (magenta):  $\sigma(s_m) \in [\underline{\sigma}, \bar{\sigma}] = [2, 20]$ ; Case 4 (green):  $\sigma(s_m) \in [\underline{\sigma}, \bar{\sigma}] = [5, 50]$ .

ficed resource efficiency) and leads to tighter ellipsoids, and 2) a larger DTP contributes to fewer data transmissions but gives rise to a larger bound width of the ellipsoidal estimate set. This further means that the designed state estimator sacrifices its confidence to provide an accurate estimate ellipsoid in exchange for a resource expenditure reduction.

## 5 Conclusions and some challenging issues

The recent advances in dynamic event-triggered control and estimation of networked systems have been reviewed. In order to cater to various control and estimation objectives, a general event-triggered control and estimation framework has been presented. Then, a focus has been placed on the introduction and motivation of DETMs, and their main features, benefits and limita-

tions, and the relevant analysis and design techniques. Two representative classes of DETMs based on ADVs and DTPs have been discussed in detail, followed by a review of the existing results on event-triggered control and estimation that use these DETMs. Furthermore, several practically motivated examples have been provided to evaluate the performance of different DETMs.

Research on dynamic event-triggered control and estimation has attracted intensive attention in the past several years. This paper covers a small proportion of the vast literature and is by no means complete. For example, we have not discussed some closely relevant topics in the field, such as dynamic event-triggered optimization [114, 115], self-triggered control and estimation [12, 116–118], and stochastic event-triggered control and estimation based on random thresholds [119–121]. In what follows, we outline some challenging issues worthy of further study for dynamic event-triggered control and estimation.

1) Novel DETMs with less or easily-tunable trigger parameters: To evaluate a traditional SETM of the form (1), the determination of the static threshold parameter  $\underline{\sigma}$  for preserving both desired control/estimation performance and satisfactory resource efficiency often requires either certain design experience or extensive simulations and experiments. This is particularly the case in a DETM, where several trigger parameters need to be suitably chosen and tuned to achieve desired control/estimation performance. Developing a suitable DETM, which involves less or easily tunable trigger parameters, for general event-triggered control/estimation applications, deserves further investigation.

2) Asynchronous DETMs over feedback (sensor-to-controller) and forward (controller-to-actuator) channels: In networked control systems, control loops are closed via communication networks, which makes clock synchronization between local sensors and remote controllers essentially challenging and expensive. On the other hand, it is not uncommon that transmission delays, data packet dropouts, and packet disorder may occur when the triggered sensor measurements and control commands are propagated over communication networks. This may result in asynchronous time series of triggered data packets. The existing literature on dynamic event-triggered control often assumes that the timing regime is the same for all system components. This may lead to inapplicability in cyber-physical scenarios, where sensors and actuators are collocated with the plant while controllers are spatially distributed and remotely configured. Hence, how to tackle asynchronous dynamic event-triggered control requires further exploration.

3) Bandwidth-aware DETMs: Some DTP-based DETMs presented in Section 4 have the potential to partially address the bandwidth-aware scheduling issue. For example, the DETMs under DTP (27b) and DTP (30) generally lead to more often data samplings and/or transmissions than their static counterparts to seek better control/estimation performance. As a result, they may be employed when the bandwidth resource is sufficient, or the system demands fast convergence during its operation. In contrast, the DETMs under DTP (31) and the ADV-based DETMs in Section 3 may find their applicability when the bandwidth appears constantly busy. Nevertheless, some novel DETMs have not been adequately explored in the event-triggered control/estimation literature. These DETMs are aware of the real-time bandwidth status, where events are generated less often if the real-time network channels are overloaded, and vice-versa. It should be mentioned that the research of event-triggered control/estimation necessitates an integrated view of both control/estimation theory and communication theory. There is a clear need to develop new dynamic event-triggered control/estimation techniques that incorporate real-time network dynamics and bandwidth status such that bandwidth allocation and controller/estimator can

be efficiently co-designed.

4) Significance-based DETMs: Under an ETM of the form (1), the data of interest  $z(\tilde{t})$  is sampled and/or transmitted only if it is deemed as significant for achieving the desired system performance instead of based on the progression of time. Specifically, the significance of data is characterized by its amplitude variation  $g(e(\tilde{t}))$  exceeding some well-defined threshold  $\underline{\sigma}f(Z(\tilde{t}))$ . The existing DETMs mostly focus on designing appropriate and flexible thresholds such that the triggers are more sensitive to the data amplitude variations. An interesting yet open question is how to develop an effective DETM for better system performance but with fewer released events. The question, however, seems paradoxical because fewer triggering events indicate fewer data packets for controller/estimator implementation and design, and thus generally degraded system performance. A possible way of addressing this question is to look closely at what data is virtually significant for better performance guarantees. For example, some attempts have been made via employing a range of previously triggered data packets at the trigger's side to characterize data amplitude variations, leading to the so-called memory-based ETMs<sup>[105, 106]</sup>. In this case, the significance can be enhanced by the suitably weighted historical data packets. However, it is shown that these memory-based ETMs may result in more triggering events near troughs/crests of the system trajectories or when the system exhibits drastic evolutions. How they can be further exploited for both better system performance and resource efficiency remains open. On the other hand, in some control systems, frequent data feedings are not preferable for desired control performance improvement. For example, it is demonstrated in [122] that intentionally discarding some control input packets can reduce heading deviation and rudder oscillation of an unmanned surface vehicle in network environments. When a DETM is adopted, this means that those data packets due to fast/drastic amplitude changes may not need to be released in order to mitigate unnecessary deviation and oscillation. In this case, the significance evaluated by the DETM may not necessarily be related to significant data amplitude variations. Hence, how to develop novel DETMs that emphasize the significance of the data of interest for better system performance and resource efficiency requires deep investigation.

5) Resilient dynamic event-triggered control/estimation approaches against malicious attacks: Compared with the traditional SETMs, most DETMs are capable of transmitting data packets much less frequently. This implies that only those data packets that are deemed as significant by the DETM are released over the network to preserve desired control/estimation performance. However, a sophisticated attacker may leverage this fact to maliciously manipulate these significant data packets during network transmission to disrupt the system performance. Typical attacks on the data transmission chan-

nels include false data injection attacks, which tamper data integrity, and denial-of-service (DoS) attacks, which cause data interruption/unavailability. This thus makes the resilient dynamic event-triggered control/estimation issue in the presence of attacks particularly important. However, to the best of the authors' knowledge, developing resilient dynamic event-triggered control/estimation approaches that consider realistic attack models and ensure the survivability of the event-triggered system despite malicious attacks has not been fully addressed, which calls for additional research effort.

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