

Assignment 2

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Download all python codes from

[https://github.com/mirhasidheek7213/
InternshipIITH/tree/main/Assignment-2/Codes](https://github.com/mirhasidheek7213/InternshipIITH/tree/main/Assignment-2/Codes)

and latex-tikz codes from

[https://github.com/mirhasidheek7213/
InternshipIITH/blob/main/Assignment-2/
Assignment2.tex](https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment-2/Assignment2.tex)

1 QUESTION NO. 1.23 - LINEAR FORMS

Find the equation of the line, which makes intercepts -3 and 2 on the x and y axes respectively.

2 SOLUTION

Given, x -intercept $= -3$, y -intercept $= 2$ (2.0.1)

Hence, the line cuts through the x -axis at $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

and the line cuts through the y -axis at $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.2)$$

Let the line equation be,

$$\mathbf{n}^T \mathbf{x} = 1 \quad (2.0.3)$$

Then,

$$\mathbf{n}^T \mathbf{A} = 1 \implies \mathbf{A}^T \mathbf{n} = 1 \quad (2.0.4)$$

$$\mathbf{n}^T \mathbf{B} = 1 \implies \mathbf{B}^T \mathbf{n} = 1 \quad (2.0.5)$$

$$\mathbf{A}^T = \begin{pmatrix} -3 & 0 \end{pmatrix}, \mathbf{B}^T = \begin{pmatrix} 0 & 2 \end{pmatrix} \quad (2.0.6)$$

We find \mathbf{n} by augmenting the matrix below and doing row reduction on it.

$$\begin{pmatrix} \mathbf{A}^T \\ \mathbf{B}^T \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.7)$$

Substituting values in 2.0.7,

$$\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.8)$$

This can be row reduced as follows

$$\begin{pmatrix} -3 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \xrightarrow[R_1 \leftarrow R_1 / -3]{R_2 \leftarrow R_2 / 2} = \begin{pmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{1}{2} \end{pmatrix} \quad (2.0.9)$$

The left part is converted into identity matrix and the normal vector(\mathbf{n}) is $\begin{pmatrix} -\frac{1}{3} \\ \frac{1}{2} \end{pmatrix}$

Multiplying \mathbf{n} with 6 ie,

$$\mathbf{n} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{n}^T = \begin{pmatrix} -2 & 3 \end{pmatrix} \quad (2.0.11)$$

The equation of the line is found out by,

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (2.0.12)$$

$$\begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -2 & 3 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$= \begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = 6 \quad (2.0.14)$$

Therefore, the equation of line is,

$$\begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = 6 \quad (2.0.15)$$

Since the line passes through the points $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, The line AB is plotted using these points as shown below.

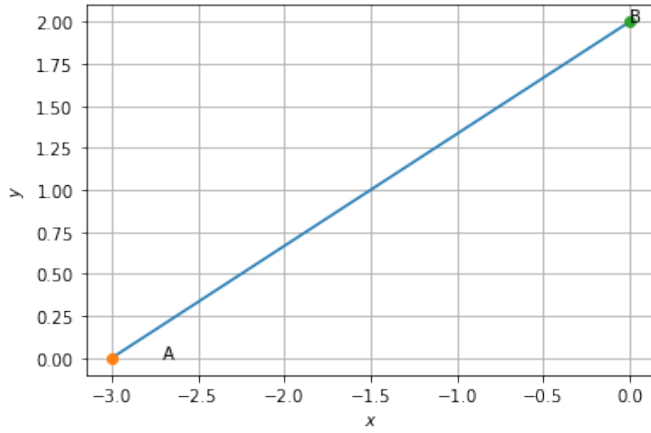


Fig. 0: The line $\begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = 6$