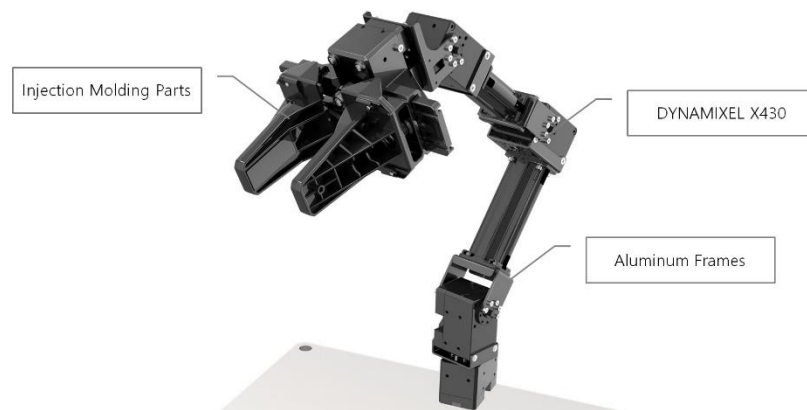


Mechatronics Course Project

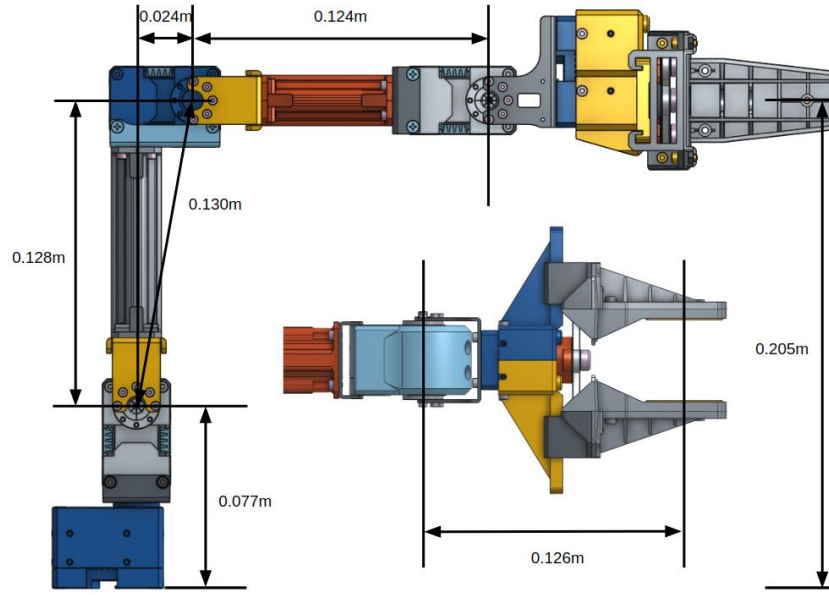
OpenmanipulatorX Robot Control



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A) Calculation of Jacobian matrix of the robot:



According to the geometric characteristics of the robot shown in the figure above, the transfer matrix is as follows:

$$P_{EE} = \begin{bmatrix} \cos(\theta_1)[0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.024\cos(\theta_2)] \\ \sin(\theta_1)[0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.024\cos(\theta_2)] \\ 0.077 + 0.126\sin(\theta_2 + \theta_3 + \theta_4) + 0.124\sin(\theta_2 + \theta_3) + 0.128\sin(\theta_2) \end{bmatrix} = \begin{bmatrix} P_{EE})_1 \\ P_{EE})_2 \\ P_{EE})_3 \end{bmatrix}$$

$$\begin{aligned} \frac{d}{dt} P_{EE})_1 &= -\dot{\theta}_1 \sin(\theta_1)[0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.024\cos(\theta_2)] \\ &\quad - \cos(\theta_1)[0.126(\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)\sin(\theta_2 + \theta_3 + \theta_4) + 0.124(\dot{\theta}_2 + \dot{\theta}_3)\sin(\theta_2 + \theta_3) + 0.024\dot{\theta}_2 \sin(\theta_2)] \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} P_{EE})_2 &= \dot{\theta}_1 \cos(\theta_1)[0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.024\cos(\theta_2)] \\ &\quad - \sin(\theta_1)[0.126(\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)\sin(\theta_2 + \theta_3 + \theta_4) + 0.124(\dot{\theta}_2 + \dot{\theta}_3)\sin(\theta_2 + \theta_3) + 0.024\dot{\theta}_2 \sin(\theta_2)] \end{aligned}$$

$$\frac{d}{dt} P_{EE})_3 = 0.126(\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)\cos(\theta_2 + \theta_3 + \theta_4) + 0.124(\dot{\theta}_2 + \dot{\theta}_3)\cos(\theta_2 + \theta_3) + 0.128(\dot{\theta}_2)\cos(\theta_2)$$

$$\frac{d}{dt} P_{EE} = \begin{bmatrix} \frac{d}{dt} P_{EE})_1 & \frac{d}{dt} P_{EE})_2 & \frac{d}{dt} P_{EE})_3 \end{bmatrix}^T$$

$${}^0J_{v1}(\theta) = \begin{bmatrix} -\sin(\theta_1)[0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.024\cos(\theta_2)] \\ \cos(\theta_1)[0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.024\cos(\theta_2)] \\ 0 \end{bmatrix}$$

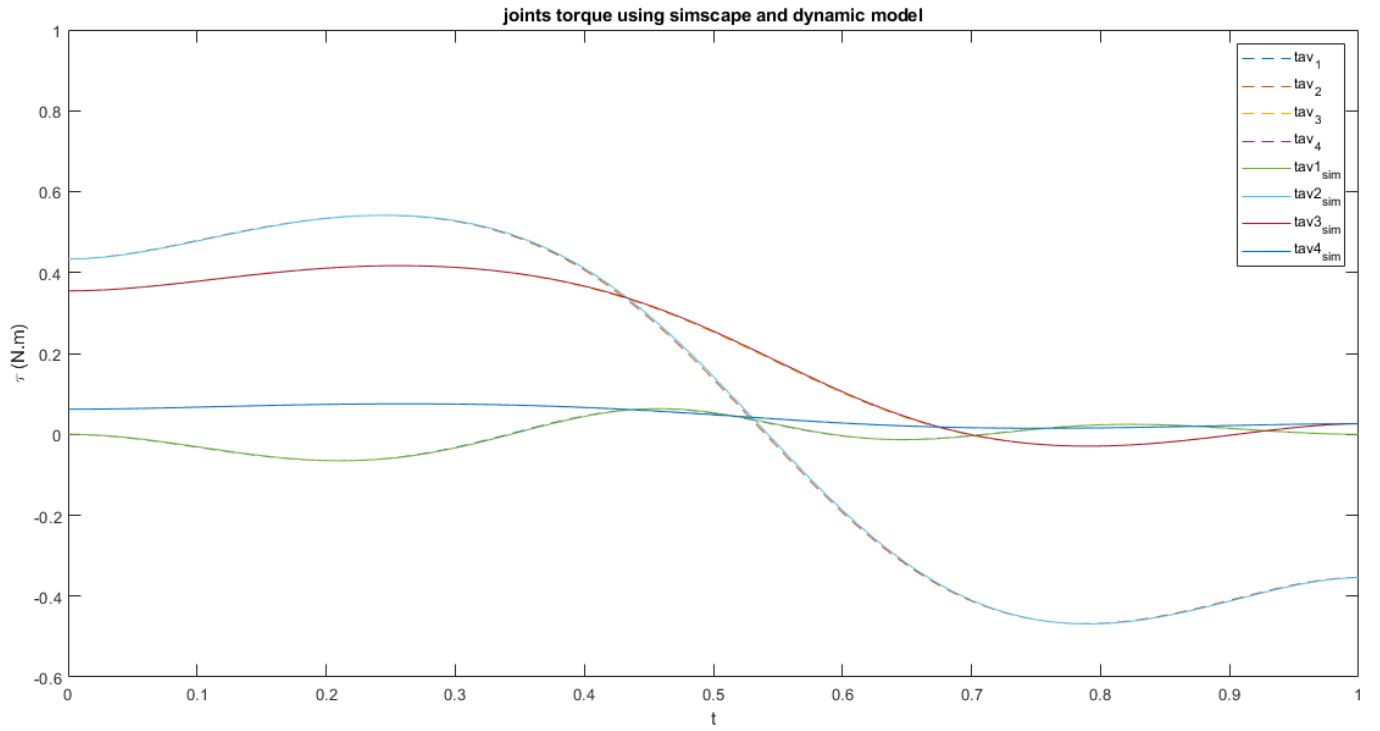
$${}^0J_{v2}(\theta) = \begin{bmatrix} -\cos(\theta_1)[0.126\sin(\theta_2 + \theta_3 + \theta_4) + 0.124\sin(\theta_2 + \theta_3) + 0.024\sin(\theta_2)] \\ -\sin(\theta_1)[0.126\sin(\theta_2 + \theta_3 + \theta_4) + 0.124\sin(\theta_2 + \theta_3) + 0.024\sin(\theta_2)] \\ 0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) + 0.128\cos(\theta_2) \end{bmatrix}$$

$${}^0J_{v3}(\theta) = \begin{bmatrix} -\cos(\theta_1)[0.126\sin(\theta_2 + \theta_3 + \theta_4) + 0.124\sin(\theta_2 + \theta_3)] \\ -\sin(\theta_1)[0.126\sin(\theta_2 + \theta_3 + \theta_4) + 0.124\sin(\theta_2 + \theta_3)] \\ 0.126\cos(\theta_2 + \theta_3 + \theta_4) + 0.124\cos(\theta_2 + \theta_3) \end{bmatrix}$$

$${}^0J_{v4}(\theta) = \begin{bmatrix} -0.126\cos(\theta_1)\sin(\theta_2 + \theta_3 + \theta_4) \\ -0.126\sin(\theta_1)\sin(\theta_2 + \theta_3 + \theta_4) \\ 0.126\cos(\theta_2 + \theta_3 + \theta_4) \end{bmatrix}$$

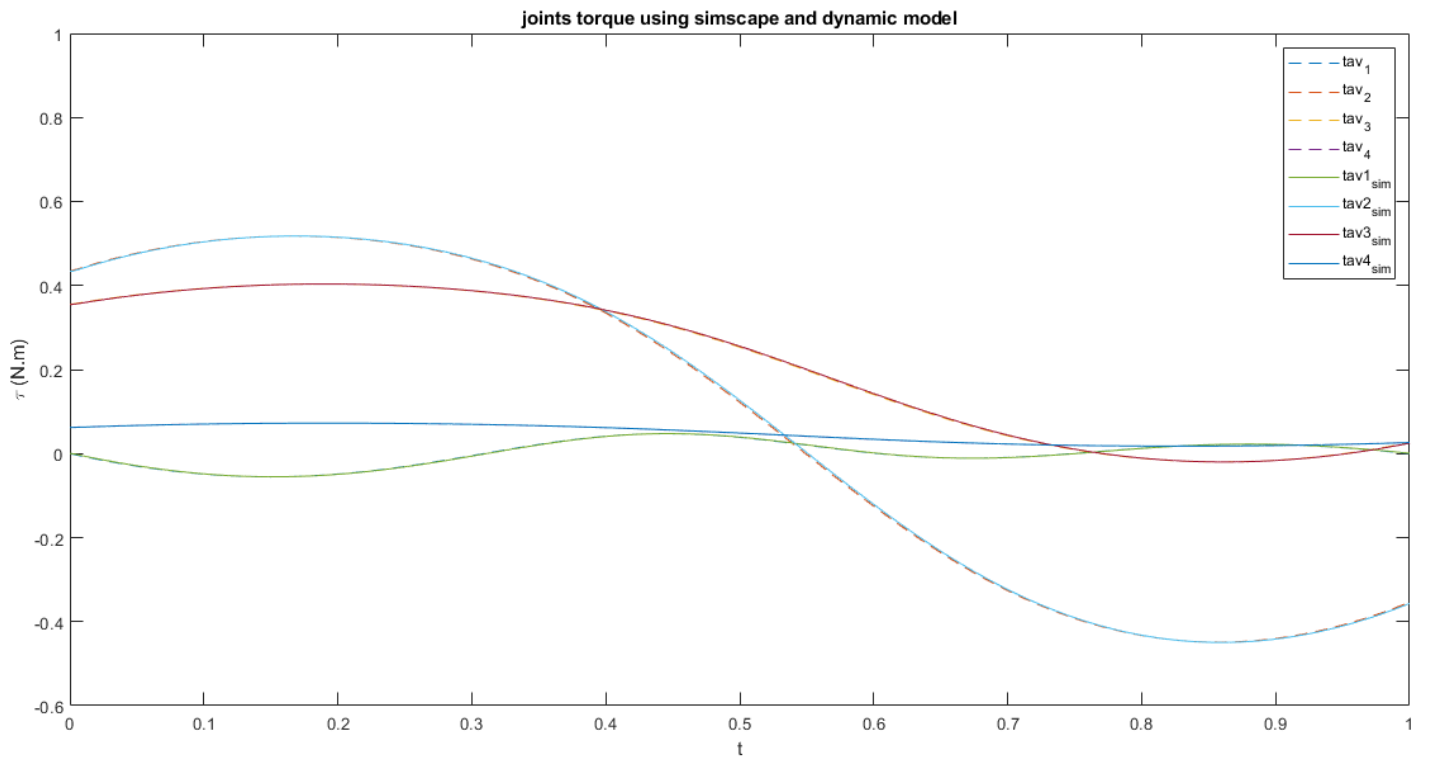
$${}^0J(\theta) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ {}^0J_{v1} & {}^0J_{v2} & {}^0J_{v3} & {}^0J_{v4} \end{bmatrix}$$

B) Torque diagram of joints moving towards $\theta_f = \left[-\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}, -\frac{\pi}{5}\right]$ in 4567 routing:



C) Torque diagram of joints moving towards $\theta_f = \left[-\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}, -\frac{\pi}{5}\right]$ in 345 routing:

$$s(\tau) = 6\tau^5 - 15\tau^4 + 10\tau^3$$



D) The calculation of the Robot end point position using *ode23s* *ode function*

```
function dp = odefun(t, p, th, dth)

i = floor(t*10^4 + 1);

th1 = th(1,i);
th2 = th(2,i);
th3 = th(3,i);
th4 = th(4,i);

dth1 = dth(1,i);
dth2 = dth(2,i);
dth3 = dth(3,i);
dth4 = dth(4,i);

dp = zeros(3,1);

J1 = [-sin(th1)*(0.126*cos(th2+th3+th4) + 0.124*cos(th2+th3) +
0.024*cos(th2));
      cos(th1)*(0.126*cos(th2+th3+th4) + 0.124*cos(th2+th3) +
0.024*cos(th2));
      0];

J2 = [-cos(th1)*(0.126*sin(th2+th3+th4) + 0.124*sin(th2+th3) +
0.024*sin(th2));
      -sin(th1)*(0.126*sin(th2+th3+th4) + 0.124*sin(th2+th3) +
0.024*sin(th2));
      0.126*cos(th2+th3+th4) + 0.124*cos(th2+th3) + 0.128*cos(th2)];

J3 = [-cos(th1)*(0.126*sin(th2+th3+th4) + 0.124*sin(th2+th3));
      -sin(th1)*(0.126*sin(th2+th3+th4) + 0.124*sin(th2+th3));
      0.126*cos(th2+th3+th4) + 0.124*cos(th2+th3)];

J4 = [-cos(th1)*0.126*sin(th2+th3+th4);
      -sin(th1)*0.126*sin(th2+th3+th4);
      0.126*cos(th2+th3+th4)];

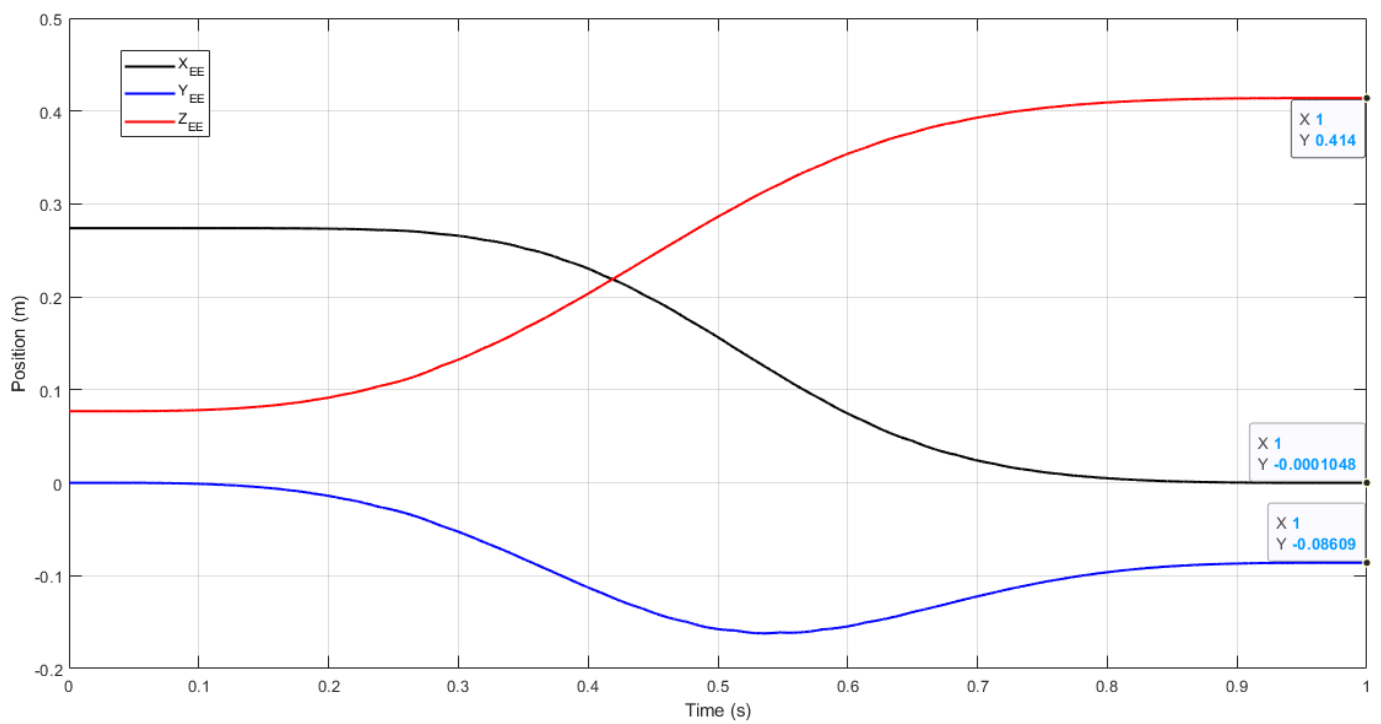
J = [J1 , J2 , J3 , J4];

v = J * [dth1;dth2;dth3;dth4];

dp(1) = v(1);
dp(2) = v(2);
dp(3) = v(3);
```

main function

```
main_4567;  
  
th = [theta1_num;theta2_num;theta3_num;theta4_num];  
dth = [theta1_dot_num;theta2_dot_num;theta3_dot_num;theta4_dot_num];  
  
[t, p] = ode23s(@(t,p) odefun(t,p,th,dth), T, [0.274;0;0.077]);  
  
plot(T,p(:,1),'k','LineWidth',1.5);  
hold on  
plot(T,p(:,2),'b','LineWidth',1.5);  
plot(T,p(:,3),'r','LineWidth',1.5);  
xlabel('Time (s)')  
ylabel('Position (m)')  
grid on  
legend('X_{EE}','Y_{EE}','Z_{EE}')
```



E) The calculation of the Robot end point position using cinematics directly:

$$\theta_f = \left[-\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}, -\frac{\pi}{5} \right]$$

$$P_{EE} = \begin{bmatrix} \cos(\theta_1) [0.126 \cos(\theta_2 + \theta_3 + \theta_4) + 0.124 \cos(\theta_2 + \theta_3) + 0.024 \cos(\theta_2)] \\ \sin(\theta_1) [0.126 \cos(\theta_2 + \theta_3 + \theta_4) + 0.124 \cos(\theta_2 + \theta_3) + 0.024 \cos(\theta_2)] \\ 0.077 + 0.126 \sin(\theta_2 + \theta_3 + \theta_4) + 0.124 \sin(\theta_2 + \theta_3) + 0.128 \sin(\theta_2) \end{bmatrix} = \begin{bmatrix} P_{EE}_1 \\ P_{EE}_2 \\ P_{EE}_3 \end{bmatrix}$$

$$x_{EE} = 0$$

$$y_{EE} = -1 \left[0.126 \cos\left(\frac{\pi}{3} + \frac{\pi}{6} - \frac{\pi}{5}\right) + 0.124 \cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right) + 0.024 \cos\left(\frac{\pi}{3}\right) \right] \\ = -0.0861$$

$$z_{EE} = 0.077 + 0.126 \sin\left(\frac{\pi}{3} + \frac{\pi}{6} - \frac{\pi}{5}\right) + 0.124 \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) + 0.128 \sin\left(\frac{\pi}{3}\right) \\ = 0.4138$$

As shown, the results from cinematics calculation matches the ones from ode23s.