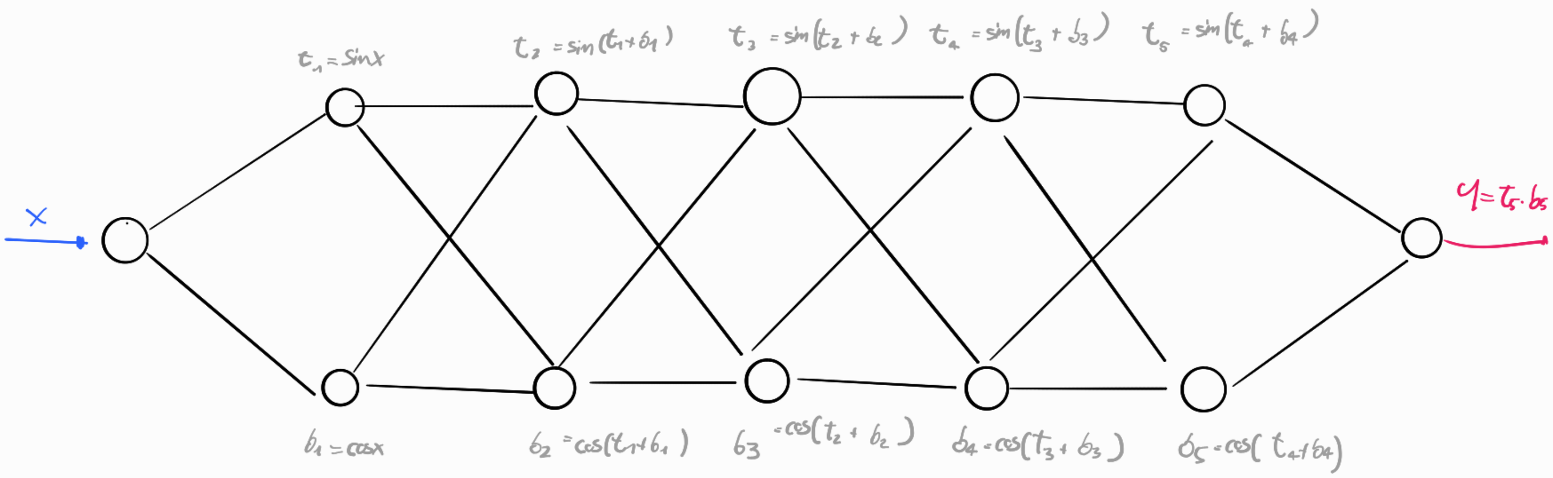
2023/07/06

Exercise 3
Consider the following computational graph:

Figure 1: Computational Graph

The upper node in each layer computes $\sin(x+y)$ and the lower computes $\cos(x+y)$ with respect to its 2 inputs. For the first hidden layer, there is only a single input x, and therefore the values $\sin(x)$ and $\cos(x)$ are computed. The final node computes the product of the two inputs. The single input is denoted by x (value in radiants). Compute the numerical value of the partial derivative of the output with respect to x for $x = 1$ using the backpropagation algorithm. Explain clearly each step you have performed.



$x = 1$

Forward pass

$h_1 \begin{cases} t_1 = \sin(1) = 0,84 \\ b_1 = \cos(1) = 0.54 \end{cases}$

$h_2 \begin{cases} t_1 + b_1 = 1,38 \\ t_2 = \sin(1,38) = 0,98 \\ b_2 = \cos(1.38) = 0,18 \end{cases}$

$h_3 \begin{cases} t_2 + b_2 = 1.16 \\ t_3 = \sin(1.16) = 0,91 \\ b_3 = \cos(1.16) = 0,40 \end{cases}$

$h_4 \begin{cases} t_3 + b_3 = 1,31 \\ t_4 = \sin(1.31) = 0,97 \\ b_4 = \cos(1.31) = 0,25 \end{cases}$

$h_5 \begin{cases} t_4 + b_4 = 1,22 \\ t_5 = \sin(1.22) = 0,94 \\ b_5 = \cos(1.22) = 0,34 \end{cases}$

output $y = t_5 \cdot b_5 = 0,32$

Backward pass $\left(\frac{d \sin x}{dx} = \cos x \, , \, \frac{d \cos x}{dx} = -\sin x \right)$ $\quad \dot{t} = \frac{\partial y}{\partial t}$

obs: T and b have same contribute on derivative (symmetry!)

$h_5 \begin{cases} \dot{t}_5 = b_5 = 0,34 \\ \dot{b}_5 = t_5 = 0,94 \end{cases}$

$h_4 \begin{cases} \dot{t}_4 = \dot{t}_5 \cdot \cos(t_4 + b_4) + \dot{b}_5 \left(-\sin(t_4+b_4)\right) = \ldots = -0,77 \\ \dot{b}_4 = -0,77 \end{cases}$

h_3 $\begin{cases} \dot{t}_3 = \dot{t}_4 \cos(t_3+\theta_3) + \dot{\theta}_4 \left(-\sin(t_3+\theta_3)\right) = \dots \; 0{,}54 \\ \dot{\theta}_3 = 0{,}54 \end{cases}$

h_2 $\begin{cases} \dot{t}_2 = \dot{t}_3 \cos(t_2+\theta_2) \cdot \dot{\theta}_3 \left(-\sin(t_2+\theta_2)\right) = \dots \; -0.28 \\ \dot{\theta}_2 = -0{,}28 \end{cases}$

h_1 $\begin{cases} \dot{t}_1 = \dot{t}_2 \cos(t_1+\theta_1) \cdot \dot{\theta}_2 \left(-\sin(t_1+\theta_1)\right) = 0{,}22 \\ \dot{\theta}_1 = 0{,}22 \end{cases}$

input $\dot{x} = \dot{t}_1 \cos(1) + \dot{\theta}_1 \left(-\sin(1)\right) = -0{,}07$