Assignment 3

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Download all python codes from

https://github.com/mirhasidheek7213/InternshipIITH/tree/main/Assignment−3/Codes

and latex-tikz codes from

https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment−3/Assignment3.tex

1 Question No. 2.2 - Quadratic forms

Find the equation of a circle with centre $\begin{pmatrix}2\\2\end{pmatrix}$ and passes through the point $\begin{pmatrix}4\\5\end{pmatrix}$

2 Solution

Given,

$$Centre = \begin{pmatrix}2\\2\end{pmatrix} \tag{2.0.1}$$

The general equation of circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{2.0.2}$$

$$f = \mathbf{u}^\top\mathbf{u} - r^2 \tag{2.0.3}$$

r is the radius and c is the centre where, $\mathbf{c} = -\mathbf{u}$

$$\mathbf{c} = \begin{pmatrix}2\\2\end{pmatrix} \tag{2.0.4}$$

$$\mathbf{u} = \begin{pmatrix}-2\\-2\end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u}^\top = \begin{pmatrix}-2 & -2\end{pmatrix} \tag{2.0.6}$$

Substituting $\mathbf{u}^\top$ in 2.0.2 and simplifying

$$\mathbf{x}^\top\mathbf{x} + \begin{pmatrix}-4 & -4\end{pmatrix}\mathbf{x} + f = 0 \tag{2.0.7}$$

The circle passes through the point $\begin{pmatrix}4\\5\end{pmatrix}$. Hence we can substitute this point for $\mathbf{x}$

$$\mathbf{x} = \begin{pmatrix}4\\5\end{pmatrix} \tag{2.0.8}$$

Substituting in 2.0.7

$$\begin{pmatrix}4 & 5\end{pmatrix}\begin{pmatrix}4\\5\end{pmatrix} + \begin{pmatrix}-4 & -4\end{pmatrix}\begin{pmatrix}4\\5\end{pmatrix} + f = 0 \tag{2.0.9}$$

$$f + \begin{pmatrix}41\end{pmatrix} + \begin{pmatrix}-36\end{pmatrix} = 0 \implies f = -5 \tag{2.0.10}$$

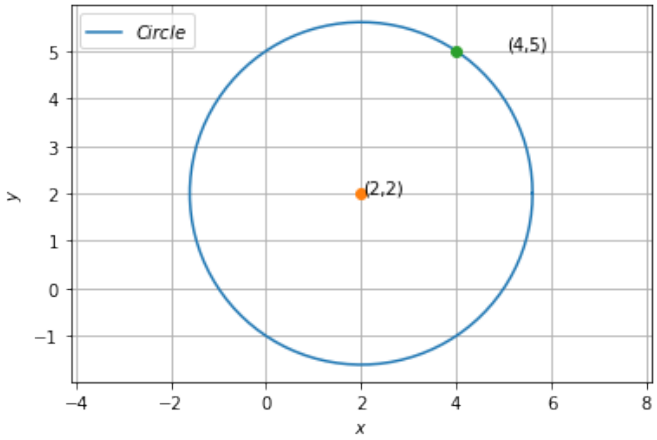

Fig. 0: Plot of the required circle

Radius of the circle can be found out using

$$f = \mathbf{u}^\top\mathbf{u} - r^2 \tag{2.0.11}$$

$$\implies -5 = \begin{pmatrix}-2 & -2\end{pmatrix}\begin{pmatrix}-2\\-2\end{pmatrix} - r^2 \tag{2.0.12}$$

$$\implies -5 = 8 - r^2 \tag{2.0.13}$$

$$\implies r = \sqrt{13} \tag{2.0.14}$$

Hence , the equation of the circle is,

$$\mathbf{x}^\top\mathbf{x} + \begin{pmatrix}-4 & -4\end{pmatrix}\mathbf{x} - 5 = 0 \tag{2.0.15}$$