the stochastic, primitive matrix $\mathbf{G}$ is

$$\mathbf{G} = .9\mathbf{H} + (.9 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + .1 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}) \, 1/6 \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/60 & 7/15 & 7/15 & 1/60 & 1/60 & 1/60 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 19/60 & 19/60 & 1/60 & 1/60 & 19/60 & 1/60 \\ 1/60 & 1/60 & 1/60 & 1/60 & 7/15 & 7/15 \\ 1/60 & 1/60 & 1/60 & 7/15 & 1/60 & 7/15 \\ 1/60 & 1/60 & 1/60 & 11/12 & 1/60 & 1/60 \end{pmatrix}.$$

Google's PageRank vector is the stationary vector of $\mathbf{G}$ and is given by

$$\begin{array}{cccccccc} & & 1 & 2 & 3 & 4 & 5 & 6 \\ \boldsymbol{\pi}^T & = & \big(.03721 & .05396 & .04151 & .3751 & .206 & .2862 \big). \end{array}$$

The interpretation of $\pi_1 = .03721$ is that 3.721% of the time the random surfer visits page 1. Therefore, the pages in this tiny web can be ranked by their importance as $(\,4 \quad 6 \quad 5 \quad 2 \quad 3 \quad 1\,)$, meaning page 4 is the most important page and page 1 is the least important page, according to the PageRank definition of importance.

4.6 COMPUTATION OF THE PAGERANK VECTOR

The PageRank problem can be stated in two ways:

1. Solve the following *eigenvector* problem for $\boldsymbol{\pi}^T$.

$$\boldsymbol{\pi}^T = \boldsymbol{\pi}^T\mathbf{G},$$
$$\boldsymbol{\pi}^T\mathbf{e} = 1.$$

2. Solve the following *linear homogeneous system* for $\boldsymbol{\pi}^T$.

$$\boldsymbol{\pi}^T(\mathbf{I} - \mathbf{G}) = \mathbf{0}^T,$$
$$\boldsymbol{\pi}^T\mathbf{e} = 1.$$

In the first system, the goal is to find the normalized *dominant left-hand eigenvector* of $\mathbf{G}$ corresponding to the *dominant eigenvalue* $\lambda_1 = 1$. ($\mathbf{G}$ is a stochastic matrix, so $\lambda_1 = 1$.) In the second system, the goal is to find the normalized left-hand null vector of $\mathbf{I} - \mathbf{G}$. Both systems are subject to the normalization equation $\boldsymbol{\pi}^T\mathbf{e} = 1$, which insures that $\boldsymbol{\pi}^T$ is a probability vector. In the example in section 4.5, $\mathbf{G}$ is a 6×6 matrix, so we used Matlab's `eig` command to solve for $\boldsymbol{\pi}^T$, then normalized the result (by dividing the vector by its sum) to get the PageRank vector. However, for a web-sized matrix like Google's, this will not do. Other more advanced and computationally efficient methods must be used. Of course, $\boldsymbol{\pi}^T$ is the stationary vector of a Markov chain with transition matrix $\mathbf{G}$, and much research has been done on computing the stationary vector for a general Markov chain. See William J. Stewart's book *Introduction to the Numerical Solution of Markov Chains* [154], which contains over a dozen methods for finding $\boldsymbol{\pi}^T$. However, the specific features of the