

# Sequential P Systems with Active Membranes Working on Sets

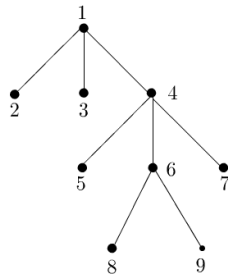
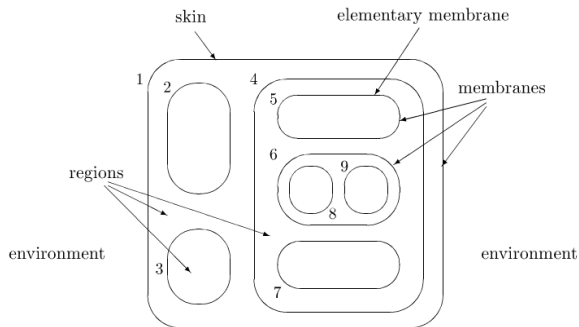
Michal Kováč, Damas Gruska

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Slovakia

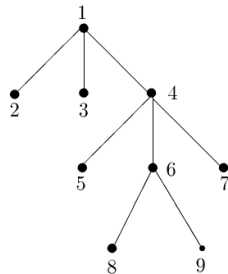
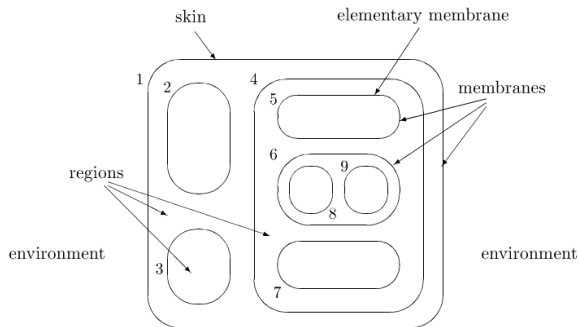
30.9.2015

- 1 Overview of formal models
  - P systems
  - Using sets instead of multisets
- 2 Sequential active set membrane systems
  - Original semantics
  - Modified membrane creation semantics

# Membrane structure

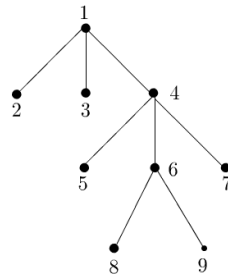
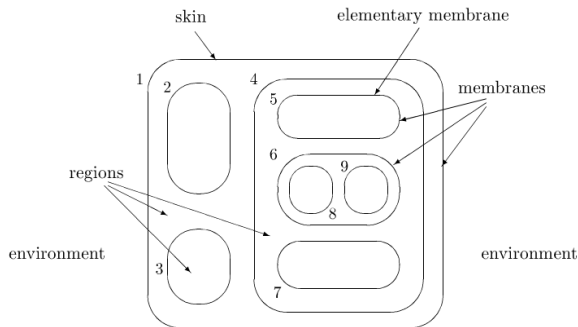


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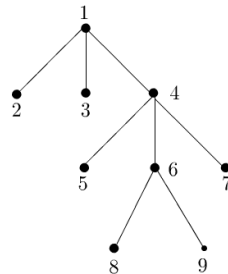
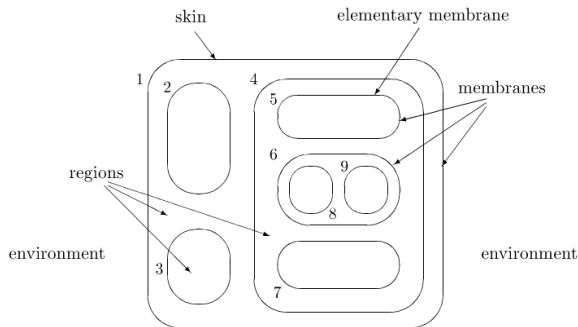
- Multisets

# Membrane structure



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- Rewriting rules

# Membrane structure



- Multisets
- Rewriting rules
- Passive vs. Active

# Computation

- Maximal parallel vs. sequential

# Computation

- Maximal parallel vs. sequential
- Language
  - Generating mode: language of sequences of objects sent out from the skin membrane
  - Accepting mode: accept the given configuration if the system can halt



# Using sets instead of multisets

- How realistic is the counting?
- Effectiveness of verification techniques

# Set membrane systems

- Alhazov [Alhazov, 2006]: multiplicities of objects are ignored  
R, with active membranes universal
- Kleijn, Koutny [Kleijn and Koutny, 2011]:  
min-enabled computational step  $\Rightarrow$  sequential R
- Maximal parallel  $\Rightarrow$  deterministic
- No conflict (objects can participate as reactants in as many rules as they want)

# Sequential active set membrane systems

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- $C = (T, l, c)$ 
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  - $c : V(T) \rightarrow 2^\Sigma$

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- $C = (T, l, c)$ 
  - $l: V(T) \rightarrow \{1, \dots, m\}$
  - $c: V(T) \rightarrow 2^\Sigma$
- Rewriting rules
  - $u \rightarrow w$
  - $u \rightarrow w\delta$
  - $u \rightarrow [{}_j v_1]_j v_2,$   
 where  $u \in \Sigma, |u| \geq 1, v_1, v_2 \in \mathbb{N}$  and  $w \in (\Sigma \times \{\cdot, \uparrow, \downarrow_j\})$

# Register machine

- Registers with non-negative values  $r_1, r_2, \dots$
- Labeled instructions  $i : op$ , where  $op$  is:
  - $add(j, k)$
  - $sub(j, k, l)$
  - $halt$
- State = (instruction pointer, values of registers)
- Step: modify the register value, move the instruction pointer
- Computation:
  - Given the input values of registers, accept if  $halt$  can be reached.
  - ...

# Proof of universality

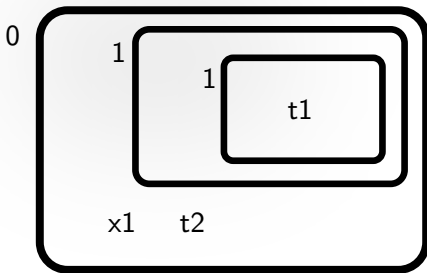
Simulation of a register machine:

- 1 : *sub*(1, 2, 3)
- 2 : *add*(2, 1)
- 3 : *halt*

# Example simulation

## Skin membrane:

- $x_1 \rightarrow x_1 \downarrow_1$  ,  $x_2 \rightarrow x_2 \downarrow_2$
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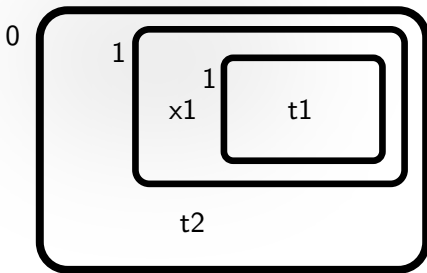
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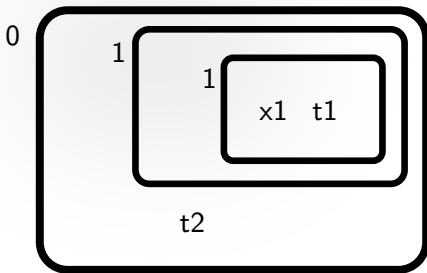
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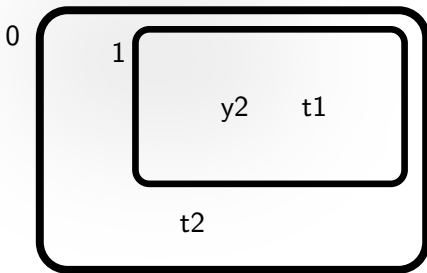
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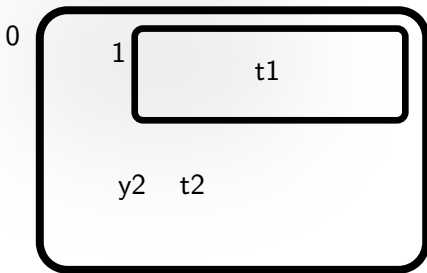
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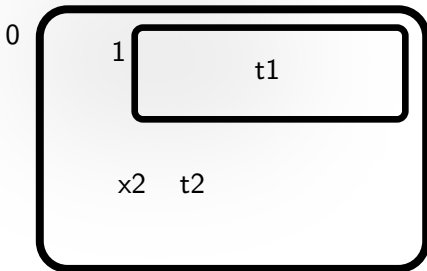
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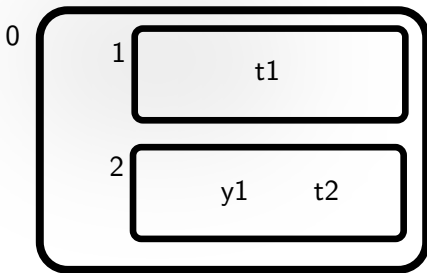
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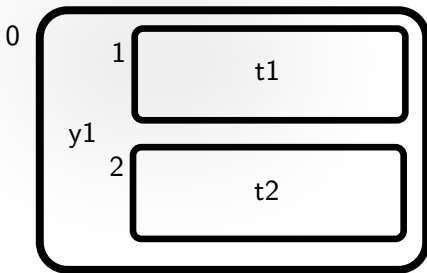
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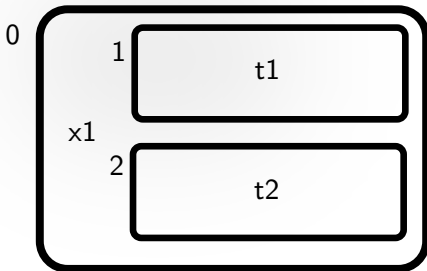
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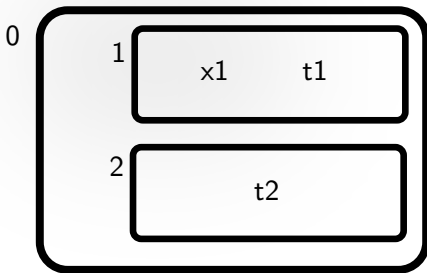
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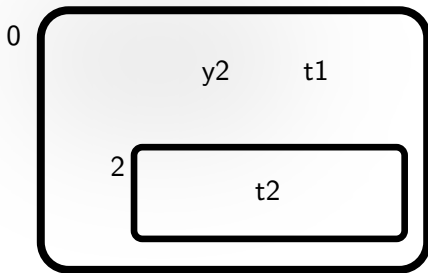
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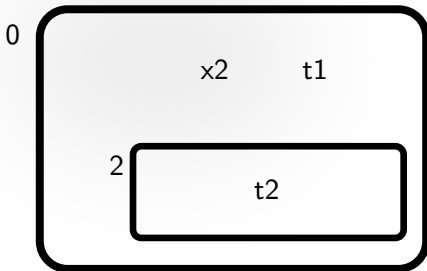
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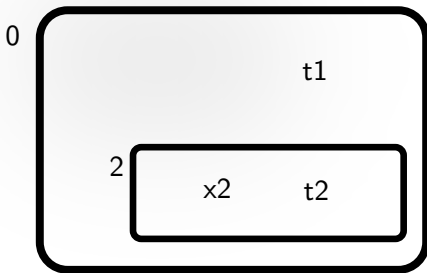
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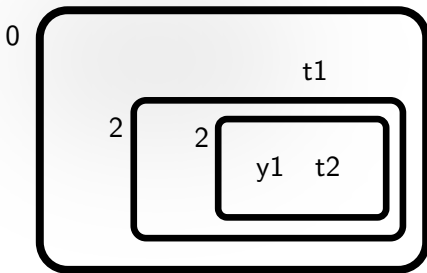
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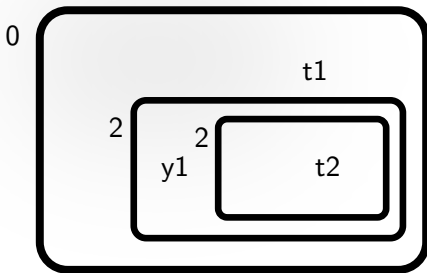
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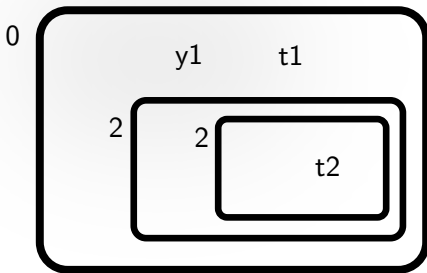
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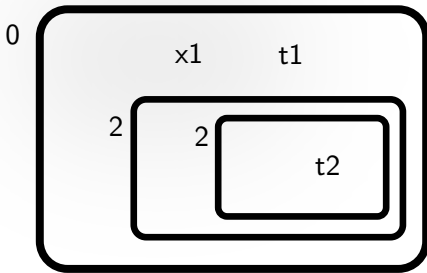
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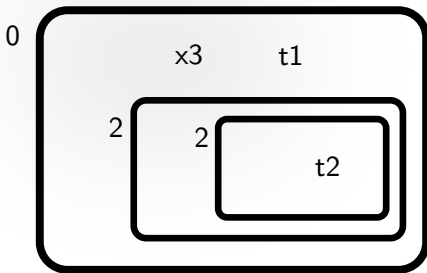
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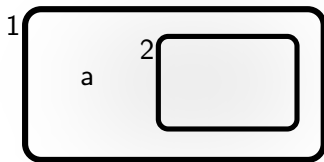
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# Issues with original semantics

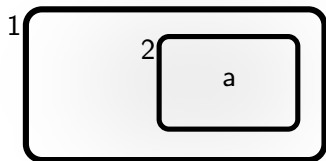
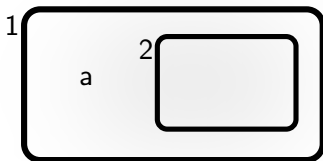
- Explicit membrane creation rule
- Sending an object to a child membrane

# Inject-or-create semantics

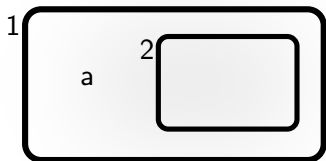


# Inject-or-create semantics

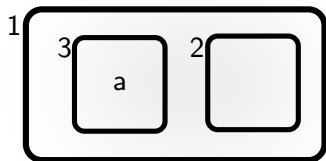
$$a \rightarrow a \downarrow_2$$



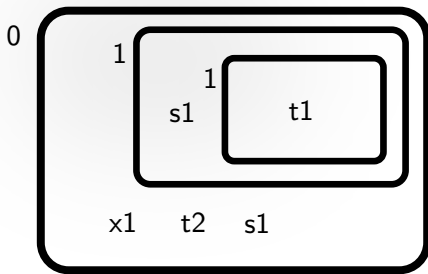
# Inject-or-create semantics



$a \rightarrow a \downarrow_3$



# Wrap-or-create semantics



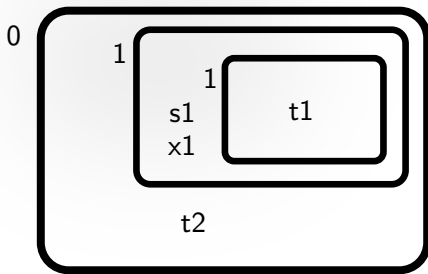
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Wrap-or-create semantics



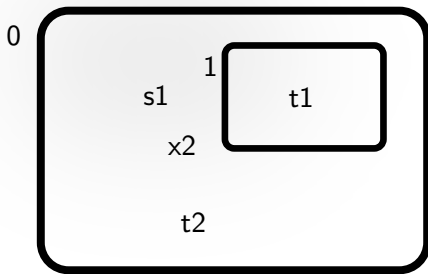
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Wrap-or-create semantics



Skin membrane:

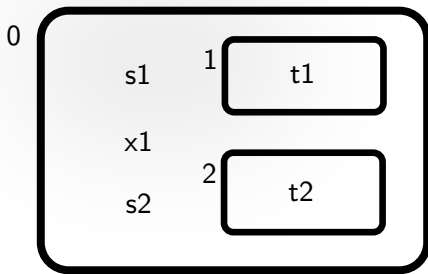
- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$



# Wrap-or-create semantics



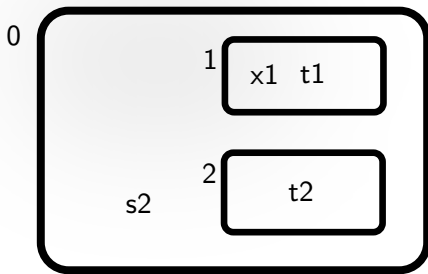
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Wrap-or-create semantics



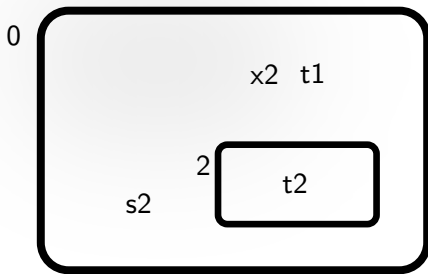
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Wrap-or-create semantics



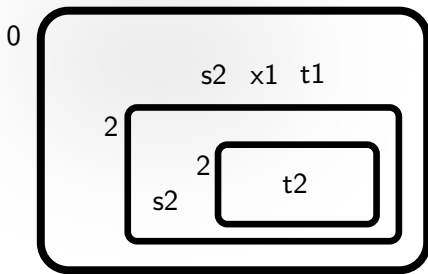
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Wrap-or-create semantics



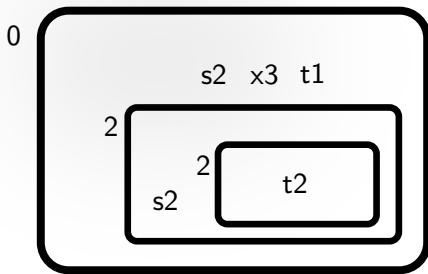
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Wrap-or-create semantics



Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

# Comparison of membrane creation semantics

	space	time
original	$O(n)$	$O(n)$
original	$O(\log(n))$	$O(\log(n))$
inject-or-create	$O(\log(n))$	$O(\log(n))$
wrap-or-create	$O(n)$	$O(1)$



Alhazov, A. (2006).

P systems without multiplicities of symbol-objects.  
*Information Processing Letters*, 100(3):124–129.



Kleijn, J. and Koutny, M. (2011).

Membrane systems with qualitative evolution rules.  
*Fundam. Inf.*, 110(1-4):217–230.

Thanks for your attention!