Parallel CRC Algorithm and Implementation with CUDA

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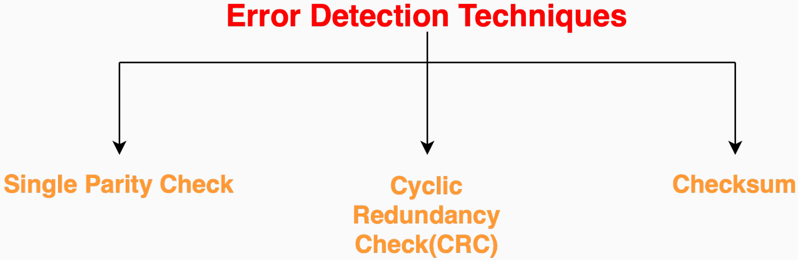
Introduction

What is CRC



Cyclic redundancy-check codes (**CRC codes**) are used for **detecting errors** that occur during trasmission **of digital (binary) information**.

Specification of a CRC code

- CRCs are based on the theory of **systematic cyclic codes**, which encode messages by adding a fixed-length check value.
- Specification of a CRC code requires definition so-called **generator polynomial**.
- This polynomial becomes the **divisor** in a **polynomial long division**, which takes the message as the **divident** and in which the **quotient** is discarded and the **remainder** becomes the result.

Math Background

- The CRC see the **message** A of **length** n as a **polynomial** $A(x)$ **of degree** $n-1$ in which every bit of message is the coefficient of the respective monomial:

$$A = [a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0]$$

$$A(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$$

- For example, the input data `0x25 = 0010 0101` is taken as:

$$A(x) = 0x^7 + 0x^6 + 1x^5 + 0x^4 + 0x^3 + 1x^2 + 0x^1 + 1x^0$$

Math Background cont.

- Then for the CRC computation is used a **polynomial generator** $G(x)$ that is **polynomial of degree** m. The major index coefficient is always 1 1, in this way the polynomial generator guarantees to always be of m degree.
- In a reverse way the than the message A, **the polynomial generator can be represented as a sequence of bit** G **of size** $m+1$ with the most significant bit always to 1.

$$G = [g_m, g_{m-1}, \dots, g_1, g_0]$$

$$G = g_m x^m + g_{m-1} x^{m-1} + \dots + g_1 x + g_0$$

- For example, Ethernet uses the following 32-bit polynomial value:

$$G(x) = 1 + x + x^2 + x^4 + x^5 + x^7 + x^8 + x^{10} + x^{11} + x^{12} + x^{16} \\ + x^{22} + x^{23} + x^{26} + x^{32}$$

Math Background cont.

- The **CRC** detection code **is the result of the reminder** after dividing the original message A concatenated with m zero bits by a polynomial generator in **binary modulo 2 arithmetic**.
- In the polynomial version this definition of CRC is equivalent to:

$$CRC[A(x)] = A(x)x^m mod2(G(x))$$

CRC algorithms

Bitwise CRC algorithm

```
// Result size: N + M.
append(message, M)

// Usually 0.
message[M-1 : 0] = crc_initial_value

// Check each bit of the message.
for i in range(0 : N)
{
   LSB = (N + M - 1) - i
   if (message[LSB] == 1)
   {
      message[LSB : LSB-(M-1)] ^=
      polynomial_generator;
   }
}
```

The CRC lookup table optimization

```
const uint32_t crc32_tab[] = {
   0x00000000, 0x77073096, 0xee0e612c, 0x990951ba, 0x076dc419, 0x706af48f,
   0xe963a535, 0x9e6495a3, 0x0edb8832, 0x79dcb8a4, 0xe0d5e91e, 0x97d2d988,
   0x09b64c2b, 0x7eb17cbd, 0xe7b82d07, 0x90bf1d91, 0x1db71064, 0x6ab020f2,
   0xf3b97148, 0x84be41de, 0x1adad47d, 0x6ddde4eb, 0xf4d4b551, 0x83d385c7,
   0x136c9856, 0x646ba8c0, 0xfd62f97a, 0x8a65c9ec, 0x14015c4f, 0x63066cd9,
   0xfa0f3d63, 0x8d080df5, 0x3b6e20c8, 0x4c69105e, 0xd56041e4, 0xa2677172,
   0x3c03e4d1, 0x4b04d447, 0xd20d85fd, 0xa50ab56b, 0x35b5a8fa, 0x42b2986c,
   0xdbbbc9d6, 0xacbcf940, 0x32d86ce3, 0x45df5c75, 0xdcd60dcf, 0xabd13d59,
   0x26d930ac, 0x51de003a, 0xc8d75180, 0xbfd06116, 0x21b4f4b5, 0x56b3c423,
   0xcfba9599, 0xb8bda50f, 0x2802b89e, 0x5f058808, 0xc60cd9b2, 0xb10be924,
   0x2f6f7c87, 0x58684c11, 0xc1611dab, 0xb6662d3d, 0x76dc4190, 0x01db7106,
   0x98d220bc, 0xefd5102a, 0x71b18589, 0x06b6b51f, 0x9fbfe4a5, 0xe8b8d433,
   0x7807c9a2, 0x0f00f934, 0x9609a88e, 0xe10e9818, 0x7f6a0dbb, 0x086d3d2d,
   0x91646c97, 0xe6635c01, 0x6b6b51f4, 0x1c6c6162, 0x856530d8, 0xf262004e,
   0x6c0695ed, 0x1b01a57b, 0x8208f4c1, 0xf50fc457, 0x65b0d9c6, 0x12b7e950,
   0x8bbeb8ea, 0xfcb9887c, 0x62dd1ddf, 0x15da2d49, 0x8cd37cf3, 0xfbd44c65,
   0x4db26158, 0x3ab551ce, 0xa3bc0074, 0xd4bb30e2, 0x4adfa541, 0x3dd895d7,
   0xa4d1c46d, 0xd3d6f4fb, 0x4369e96a, 0x346ed9fc, 0xad678846, 0xda60b8d0,
   0x44042d73, 0x33031de5, 0xaa0a4c5f, 0xdd0d7cc9, 0x5005713c, 0x270241aa,
   0xbe0b1010, 0xc90c2086, 0x5768b525, 0x206f85b3, 0xb966d409, 0xce61e49f,
   0x5edef90e, 0x29d9c998, 0xb0d09822, 0xc7d7a8b4, 0x59b33d17, 0x2eb40d81,
   0xb7bd5c3b, 0xc0ba6cad, 0xedb88320, 0x9abfb3b6, 0x03b6e20c, 0x74b1d29a,
```

The CRC lookup table optimization

```
    0xead54739, 0x9dd277af, 0x04db2615, 0x73dc1683, 0xe3630b12, 0x94643b84,
    0x0d6d6a3e, 0x7a6a5aa8, 0xe40ecf0b, 0x9309ff9d, 0x0a00ae27, 0x7d079eb1,
    0xf00f9344, 0x8708a3d2, 0x1e01f268, 0x6906c2fe, 0xf762575d, 0x806567cb,
    0x196c3671, 0x6e6b06e7, 0xfed41b76, 0x89d32be0, 0x10da7a5a, 0x67dd4acc,
    0xf9b9df6f, 0x8ebeeff9, 0x17b7be43, 0x60b08ed5, 0xd6d6a3e8, 0xa1d1937e,
    0x38d8c2c4, 0x4fdff252, 0xd1bb67f1, 0xa6bc5767, 0x3fb506dd, 0x48b2364b,
    0xd80d2bda, 0xaf0a1b4c, 0x36034af6, 0x41047a60, 0xdf60efc3, 0xa867df55,
    0x316e8eef, 0x4669be79, 0xcb61b38c, 0xbc66831a, 0x256fd2a0, 0x5268e236,
    0xcc0c7795, 0xbb0b4703, 0x220216b9, 0x5505262f, 0xc5ba3bbe, 0xb2bd0b28,
    0x2bb45a92, 0x5cb36a04, 0xc2d7ffa7, 0xb5d0cf31, 0x2cd99e8b, 0x5bdeae1d,
    0x9b64c2b0, 0xec63f226, 0x756aa39c, 0x026d930a, 0x9c0906a9, 0xeb0e363f,
    0x72076785, 0x05005713, 0x95bf4a82, 0xe2b87a14, 0x7bb12bae, 0x0cb61b38,
    0x92d28e9b, 0xe5d5be0d, 0x7cdcefb7, 0x0bdbdf21, 0x86d3d2d4, 0xf1d4e242,
    0x68ddb3f8, 0x1fda836e, 0x81be16cd, 0xf6b9265b, 0x6fb077e1, 0x18b74777,
    0x88085ae6, 0xff0f6a70, 0x66063bca, 0x11010b5c, 0x8f659eff, 0xf862ae69,
    0x616bffd3, 0x166ccf45, 0xa00ae278, 0xd70dd2ee, 0x4e048354, 0x3903b3c2,
    0xa7672661, 0xd06016f7, 0x4969474d, 0x3e6e77db, 0xaed16a4a, 0xd9d65adc,
    0x40df0b66, 0x37d83bf0, 0xa9bcae53, 0xdebb9ec5, 0x47b2cf7f, 0x30b5ffe9,
    0xbdbdf21c, 0xcabac28a, 0x53b39330, 0x24b4a3a6, 0xbad03605, 0xcdd70693,
    0x54de5729, 0x23d967bf, 0xb3667a2e, 0xc4614ab8, 0x5d681b02, 0x2a6f2b94,
    0xb40bbe37, 0xc30c8ea1, 0x5a05df1b, 0x2d02ef8d
};
```

Bytewise CRC algorithm

```
// Result size: N + M.
append(message, M)

// Usually 0
message[M-1 : 0] = crc_initial_value

// Check each byte of the message.
for i in range(0:N / 8-1)
{
   LSB = (N + M - 1) - (i * 8);
   message[LSB-8 : LSB-8-(M-1)] ^=
   lookup_table[message[LSB : LSB-7]];
}

return message[M-1 : 0];
```

Implementation

Parallel CRC algorithm

- For a given message with any length, we **first chunk the message into blocks**, each of which has a fixed size equal to the degree of the generator polynomial.
- The message is seen as divided in M bit size chunk and the following is its polynomial representation:

$$A(x) = W_{n-1}(x)x^{(n-1)M} + \ldots + W_1(x)x^M + W_0(x)$$

- Where each $W_i(x)$ polynomial is a chunk of the message. From this equation and the CRC definition, we can compute the CRC for the chunked message by:

$$CRC[A(x)] = W_{n-1}x^{nM} mod G(x) + \ldots + W_0 x^M mod G(x)$$

Parallel CRC algorithm cont.

- Furthermore, from the Galois Field operations lemma, we obtain that:

$$W_i(x)x^{(i+1)M} modG(x) = (W_i(x) modG(x) \\ x^{(i+1)M} modG(x)) modG(x)$$

- The degree of the polynomial $W_i(x)$ for each chunk is $M-1$, therefore it is less than M, it means that $W_i(x) modG(x) = W_i(x)$. On the other hand the beta coefficients are defined as $\beta_i = x^{(i+1)*M} modG(x)$ for $i = 0, 1, \ldots, n-1$. Therefore we have:

$$CRC[A(x)] = W_{n-1} \otimes \beta_{n-1} \oplus \ldots \oplus W_0 \otimes \beta_0$$

Note that the operations $\otimes$, $\oplus$ in the above equation is Galois Field multiplication, addition over $GF(2^M)$, respectively.

Parallel CRC algorithm cont.

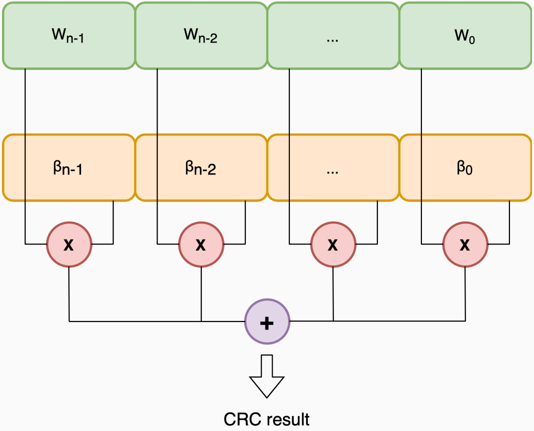


Figure 1: ILLUSTRATION OF PARALLEL CRC ALGORITHM OVER CHUNK OF M BITS

Threads execution

```
// Get the data of this thread.
W_i = orginal_message[global_index];
beta_i = beta[global_index];

// Perform binary modulo 2 multiplication.
mul = mod2_mul(W_i, beta_i);

// Perform binary modulo 2 reminder.
mod = mod2_mod(mul, generator_poly);

// Copy in shared memory.
shared_memory[threadX] = mod;
sync();

// XOR all data in shared memory.
return xored(shared_memory);
```

Threads execution cont.

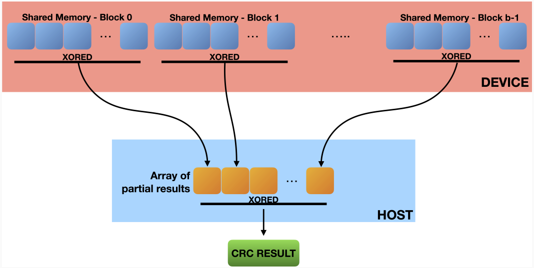


Figure 2: XOR OPERATIONS PERFORM BY DEVICE IN RELATION WITH THE ONE PERFORM BY HOST

Performance analysis

Test PCRC 64 blocksize

$N = 2^{16}$. $BlockSize = 64$. $StreamDim = 4$. $SegSize = N/StreamDim$

Test PCRC 64 blocksize	Speedup
PCRC8	34.71340
PCRC8 with reduction	37.40804
PCRC8 with task parallelism	27.86828
PCRC8 bytewise comparison	2.58712
PCRC8 bytewise comparison with reduction:	2.61057
PCRC8 bytewise comparison with task parallelism	1.93434
PCRC16	34.98040
PCRC16 with reduction	40.59672
PCRC16 with task parallelism	26.11727
PCRC16 bytewise comparison	3.44060
PCRC16 bytewise comparison with reduction:	3.85280
PCRC16 bytewise comparison with task parallelism	1.51222
PCRC32	20.23205
PCRC32 with reduction	20.01593
PCRC32 with task parallelism	8.46386
PCRC32 bytewise comparison	1.78715
PCRC32 bytewise comparison with reduction:	1.86166
PCRC32 bytewise comparison with task parallelism	0.71914

Test PCRC 128 blocksize

$N = 2^{16}$. $BlockSize = 128$. $StreamDim = 4$. $SegSize = N/StreamDim$

Test PCRC 128 blocksize	**Speedup**
PCRC8	34.63720
PCRC8 with reduction	37.41059
PCRC8 with task parallelism	28.19650
PCRC8 bytewise comparison	2.24552
PCRC8 bytewise comparison with reduction:	2.70207
PCRC8 bytewise comparison with task parallelism	2.02773
PCRC16	34.14637
PCRC16 with reduction	40.48397
PCRC16 with task parallelism	25.37525
PCRC16 bytewise comparison	3.57066
PCRC16 bytewise comparison with reduction:	3.60613
PCRC16 bytewise comparison with task parallelism	1.49754
PCRC32	20.41607
PCRC32 with reduction	21.54121
PCRC32 with task parallelism	8.51250
PCRC32 bytewise comparison	1.89645
PCRC32 bytewise comparison with reduction:	1.85455
PCRC32 bytewise comparison with task parallelism	0.79350

Test PCRC 256 blocksize

$N = 2^{16}$. $BlockSize = 256$. $StreamDim = 4$. $SegSize = N/StreamDim$

Test PCRC 256 blocksize	**Speedup**
PCRC8	35.11670
PCRC8 with reduction	37.36065
PCRC8 with task parallelism	29.85230
PCRC8 bytewise comparison	2.66612
PCRC8 bytewise comparison with reduction:	2.71714
PCRC8 bytewise comparison with task parallelism	2.00425
PCRC16	33.09628
PCRC16 with reduction	39.25975
PCRC16 with task parallelism	27.58368
PCRC16 bytewise comparison	3.53824
PCRC16 bytewise comparison with reduction:	3.66416
PCRC16 bytewise comparison with task parallelism	1.54122
PCRC32	20.29566
PCRC32 with reduction	21.39834
PCRC32 with task parallelism	7.98474
PCRC32 bytewise comparison	1.75776
PCRC32 bytewise comparison with reduction:	1.80291
PCRC32 bytewise comparison with task parallelism	0.68456

Test PCRC 128 blocksize

$N = 2^{16}$. $BlockSize = 128$. $StreamDim = 2$. $SegSize = N/StreamDim$

Test PCRC 128 blocksize	**Speedup**
PCRC8	21.32339
PCRC8 with reduction	39.41265
PCRC8 with task parallelism	32.97171
PCRC8 bytewise comparison	2.70535
PCRC8 bytewise comparison with reduction:	2.64768
PCRC8 bytewise comparison with task parallelism	2.48787
PCRC16	33.66302
PCRC16 with reduction	38.77745
PCRC16 with task parallelism	30.44083
PCRC16 bytewise comparison	3.46636
PCRC16 bytewise comparison with reduction:	3.59283
PCRC16 bytewise comparison with task parallelism	2.80089
PCRC32	20.42403
PCRC32 with reduction	21.38471
PCRC32 with task parallelism	7.35099
PCRC32 bytewise comparison	1.87434
PCRC32 bytewise comparison with reduction:	1.86528
PCRC32 bytewise comparison with task parallelism	0.72902

Test PCRC 128 blocksize

$N = 2^{16}$. $BlockSize = 128$. $StreamDim = 8$. $SegSize = N/StreamDim$

Test PCRC 128 blocksize	Speedup
PCRC8	35.05756
PCRC8 with reduction	38.59024
PCRC8 with task parallelism	11.97966
PCRC8 bytewise comparison	2.65855
PCRC8 bytewise comparison with reduction:	2.73349
PCRC8 bytewise comparison with task parallelism	1.20074
PCRC16	34.06469
PCRC16 with reduction	38.96202
PCRC16 with task parallelism	11.68695
PCRC16 bytewise comparison	3.87572
PCRC16 bytewise comparison with reduction:	3.22093
PCRC16 bytewise comparison with task parallelism	1.58960
PCRC32	20.83102
PCRC32 with reduction	22.14467
PCRC32 with task parallelism	6.38649
PCRC32 bytewise comparison	1.94807
PCRC32 bytewise comparison with reduction:	1.91344
PCRC32 bytewise comparison with task parallelism	0.65893

Test PCRC 128 blocksize

$N = 2^{16}$. $BlockSize = 128$. $StreamDim = 4$. $SegSize = N/8$

Test PCRC 128 blocksize	**Speedup**
PCRC8	35.41841
PCRC8 with reduction	40.88182
PCRC8 with task parallelism	17.13281
PCRC8 bytewise comparison	2.40438
PCRC8 bytewise comparison with reduction:	2.97784
PCRC8 bytewise comparison with task parallelism	0.47928
PCRC16	34.66451
PCRC16 with reduction	42.29803
PCRC16 with task parallelism	16.70620
PCRC16 bytewise comparison	4.04358
PCRC16 bytewise comparison with reduction:	3.76574
PCRC16 bytewise comparison with task parallelism	1.51226
PCRC32	21.27053
PCRC32 with reduction	21.97304
PCRC32 with task parallelism	7.47568
PCRC32 bytewise comparison	1.92987
PCRC32 bytewise comparison with reduction:	1.90569
PCRC32 bytewise comparison with task parallelism	0.52763

Conclusion

The implementation with tasks parallelism does not give better performance because the overhead of the technique is not positively compensated on the improvement that leads to performance. So the best result with parallelism tasks is obtained for $StreamDim = 4$ and $SegSize = N/StreamDim$.

In general the best performances are with $BlockSize = 128$, while in the case of parallelism tasks the best performances are with $StreamDim = 2$ and $SegSize = N/StreamDim$ using the reduction algorithm that reduces the divergence of the threads, which on average increases the performance compared to the standard solution of some speedup units.