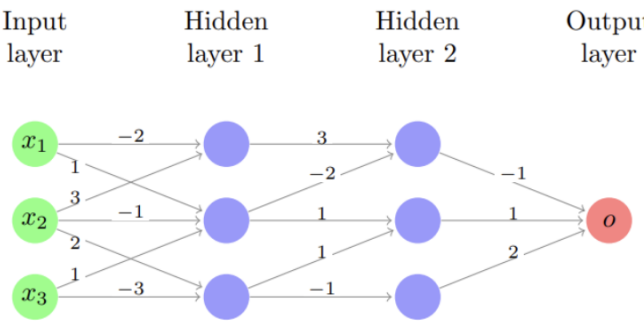
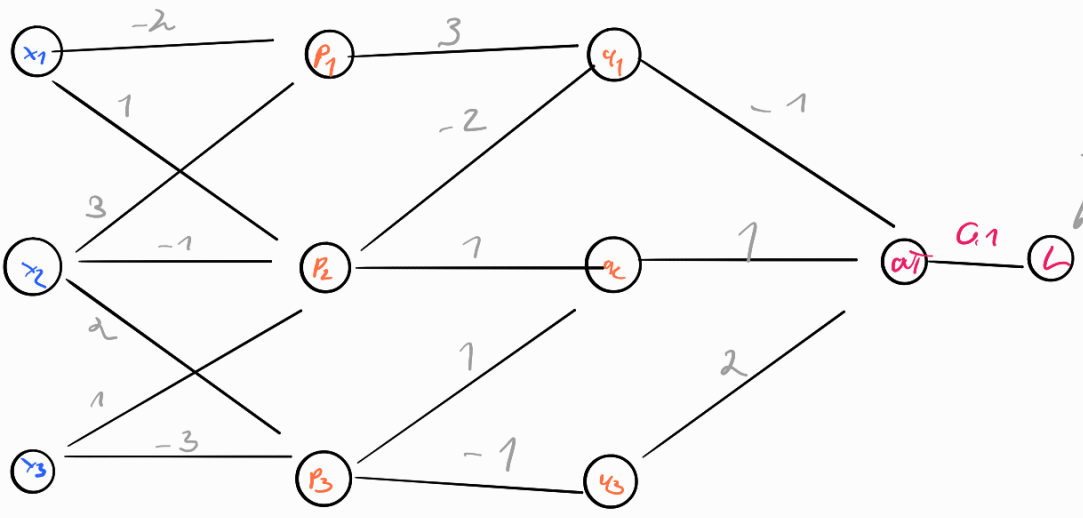
2022/06/16

Exercise 3

Consider the following network where on each edge (i, j) the value of $\frac{\partial y(j)}{\partial y(i)}$ is given; $y(k)$ denotes the activation of node k.



The output o is equal to 0.1 and the loss function is $L = -log(o)$. Compute the value of $\frac{\partial L}{\partial x_i}$ for each input x_i using the backpropagation method.



Backpropagation

$$\frac{\partial L}{\partial out} = -\frac{1}{out} = \frac{1}{0.1} = -10$$

$$\frac{\partial L}{\partial q_1} = \frac{\partial L}{\partial out} \cdot \frac{\partial out}{\partial q_1} = -10 \cdot -1 = +10$$

$$\frac{\partial L}{\partial q_2} = \frac{\partial L}{\partial out} \cdot \frac{\partial out}{\partial q_2} = -10 \cdot 1 = -10$$

$$\frac{\partial L}{\partial q_3} = \frac{\partial L}{\partial out} \cdot \frac{\partial out}{\partial q_3} = -10 \cdot 2 = -20$$

$$\frac{\partial L}{\partial p_1} = \frac{\partial L}{q_1} \cdot \frac{\partial q_1}{\partial p_1} = 10 \cdot 3 = 30$$

$$\frac{\partial L}{\partial p_2} = \frac{\partial L}{q_1} \cdot \frac{\partial q_1}{\partial p_2} + \frac{\partial L}{\partial q_2} \cdot \frac{\partial q_2}{\partial p_2} = (10 \cdot -2) + (-10 \cdot 1) = -30$$

$$\frac{\partial L}{\partial p_3} = \frac{\partial L}{\partial y_2} \cdot \frac{\partial y_2}{\partial p_3} + \frac{\partial L}{\partial y_3} \cdot \frac{\partial y_3}{\partial p_3} = (-10 \cdot 1) + (-20 \cdot -1) = 10$$

$$\frac{\partial L}{x_1} = \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial x_1} + \frac{\partial L}{\partial p_2} \cdot \frac{\partial p_2}{\partial x_1} = (30 \cdot -2) + (-30 \cdot 1) = -90$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial p_1} \cdot \frac{\partial p_1}{\partial x_2} + \frac{\partial L}{\partial p_2} \cdot \frac{\partial p_2}{\partial x_3} + \frac{\partial L}{\partial p_3} \cdot \frac{\partial p_3}{\partial x_3} = (30 \cdot 3) + (-30 \cdot -1) + (10 \cdot 2)$$
$$= 160$$

$$\frac{\partial L}{\partial x_3} = \frac{\partial L}{\partial p_2} \cdot \frac{\partial p_2}{\partial x_3} + \frac{\partial L}{\partial p_3} \cdot \frac{\partial p_3}{\partial x_3} = (-30 \cdot 1) + (10 \cdot -3) = -60$$