Name: ID:

4. **(Baseball Elimination)**

Suppose there are n teams in major league baseball, After the season is most of the way through, m games remain to be played. (Denote by g_{ij} a game scheduled between certain teams t_i and t_j.) Assume that each game ends in a win or a loss (no ties). We wish to know whether it is still possible for the Angels to end up as the best team in the league.

The result will depend not only on how the Angels play (in the optimal case, they will win all their games) but also on how well the other teams play. If team t_i is a_i games ahead of the Angels, that team must lose at least a_i games if the Angels are going to come out ahead. Conversely if team t_i does lose a_i games and the Angels win all their games, the Angels will come out ahead of t_i. But obviously if team t_i loses g_{ij}, team t_j wins, and t_j may instead need to lose some different game.

Show how to formulate this as a maximum flow problem. The object is to create a graph and specify its edge capacities, in such a way that the maximum flow will be at least some value F if and only if it is possible to specify who wins and loses each game g_{ij} in a way that makes each team t_i lose at least a_i games. (Hint: Make a graph that has vertices not only for each team but also for each scheduled game not involving the Angels. Set up your capacities so that each game receives one unit of flow, representing one loss, which it can pass along to either team playing the game.)

You should describe this construction in general terms, including a list of all vertices and edges in the graph, the edge capacities, and the value of F. You may draw an example if you think that would make your description easier to understand, but you should not actually compute any maximum flows, nor do you need to prove that your construction works; I just want you to say how you would set up the problem.