

Assignment-6

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Download all python codes from

<https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment-6/codes>

and latex-tikz codes from

<https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment-6/Assignment6.tex>

$$\text{Maxima} = 112.99999999999876 \approx 113 \quad (2.0.8)$$

$$\text{Maxima Point} = -1.9999997364868565 \approx -2 \quad (2.0.9)$$

verifying this by the derivative test. Since $p(x)$ is a concave function it has a maxima.

1 OPTIMIZATION 2.2

Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$

2 SOLUTION

The profit of the company is given as;

$$p(x) = 41 - 72x - 18x^2 \quad (2.0.1)$$

Lemma 2.1. A function $p(x)$ is said to be concave if following inequality is true for $\lambda \in [0, 1]$:

$$\lambda p(x_1) + (1 - \lambda)p(x_2) \leq p(\lambda x_1 + (1 - \lambda)x_2) \quad (2.0.2)$$

Checking if $p(x)$ is convex:

$$\begin{aligned} & \lambda(41 - 72x_1 - 18x_1^2) + (1 - \lambda)(41 - 72x_2 - 18x_2^2) \\ & \leq (41 - 72(\lambda x_1 + (1 - \lambda)x_2) - 18(\lambda x_1 + (1 - \lambda)x_2)^2) \end{aligned} \quad (2.0.3)$$

resulting in

$$18\lambda(\lambda - 1)(x_1 - x_2)^2 \leq 0 \quad (2.0.4)$$

$$\Rightarrow \lambda(\lambda - 1) \leq 0 \quad (2.0.5)$$

is true .

\Rightarrow The function is concave.

Using gradient ascent method we can find its maxima,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (2.0.6)$$

$$\Rightarrow x_{n+1} = x_n + \alpha (-36x_n - 72) \quad (2.0.7)$$

Taking $x_0 = 2, \alpha = 0.001$ and precision= 0.00000001, values obtained using python are:

$$\frac{dp(x)}{dx} = -36x - 72 \quad (2.0.10)$$

Critical point :

$$\frac{dp(x)}{dx} = 0 \quad (2.0.11)$$

$$-36x - 72 = 0 \quad (2.0.12)$$

$$x = -2 \quad (2.0.13)$$

is a critical point. And since $p(x)$ is a concave function there will be a maxima at $x=-2$. And the maxima is

$$p(-2) = 113 \quad (2.0.14)$$

This is shown in the Fig.2.1

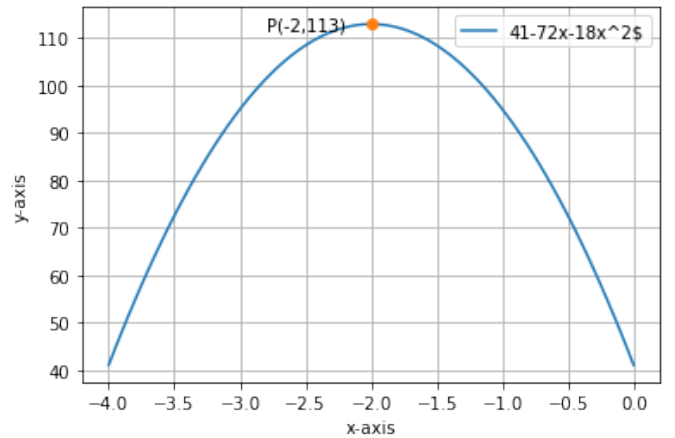


Fig. 2.1: $p(x) = 41 - 72x - 18x^2$