Assignment-6

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Download all python codes from

$$\boxed{\text{https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment-6/codes}}$$

and latex-tikz codes from

$$\boxed{\text{https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment-6/Assignment6.tex}}$$

1 OPTIMIZATION 2.2

Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$

2 SOLUTION

The profit of the company is given as;

$$p(x) = 41 - 72x - 18x^2 \tag{2.0.1}$$

Lemma 2.1. *A function p(x) is said to be concave if following inequality is true for* $\lambda \in [0, 1]$:

$$\lambda p(x_1) + (1 - \lambda)p(x_2) \leq p(\lambda x_1 + (1 - \lambda)x_2) \tag{2.0.2}$$

Checking if $p(x)$ is convex:

$$\lambda\left(41 - 72x_1 - 18{x_1}^2\right) + (1 - \lambda)\left(41 - 72x_2 - 18{x_2}^2\right) \leq \left(41 - 72(\lambda x_1 + (1 - \lambda)x_2) - 18\left(\lambda x_1 + (1 - \lambda)x_2\right)^2\right) \tag{2.0.3}$$

resulting in

$$18\lambda(\lambda - 1)(x_1 - x_2)^2 \leq 0 \tag{2.0.4}$$
$$\implies \lambda(\lambda - 1) \leq 0 \tag{2.0.5}$$

is true .
$\implies$ The function is concave.
Using gradient ascent method we can find its maxima,

$$x_{n+1} = x_n + \alpha\nabla f(x_n) \tag{2.0.6}$$
$$\implies x_{n+1} = x_n + \alpha\left(-36x_n - 72\right) \tag{2.0.7}$$

Taking $x_0 = 2, \alpha = 0.001$ and precision= 0.00000001,values obtained using python are:

$$\boxed{\text{Maxima} = 112.99999999999876 \approx 113} \tag{2.0.8}$$

$$\boxed{\text{Maxima Point} = -1.9999997364868565 \approx -2} \tag{2.0.9}$$

verifying this by the derivative test. Since $p(x)$ is a concave function it has a maxima.

$$\frac{dp(x)}{dx} = -36x - 72 \tag{2.0.10}$$

Critical point :

$$\frac{dp(x)}{dx} = 0 \tag{2.0.11}$$
$$-36x - 72 = 0 \tag{2.0.12}$$
$$x = -2 \tag{2.0.13}$$

is a critical point.And since $p(x)$ is a concave function there will be a maxima at x=-2. And the maxima is

$$p(-2) = 113 \tag{2.0.14}$$

This is shown in the Fig.2.1

assignment14.png

Fig. 2.1: $p(x) = 41 - 72x - 18x^2$