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cs477

hw1

problems

1-5

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CS 477

HW #1

1.) List Functions in Increasing Growth

Increasing Growth ↓

A) $5\lg(n+100)^n$
B) $\sqrt[3]{n}$
C.) $\ln^2 n$
d) $.001n^4+3n^3+1$
e) 2^{2n}
f.) 3^n
g) $(n-2)!$

Order for big O

$O(1) < O(\log n) < O(n) < O(n\log n) < O(n^c) < O(c^n) < O(n!)$

↳ $f_2(n) < f_6(n) < f_5(n) < f_4(n) < f_3(n) < f_7(n) < f_1(n)$

2.) Find Correct Θ notations for the expressions

Order: $O(1) < O(\log n) < O(n) < O(n\log n) < O(n^c) < O(c^n) < O(n!)$

A) $2(\mathrm{Log}(n))^2 + 4n + 3n^2\,\mathrm{Log}(n)$
↳ order for dominating term
$\mathrm{Log}(n)^2 < (n) < n^2\log(n)$
↳ Dominating Term is $n^2\log(n)$

B.) $6n^3(\log(n)+4) + (10+n)$
$= 60n^3\log(n) + 6n^4\,\mathrm{Log}(n) + 40 + 4n$
$\log(n) < n < n^3 < n^4\log(n)$
↳ Dominating Term is $n^4\,\mathrm{Log}(n)$

C.) $\dfrac{n^2+\text{Log}(n)(n+1)}{n+n^2}$

$= \dfrac{n^2+\text{Log}(n)\,\cancel{(n+1)}}{n\cancel{(n+1)}} = \dfrac{n^2+\text{Log}(n)}{n} \quad n + \dfrac{\text{Log}(n)}{n}$

$= \dfrac{\text{Log}(n)}{n} < n$

↳ n is dominating Term

D.) $2+4+8+16 \ldots + 2^n$

$= a = \dfrac{(r^n-1)}{r-1}$

$= \dfrac{2(2^n-1)}{2-1} = 2(2^n-1)$

↳ Dominating Term is 2^n

E.) $8^{\text{Log}(n)} = n^{\text{Log}\,8} = n^{\text{Log}_2 8} = n^{2.079}$

Dominating Term is $n^{2.079}$

3.) Prove Using Mathematical Induction

A.) $\sum_{i=1}^{n} (-1)^{i+1}\, i^2 = \dfrac{(-1)^{n+1}\, n(n+1)}{2}$

Base case: $n=1$

$(-1)^{1+1}\, 1^2 = -1^2\ \dfrac{1(1+1)}{2}$

$= 1 = 1$ ✓

Inductive

$\dfrac{(-1)^{n+1}\,(k+1)^2}{2}$

$= \dfrac{(-1)^{k+1}\, k(k+1)}{2} + (-1)^{k+2}\,(k+1)^2$

$= \dfrac{(-1)^{k+1}(k+1)}{2}\left[k - 2(k+1)\right]$

$= \dfrac{(-1)^{k+1}}{2}(k+1)(-k-2)$

Proved ✓

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3) Continued

B) $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = n/_{2n+1}$

Base case:

$1/_{(2-1)(2+1)} = 1/_{2+1} = 1/_3 = 1/_3$ ✓

$n=k$

$\hookrightarrow 1/_{(2i-1)(2i+1)} = k/_{2k+1}$

$n=k+1$

$1/_{(2i-1)(2i+1)} = k+1/_{2(k+1)+1} = 1/_{2(k+1)-1} + 1/_{2(k+1)+1}$

$= k/_{2k+1} + 1/_{(2k+1)(2k+3)}$

$= \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}$

Proved ✓

4.) Prove using asympotic notions

A) $10n^2 + 1 \in O(n^3)$

=> Dominating Term in n^2

$O(n^2) \in O(n^3)$ ✓ True

$n^2 < n^3$ ✓

O : $f(n) \leq g(n)$

Ω : $f(n) \geq g(n)$

Θ : $f(n) = g(n)$

B) $5n^2 + 10 \in \Omega(n)$

$f(n) = 5n^2 + 10$

Dominating Term is $5n^2 \Rightarrow n^2$

$n^2 \in \Omega(n) \Rightarrow n^2 \geq n$ ✓ Proved :)

5.) Extra Credit

A) Order of Growth $\sum_{i=0}^{n-1} (i+2)^2$

$= \sum_{i=0}^{n-1} [i^2 + 4i + 4] = \sum_{i=0}^{n-1} (i^2) + 4\sum_{i=0}^{n-1} (i) + 4\sum_{i=0}^{n-1} (1)$

Sorry for messy work

$= \sum_{i=0}^{(n-1)} i^2 + 4\sum_{i=0}^{(n-1)} i + 4(n-1)$

$= \frac{(n-1)(n-1+1)(2(n-1)+1)}{6} + \frac{4(n-1)(n)}{2} + 4(n-1)$

$= \frac{2n^3 + 3n^2 + n}{6} + \frac{2(n^2 - n)}{1} + \frac{4n - 4}{1} = \frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6} + 2n^2 - 2n + 4n - 4$

$= \frac{n^3}{3} + \frac{3n^2}{2} + \frac{13n}{6} - 4 \Rightarrow n < n^2 < n^3 \Rightarrow n^3$ is dominating term

n^3 is order of growth

B) $30n^2 + 100 \notin \Omega(n^3) \Rightarrow f(n) \geq g(n)$

Proof by Contradiction $\Rightarrow$

$\hookrightarrow 0 \leq g(n) < f(n)$

$0 \leq cn^3 \leq 30n + 100n^2$

$\hookrightarrow 0 \leq cn^3 < 130n \Rightarrow (cn^3 \leq 130n^2) \Rightarrow n^2 (n \leq 130)$

$\Rightarrow n < 130$

$\Rightarrow$ 130 is constant so it'd be faster than n

$\Rightarrow n^2 \in n^3 \Rightarrow n^2 \neq n^3$ ✓ (Proven) 4