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Assignment 2

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Download all python codes from

https://github.com/mirhasidheek7213/ InternshipIITH/tree/main/Assignment-2/Codes

and latex-tikz codes from

https://github.com/mirhasidheek7213/ InternshipIITH/blob/main/Assignment-2/ Assignment2.tex

1 Question No. 1.23 - Linear forms

Find the equation of the line, which makes intercepts -3 and 2 on the x and y axes respectively.

2 Solution

Given, x-intercept = -3, y-intercept = 2 (2.0.1)

Hence , the line cuts through the x-axis at $\binom{-3}{0}$ and the line cuts through the y-axis at $\binom{0}{2}$

$$\mathbf{A} = \begin{pmatrix} -3\\0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0\\2 \end{pmatrix} (2.0.2)$$

Let the line equation be,

$$\mathbf{n}^{\mathbf{T}}\mathbf{x} = 1 \tag{2.0.3}$$

Then,

$$\mathbf{n}^{\mathbf{T}}\mathbf{A} = 1 \implies \mathbf{A}^{\mathbf{T}}\mathbf{n} = 1 \tag{2.0.4}$$

$$\mathbf{n}^{\mathbf{T}}\mathbf{B} = 1 \implies \mathbf{B}^{\mathbf{T}}\mathbf{n} = 1 \tag{2.0.5}$$

$$\mathbf{A}^{\mathbf{T}} = \begin{pmatrix} -3 & 0 \end{pmatrix}, \mathbf{B}^{\mathbf{T}} = \begin{pmatrix} 0 & 2 \end{pmatrix} \tag{2.0.6}$$

We find n by augmenting the matrix below and doing row reduction on it.

$$\begin{pmatrix} \mathbf{A}^{\mathbf{T}} \\ \mathbf{B}^{\mathbf{T}} \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.7}$$

Substituting values in 2.0.7,

$$\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.8}$$

This can be row reduced as follows

$$\begin{pmatrix} -3 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} = \begin{pmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{1}{2} \end{pmatrix}$$
 (2.0.9)

The left part is converted into identity matrix and the normal vector(n) is $\begin{pmatrix} \frac{-1}{3} \\ \frac{1}{2} \end{pmatrix}$ Multiplying n with 6 ie,

$$\mathbf{n} = \begin{pmatrix} -2\\3 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{n}^{\mathbf{T}} = \begin{pmatrix} -2 & 3 \end{pmatrix} \tag{2.0.11}$$

The equation of the line is found out by,

$$\mathbf{n}^{\mathbf{T}}\mathbf{x} = \mathbf{n}^{\mathbf{T}}\mathbf{A} \tag{2.0.12}$$

$$\left(-2\ 3\right)\mathbf{x} = \left(-2\ 3\right) \begin{pmatrix} -3\\0 \end{pmatrix} \tag{2.0.13}$$

$$= (-2 \ 3) \mathbf{x} = 6 \tag{2.0.14}$$

Therefore, the equation of line is,

$$(-2\ 3)\mathbf{x} = 6$$
 (2.0.15)

Since the line passes through the points $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, The line AB is plotted using these points as shown below.

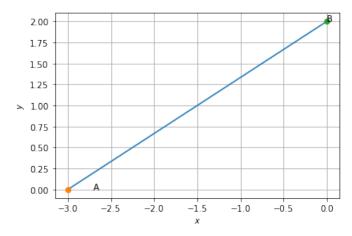


Fig. 0: The line $(-2 \ 3)\mathbf{x} = 6$