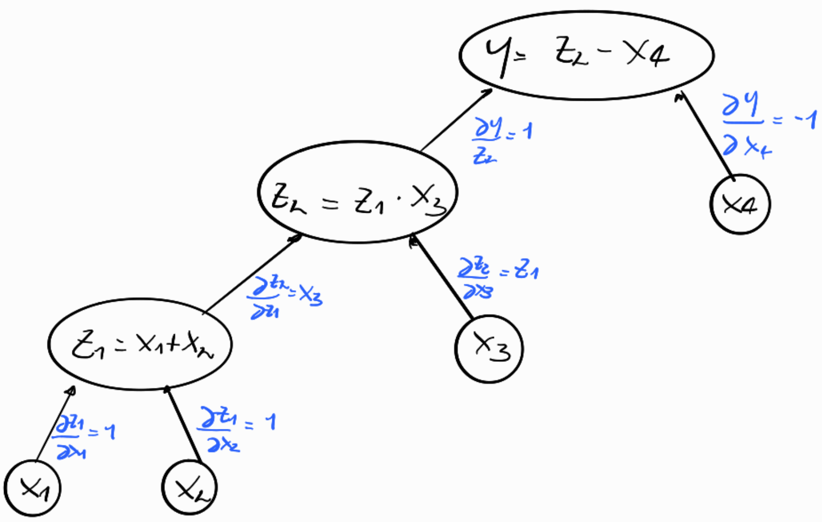
2024/09/03

Exercise 3 (6 points)
Consider the expression

$$y = f(x_1, x_2, x_3, x_4) = (x_1 + x_2) * x_3 - x_4. \tag{5}$$

1. **(1 point)** Draw the computational graph and write the Wengert list corresponding to equation (5). How many intermediate variables do you need ?
2. **(1 point)** Compute the values of the *derivatives on the edges*; report these values on the computational graph.
3. **(1 point)** Assume $x_1 = 1, x_2 = -2, x_3 = -1$ and $x_4 = 5$. Write the Wengert list for these input values.
4. **(1 point)** By using the *derivatives on the edges* compute the values of $\frac{\partial y}{\partial x_i}$ for $i = 2$ and $i = 4$. For the same cases write the Wengert list for the derivatives.
5. **(2 points)** Write the reverse list and use it to compute $\frac{\partial y}{\partial x_i}$ for $i = 1, 2, 3, 4$.



x_1
x_2
x_3
x_4 } input

$z_1 = x_1 + x_2$
$z_2 = z_1 \cdot x_3$ } intermediate

$y = z_2 - x_4$ } output

$\frac{\partial z_1}{\partial x_1} = 1$ $\quad$ $\frac{\partial z_1}{\partial x_2} = 1$

$\frac{\partial z_2}{\partial z_1} = x_3$ $\quad$ $\frac{\partial z_2}{\partial x_3} = z_1$

$\frac{\partial y}{\partial z_2} = 1$ $\quad$ $\frac{\partial y}{\partial x_4} = -1$

→ Numerical example $\quad x_1 = 1 \quad x_2 = -2 \quad x_3 = -1 \quad x_4 = 5$

$z_1 = -1$

$z_2 = 1$

$y = -4$

→ Compute $\frac{\partial y}{\partial x_2}$ $\frac{\partial y}{\partial x_2} = 1 \cdot x_3 \cdot 1 = x_3 = -1$

→ Compute $\frac{\partial y}{\partial x_4}$ $\frac{\partial y}{\partial x_4} = -1$

↳ Reverse Wenger list (Backpropagation)

define $\dot{v} = \frac{\partial y}{\partial v}$ initialize $\dot{y} = 1$ (because $\frac{\partial y}{\partial y} = 1$)

$$\dot{z}_2 = \dot{y} \cdot \frac{\partial y}{\partial z_2} = 1 \cdot 1 = 1$$
$$\dot{x}_4 = \dot{y} \cdot \frac{\partial y}{\partial x_4} = 1 \cdot -1 = -1$$

$$\dot{z}_1 = \dot{z}_2 \cdot \frac{\partial z_2}{\partial z_1} = 1 \cdot x_3 = x_3$$
$$\dot{x}_3 = \dot{z}_2 \frac{\partial z_2}{\partial x_3} = 1 \cdot z_1 = z_1$$

$$\dot{x}_1 = \dot{z}_1 \cdot \frac{\partial z_1}{\partial x_1} = \dot{z}_1 \cdot 1 = \dot{z}_1$$
$$\dot{x}_2 = \dot{z}_1 \cdot \frac{\partial z_1}{\partial x_2} = \dot{z}_1 \cdot 1 = \dot{z}_1$$

Numerical example $x_1 = 1$ $x_2 = -2$ $x_3 = -1$ $x_4 = 5$

$z_1 = -1$ $z_2 = 1$ $y = -4$

$\dot{y} = 1$

$\dot{z}_2 = 1$

$\dot{x}_4 = -1$

$\dot{z}_1 = 1 \cdot x_3 = -1$

$\dot{x}_3 = 1 \cdot z_1 = -1$

$\dot{x}_1 = \dot{z}_1 = -1$

$\dot{x}_2 = \dot{z}_1 = -1$