Assignment 2

Mirha Sidheek

Download all python codes from

https://github.com/mirhasidheek7213/InternshipIITH/tree/main/Assignment−2/Codes

and latex-tikz codes from

https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment−2/Assignment2.tex

1 Question No. 1.23 - Linear forms

Find the equation of the line, which makes intercepts -3 and 2 on the x and y axes respectively.

2 Solution

$$Given, x-intercept = -3, y-intercept = 2 \quad (2.0.1)$$

Hence , the line cuts through the x-axis at $\begin{pmatrix}-3\\0\end{pmatrix}$ and the line cuts through the y-axis at $\begin{pmatrix}0\\2\end{pmatrix}$

$$\mathbf{A} = \begin{pmatrix}-3\\0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\2\end{pmatrix} \quad (2.0.2)$$

Let the line equation be,

$$\mathbf{n^T}x = 1 \quad (2.0.3)$$

Then,

$$\mathbf{n^T}A = 1 \implies \mathbf{A^T}n = 1 \quad (2.0.4)$$

$$\mathbf{n^T}B = 1 \implies \mathbf{B^T}n = 1 \quad (2.0.5)$$

$$\mathbf{A^T} = \begin{pmatrix}-3 & 0\end{pmatrix}, \mathbf{B^T} = \begin{pmatrix}0 & 2\end{pmatrix} \quad (2.0.6)$$

We find n by augmenting the matrix below and doing row reduction on it.

$$\begin{pmatrix}A^T\\B^T\end{pmatrix}n = \begin{pmatrix}1\\1\end{pmatrix} \quad (2.0.7)$$

Substituting values in 2.0.7 ,

$$\begin{pmatrix}-3 & 0\\0 & 2\end{pmatrix}n = \begin{pmatrix}1\\1\end{pmatrix} \quad (2.0.8)$$

This can be row reduced as follows

$$\begin{pmatrix}-3 & 0 & 1\\0 & 2 & 1\end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_1/-3]{R_2 \leftarrow R_2/2} = \begin{pmatrix}1 & 0 & \frac{-1}{3}\\0 & 1 & \frac{1}{2}\end{pmatrix} \quad (2.0.9)$$

The left part is converted into identity matrix and the normal vector(n) is $\begin{pmatrix}\frac{-1}{3}\\\frac{1}{2}\end{pmatrix}$ ie,

$$\mathbf{n} = \begin{pmatrix}\frac{-1}{3}\\\frac{1}{2}\end{pmatrix} \quad (2.0.10)$$

$$\mathbf{n^T} = \begin{pmatrix}\frac{-1}{3} & \frac{1}{2}\end{pmatrix} \quad (2.0.11)$$

The equation of the line is found out by,

$$\mathbf{n^T}x = \mathbf{n^T}A \quad (2.0.12)$$

$$\begin{pmatrix}\frac{-1}{3} & \frac{1}{2}\end{pmatrix}x = \begin{pmatrix}\frac{-1}{3} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}-3\\0\end{pmatrix} \quad (2.0.13)$$

$$= \begin{pmatrix}\frac{-1}{3} & \frac{1}{2}\end{pmatrix}x = 1 \quad (2.0.14)$$

Therefore, the equation of line is,

$$\begin{pmatrix}\frac{-1}{3} & \frac{1}{2}\end{pmatrix}x = 1 \quad (2.0.15)$$

Since the line passes through the points $\begin{pmatrix}-3\\0\end{pmatrix}$ and $\begin{pmatrix}0\\2\end{pmatrix}$, The line AB is plotted using these points as shown below.

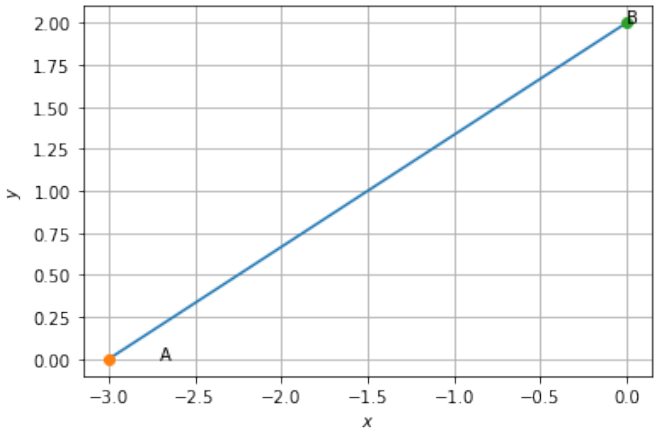

Fig. 0: The line $\begin{pmatrix}2 & -3\end{pmatrix} x = -6$