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CS477

HW7

1. Customer =  $i$

time job =  $t_i$

weight =  $w_i$

Goal: Minimize  $\sum_{i=1}^n w_i \cdot c_i$

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Observation: Go for the greedy approach, the greatest weight/time should be finished first

Algorithm:  $\frac{w_1}{t_1} > \frac{w_2}{t_2} > \frac{w_3}{t_3} \dots \frac{w_n}{t_n}$ , most important jobs are completed first, so we would have the most in customers possible

Proof:  $\sum_{i=1}^n w_i \cdot c_i = w_1 \cdot c_1 + w_2 \cdot c_2 + w_3 \cdot c_3 \dots w_n \cdot c_n$

↳ weight to processing time ratio.

The algorithm has to take properties from a Priority Queue, so the most important jobs are completed first

Greedy Solution:

$T_1$ : First Page prints in 1 second

$T_2$ : Second Page prints in 2 seconds

Job A = 10 (weight)  
Job B = 2 (weight)

$$A_{\text{greedy}} = (Job A \cdot T_1) + (Job B \cdot T_2) = (10 \cdot 1) + (2 \cdot 2) = 10 + 4 = 14$$

$$A_{\text{other}} = (Job B \cdot T_1) + (Job A \cdot T_2) = (2 \cdot 1) + (10 \cdot 2) = 2 + 20 = 22$$

①

Next Page  
=>

Agreeing = 14; Aother = 22

Agreeing < A other

Conclusion: The Greedy solution (Agreeedy) will always be the most efficient solution. If the Printer were to print a page every second, then time would be following a linear growth. By sorting the print jobs as a Priority Queue, where the most important jobs are printed first, then we will have the smallest weighted sum possible.

2) Workers =  $n$

- one shift every week
- Shifts are contiguous [start, final]

• Multiple shifts possible

\* Goals: Create an Advisory Committee that can observe workers. Most amount of workers w/ least amount of Advisory workers

Key:  $C$  = Advisory Committee

$W$  = Workers

$P$  = Set of workers w/ overlap

Observation: Go for Greedy Solution by Utilizing Interval Partitioning, maximize number of workers while minimizing number of Advisors

Algorithm:

1)  $W$  = workers

2) Sort(Shifts) // earliest finishing Shift  $\rightarrow$  Latest finishing Shift

3) for ( $i$  > Shifts) //  $i$  in set  $W = \{ \text{worker}_1, \text{worker}_2, \text{worker}_3 \}$

4)  $P = \{ w \in W \mid \text{overlaps in interval } i \}$  //  $P$  is the set

5)  $P \in P$  // latest finish time, and add that to set  $C$   
( $C$  = Committee)

6) Delete interval  $i$  in set  $W$  that has overlap with  $P \in P$

7) Loop( $w \in W$ ) // Loop till  $W$  is empty

②

Conclusion:

Let  $C = \{C_1, C_2, C_3, \dots, C_j\}$ , where  $C$  represents advisors ordered from latest earliest shift (mid shift) to latest shift. The optimal solution must be  $k < j$  advisors. Using proof by contradiction, we try to prove the greedy solution is not the optimal solution.

Let  $O = \{O_1, O_2, O_3, \dots, O_n\}$ , where  $O$  represents the "other" solution where we order the earliest finish time to latest finish time. Assume  $\{C_n\}$  has a later or equal finish time than  $\{O_n\}$ :

$\{C_1, C_2, C_3, C_4, \dots, C_n\} \geq \{O_1, O_2, O_3, O_4, \dots, O_n\}$  For every  $i < k$ , the  $(C_i)$  advises  $(O_i)$  &  $C_k$  advises  $(O_k)$ .

Since  $C = \{C_1, C_2, C_3, \dots, C_k\}$ , then this implies that  $j = k$ . However the optimal solution is  $k < j$ . Because of proof by contradiction, greedy solution is optimal solution.



3)  $I=75$ ,  $U=200$ ,  $B=25$ ,  $S=275$ ,  $C=50$ ,  $H=100$ ,  $M=25$ ,  $P=25$

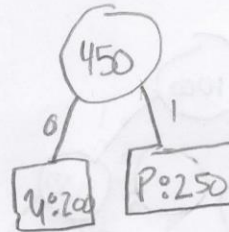
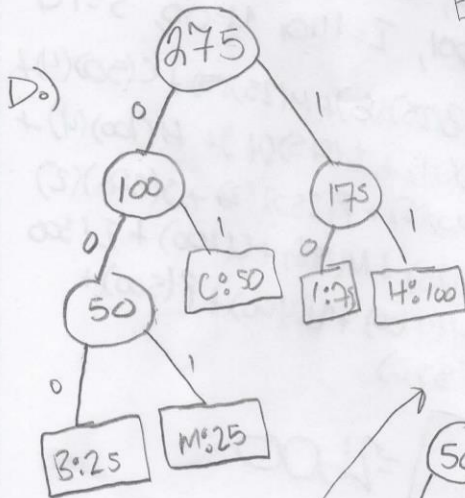
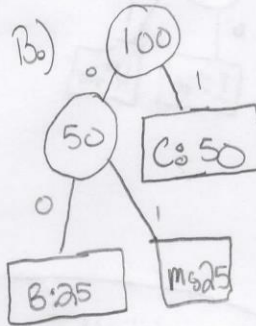
$C$  = Alphabet of Chars

$f(c)$  = frequency of  $C$

$d_t(c)$  = depth of  $C$ 's leaf in tree

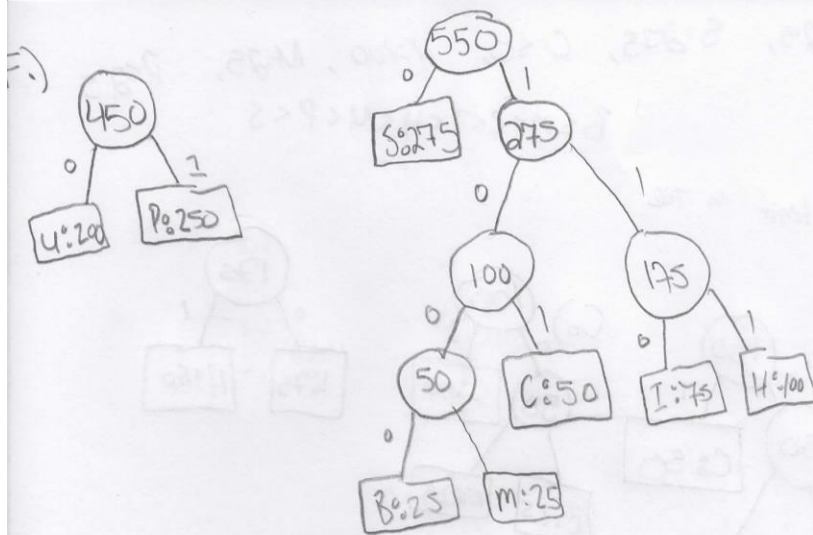
$$B(T) = \sum f(c) d_t(c)$$

$B \leq M < C < I < H < U < P < S$

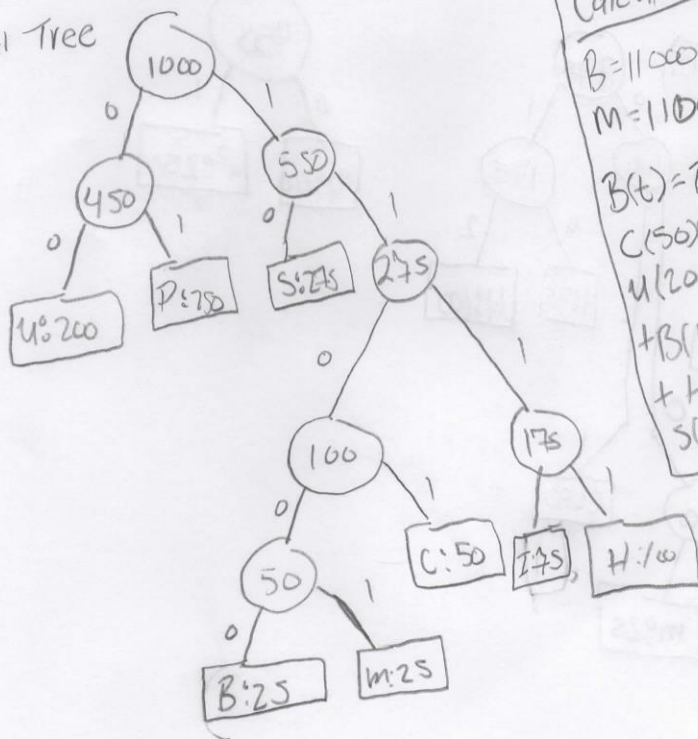


Didn't think about spacing

(3)



Final Tree



Calculation:

$$\begin{aligned}
 &B=11000, C=1101, H=1111, P=01, \\
 &M=11001, I=1110, U=00, S=10 \\
 &B(4) = B(25)(8) + M(25)(8) + C(50)(4) + \\
 &C(50)(4) + I(75)(4) + H(100)(4) + \\
 &U(200)(2) + P(250)(2) + S(275)(2) \\
 &+ B(125) + M(125) + C(200) + I(300) \\
 &+ H(400) + U(400) + P(500) + \\
 &S(550)
 \end{aligned}$$

=2400  
bits to  
encode

(4)

4a) Prof. Gigabyte is correct, the way he stores the characters is more ~~space~~ space efficient than the optimal Huffman code.

However the way the Prof stores the characters is not feasible for an actual application. Due the ambiguity of ~~the~~ the Prof's method, too many things can go wrong. For example, the binary string 10 can represent AE or D. While the method is space efficient, it is also useless.



Extra Credit