Assignment-6

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Download all python codes from

https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment−6/codes

and latex-tikz codes from

https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment−6/Assignment6.tex

1 Optimization 2.11

Maximise $Z = x + y$, subject to the constraints: $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.

2 Solution

In order to obtain the maximum value we need to solve the system of inequalities by adding slack variables. The equations become:

$$Z - x - y = 0 \tag{2.0.1}$$

$$x - y + S_1 = -1 \tag{2.0.2}$$

$$-x + y + S_2 = 0 \tag{2.0.3}$$

The simplex tableau can be formed as

$$\left(\begin{array}{ccccc} x & y & s_1 & s_2 & c \\ 1 & -1 & 1 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 \\ \hline -1 & -1 & 0 & 0 & 0 \end{array}\right) \tag{2.0.4}$$

Using Gauss jordan elimination by keeping the pivot element as -1 and the reduced form is given below.

$$\left(\begin{array}{ccccc} x & y & s_1 & s_2 & c \\ 1 & 0 & 0 & \frac{-1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 \\ \hline 0 & 0 & 1 & 1 & -1 \end{array}\right) \tag{2.0.5}$$

In this tableau, there are no negative elements in the bottom row.Therefore, the optimal solution is determined as:

$$(x, y, s_1, s_2) = \left(\frac{1}{2}, \frac{-1}{2}, 0, 0\right) \tag{2.0.6}$$

$$Z = x + y \tag{2.0.7}$$

$$Z = \frac{1}{2} + \frac{-1}{2} \tag{2.0.8}$$

$$Z = 0 \tag{2.0.9}$$

Since $Z = 0$, there is no maximum value hence no feasible region

This can be solved in Python which generates the result as shown in the Figure.

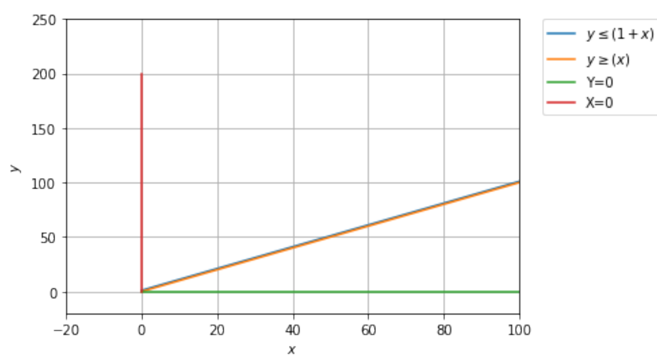

Fig. 1: Plot from python code

The given problem can be expressed in general as matrix inequality as:

$$\max_{\{x\}} \mathbf{c}^T \mathbf{x} \tag{2.0.10}$$

$$s.t \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \tag{2.0.11}$$

$$\mathbf{x} \geq 0 \tag{2.0.12}$$

$$\mathbf{y} \geq 0 \tag{2.0.13}$$

where,

$$\mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{b} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{2.0.16}$$

Solving for Z by this reduction method we get

$$MaxZ = None \tag{2.0.17}$$