MATLAB Final

Introduction to Linear Algebra **Spring, 2021**

10. (MATLAB Programming Problem)

(1) (10 points, 2 points for each)

Let A be a 7×8 matrix given by

$$A = \begin{bmatrix} 1 & 8 & 0 & 0 & 0 & 0 & 0 & 50 \\ 2 & 9 & 0 & 0 & 0 & 0 & 44 & 51 \\ 3 & 10 & 0 & 0 & 0 & 0 & 45 & 52 \\ 4 & 11 & 0 & 0 & 0 & 0 & 46 & 53 \\ 5 & 12 & 0 & 0 & 0 & 0 & 47 & 54 \\ 6 & 13 & 0 & 0 & 0 & 0 & 48 & 55 \\ 7 & 0 & 0 & 0 & 0 & 0 & 49 & 56 \end{bmatrix}.$$

Choose **True** for codes that generate the same matrix A as above, or **False** for codes that do not.

(a)
```
A = reshape(1:56, 7, 8);
A(14 <= A & 43 >= A) = 0;
```

(b)
```
A = reshape(1:56, 7, 8);
A = A .* ((14 > A) + (43 < A));
```

(c)
```
A(1:13) = 1:13;   A(44:56) = 44:56;
A = reshape(A, 7, 8);
```

(d)
```
A = reshape(1:56, 7, 8);
A = A(14 <= A) .* A(43 >= A);
```

(e)
```
A = zeros(7, 8);
A(:) = [1:13, zeros(1, 30), 44:56];
```

(2) (10 potins)

Let A be an $m \times n$ matrix with full column rank. The following MATLAB function performs a Gram-Schmidt process and produces an $m \times n$ orthogonal matrix Q whose columns form an orthonormal basis for col(A). Fill in the blanks (1) - (5).
(In this problem, assume that the input A is a matrix that gives the same result for the both of classical Gram-Schmidt and the modified Gram-Schmidt.)

```
% ------- The following is the script file 'GramSchmidt.m'. -------
___(1)___ Q = GramSchmidt(A)
[m, n] = size(A);

% Initialize the matrix Q as an m*n zero matrix.
Q = zeros(m, n);
for i = 1 : n
    % v starts with a column of A.
    v = ___(2)___;
    for j = 1 : i-1
        % Subtract orthogonal projections of v onto the subspaces
        % spanned by the previously generated orthonormal vectors.
        q = ___(3)___;
        v = v - ___(4)___;
    end
    % Normalize v by its Euclidean norm.
    Q(:, i) = v / ___(5)___;
end
```

(a) (1) `def`, (2) `A(i, :)`, (3) `Q(:, j)`, (4) `(q' * A(:, i)) * q`, (5) `norm(v)`
(b) (1) `function`, (2) `A(:, i)`, (3) `Q(:, i)`, (4) `(q' * v) * q`, (5) `norm(v)`
(c) (1) `def`, (2) `A(:, i)`, (3) `Q(:, i)`, (4) `(q' * v) * q`, (5) `sqrt(sum(v^2))`
(d) (1) `function`, (2) `A(:, i)`, (3) `Q(:, j)`, (4) `(q' * A(:, i)) * q`, (5) `norm(v)`
(e) (1) `function`, (2) `A(:, i)`, (3) `Q(:, j)`, (4) `(q' * v) * q`, (5) `sqrt(sum(v^2))`

Solution.

(1) (a) True, (b) True, (c) True, (d) False, (e) True

An explain for (d):

```
A = reshape(1:56, 7, 8);
A = A(14 <= A) .* A(43 >= A);
```

Here, `A(14 <= A)` and `A(43 >= A)` are just logical statments. Thus the `A` gernerated in (d) consists of only 0s and 1s. To achieve the goal, the task of selecting the correct values of `A` using the generated logical variables should be added.

(2) answer : (d)

(1) `function`

(2) `A(:, i)`

(3) `Q(:, j)`

(4) `(q' * A(:, i)) * q`

(5) `norm(v)`