1 Pokus zavést hry formálně

V_X is a set of all valuations; Φ_X is a set of all formulas over variables X.

Definition 1. A *code-breaking game* is a triple $\mathcal{G} = (X, \iota, E, \rho)$, where X is a finite set of propositional variables, $\iota \in \Phi_X$ is an initial prepositional formula, E is a finite set of possible experiments and $\rho : V_X \times E \to \Phi_X$ is the inference function such that $v(\rho(v, e)) = 1$ for every $v \in V$, $e \in E$.

Example 1 (Fake-coin problem). Consider a fake-coin problem with n coins, one of which is fake. This game can be formalized as follows:

- $X = \{x_1, x_2, \ldots, x_n, y\}$
  Intuitively, variable x_i tells weather the coin i is fake. Variable y tells weather it's lighter or heavier.
- $\iota = \text{Exactly-1}\left(\{x_1, \ldots, x_n\}\right)$
  This is to ensure that exactly one coin is fake.
- $E = 2^{\{1..n\}} \times 2^{\{1..n\}}$
  Experiment consist of two sets of coins. For the sake of simplicity, we allow non-disjoint sets and sets of different size, however such experiments would give no new information.
- $$\rho(v, (A, B)) = \begin{cases} \varepsilon & \text{if r = error} \\ (\bigvee\{x_c \mid c \in A\} \wedge \neg y) \vee (\bigvee\{x_c \mid c \in B\} \wedge y) & \text{if r = lighter} \\ (\bigvee\{x_c \mid c \in A\} \wedge y) \vee (\bigvee\{x_c \mid c \in B\} \wedge \neg y) & \text{if r = heavier} \\ \neg \bigvee\{x_c \mid c \in A \cup B\} & \text{if r = equal} \end{cases}$$
  The conditions correspond to the result r of the experiment:
  - r = error if $|A| \neq |B|$ or $A \cap B \neq \emptyset$
  - r = lighter if $(v(c) = 1$ for some $c \in A$ and $v(y) = 0)$ or $(v(c) = 1$ for some $c \in B$ and $v(y) = 1)$
  - r = heavier if $(v(c) = 1$ for some $c \in A$ and $v(y) = 1)$ or $(v(c) = 1$ for some $c \in B$ and $v(y) = 0)$
  - r = equal if $v(c) = 0$ for every $c \in A \cup B$

Example 2 (Mastermind). Consider a mastermind puzzle with p pegs and color set C. This game can be formalized as follows:

- $X = \{x_{i,c} \mid 1 \leq i \leq p, c \in C\}$.
  Variable $x_{i,c}$ tells whether there is color c at position i.

- $\iota = \bigwedge \{\text{Exactly-1}\ \{x_{i,c} \mid c \in C\} \mid 1 \leq i \leq p\}$.
  This guarantees that there is exactly one color at each position.
- $E = C^p$.
  Experiment is just a sequence of colors.
- Inference function is defined by

$$\begin{aligned}\rho(v,g) = {} & \text{Exactly-b}\ \{x_{i,g[i]} \mid 1 \leq i \leq p\} \wedge \\ & \text{Exactly-t} \bigcup \{\{\text{AtLeast-n}\ \{x_{i,c} | 1 \leq i \leq p\} \mid 1 \leq n \leq \#i.(g[i] = c)\} \mid c \in C\}\end{aligned}$$

where $b = \#i.(v(x_{i,g[i]}) = 1)$ captures the number of black pegs in the response for the experiment and $t = \sum_{c \in C} \min(\#i.(v(x_{i,c}) = 1), \#i.(g[i] = c))$ is the total number of pegs (black + white).

Bibliography