Prova MUGA {Par}

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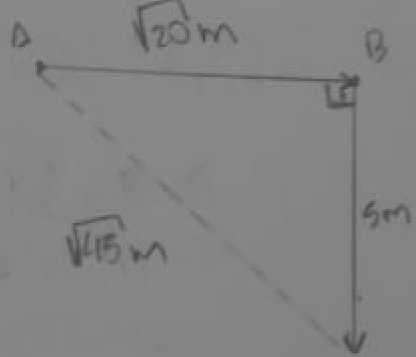
1) 

$\overrightarrow{AB} = B - A = (3,4) - (1,0) = (2,4)$

Sabe-se que

$|\overrightarrow{AB}| = \sqrt{2^2 + 4^2} = \sqrt{20}$

Por pitágoras, tem-se

$(\sqrt{20})^2 + 5^2 = (\overrightarrow{AC})^2$

$\rightarrow \overrightarrow{AC} = \sqrt{45}$

Para achar as coordenadas de C:

$|\overrightarrow{AB}| = \sqrt{20} = (2,4)$

$\hookrightarrow B - A = 3 - 1 = 2,\ 4 - 0 = 4$

$|\overrightarrow{AC}| = \sqrt{45} = (\sqrt{20}, 5)$

$\hookrightarrow C - A = x - 1 = \sqrt{20},\ \underline{y - 0 = 5}$

$|\overrightarrow{BC}| = \sqrt{25} = (5,1)$

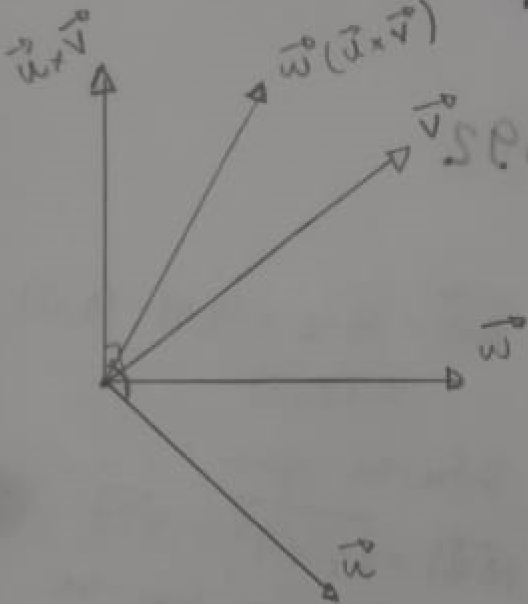
$\hookrightarrow C - B = \underline{x - 3 = 5},\ 5 - 4 = 1$

Portanto:

$x = 2 \quad y = 5$

$\boxed{C\ (2,5)}$

2) a) $\vec{w} \times (\vec{u} \times \vec{v})$



b) $\vec{w} \times (\vec{u} \times \vec{v}) \rightarrow \vec{u}(x_1, y_1, z_1), \vec{v}(x_2, y_2, z_2), \vec{w}(x_3, y_3, z_3)$

(I) $\vec{u} \times \vec{v} = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} \vec{i} - \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} \cdot \vec{j} + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \cdot \vec{k}$

$\vec{u} \times \vec{v} = (y_1 z_2 - z_1 y_2)\vec{i} - (x_1 . z_2 - z_1 x_2)\vec{j} + (x_1 y_2 - y_1 x_2) \cdot \vec{k}$

(II) $\vec{w} \times (\vec{u} \times \vec{v}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_3 & y_3 & z_3 \\ (y_1 z_2 - y_2 z_1) & (x_1 z_2 - x_2 z_1) & (x_1 y_2 - x_2 y_1) \end{vmatrix}$

$\vec{w} \times (\vec{u} \times \vec{v}) = [y_3(x_1 y_2 - x_2 y_1) - z_3(x_1 z_2 - x_2 z_1)] \cdot \vec{i}$

$- [x_3(x_1 y_2 - x_2 y_1) - z_3(y_1 z_2 - y_2 z_1)] \cdot \vec{j}$

$+ [x_3(x_1 z_2 - x_2 z_1) - y_3(y_1 z_2 - y_2 z_1)] \cdot \vec{k}$

4) a) $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & -1 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow{\text{det}} \begin{vmatrix} 2 & 1 & 0 \\ 0 & 3 & -1 \\ 1 & 1 & 4 \end{vmatrix} = \underline{25}$

MATRIZ DOS COFATORES:

$$\begin{pmatrix} 13 & -1 & -3 \\ -4 & 8 & -1 \\ 1 & 2 & 6 \end{pmatrix} \xrightarrow{\text{cof}^T} \begin{pmatrix} 13 & -4 & -1 \\ -1 & 8 & 2 \\ -3 & -1 & 6 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 13/25 & -4/25 & -1/25 \\ -1/25 & 8/25 & 2/25 \\ -3/25 & -1/25 & 6/25 \end{pmatrix}$$

$$A^{-2} = A^{-1} \cdot A^{-1} = \begin{pmatrix} 176/625 & -83/625 & -27/625 \\ -27/625 & 66/625 & 29/625 \\ -56/625 & -2/625 & 37/625 \end{pmatrix}$$

b) $[(A-I)^{-1} B]^{-1}$

$$(A-I)^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \\ 1 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 7/6 & -1/2 & -1/6 \\ -1/6 & 1/2 & 1/6 \\ -1/3 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/6 & -4/3 \\ 0 & 5/6 & 4/3 \\ 0 & -1/3 & 2/3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 2/3 & -4/3 \\ 0 & 1/3 & 5/6 \end{bmatrix}$$

5) $\begin{vmatrix} 3 & 2 & -4 & 1 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -3 & -3 \\ 3 & 3 & -5 & 0 \\ 1 & -1 & -1 & -1 \end{vmatrix} \sim \begin{vmatrix} 3 & 2 & -4 & 1 \\ 0 & -5 & 7 & 8 \\ 0 & -5 & -5 & -10 \\ 0 & 1 & -1 & -1 \\ 0 & -5 & 1 & -4 \end{vmatrix}$

$-1L_1 + 3L_2$
$-1L_1 + 3L_3$
$-1L_1 + L_4$
$-1L_2 + 3L_5$

$-1L_2 + L_3$
$-1L_2 + 3L_4$
$-1L_2 + L_5$

$\sim \begin{vmatrix} 3 & 2 & -4 & 1 \\ 0 & -5 & 7 & 5 \\ 0 & 0 & -12 & -18 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -6 & -9 \end{vmatrix} \sim \begin{vmatrix} 3 & 2 & -4 & 1 \\ 0 & 5 & 7 & 5 \\ 0 & 0 & -12 & -18 \\ 0 & 0 & 0 & -18 \\ 0 & 0 & 0 & -3 \end{vmatrix} \sim \begin{vmatrix} 3 & 2 & -4 & 1 \\ 0 & 5 & 7 & 5 \\ 0 & 0 & -12 & -18 \\ 0 & 0 & 0 & -18 \\ 0 & 0 & 0 & 0 \end{vmatrix}$

$1L_3 + 6L_4$
$-L_3 + 2L_5$

$-1L_4 + 6L_5$

$\sim \begin{matrix} 3 & 2 & -4 & 1 \\ 0 & 5 & 7 & 5 \\ 0 & 0 & -12 & -18 \\ 0 & 0 & 0 & -18 \end{matrix} \rightarrow \begin{cases} 3x_1 + 2x_2 - 4x_3 = 1 \\ 5x_2 + 7x_3 = 5 \\ -12x_3 = -18 \\ 0 = -18 \text{ (F)} \end{cases}$

$\Rightarrow$ Logo o sistema é <u>IMPOSSIVEL</u>.