

Assignment 7

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Download all python codes from

[https://github.com/mirhasidheek7213/
InternshipIITH/tree/main/Assignment-7/Codes](https://github.com/mirhasidheek7213/InternshipIITH/tree/main/Assignment-7/Codes)

and latex-tikz codes from

[https://github.com/mirhasidheek7213/
InternshipIITH/tree/main/Assignment-7/
Assignment7.tex](https://github.com/mirhasidheek7213/InternshipIITH/tree/main/Assignment-7/Assignment7.tex)

1 QUESTION NO. 2.2 - LINEAR INEQUALITIES

Solve the following system of inequalities graphically.

$$5x+4y \leq 40, x \geq 2, y \geq 3$$

2 SOLUTION

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -5 & -4 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -40 \\ 2 \\ 3 \end{pmatrix} \quad (2.0.1)$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \quad (2.0.2)$$

The first pair of inequality can be solved as,

1)

$$\begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -40 \\ 2 \end{pmatrix} \quad (2.0.3)$$

$$\Rightarrow \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -40 \\ 2 \end{pmatrix} + \mathbf{u} \quad (2.0.4)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -40 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.5)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 2 \\ \frac{15}{2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{-1}{4} & \frac{-5}{4} \end{pmatrix} \mathbf{u} \quad (2.0.6)$$

Similarly, solving 2nd pair of inequality

2)

$$\begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -40 \\ 3 \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -40 \\ 3 \end{pmatrix} + \mathbf{u} \quad (2.0.8)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -40 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.9)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} \frac{28}{5} \\ 3 \end{pmatrix} + \begin{pmatrix} \frac{-1}{5} & \frac{-4}{5} \\ 0 & 1 \end{pmatrix} \mathbf{u} \quad (2.0.10)$$

Now, solution region which is common to regions of eq. (2.0.6) and eq. (2.0.10), is given by

$$\mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{1}{20} & \frac{-21}{20} \end{pmatrix} \mathbf{u} \quad (2.0.11)$$

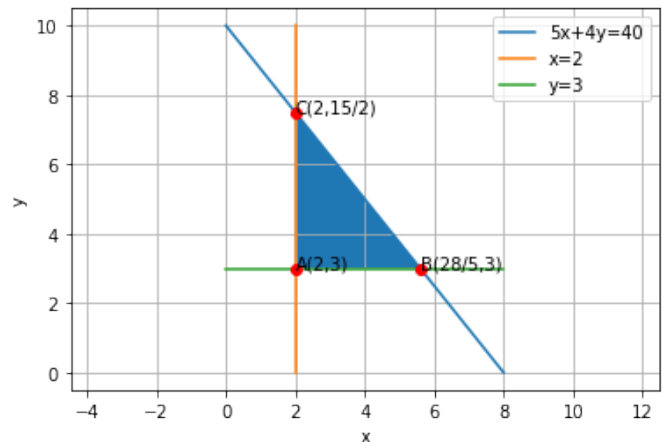


Fig. 2.1: Solution Region

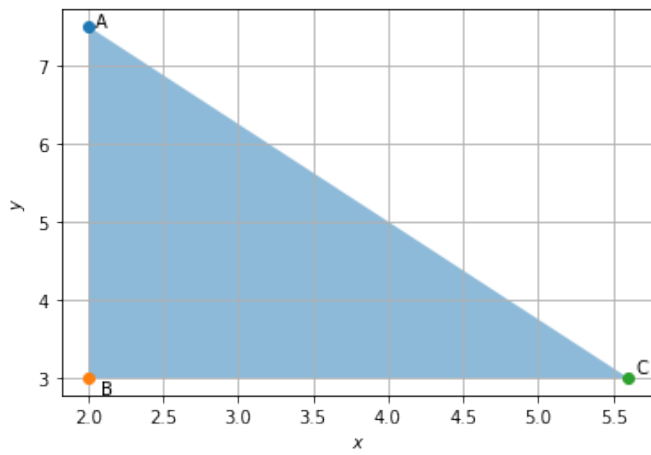


Fig. 2.2: Magnified Solution Region