1 Pokus zavést hry formálně

V_X is a set of all valuations; Φ_X is a set of all formulas over variables X.

Definition 1. *A* code-breaking game *is a triple* $\mathcal{G} = (X, \iota, E, \rho)$*, where* X *is a finite set of propositional variables,* $\iota \in \Phi_X$ *is an initial prepositional formula,* E *is a finite set of possible experiments and* $\rho : V_X \times E \to \Phi_X$ *is the inference function such that* $v(\rho(v,e)) = 1$ *for every* $v \in V$*,* $e \in E$*.*

Example 1 (Fake-coin problem). *Consider a fake-coin problem with* n *coins, one of which is fake. This game can be formalized as follows:*

- $X = \{x_1, x_2, \ldots, x_n, y\}$
  Intuitively, variable x_i *tells weather the coin* i *is fake. Variable* y *tells weather it's lighter or heavier.*
- $\iota = \mathit{Exactly\text{-}1}\,(\{x_1, \ldots, x_n\})$
  This is to ensure that exactly one coin is fake.
- $E = 2^{\{1..n\}} \times 2^{\{1..n\}}$
  Experiment consist of two sets of coins. For the sake of simplicity, we allow non-disjoint sets and sets of different size, however such experiments would give no new information.
- $\rho(v,(A,B)) = \begin{cases} \varepsilon & \text{if } r = \mathit{error} \\ (\bigvee\{x_c \mid c \in A\} \wedge \neg y) \vee (\bigvee\{x_c \mid c \in B\} \wedge y) & \text{if } r = \mathit{lighter} \\ (\bigvee\{x_c \mid c \in A\} \wedge y) \vee (\bigvee\{x_c \mid c \in B\} \wedge \neg y) & \text{if } r = \mathit{heavier} \\ \neg \bigvee\{x_c \mid c \in A \cup B\} & \text{if } r = \mathit{equal} \end{cases}$
  The conditions correspond to the result r *of the experiment:*
  - $r = \mathit{error}$ *if* $|A| \neq |B|$ *or* $A \cap B \neq \emptyset$
  - $r = \mathit{lighter}$ *if* $(v(c) = 1$ *for some* $c \in A$ *and* $v(y) = 0)$ *or* $(v(c) = 1$ *for some* $c \in B$ *and* $v(y) = 1)$
  - $r = \mathit{heavier}$ *if* $(v(c) = 1$ *for some* $c \in A$ *and* $v(y) = 1)$ *or* $(v(c) = 1$ *for some* $c \in B$ *and* $v(y) = 0)$
  - $r = \mathit{equal}$ *if* $v(c) = 0$ *for every* $c \in A \cup B$

Example 2 (Mastermind). *Consider a mastermind puzzle with* p *pegs and color set* C*. This game can be formalized as follows:*

- $X = \{x_{i,c} \mid 1 \leq i \leq p, c \in C\}$.
  Variable $x_{i,c}$ *tells whether there is color* c *at position* i*.*

- $\iota = \bigwedge \{\textit{Exactly-1}\,\{x_{i,c} \mid c \in C\} \mid 1 \leq i \leq p\}$.
  This guarantees that there is exactly one color at each position.
- $E = C^p$.
  Experiment is just a sequence of colors.
- *Inference function is defined by*

  $$\begin{aligned}\rho(v,g) = {} & \textit{Exactly-b}\,\{x_{i,g[i]} \mid 1 \leq i \leq p\} \wedge \\ & \textit{Exactly-t} \bigcup \{\{\textit{AtLeast-n}\,\{x_{i,c} | 1 \leq i \leq p\} \mid 1 \leq n \leq \#i.(g[i] = c)\} \mid c \in C\}\end{aligned}$$

  where $b = \#i.(v(x_{i,g[i]}) = 1)$ *captures the number of black pegs in the response for the experiment and* $t = \sum_{c \in C} \min(\#i.(v(x_{i,c}) = 1), \#i.(g[i] = c))$ *is the total number of pegs (black + white).*

Bibliography