Assignment 4

Mirha Sidheek

Download all python codes from

https://github.com/mirhasidheek7213/InternshipIITH/tree/main/Assignment−4/Codes

and latex-tikz codes from

https://github.com/mirhasidheek7213/InternshipIITH/blob/main/Assignment−4/Assignment4.tex

1 Question No. 2.6 - Vectors

Are the points $\mathbf{A} = \begin{pmatrix}3\\6\\9\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}10\\20\\30\end{pmatrix}$, $\mathbf{C} = \begin{pmatrix}25\\-41\\5\end{pmatrix}$, the vertices of a right angled triangle?

2 Solution

Given,

$$\mathbf{A} = \begin{pmatrix}3\\6\\9\end{pmatrix}, \mathbf{B} = \begin{pmatrix}10\\20\\30\end{pmatrix}, \mathbf{C} = \begin{pmatrix}25\\-41\\5\end{pmatrix} \tag{2.0.1}$$

To check if the vertices are of a right angled triangle we have to prove $(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = 0$, $(\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = 0$,$(\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C}) = 0$, If the triangle is right angled at A, B and C respectively

2.1 Checking if $\angle A = 90^\circ$

We have to prove,

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = 0 \tag{2.1.1}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}7\\14\\21\end{pmatrix} \tag{2.1.2}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}22\\-47\\-4\end{pmatrix} \tag{2.1.3}$$

Now,

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}7 & 14 & 21\end{pmatrix}\begin{pmatrix}22\\-47\\-4\end{pmatrix} \tag{2.1.4}$$

$$\implies 154 - 658 - 17 = -521 \tag{2.1.5}$$

Hence, $(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) \neq 0$. Therefore the triangle is not right angled at A.

2.2 Checking if $\angle B = 90^\circ$

We have to prove,

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = 0 \tag{2.2.1}$$

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix}-7\\-14\\-21\end{pmatrix} \tag{2.2.2}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}15\\-61\\-25\end{pmatrix} \tag{2.2.3}$$

Now,

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix}-7 & -14 & -21\end{pmatrix}\begin{pmatrix}15\\-61\\-25\end{pmatrix} \tag{2.2.4}$$

$$\implies -105 + 854 + 525 = 1274 \tag{2.2.5}$$

Hence, $(\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) \neq 0$. Therefore the triangle is not right angled at B.

2.3 Checking if $\angle C = 90^\circ$

We have to prove,

$$(\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C}) = 0 \tag{2.3.1}$$

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix}-22\\47\\4\end{pmatrix} \tag{2.3.2}$$

$$\mathbf{B}-\mathbf{C}=\begin{pmatrix}-15\\61\\25\end{pmatrix} \tag{2.3.3}$$

Now,

$$(\mathbf{A}-\mathbf{C})^{\top}(\mathbf{B}-\mathbf{C})=\begin{pmatrix}-22 & 47 & 4\end{pmatrix}\begin{pmatrix}-15\\61\\25\end{pmatrix} \tag{2.3.4}$$

$$\implies 330+2867+200=3397 \tag{2.3.5}$$

Hence, $(\mathbf{A}-\mathbf{C})^{\top}(\mathbf{B}-\mathbf{C}) \neq 0$. Therefore the triangle is not right angled at B.
Since these vertices do not form right angles,they are not vertices of a right angled triangle

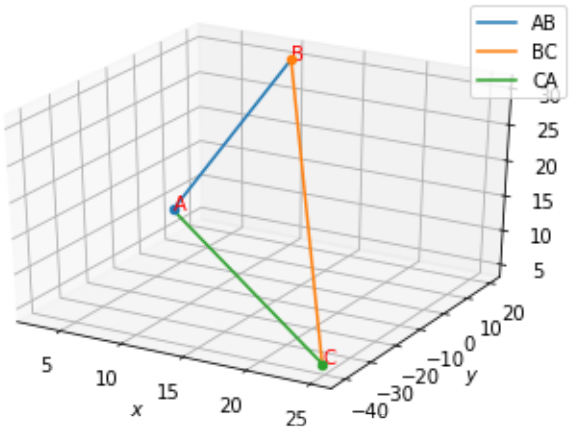

Fig. 0: Plot of the triangle