EXAMPLE 3.6

For the two-bar truss shown in Figure 3–14, determine the displacement in the y direction of node 1 and the axial force in each element. A force of $P = 1000$ kN is applied at node 1 in the positive y direction while node 1 settles an amount $\delta = 50$ mm in the negative x direction. Let $E = 210$ GPa and $A = 6.00 \times 10^{-4}$ m^2 for each element. The lengths of the elements are shown in the figure.

SOLUTION:

We begin by using Eq. (3.4.23) to determine each element stiffness matrix.

Element 1

$$\cos\theta^{(1)} = \frac{3}{5} = 0.60 \qquad \sin\theta^{(1)} = \frac{4}{5} = 0.80$$

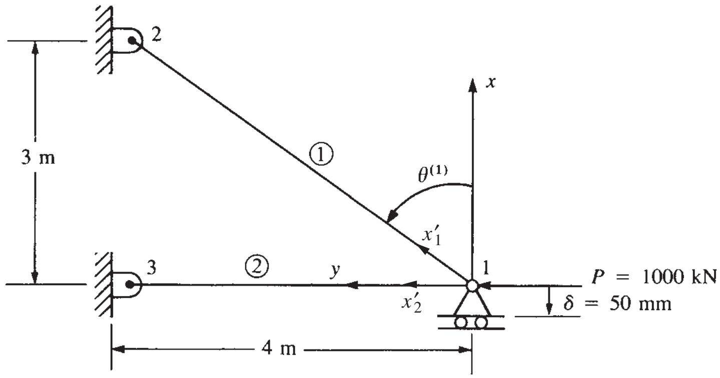

Figure 3–14 Two-bar truss

$$[k^{(1)}] = \frac{(6.0 \times 10^{-4}\,\text{m}^2)(210 \times 10^6\ \text{kN/m}^2)}{5\ \text{m}} \begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ & 0.64 & -0.48 & -0.64 \\ & & 0.36 & 0.48 \\ \text{Symmetry} & & & 0.64 \end{bmatrix} \tag{3.6.7}$$

Simplifying Eq. (3.6.7), we obtain

$$[k^{(1)}] = (25{,}200) \begin{array}{c} \begin{matrix} u_1 & \quad v_1 & \quad u_2 & \quad v_2 \end{matrix} \\ \begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ & 0.64 & -0.48 & -0.64 \\ & & 0.36 & 0.48 \\ \text{Symmetry} & & & 0.64 \end{bmatrix} \end{array} \tag{3.6.8}$$

Element 2

$$\cos\theta^{(2)} = 0.0 \qquad \sin\theta^{(2)} = 1.0$$

$$[k^{(2)}] = \frac{(6.0 \times 10^{-4})(210 \times 10^6)}{4} \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ \text{Symmetry} & & & 1 \end{bmatrix} \tag{3.6.9}$$

$$[k^{(2)}] = (25{,}200) \begin{array}{c} \begin{matrix} u_1 & \quad v_1 & \quad u_3 & \quad v_3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1.25 & 0 & -1.25 \\ & & 0 & 0 \\ \text{Symmetry} & & & 1.25 \end{bmatrix} \end{array} \tag{3.6.10}$$

where, for computational simplicity, Eq. (3.6.10) is written with the same factor (25,200) in front of the matrix as Eq. (3.6.8). Superimposing the element stiffness matrices, Eqs. (3.6.8) and (3.6.10), we obtain the global $[K]$ matrix and relate the global forces to global displacements by

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix} = (25{,}200)\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 & 0 & 0 \\ & 1.89 & -0.48 & -0.64 & 0 & -1.25 \\ & & 0.36 & 0.48 & 0 & 0 \\ & & & 0.64 & 0 & 0 \\ & & & & 0 & 0 \\ \text{Symmetry} & & & & & 1.25 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \tag{3.6.11}$$

We can again partition equations with known displacements and then simultaneously solve those associated with unknown displacements. To do this partitioning, we consider the boundary conditions given by

$$u_1 = \delta \qquad u_2 = 0 \qquad v_2 = 0 \qquad u_3 = 0 \qquad v_3 = 0 \tag{3.6.12}$$

Therefore, using Eqs. (3.6.12), we partition equation 2 from equations 1, 3, 4, 5, and 6 of Eq. (3.6.11) and are left with

$$P = 25{,}200(0.48\delta + 1.89v_1) \tag{3.6.13}$$

where $F_{1y} = P$ and $u_1 = \delta$ were substituted into Eq. (3.6.13). Expressing Eq. (3.6.13) in terms of P and δ allows these two influences on v_1 to be clearly separated. Solving Eq. (3.6.13) for v_1, we have

$$v_1 = 0.000021P - 0.254\delta \tag{3.6.14}$$

Now, substituting the numerical values $P = 1000$ kN and $\delta = -0.05$ m into Eq. (3.6.14), we obtain

$$v_1 = 0.0337 \text{ m} \tag{3.6.15}$$

where the positive value indicates horizontal displacement to the left.

The local element forces are obtained by using Eq. (3.4.11). We then have the following.

Element 1

$$\begin{Bmatrix} f'_{1x} \\ f'_{2x} \end{Bmatrix} = (25{,}200)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0.60 & 0.80 & 0 & 0 \\ 0 & 0 & 0.60 & 0.80 \end{bmatrix}\begin{Bmatrix} u_1 = -0.05 \\ v_1 = 0.0337 \\ u_2 = 0 \\ v_2 = 0 \end{Bmatrix} \tag{3.6.16}$$

Performing the matrix triple product in Eq. (3.6.16) yields

$$f'_{1x} = -76.6 \text{ kN} \qquad f'_{2x} = 76.6 \text{ kN} \tag{3.6.17}$$

Element 2

$$\begin{Bmatrix} f'_{1x} \\ f'_{3x} \end{Bmatrix} = (31{,}500)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} u_1 = -0.05 \\ v_1 = 0.0337 \\ u_3 = 0 \\ v_3 = 0 \end{Bmatrix} \quad (3.6.18)$$

Performing the matrix triple product in Eq. (3.6.18), we obtain

$$f'_{1x} = 1061 \text{ kN} \qquad f'_{3x} = -1061 \text{ kN} \quad (3.6.19)$$

Verification of the computations by checking that equilibrium is satisfied at node 1 is left to your discretion.