

Sequential P Systems with Active Membranes Working on Sets

Michal Kováč

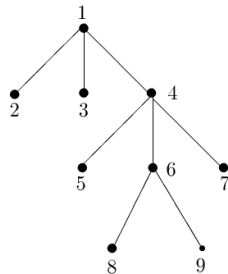
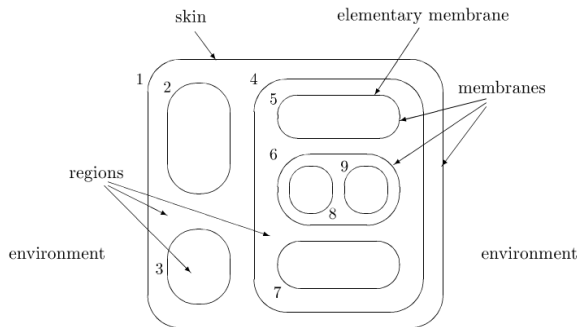
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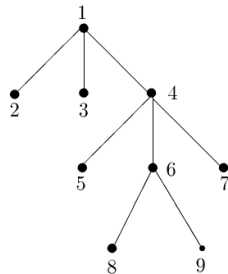
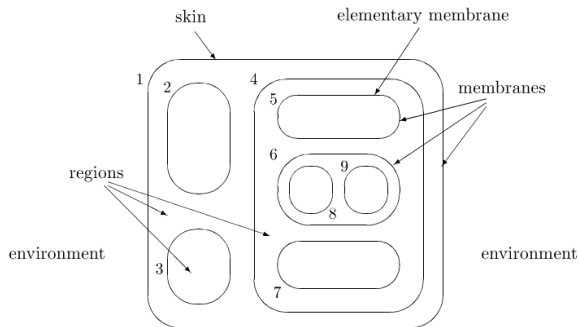
- 1 Overview of formal models
 - P systems
 - Using sets instead of multisets

- 2 Sequential active set membrane systems
 - Original semantics
 - Modified membrane creation semantics

Membrane structure

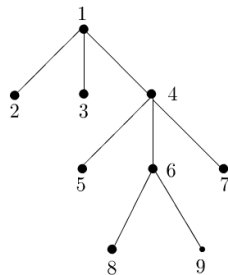
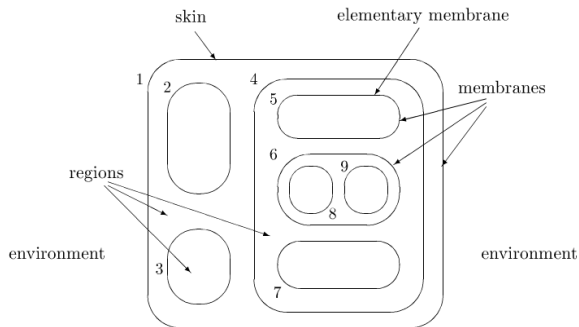


Membrane structure



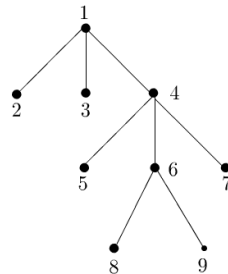
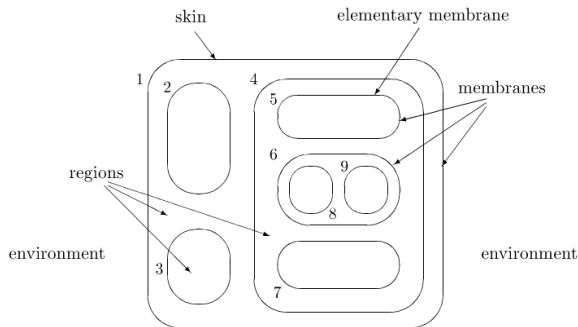
- Multisets

Membrane structure



- Multisets
- Rewriting rules

Membrane structure



- Multisets
- Rewriting rules
- Passive vs. Active

Computation

- Maximal parallel vs. sequential

Computation

- Maximal parallel vs. sequential
- Language
 - Generating mode: language of sequences of objects sent out from the skin membrane
 - Accepting mode: accept the given configuration if the system can halt

Using sets instead of multisets

- How realistic is the counting?
- Effectiveness of verification techniques

Set membrane systems

- Alhazov [Alhazov, 2006]: multiplicities of objects are ignored
R, with active membranes universal
- Kleijn, Koutny [Kleijn and Koutny, 2011]:
min-enabled computational step \Rightarrow sequential R
- Maximal parallel \Rightarrow deterministic
- No conflict (objects can participate as reactants in as many rules as they want)

Sequential active set membrane systems

- $\Pi = (\Sigma, C_0, R_1, \dots, R_m)$

Sequential active set membrane systems

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- $C = (T, l, c)$
 - $l: V(T) \rightarrow \{1, \dots, m\}$
 - $c: V(T) \rightarrow 2^\Sigma$

Sequential active set membrane systems

- $\Pi = (\Sigma, C_0, R_1, \dots, R_m)$
- $C = (T, l, c)$
 - $l: V(T) \rightarrow \{1, \dots, m\}$
 - $c: V(T) \rightarrow 2^\Sigma$
- Rewriting rules
 - $u \rightarrow w$
 - $u \rightarrow w\delta$
 - $u \rightarrow [{}_j v_1]_j v_2,$
 where $u \in \Sigma, |u| \geq 1, v_1, v_2 \in \mathbb{N}$ and $w \in (\Sigma \times \{\cdot, \uparrow, \downarrow_j\})$

Register machine

- Registers with non-negative values r_1, r_2, \dots
- Labeled instructions $i : op$, where op is:
 - $add(j, k)$
 - $sub(j, k, l)$
 - $halt$
- State = (instruction pointer, values of registers)
- Step: modify the register value, move the instruction pointer
- Computation:
 - Given the input values of registers, accept if $halt$ can be reached.
 - ...

Proof of universality

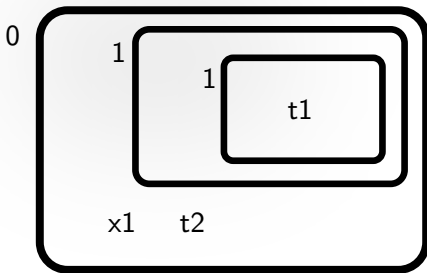
Simulation of a register machine:

- 1 : *sub*(1, 2, 3)
- 2 : *add*(2, 1)
- 3 : *halt*

Example simulation

Skin membrane:

- $x_1 \rightarrow x_1 \downarrow_1$, $x_2 \rightarrow x_2 \downarrow_2$
- $x_2 t_2 \rightarrow [{}_2 y_1 t_2]_2$
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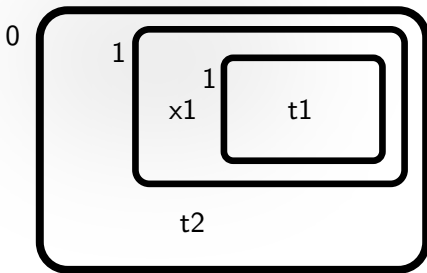
Membrane 2:

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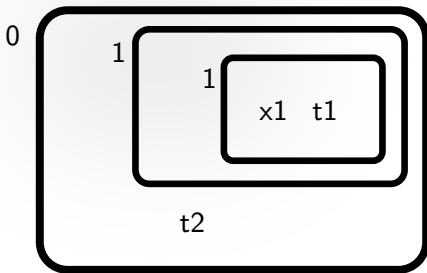
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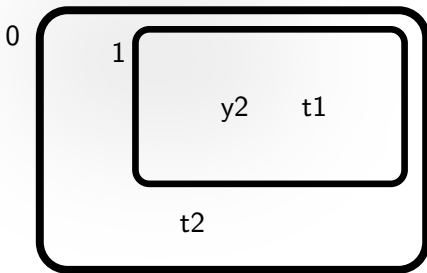
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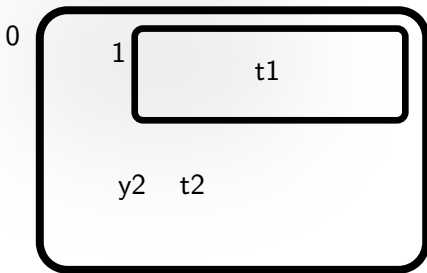
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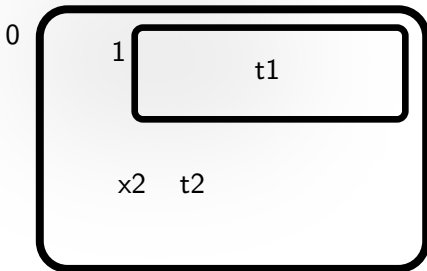
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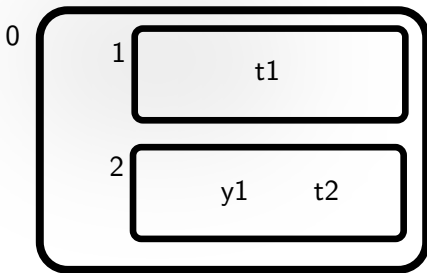
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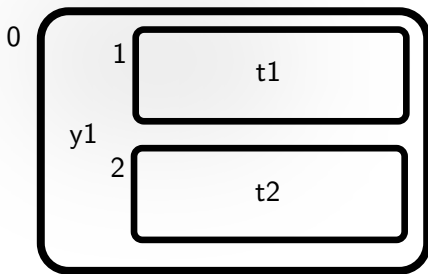
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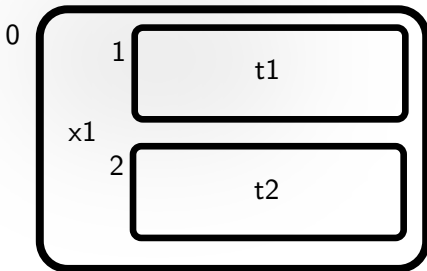
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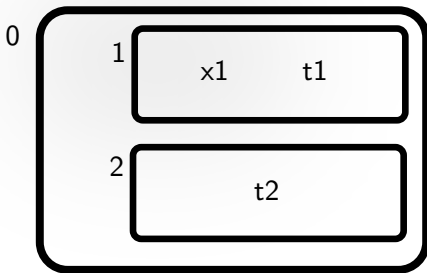
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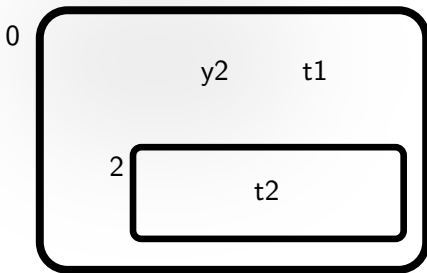
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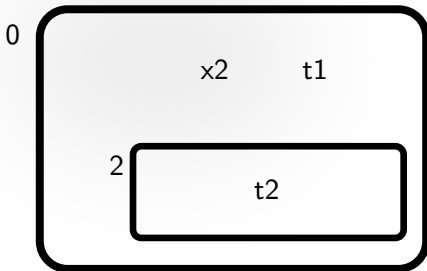
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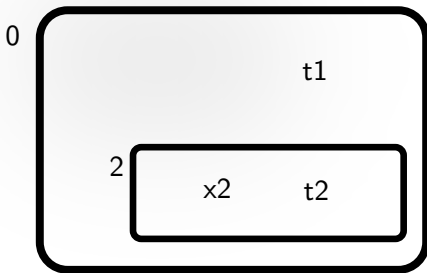
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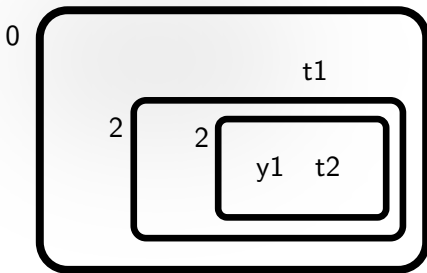
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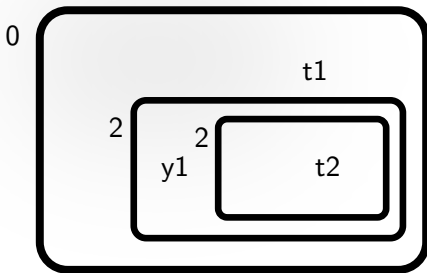
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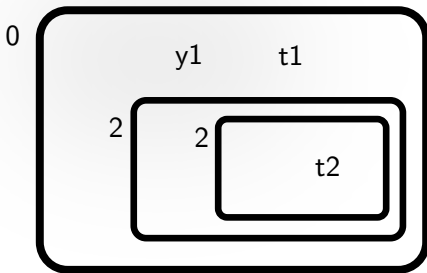
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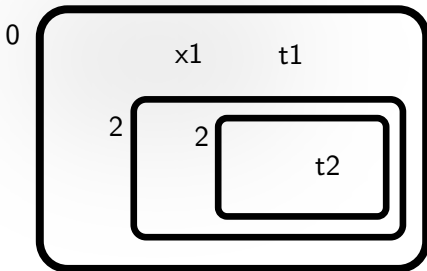
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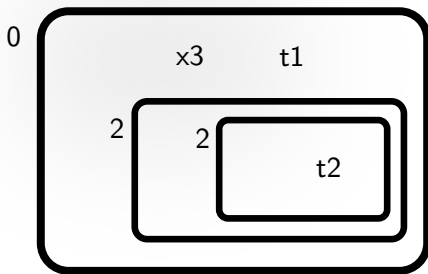
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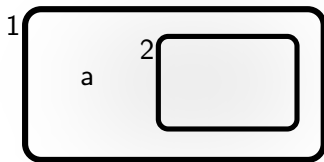
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Issues with original semantics

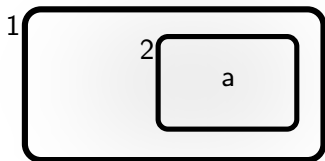
- Explicit membrane creation rule
- Sending an object to a child membrane

Inject-or-create semantics

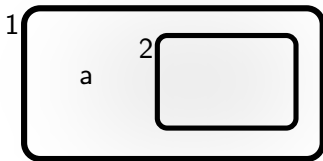


Inject-or-create semantics

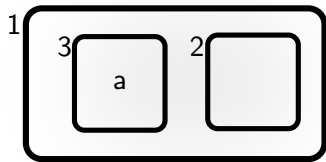
$$a \rightarrow a \downarrow_2$$



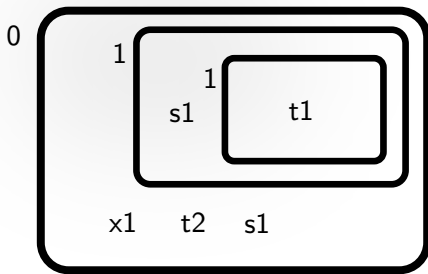
Inject-or-create semantics



$a \rightarrow a \downarrow_3$



Wrap-or-create semantics



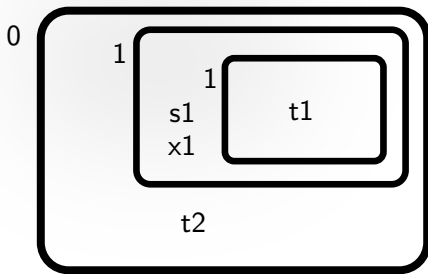
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



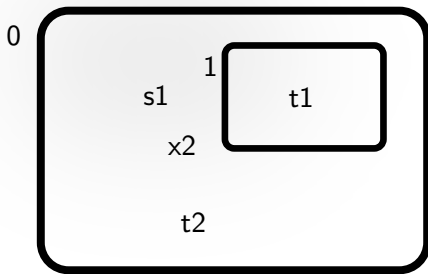
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



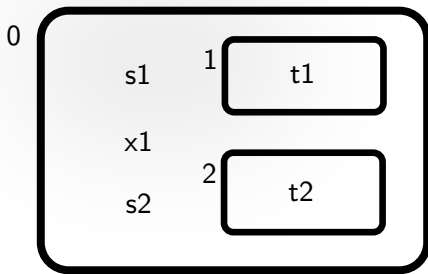
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



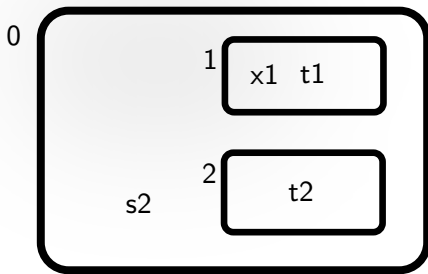
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



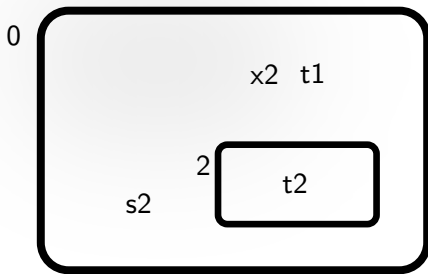
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



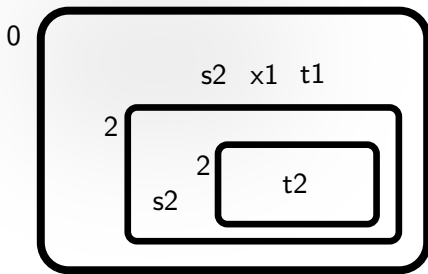
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



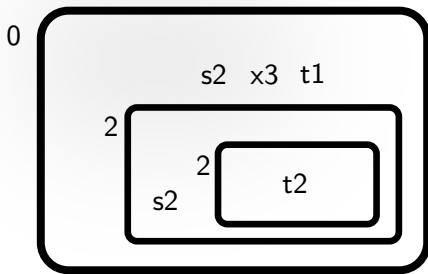
Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [{}_2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [{}_2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$

Wrap-or-create semantics



Skin membrane:

- $x_1 t_1 \rightarrow x_3 t_1$
- $x_1 s_1 \rightarrow x_1 \downarrow_1$
- $x_2 s_2 \rightarrow [2 s_2]_2 s_2 x_1$
- $x_2 t_2 \rightarrow [2 t_2]_2 s_2 x_1$

Membrane 1:

- $x_1 \rightarrow x_2 \delta$



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Thanks for your attention!