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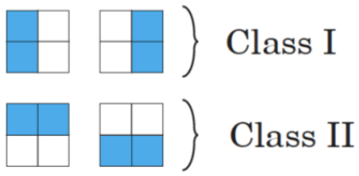
Exercise 3

Consider the two classes of patterns that are shown in the following figure where Class I represents vertical lines and Class II represents horizontal lines.



1. Are these categories linearly separable ?
2. Design a multilayer network to distinguish these categories.

- Encode input as $[x_1, x_2, x_3, x_4]$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$
$[1\ 0\ 1\ 0] \to$ class 1
$[0\ 1\ 0\ 1] \to$ class 1
$[1\ 1\ 0\ 0] \to$ class 2
$[0\ 0\ 1\ 1] \to$ class 2

↳ Linear decision boundary in the form $W^T x + b > 0$ (e.g. $> 0 \to$ class 1)

$[1\ 0\ 1\ 0] \to w_1 + w_3 > 0$ (1)
$[0\ 1\ 0\ 1] \to w_2 + w_4 > 0$ (2)
$[1\ 1\ 0\ 0] \to w_1 + w_2 \leq 0$ (3)
$[0\ 0\ 1\ 1] \to w_3 + w_4 \leq 0$ (4)

→ It raises inconsistencies :

From (1) and (3) $\to \begin{cases} w_1 + w_3 > 0 \\ -w_1 > w_2 \end{cases} \xrightarrow{\text{sum}} w_3 > w_2$ ⚠

From (2) and (4) $\to \begin{cases} w_2 + w_4 > 0 \\ -w_4 > w_3 \end{cases} \xrightarrow{\text{sum}} w_2 > w_3$

- Multi-layer perceptron

vertical detector : $h_1 = \sigma(x_1 + x_3 - x_2 - x_4)$

horizontal detector : $h_2 = \sigma(x_1 + x_2 - x_3 - x_4)$

output : $y = \sigma(w_1 h_1 + w_2 h_2 + b)$

loss : cross-entropy