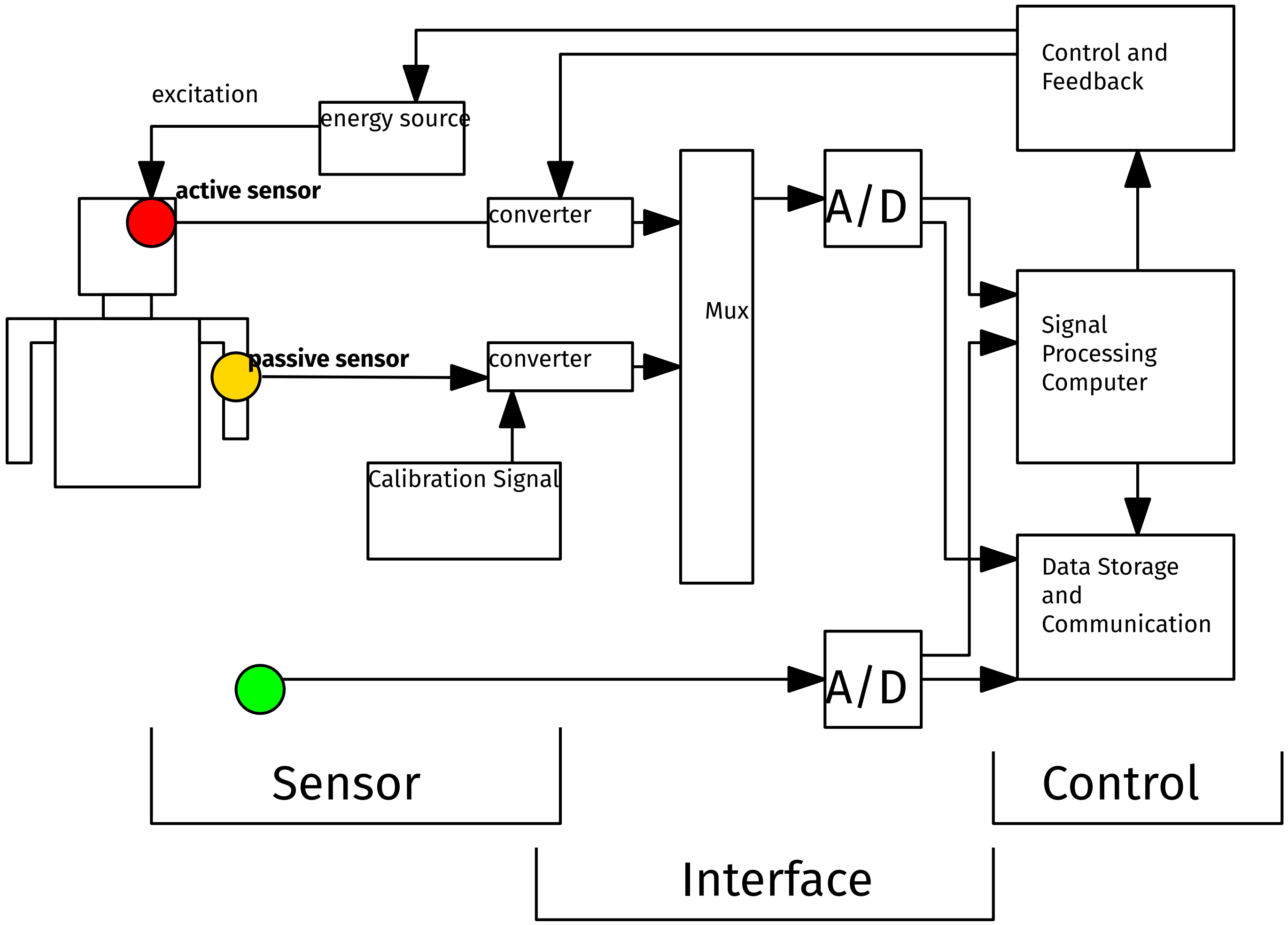
A Schematic of a Medical Device



Medical and Physiological Measurands

Parameter or Measuring Technique	Principal Measurement Range of Parameter	Signal Frequency Range, Hz	Standard Sensor or Method
Ballistocardiography (BCG)	0–7 mg	dc–40	Accelerometer, strain gage
	0–100 μm	dc–40	Displacement linear variable differential transformer (LVDT)
Bladder pressure	1–100 cm H_2O	dc–10	Strain-gage manometer
Blood flow	1–300 ml/s	dc–20	Flowmeter (electromagnetic or ultrasonic)
Blood pressure, arterial			
Direct	10–400 mm Hg	dc–50	Strain-gage manometer
Indirect	25–400 mm Hg	dc–60	Cuff, auscultation
Blood pressure, venous	0–50 mm Hg	dc–50	Strain gage
Blood gases			
P_{O_2}	30–100 mm Hg	dc–2	Specific electrode, volumetric or manometric
P_{CO_2}	40–100 mm Hg	dc–2	Specific electrode, volumetric or manometric
P_{N_2}	1–3 mm Hg	dc–2	Specific electrode, volumetric or manometric
P_{CO}	0.1–0.4 mm Hg	dc–2	Specific electrode, volumetric or manometric
Blood pH	6.8–7.8 pH units	dc–2	Specific electrode
Cardiac output	4–25 liter/min	dc–20	Dye dilution, Fick
Electrocardiography (ECG)	0.5–4 mV	0.01–250	Skin electrodes
Electroencephalography (EEG)	5–300 μV	dc–150	Scalp electrodes
(Electrocorticography and brain depth)	10–5000 μV	dc–150	Brain-surface or depth electrodes
Electrogastrography (EGG)	10–1000 μV	dc–1	Skin-surface electrodes

Resistance Measurement

Electrode Measurement

Medical and Physiological Measureands

Parameter or Measuring Technique	Principal Measurement Range of Parameter	Signal Frequency Range, Hz	Standard Sensor or Method
Gastrointestinal pressure	0–100 cm H_2O	dc–10	Strain-gage manometer
Gastrointestinal forces	1–50 g	dc–1	Displacement system, LVDT
Nerve potentials	0.01–3 mV	dc–10,000	Surface or needle electrodes
Phonocardiography	Dynamic range 80 dB, threshold about 100 μPa	5–2000	Microphone
Plethysmography (volume change)	Varies with organ measured	dc–30	Displacement chamber or impedance change
Circulatory	0–30 ml	dc–30	Displacement chamber or impedance change
Respiratory functions Pneumotachography (flow rate)	0–600 liter/min	dc–40	Pneumotachograph head and differential pressure
Respiratory rate	2–50 breaths/min	0.1–10	Strain gage on chest, impedance, nasal thermistor
Tidal volume	50–1000 ml/breath	0.1–10	Above methods
Temperature of body	32–40°C 90–104 °F	dc–0.1	Themistor, thermocouple

Stimulus Response Relationship

The input (stimulus, x) and output (response, y) can be related by simple functions:

- **linear**: $y = y_0 + B(x - x_0)$
- **logarithmic**: $y = A + Bln(x)$
- **exponent**: $y = A * exp(kx)$
- **power law**: $y = A + B * s^k$

All these relationships involved only a small number of parameters, therefore they are easy to work with.

How do we use this relationship (the **transfer function**)?

- What kind of T.F. appear in sensor measurement and how can we approximate them?
- What are the effects of statistical variation or non-ideal behaviour of sensors and how to account for them?
- What are the measures to characterise and utilise the noise present in the measurement process?

Approximations of the transfer function

Most of the time the real sensor will show non-idealities or there can be a range of statistical variations, which will lead to a non-linear, complicated response. In such cases, the T.F. can be approximated to desired accuracy using an interpolation process:

- Truncated taylor series or polynomial interpolation: $y = \Sigma a_j x^j$
- Using Lagrange polynomials $L_n(x) = \sum_{i=0}^{n} y_i l_i(x)$

$$l_i(x) = \prod_{j=0,\ j\neq i}^{n} \frac{x - x_j}{x_i - x_j} = \frac{(x-x_0)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)} \quad (1)$$

- Newton's interpolation

$$N_n(x) = \sum_{i=0}^{n} c_i n_i(x) = c_0 + \sum_{i=1}^{n} c_i \left(\prod_{j=0}^{i-1} (x - x_j) \right) \quad (2)$$

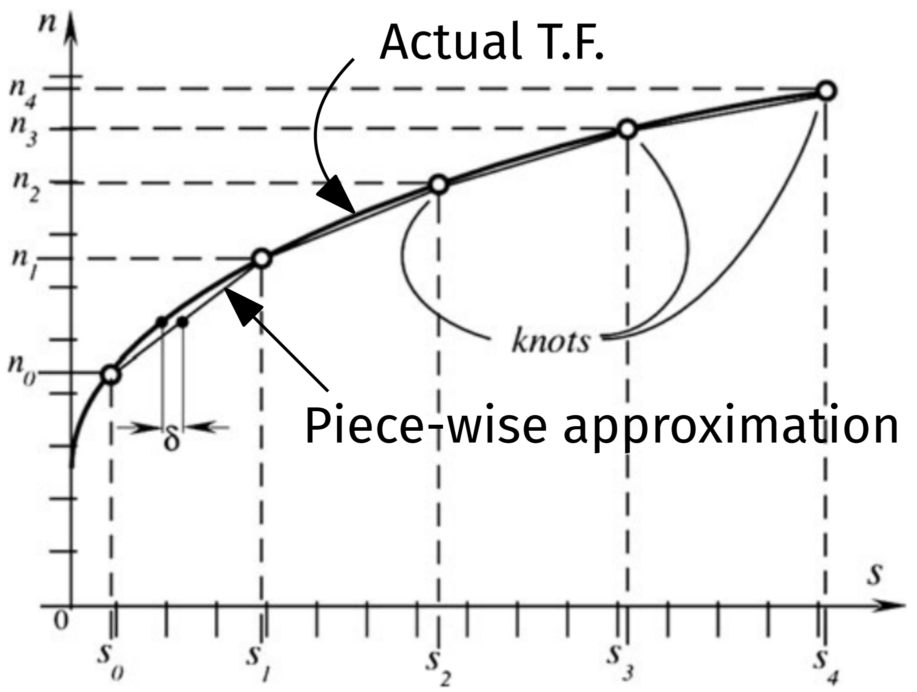
Newton's interpolation can add additional datapoints without recalculations.

Approximations of the transfer function

The polynomial approximations can have large erros at the end points, even if more data points are added (Runge's Phenomenon).

Piece-wise approximations avoid this problem, with lower computational complexity and scalability. The most popular ones are:

- Piece-wise Linear interpolation
- Piece-wise Cubic Spine interpolation (matches 2nd-order derivatives at the end points)



How do we choose locations and number of the knots to minimize δ?

Calibration finds parameters of the T.F.

It finds the unknown coefficients of the inverted transfer function for estimating stimulus based on measured signal.

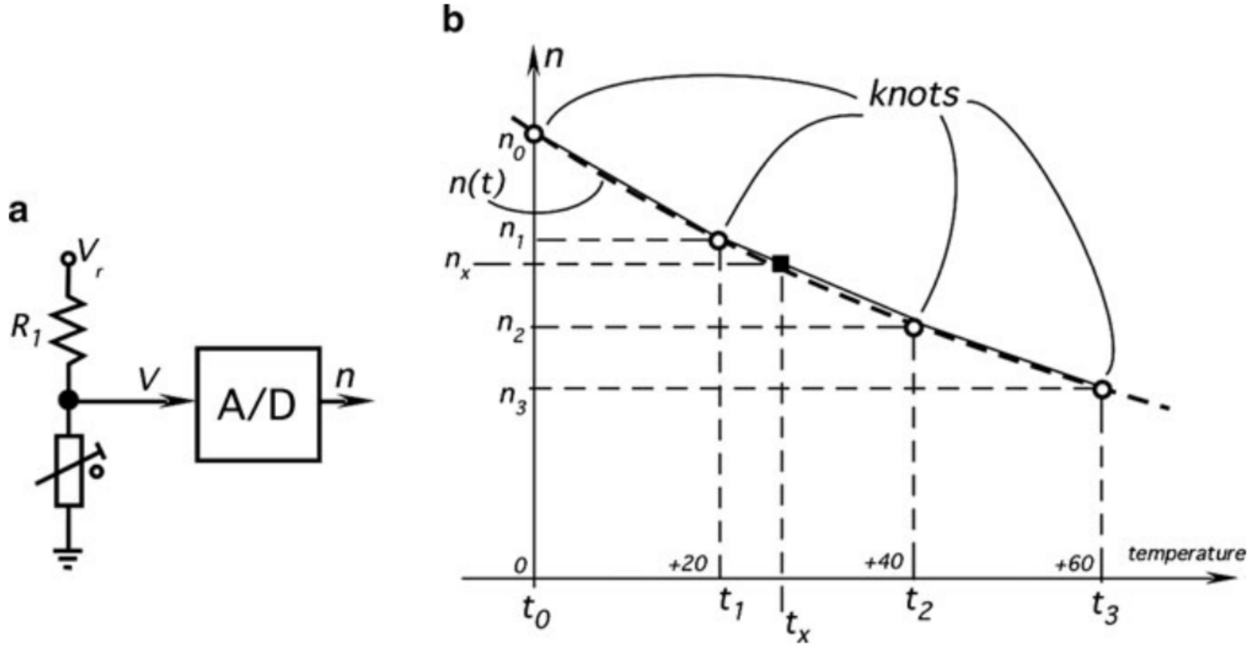
Only a small number of stimulus-response pairs are used to find the parameters. Once fixed, the functional form can be used to find stimulus for arbitrary response.

Calibration can be done in many ways:

- By fitting stimulus-response to the model transfer function
- by adjusting the data acquisition system parameters to fit measured data into a predetermined trasnfer function, with known coefficient.
- by modifying sensor's properties to fit the predetermined transfer frunction

An example

Consider a thermistor circuit (resistance changes with temperature) to measure temperature:



$$n_x = N_0 \frac{R_0 e^{\beta(T^{-1} - T_0^{-1})}}{R_1 + R_0 e^{\beta(T^{-1} - T_0^{-1})}} \qquad T_x = \left(\frac{1}{T_0} + \frac{1}{\beta} \ln\left(\frac{n_x}{N_0 - n_x} \frac{R_1}{R_0}\right)\right)^{-1}$$

n_x is the count coming out of the ADC, R_1 is the reference resistance (10.00 $k\Omega$). The unknown parameters are β and R_0, the resistance of thermister at $T = T_0$ (273.15 K).

An example

To estimate the parameters, the sensor output is measured in two baths of known temperature:
at $T_1 = 293.15K, n_1 = 1863$
at $T_2 = 313.15K, n_2 = 1078$

Which give β = 3895 K, R_0 = 8.350 $k\Omega$.

To apply a piece-wise linear approximation to this function, we can have knots at T_0 (273.15 K, n_0 = 2819), and T_3 (333.15 K, n_3 = 593).

Knot	0	1	2	3
Counts	2,819	1,863	1,078	593
Temperature (°C)	0	20	40	60

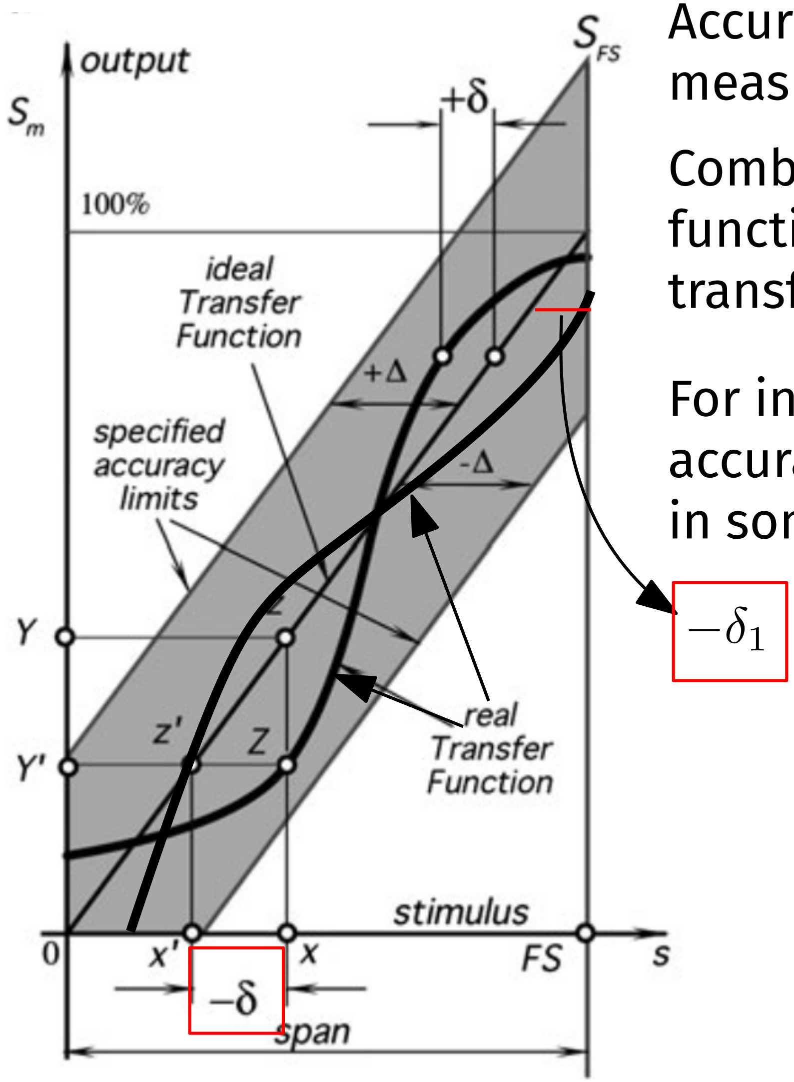
At n_x = 1505, we can calculate the temperature by linear interpolation (temperature t_x are in ^oC):

$$t_x = t_1 + \frac{n_x - n_1}{n_2 - n_i}(t_2 - t_1) = 20 + \frac{1505 - 1863}{1078 - 1863}(40 - 20) = 29.12$$

From the actual formula above, t_{1505} = 28.22 ^oC.

Statistical Variations: Accuracy

Statistical variations occur everywhere from fabrication to the signal measurements. To deal with them, certain quantities are used to characterise the T.F.



Accuracy (Δ) is the largest deviation of measured value from the calculated value.

Combines the effect all the real transfer functions, here we have shown two real transfer functions.

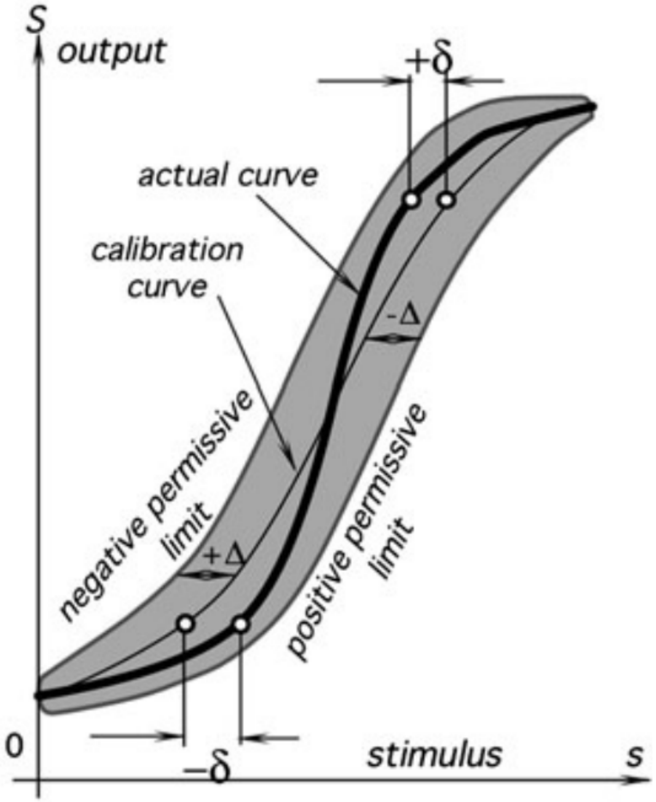
For individual transfer function the accuracy may be smaller! Can we use this in some manner?

Accuracy

Yes! by calibrating the individual sensor to discover its real transfer function.
Then $\delta < \Delta$

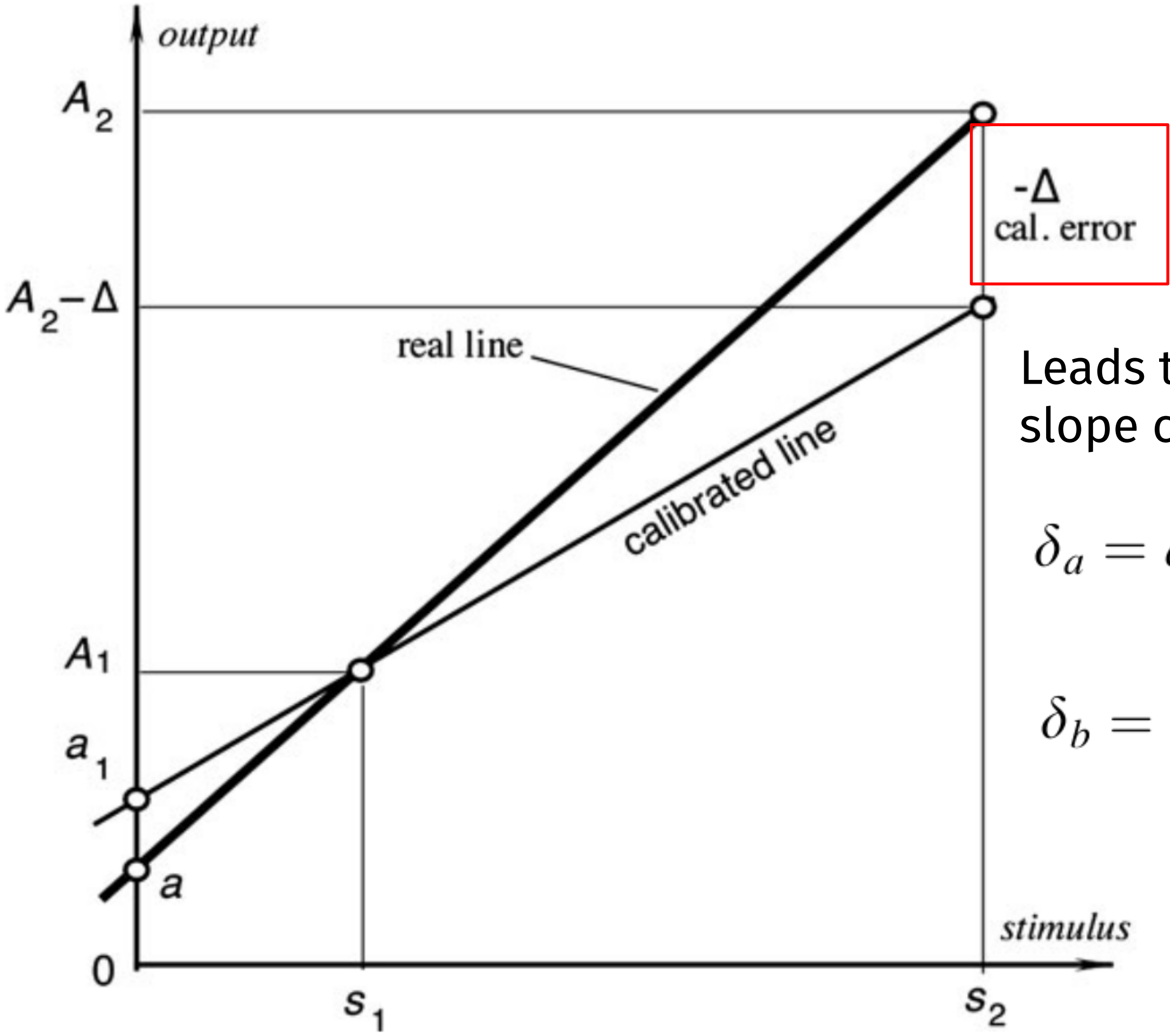
Accuracy can be specified in many ways:

- In terms of absolute measured value
- in % of the full scale value
- in % of the measured signal
- in terms of the output signal



Calibration Error

It is inaccuracy permitted by a manufacturer when a sensor is calibrated in the factory



Calibration error

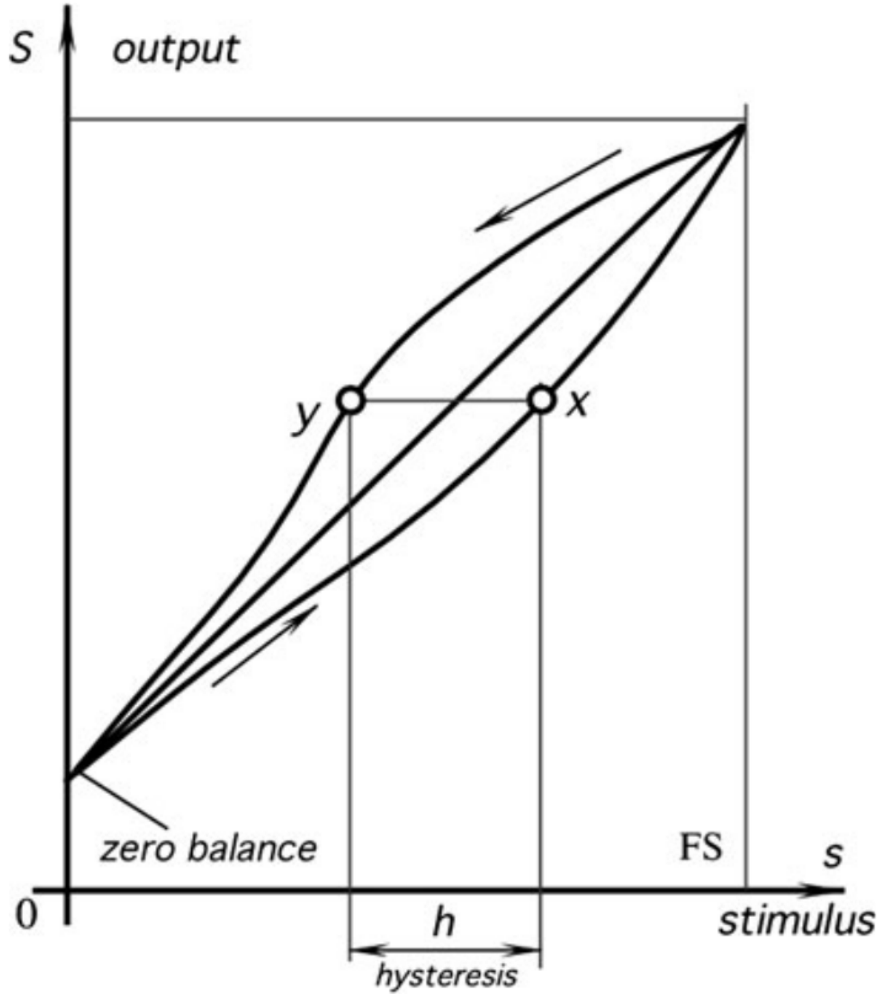
Leads to errors in intercept and slope calculations:

$$\delta_a = a_1 - a = \frac{\Delta}{s_2 - s_1}$$

$$\delta_b = -\frac{\Delta}{s_2 - s_1}$$

Hysteresis

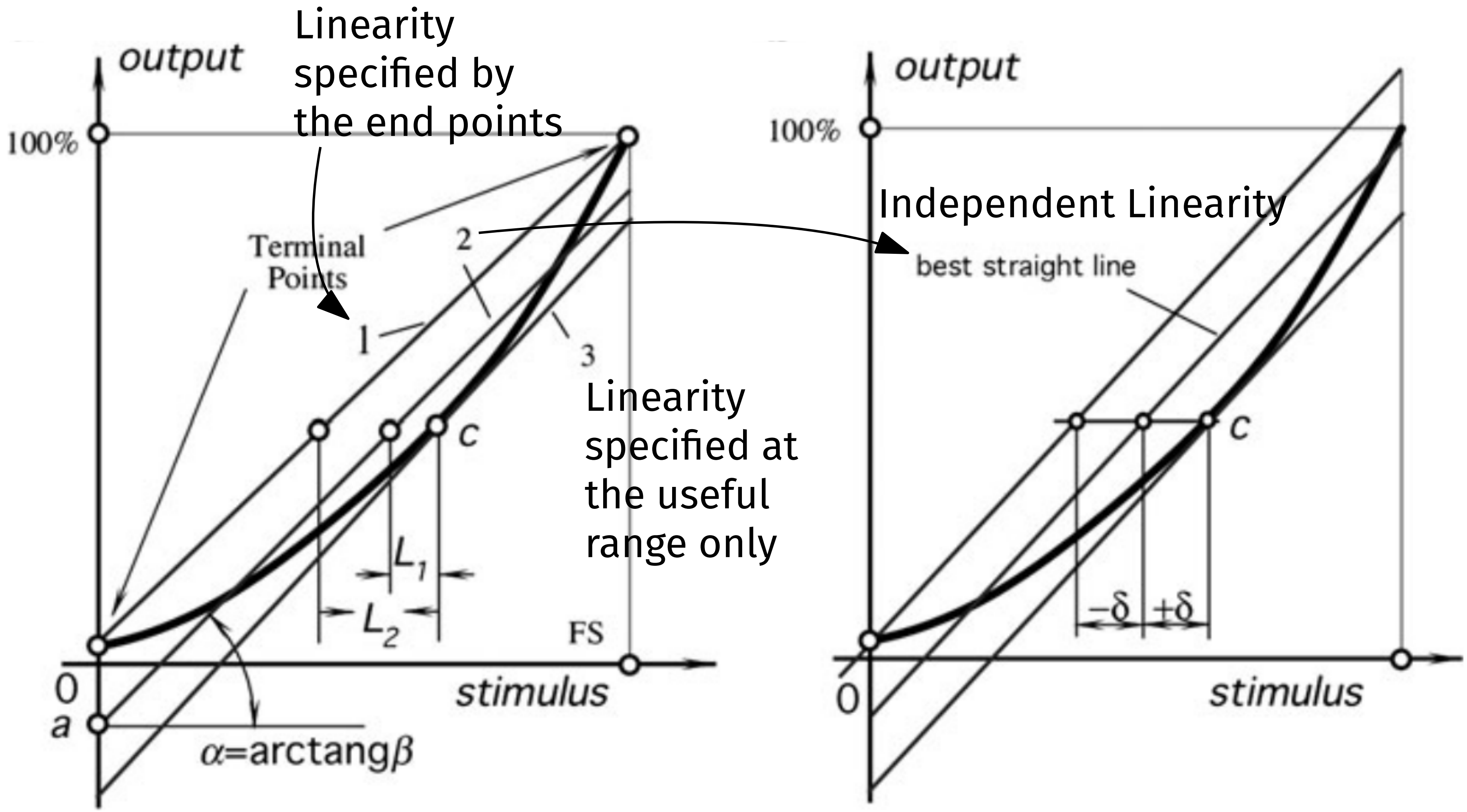
It is a deviation of the sensorffs output at a specified point of the input signal when it is approached from the opposite directions.



It is mostly caused by the material or setup properties.

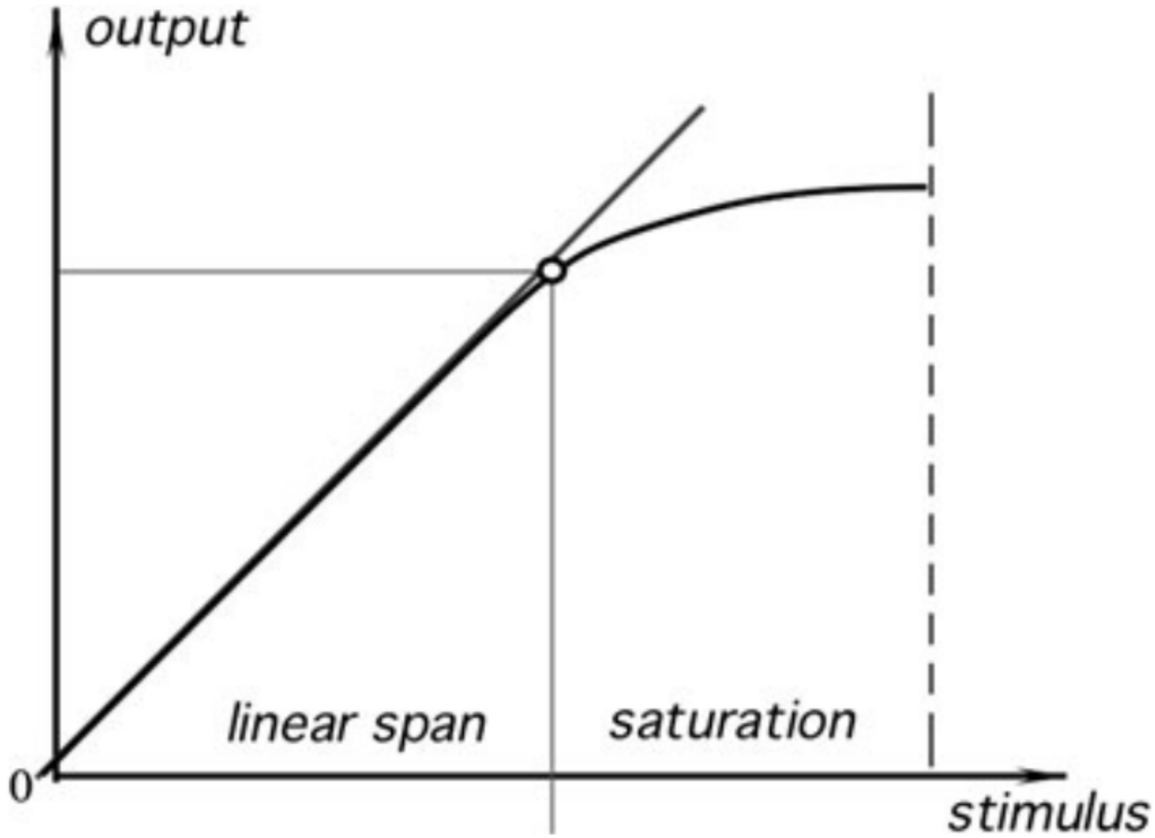
Non-linearities

A nonlinearity is a maximum deviation (L) of a real transfer function from the approximation straight line.



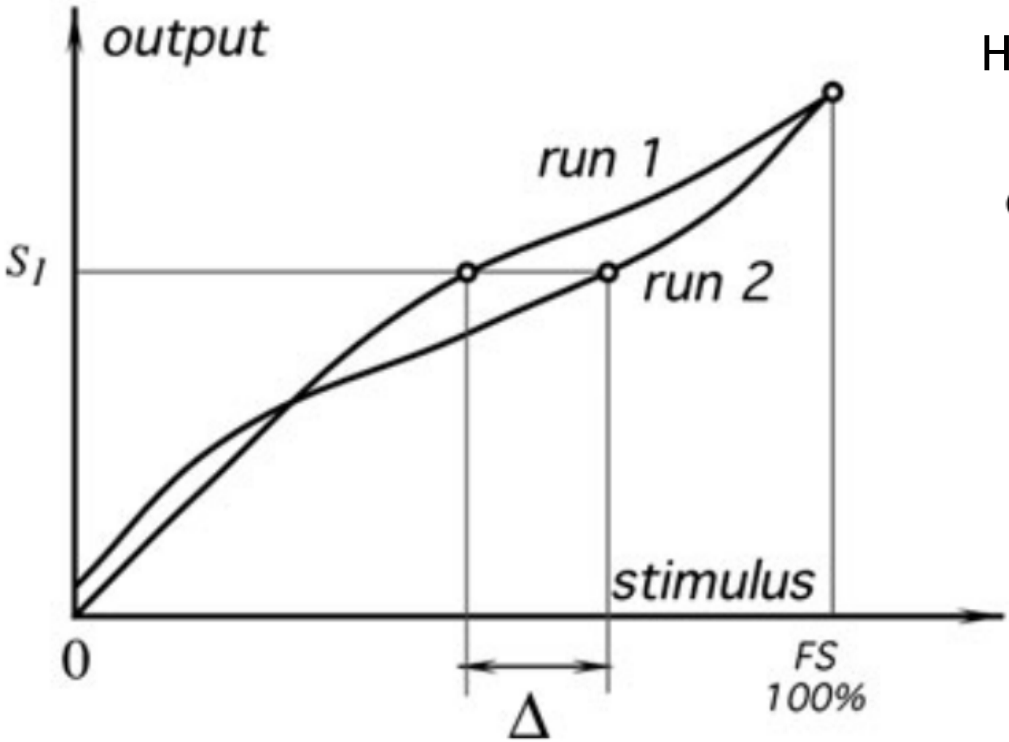
Saturation

The deviation from the linear behaviour at the end points.



Repeatability Error

The repeatability is expressed as a maximum difference between the output readings as determined by two calibrating cycles.

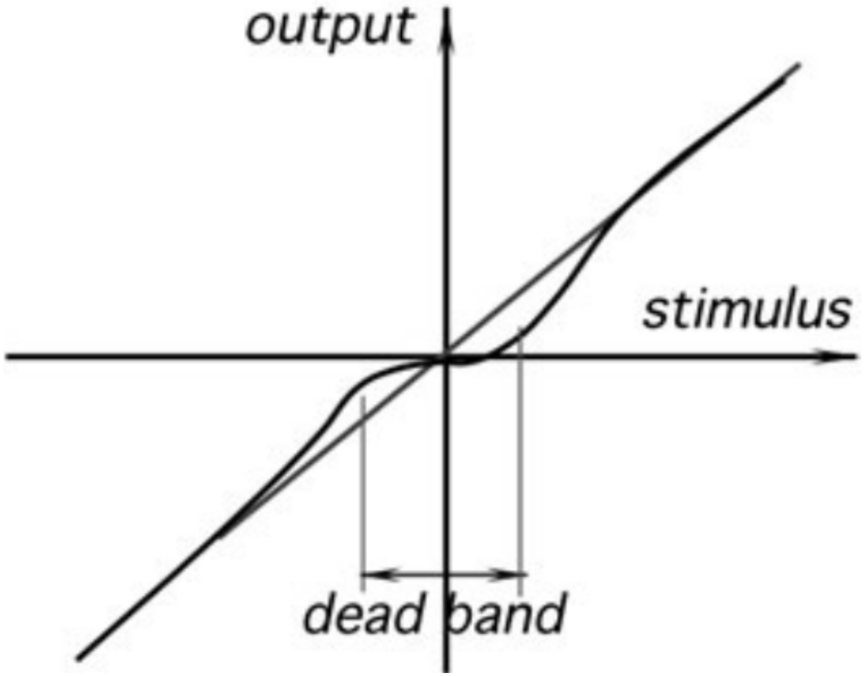


Here, the repeatability error:

$$\delta_r = \frac{\Delta}{FS} 100\%$$

Dead Band

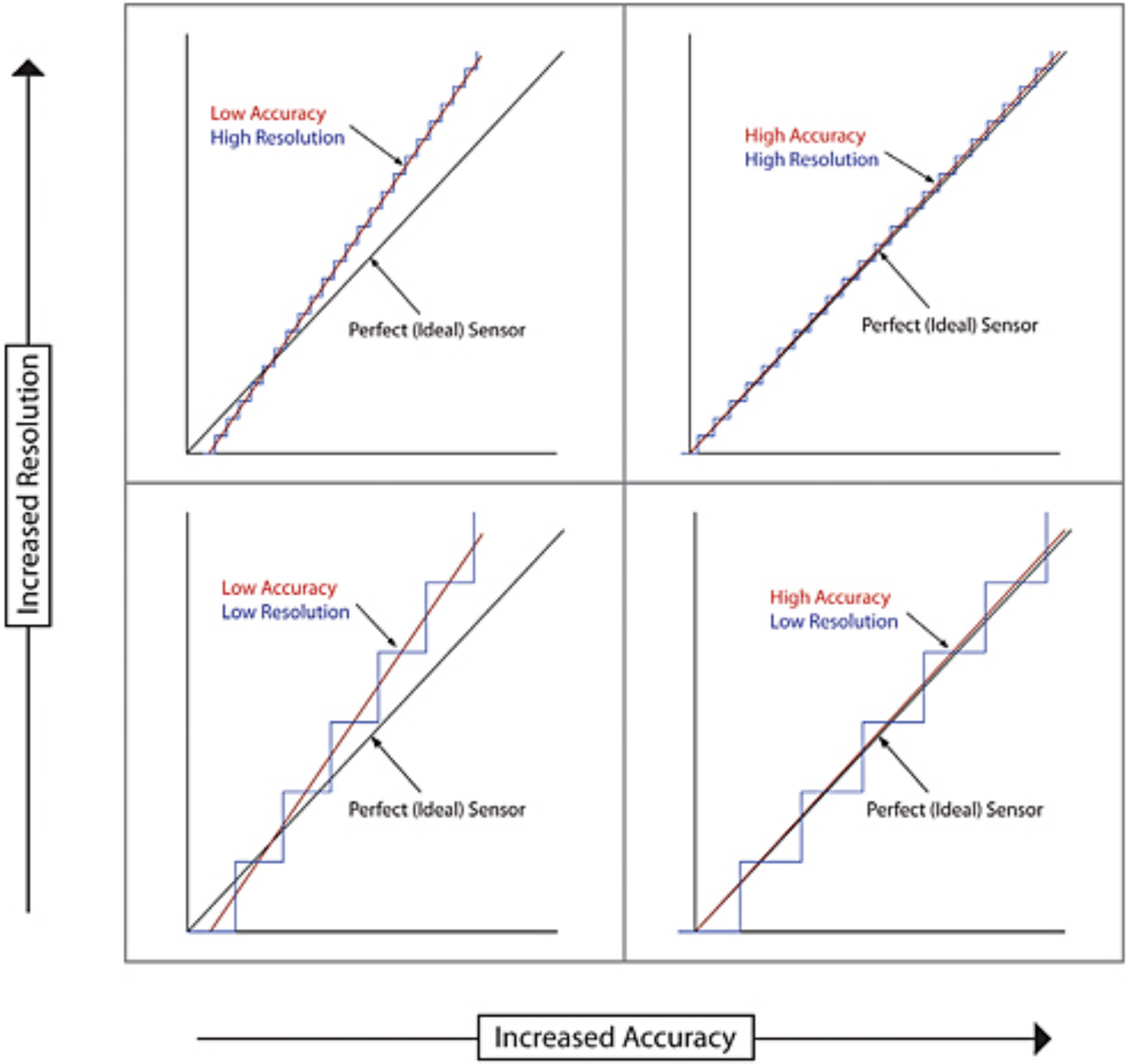
Dead band is insensitivity of a sensor in a specific range of the input signals



Resolution

The smallest incremental stimulus that can be measured with certainity is called resolution.

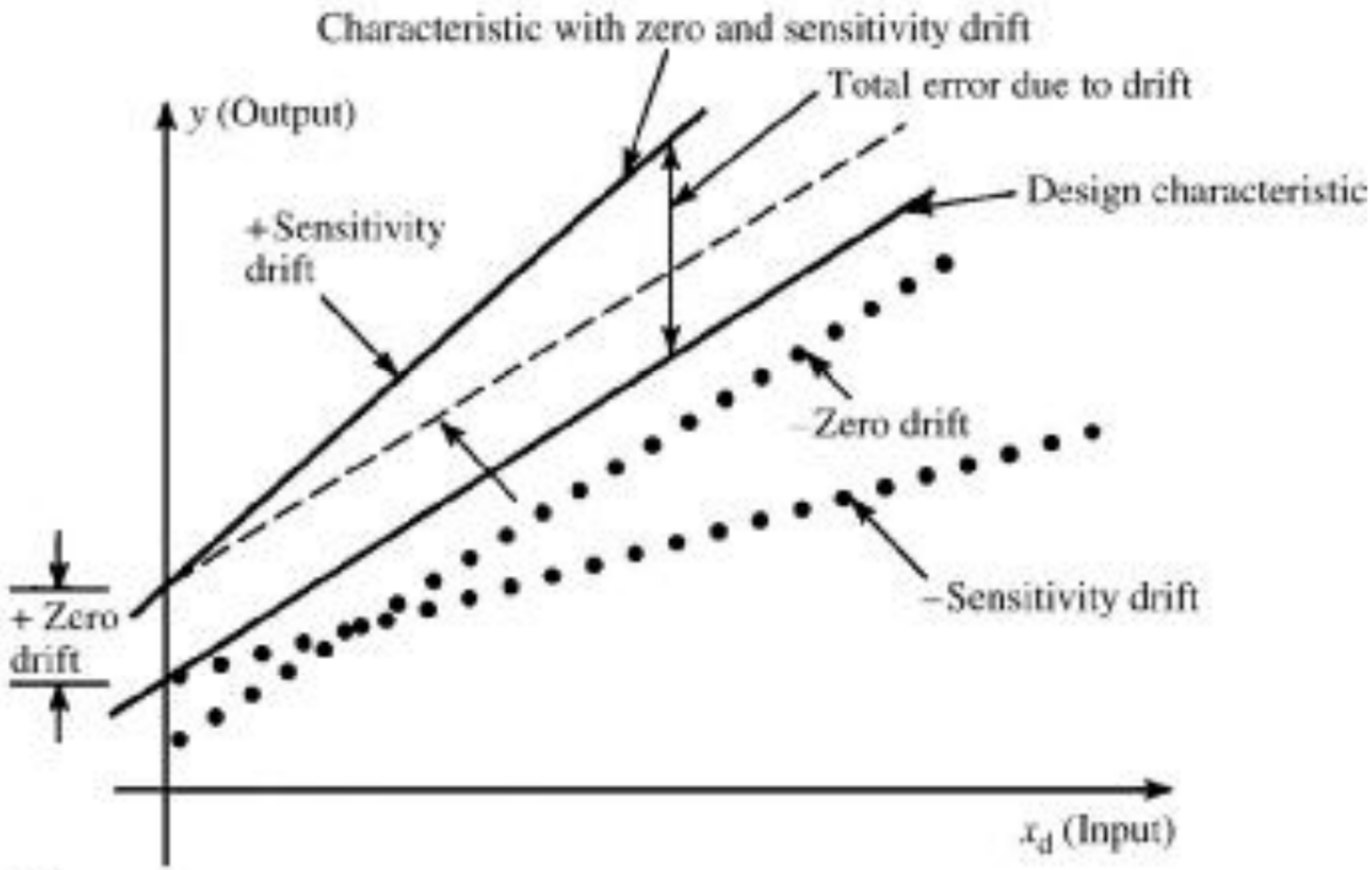
It can vary over the span of the sensor.



Zero and sensitivity drift

Zero drift occurs when all the output values increase or decrease by a constant amount.

Sensitivity drift causes error that is propoortional to the magnitude of the input, i.e. it appears as change in slope of the response curve.

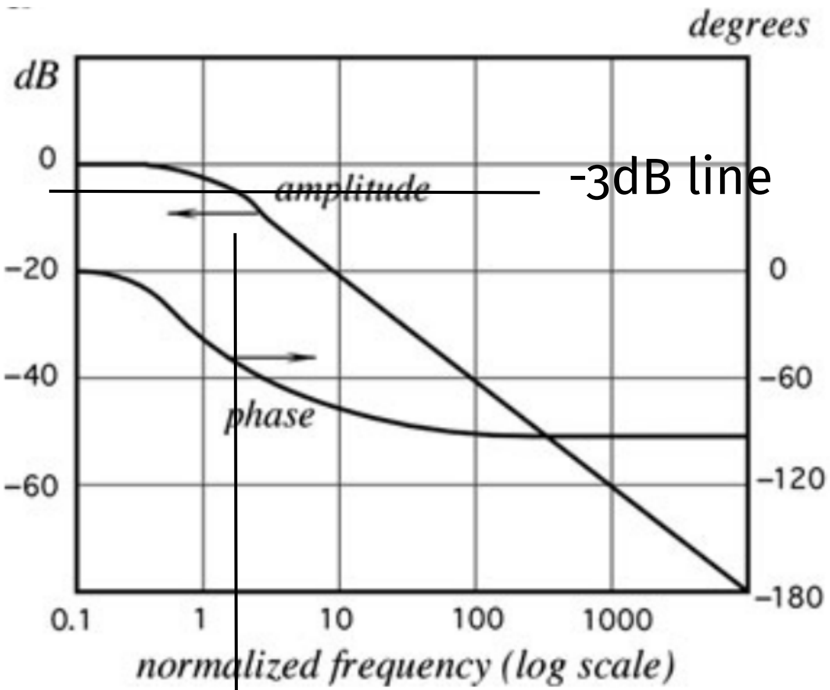


Dynamic Characteristics

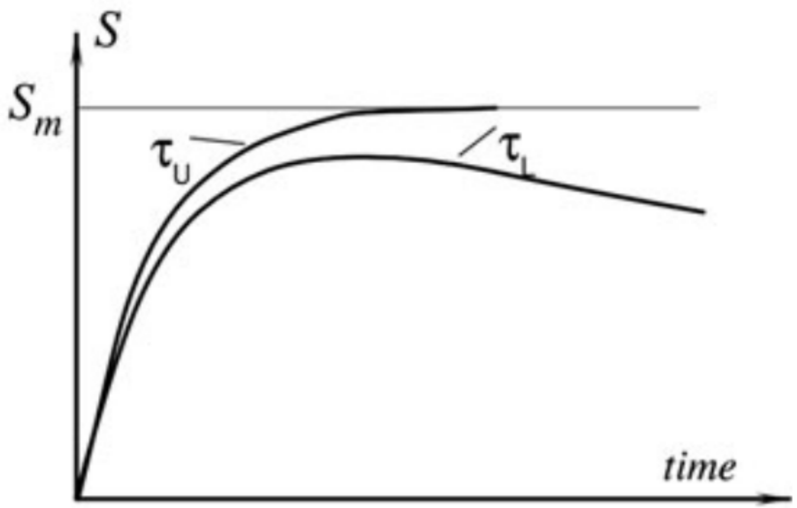
Zero-order sensors respond without delay, which is not the case most of the time.

The sensors described by a first order differential equation involve some kind of delay.

$$b_1 \frac{\mathrm{d}S(t)}{\mathrm{d}t} + b_0 S(t) = s(t)$$



After one time constant (τ), the output changes by 63% of the steady state level.



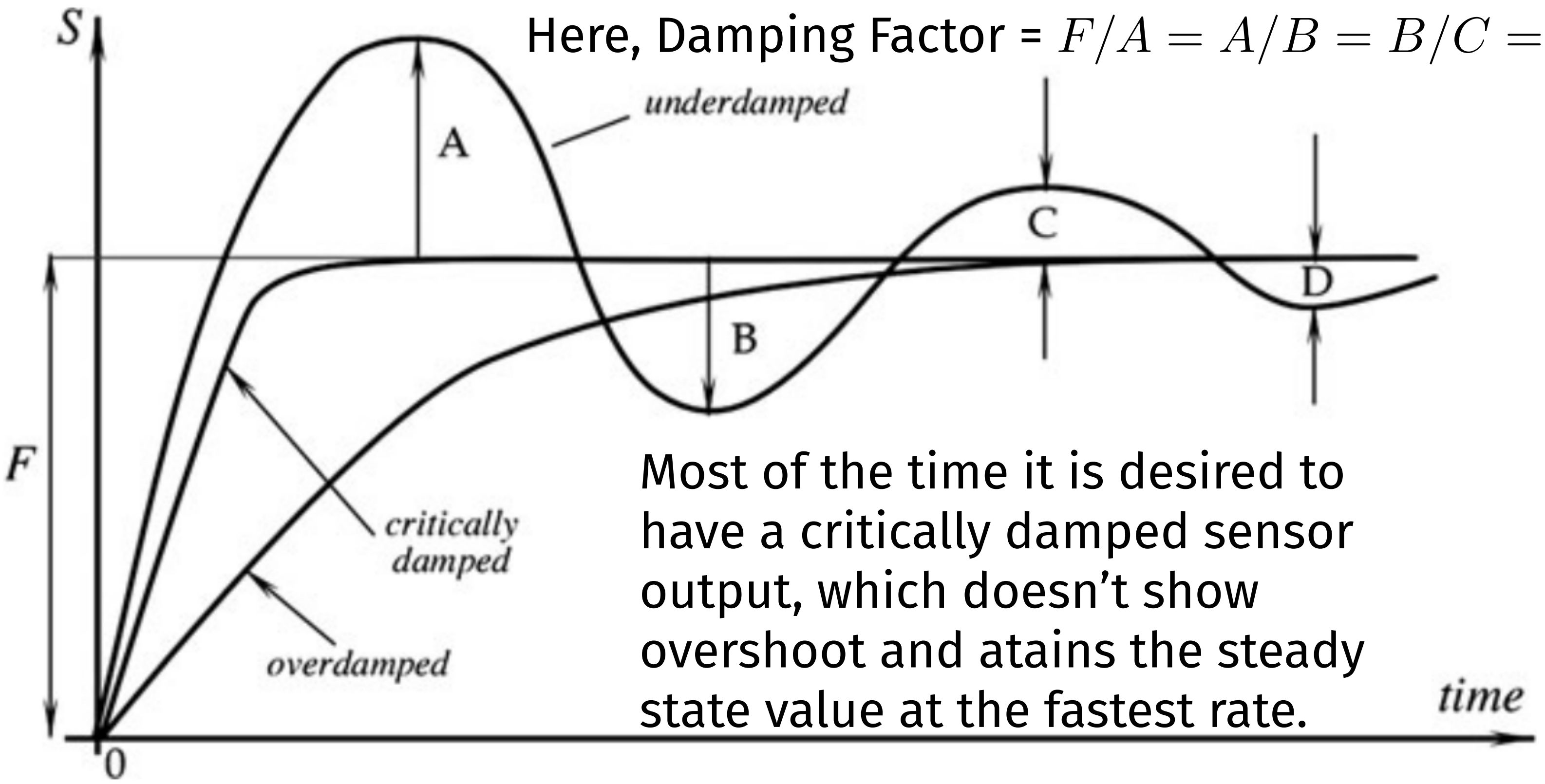
The cutoff frequency, $f_c = 0.159/\tau$

Second-order sensors

Many sensors can be modeled as second order systems:

$$b_2 \frac{d^2 S(t)}{dt^2} + b_1 \frac{dS(t)}{dt} + b_0 S(t) = s(t)$$

Here, Damping Factor = $F/A = A/B = B/C = ..$



Most of the time it is desired to have a critically damped sensor output, which doesn't show overshoot and atains the steady state value at the fastest rate.

Undamped natural frequency and damping ratio characterise such systems.

Reliability

It is expressed in statistical terms as a probability that the device will function without failure over a specified time or a number of uses.

One way to calculate it is to use MTBF (mean-time- between-failure) calculation described in MIL-HDBK-217 standard. It combines failure rates of individual components to yield the failure rate of overall system.

For ease, 1000 h of maximum temperature loading can be used to calculate failure rates. With accelerated life qualification, the time can be compressed.

To estimate number of test cycles, empirical formula can be used, e.g.:

$$n = N\left(\frac{\Delta T_{\max}}{\Delta T_{test}}\right)^{2.5}$$

where N is the estimated number of cycles per lifetime, ΔT_{max} is the maximum specified temperature fluctuation, and ΔT_{test} maximum cycled temperature fluctua- tion during the test.

Example: if the normal temperature is 25 ^oC, the maximum specified temperature is 50 ^oC cycling was up to 100 ^oC, and over the life time the sensor was estimated will be subjected to 20,000 cycles, then the number of test cycles becomes 1283.

Other tests like high rate heating/cooling, shock, high humidity and corrosive environments can be used to find the relevant reliability of the devices.

Noise Sources

Noise power

$$N = \lim_{T\to\infty} \sqrt{\frac{1}{2T}\int_{-T}^{T}(n(\tau))^2 d\tau}$$

Johnson noise: $e_n^{\,2} = 4kTRB$

Shot Noise: $i_n^{\,2} = 2qIB$

Pink Noise: $\sim 1/B^a,\ 0 < a < 2$

$$e_E = \sqrt{e_{n1}^2 + e_{n2}^2 + \cdots + (R_1 i_{n1})^2 + (R_1 i_{n2})^2 + \cdots}$$

Example: Find the SNR for a sensor which measures a voltage signal in the range of 0 to 1 V with a resolution of 3 bits. The noise is assumed to be distributed uniformly.

Noise Sources

External source	Typical magnitude	Typical cure
60/50 Hz power	100 pA	Shielding; attention to ground loops; isolated power supply
120/100 Hz supply ripple	3 μV	Supply filtering
180/150 Hz magnetic pickup from saturated 60/50 Hz transformers	0.5 μV	Reorientation of components
Radio broadcast stations	1 mV	Shielding
Switch-arcing	1 mV	Filtering of 5 to 100 MHz components; attention to ground loops and shielding
Vibration	10 pA (10–100 Hz)	Proper attention to mechanical coupling; elimination of leads with large voltages near input terminals and sensors
Cable vibration	100 pA	Use a low noise (carbon coated dielectric) cable
Circuit boards	$0.01 - 10\,\text{pA}/\sqrt{\text{Hz}}$ below 10 Hz	Clean board thoroughly; use Teflon insulation where needed and guard well