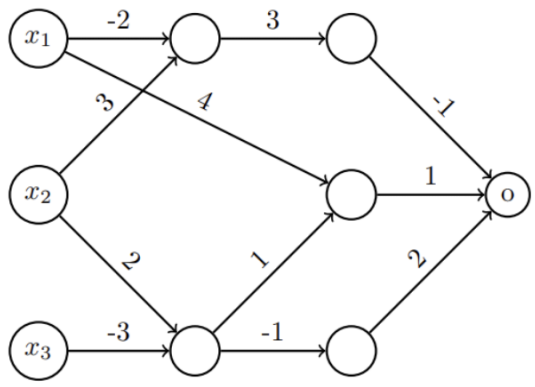
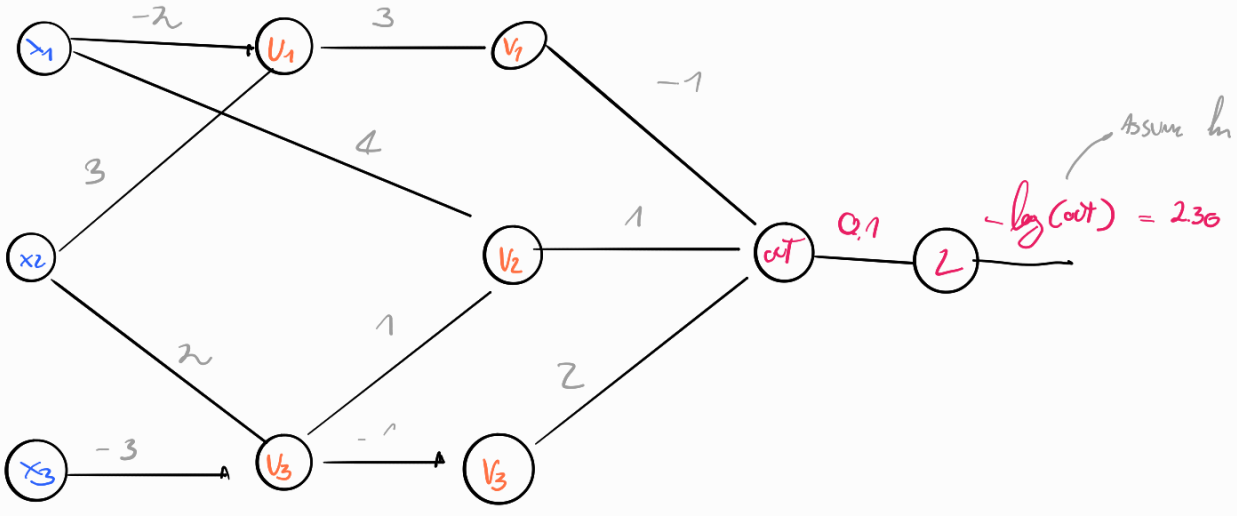
2022/09/01

Exercise 3

Consider the following network where on each edge (i, j) the value of $\frac{\partial y(j)}{\partial y(i)}$ is given; $y(k)$ denotes the activation of node k.



The output o is equal to 0.1 and the loss function is $L = -log(o)$. Compute the value of $\frac{\partial L}{\partial x_i}$ for each input x_i using the backpropagation method.



$$\frac{\partial L}{\partial out} = -\frac{1}{out} = -10$$

$$\frac{\partial L}{\partial V_1} = \frac{\partial L}{\partial out} \cdot \frac{\partial out}{\partial V_1} = -10 \cdot -1 = 10$$

$$\frac{\partial L}{\partial V_2} = \frac{\partial L}{\partial out} \cdot \frac{\partial out}{\partial V_2} = -10 \cdot 1 = -10$$

$$\frac{\partial L}{\partial V_3} = \frac{\partial L}{\partial out} \cdot \frac{\partial out}{\partial V_3} = -10 \cdot 2 = -20$$

$$\frac{\partial L}{\partial U_1} = \frac{\partial L}{\partial V_1} \cdot \frac{\partial V_1}{\partial U_1} = 10 \cdot 3 = 30$$

$$\frac{\partial L}{\partial U_3} = \frac{\partial L}{\partial V_2} \cdot \frac{\partial V_2}{\partial U_3} + \frac{\partial L}{\partial V_3} \cdot \frac{\partial V_3}{\partial U_3} = (-10 \cdot 1) + (-20 \cdot -1) = 10$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial u_1} \cdot \frac{\partial u_1}{\partial x_1} + \frac{\partial L}{\partial v_2} \cdot \frac{\partial v_1}{\partial u_1} = (30 \cdot -2) + (-10 \cdot 4) = -60 - 40 = -100$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial u_1} \cdot \frac{\partial u_1}{\partial x_2} + \frac{\partial L}{\partial v_3} \cdot \frac{\partial v_3}{\partial x_2} = (30 \cdot 3) + (10 \cdot 2) = 90 + 20 = 110$$

$$\frac{\partial L}{\partial x_3} = \frac{\partial L}{\partial v_3} \cdot \frac{\partial v_3}{\partial x_3} = 10 \cdot -3 = -30$$