# pencal: an R package for the dynamic prediction of survival with many longitudinal predictors

Mirko Signorelli

★: mirkosignorelli.github.io★: @signormirko

Mathematical Institute Leiden University

December 18, 2023 - CMStatistics 2023





pencal

The method: Penalized Regression Calibration

The R package: pencal

Prediction of survival

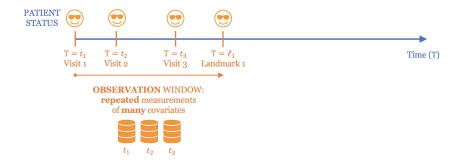
Evaluation of predictive performance

Package overview

**Appendix** 

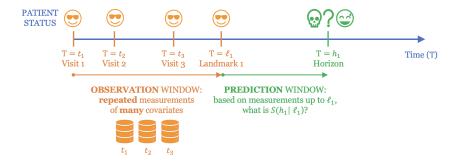
#### Dynamic prediction of survival





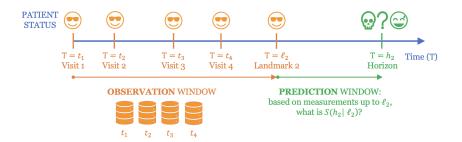
#### Dynamic prediction of survival





#### Dynamic prediction of survival





#### Goal of dynamic prediction



- Goals of dynamic prediction:
  - 1. predict future survival  $S(t|\ell_1)$  using repeated measurements collected over the observation period  $[0,\ell_1]$
  - 2. dynamically update predictions once more information becomes available, i.e. predict  $S(t|\ell_2)$  given repeated measurements over  $[0,\ell_2],\ \ell_2>\ell 1$

#### Example datasets



► Two datasets we worked with:

	ROSMAP	Mark-MD
Outcome	Alzheimer's disease diagnosis	Loss of ambulation in Duchenne patients
n	3757	157
Max follow-up	Up to 30 years	Up to 7.4 years
Baseline covariates	5	3
Longitudinal covariates	30	240 antibodies that target 118 proteins

▶ Datasets can differ substantially wrt *n* and *p* 

#### Methodological problem



- ► Traditional methods for dynamic prediction:
  - ioint models: very computationally-intensive. Can't usually be estimated with more than 3-5 longitudinal predictors!
  - ▶ landmarking with LOCF¹: no modelling of the longitudinal trajectories + no measurement error correction (important for biomarkers)
- Methodological problem: how to do dynamic prediction of survival when the predictors are
  - 1. measured longitudinally (ROSMAP, Mark-MD)
  - 2. many potentially high-dimensional setting (ROSMAP, Mark-MD)
  - 3. and potentially highly-correlated with each other (Mark-MD)





The method: Penalized Regression Calibration

The R package: pencal

Prediction of survival

Evaluation of predictive performance

Package overview

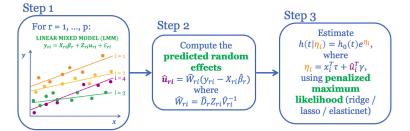
Appendix

#### Notation

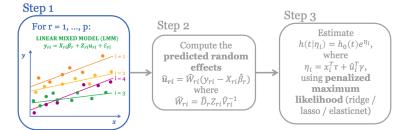


- $\triangleright$   $\ell$  = landmark time
- $i \in \{1, ..., n_{\ell}\}$  subjects who survived up until  $t = \ell$
- ▶ Observation window  $t \in [0, \ell]$ :
  - $\blacktriangleright$  k baseline covariates  $x_{qi}$  measured at  $t_i = 0$  (study entry)
  - **p** longitudinal covariates  $y_{rij}$  measured at  $t_{i1},...,t_{im_i} \in [0,\ell]$
- ▶ Prediction window  $t \in [\ell, h]$ :
  - $ightharpoonup T_i^*$  true survival time
  - C<sub>i</sub> censoring time
  - $ightharpoonup T_i = \min(T_i^*, C_i)$  observed survival time
  - $\delta_i = I(T_i = T_i^*)$  event indicator









# Step 1: model the evolution of the longitudinal covariates

- Step 1: model longitudinal predictors with mixed-effects models
- Two alternatives:
  - 1. Linear Mixed Models (LMM)
  - 2. Multivariate Latent Process Mixed Model (MLPMM, Proust-Lima et al. (2013))

#### LMM approach



- Fit to each longitudinal  $Y_r$  a LMM:  $y_{ri} = X_{ri}\beta_r + Z_{ri}u_{ri} + \varepsilon_{ri}$
- Example:

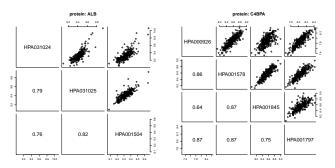
$$y_{rij} = \beta_{r0} + u_{r0i} + (\beta_{r1} + u_{r1i})a_{ij} + \varepsilon_{rij},$$

where  $u_{ri} = (u_{r0i}, u_{r1i}) \sim \textit{N}(0, D_r)$  and  $\varepsilon_{ri} \sim \textit{N}(0, \sigma_r^2 \textit{I}_{m_i})$ 

### How to handle groups of highly-correlated biomarkers?



- Issue:
  - 1. LMM approach assumes longitudinal markers to be independent
  - what if you have groups of highly-correlated biomarkers, like in Signorelli et al. (2020)?



#### MLPMM approach



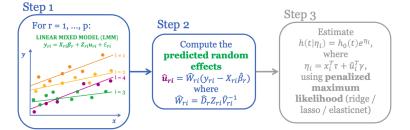
- Suppose that  $r_s$  covariates are employed to measure the same underlying phenomenon  $y_s$  that cannot be measured directly:  $(\mathbf{y_{1s}},...,\mathbf{y_{r_ss}})$
- ightharpoonup Example:  $r_s$  antibodies measured as proxies for protein s
- $\blacktriangleright$  We can specify a MLPMM for  $(y_{1s},....,y_{r_ss})$  where

$$y_{qsij} = \beta_{qs0} + u_{s0i} + \frac{b_{qsi}}{q_{s1}} + (\beta_{qs1} + u_{s1i})a_{ij} + \varepsilon_{qsij} \quad (\forall q = 1, ..., r_s),$$

with  $\varepsilon_{qsij} \sim N_1(0, \sigma_{\varepsilon qs}^2)$ , and

- $\mathbf{u}_{si} = (u_{s0i}, u_{s1i}) \sim N_2(0, \Sigma_{us})$ : shared random intercept and slope that refer to (latent) underlying quantity ( $\rightarrow$  protein)
- $lacktriangledown_{dsi} \sim N_1(0,\sigma_{bqs}^2)$  covariate-specific random intercepts (ightarrow antibodies)
- ► Latent variable interpretation (reconstruct latent protein info from measurable antibodies variables)







- Derive subject-specific summaries of the longitudinal trajectories from the mixed-effects models
  - ightharpoonup random intercepts pprox different starting levels across subjects
  - lacktriangle random slopes pprox different progression rates between subjects



- Derive subject-specific summaries of the longitudinal trajectories from the mixed-effects models
  - lacktriangle random intercepts pprox different starting levels across subjects
  - lacktriangle random slopes pprox different progression rates between subjects
- For the LMM:

$$\hat{u}_{ri} = E(u_{ri}|Y_{ri} = y_{ri}) = \hat{D}_r Z_i^T \hat{V}_{ri}^{-1} (y_{ri} - X_i \hat{\beta}_r),$$

where  $V_{ri} = Z_i D_r Z_i^T + \sigma_r^2 I_{m_i}$  is the marginal covariance matrix of subject i



#### ► For the MLPMM:

$$\left(\hat{u}_{si},\hat{b}_{si}\right) = E\left(u_{si},b_{si}|Y_{si} = y_{si}\right) = \begin{bmatrix} Z_i \Sigma_{u_s} \\ \Sigma_{b_s} I_{r_s} \otimes \mathbb{1}_{m_i,1} \end{bmatrix} \Sigma_{y_{si}}^{-1} \dot{y}_{si},$$

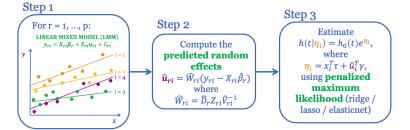
where  $y_{si} = (y_{1si1}, ..., y_{1sim_i}, ..., y_{r_ssi1}, ..., y_{r_ssim_i})^T$ ,  $\dot{y}_{si}$  is the equivalent of  $y_{si}$  with  $\dot{y}_{qsij} = y_{qsij} - \beta_{qs0} - \beta_{qs1}a_{ij}$  as entries,  $Z_i$  is the random-effects

design matrix associated to 
$$y_{si}$$
,  $\Sigma_{b_s} = \begin{bmatrix} \sigma_{b1s}^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{br_ss}^2 \end{bmatrix}$ ,

$$\Sigma_{\varepsilon_{s}} = \begin{bmatrix} \sigma_{\varepsilon_{1s}}^{2} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{\varepsilon_{r_{s}s}}^{2} \end{bmatrix}, \Sigma_{u_{s}} = \begin{bmatrix} \sigma_{us0}^{2} & \sigma_{us0,us1} \\ \sigma_{us0,us1} & \sigma_{us1}^{2} \end{bmatrix} \text{ and }$$

$$\Sigma_{v_{s}} = Z \Sigma_{v_{s}} Z_{v_{s}}^{T} + I_{v_{s}} \Sigma_{v_{s}} \Sigma_{v_{s}} I_{v_{s}} + I_{v_{s}} \Sigma_{v_{s}} I_{v_{s}} I_{v_{s}}$$





#### Step 3: prediction of survival



Cox model linking survival outcome to baseline covariates and summaries of longitudinal covariates:

$$h(t_i|x_i, \hat{u}_{0i}, \hat{u}_{1i}) = h_0(t_i) \exp(\eta_i),$$
 (1)

$$\eta_i = \sum_{q=1}^k \theta_{\mathbf{q}} \mathbf{x}_{\mathbf{q}i} + \sum_{r=1}^p \gamma_r \hat{u}_{r0i} + \sum_{r=1}^p \delta_r \hat{u}_{r1i}$$

 $(\theta, \gamma, \delta)$  large, potentially high-dimensional  $\Rightarrow$  we estimate it using penalized maximum likelihood

$$\max_{\xi,\gamma,\delta} \ \ell(\xi,\gamma,\delta) - \lambda p(\xi,\gamma,\delta;\alpha)$$

- ▶ Penalty functions: ridge ( $\ell^2$ , recommended), elastic net, lasso ( $\ell^1$ )
- ▶ Predicted survival:  $\hat{S}(h|\ell) = \hat{S}(h) = e^{-\int_0^h \hat{h}_0(s)e^{\hat{\eta}_i}ds}$

The problem: dynamic prediction of surviva

pencal

The method: Penalized Regression Calibration

The R package: pencal

Prediction of survival

Evaluation of predictive performance

Package overview

Appendix

#### Where to find the package



- Method implemented in the R package pencal
- Available on CRAN:

pencal: Penalized Regression Calibration (PRC) for the Dynamic Prediction of Survival

Computes penalized regression calibration (PRC), a statistical method for the dynamic prediction of survival when many longitudinal predictors are available. PRC is described in Signorelli et al. (2021) <a href="https://doi.org/10.1002/sim.9178">doi:10.1002/sim.9178</a>> and Signorelli (2023) <a href="https://doi.org/10.1002/sim.9178">doi:10.1002/sim.9178</a> and Signorelli (2023) <a href="https://doi.org/10.10

 $\begin{tabular}{ll} Version: & 2.1.1 \\ Depends: & R ($\geq$ 4.1.0) \\ \end{tabular}$ 

Imports: doParallel, dplyr, foreach, glmnet, lcmm, magic, MASS, Matrix, methods, nlme, purrr, riskRegression, stats, survcomp,

survival, survivalROC

Suggests: knitr, ptmixed, rmarkdown, survminer

Published: 2023-10-27

Author: Mirko Signorelli [6] [aut, cre, cph], Pietro Spitali [ctb], Roula Tsonaka [ctb], Barbara Vreede [ctb]

Maintainer: Mirko Signorelli <msignorelli.rpackages at gmail.com>

License:  $\underline{GPL} (\geq 3)$ 

URL: <a href="https://mirkosignorelli.github.io/r">https://mirkosignorelli.github.io/r</a>

NeedsCompilation: no

Citation: pencal citation info

Materials: NEWS

CRAN checks: pencal results

Documentation:

Reference manual: pencal.pdf

Vignettes: pencal: an R Package for the Dynamic Prediction of Survival with Many Longitudinal Predictors

#### Example dataset



```
library(pencal) |> suppressWarnings()
data(pbc2data)
sdata = pbc2data$baselineInfo
ldata = pbc2data$longitudinalInfo
```

- ▶ Data from the PBC2 clinical trial (1974-1984)
  - ightharpoonup n = 312, k = 3, p = 7
  - Outcome: time to death
  - Follow-up up to 14.3 years

#### Data preparation



Let's choose  $\ell = 2$  as landmark:

```
# remove subjects with event / censoring before landmark
lmark = 2
sdata = subset(sdata, time > lmark)
ldata = subset(ldata, id %in% sdata$id)

# remove repeated measurements taken after landmark
ldata = subset(ldata, fuptime <= lmark)</pre>
```

We log-transform some highly-skewed predictors:

```
ldata$logSerBil = log(ldata$serBilir)
ldata$logSerChol = log(ldata$serChol)
ldata$logAlk = log(ldata$alkaline)
ldata$logSGOT = log(ldata$SGOT)
ldata$logProthr = log(ldata$prothrombin)
```

#### Inputs



1. A dataset (ldata) with the longitudinal covariates measured up to the landmark time:

```
age fuptime logSerBil logSerChol albumin logAlk
##
      id
## 3
       2 56.45
                  0.00
                             0.10
                                         5.71
                                                 4.14
                                                        8.91
## 4
       2 56.95
                  0.50
                            -0.22
                                           NA
                                                 3.60
                                                        7.65
## 5
       2 57.45 1.00
                             0.00
                                           NA
                                                 3.55
                                                        7.44
## 16
       4 54.74
                0.00
                             0.59
                                         5.50
                                                 2.54
                                                        8.72
## 17
       4 55.26
                0.51
                             0.47
                                           NA
                                                 2.88
                                                        7.07
## 18
       4 55.76
                  1.02
                             0.53
                                           NA
                                                 2.80
                                                        7.05
## 19
       4 56.74
                  2.00
                             1.16
                                           NA
                                                 2.92
                                                        7.07
      logSGOT platelets logProthr
##
## 3
         4.73
                     221
                              2.36
## 4
         4.94
                     188
                              2.40
## 5
         4.97
                     161
                              2.45
## 16
         4.10
                     183
                              2.33
## 17
         5.13
                     240
                              2.94
## 18
         5.11
                     251
                              2.45
## 19
         5.12
                     220
                              2.38
```

#### Inputs

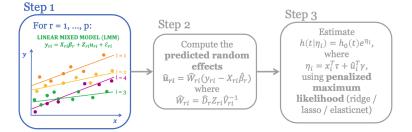


2. A dataset (sdata) with the survival outcome, and baseline covariates:

```
time event baselineAge
##
      id
                                        sex treatment
       2 14.152338
## 3
                            56.44782 female D-penicil
##
          2.770781
                                       male D-penicil
                            70.07447
  16
          5.270507
                            54.74209 female D-penicil
##
  23
      5 4.120578
                            38.10645 female placebo
##
## 29
         6.853028
                            66.26054 female
                                              placebo
## 35
          6.847552
                            55.53609 female
                                              placebo
```

#### Step 1: estimating the LMMs





#### Step 1: estimating the LMMs

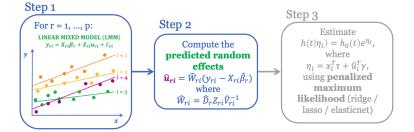


#### Extracting output from the fitted LMMs



```
getlmm(step1, yname = 'logSerBil', what = 'betas') |> round(6)
## (Intercept) age
##
     0.518320 -0.001045
getlmm(step1, yname = 'logSerBil', what = 'tTable') |> round(4)
##
        Value Std.Error DF t-value p-value
## (Intercept) 0.5183 0.2788 566 1.8590 0.0636
## age -0.0010 0.0055 566 -0.1884 0.8506
getlmm(step1, yname = 'logSerBil', what = 'variances')
## id = pdLogChol(age)
             Variance StdDev Corr
##
## (Intercept) 7.332118e-01 0.856277849 (Intr)
## age 4.731627e-05 0.006878682 0.103
## Residual 1.437622e-01 0.379159888
```







```
step2 = summarize_lmms(step1, verbose = F)
```

► Handy: summarize\_lmms automatically inherits relevant arguments from fit\_lmms ©

#### Step 2: sample output

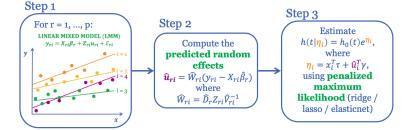


#### round(step2\$ranef.orig[1:5, 1:6], 6)

```
##
    logSerBil_b_int logSerBil_b_age logSerChol_b_int
## 2
         -0.382988
                   -0.001661
                               -0.071154
## 3
         -0.117107 -0.000584
                                   -0.598453
## 4
         0.168600
                     0.000922
                                   -0.370434
## 5
                     0.001170
                                   -0.291031
          0.380035
## 6
         -0.473763 -0.002305 -0.248214
##
    logSerChol_b_age albumin_b_int albumin_b_age
## 2
          0.000660
                   0.179725
                                   3.0e-06
## 3
          0.004916
                  0.018124
                                   1.0e-06
## 4
          0.003468 -0.529776 -7.0e-06
## 5
          0.002886
                     -0.148329
                                  8.0e-06
## 6
          0.002136
                      0.292353
                                   1.7e-05
```

#### Step 3: estimate the penalized Cox model





#### Step 3: estimate the penalized Cox model



```
step3 = fit_prclmm(step2, surv.data = sdata,
    baseline.covs = ~ baselineAge + sex + treatment,
    penalty = 'ridge', standardize = T, verbose = F)
```

## Step 3: fitted model



#### print(step3, digits = 3)

```
## Fitted model: PRC-LMM
## Penalty function used: ridge
## Sample size: 278
## Number of events: 107
## Bootstrap optimism correction: not computed
## Penalized likelihood estimates (rounded to 3 digits):
     baselineAge sexfemale treatmentD-penicil logSerBil_b_int
##
## 1
            0.05
                    -0.296
                                       -0.002
                                                         0.514
##
     logSerBil b age logSerChol b int logSerChol b age
## 1
             130.985
                                0.082
                                                 -9.015
##
     albumin b int albumin b age logAlk b int logAlk b age
## 1
             -1.36
                        29474.99
                                        0.084
                                                    -6.839
     logSGOT_b_int logSGOT_b_age platelets_b_int
##
## 1
             0.207
                         265,467
                                          -0.001
##
     platelets_b_age logProthr_b_int logProthr_b_age
## 1
               -0.13
                               2.934
                                            -366, 168
```



The problem: dynamic prediction of surviva

The method: Penalized Regression Calibration

The R package: pencal

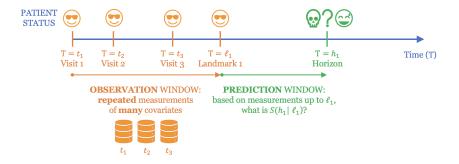
Prediction of survival

Evaluation of predictive performance

Package overview

# Back to our goal: predicting survival





#### Prediction of survival



```
Shat = survpred_prclmm(step1, step2, step3, times = 3:5)
```

► This will compute  $\hat{S}(t|2)$ , t = 3, 4, 5:

#### head(Shat\$predicted\_survival, 4)

```
## id S(3) S(4) S(5)

## 2 2 0.9517590 0.9065357 0.8590626

## 3 3 0.8623498 0.7453465 0.6344383

## 4 4 0.8181530 0.6714482 0.5397427

## 5 5 0.9449673 0.8937427 0.8403668
```

Prediction for new subjects? Possible through additional arguments new.longdata and new.basecovs



The problem: dynamic prediction of surviva

The method: Penalized Regression Calibration

The R package: pencal

Prediction of survival

Evaluation of predictive performance

Package overview



#### Internal validation



- ▶ Performance measures: time-dependent AUC, C index, Brier score
- ▶ Internal validation of predictive performance:
  - cluster bootstrap optimism correction procedure (Signorelli et al. (2021))  $\rightarrow$  appendix
  - repeated cross-validation also possible as an alternative

## Computing the CBOCP



➤ To compute the cluster bootstrap optimism correction procedure, rerun steps 1, 2 and 3 specifying nboots = B > 0 inside fit\_lmms:

- ▶ NB: n.boots needs to be specified just in step 1, but it is used also in steps 2 and 3
- ▶ n.cores allows you to parallelize computations within each step!

### Computing the performance measures



### Predictive performance



#### predPerf

```
## $call
## performance_prc(step2 = step2b, step3 = step3b, metric = c("tdauc",
       "brier"), times = 3:5, n.cores = 8, verbose = F)
##
##
## $tdAUC
##
     pred.time tdAUC.naive optimism.correction tdAUC.adjusted
## 1
             3
                    0.9434
                                        -0.0065
                                                        0.9369
## 2
             4
                    0.9348
                                        -0.0155
                                                       0.9193
## 3
                   0.9273
                                        -0.0133
                                                       0.9140
##
## $Brier
##
     pred.time Brier.naive optimism.correction Brier.adjusted
## 1
             3
                    0.0556
                                         0.0130
                                                        0.0686
## 2
             4
                    0.0679
                                         0.0243
                                                        0.0922
## 3
                    0.0823
                                         0.0291
                                                        0.1114
```



The problem: dynamic prediction of surviva

The method: Penalized Regression Calibration

The R package: pencal

Prediction of survival

Evaluation of predictive performance

Package overview



# Package overview table



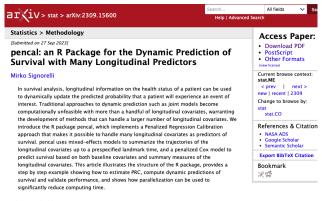
**Table 1:** Overview of the pencal functions that implement the different modelling steps for the PRC LMM and PRC MLPMM approaches.

Task	PRC LMM	PRC MLPMM
Step 1: estimate the mixed-effects models	fit_lmms	fit_mlpmms
Step 2: compute the predicted random effects	summarize_lmms	summarize_mlpmms
Step 3: estimate the penalized Cox model	fit_prclmm	fit_prcmlpmm
Computation of predicted survival probabilities	survpred_prclmm	survpred_prcmlpmm
Evaluation of predictive performance	performance_prc	performance_prc

### More about pencal



▶ Vignette (Signorelli (2023)) available at 🗹 arXiv:2309.15600:



Subjects: Methodology (stat.ME); Computation (stat.CO)
Cite as: arXiv:2309.15600 [stat.ME]
(or arXiv:2309.15600v1 [stat.ME] for this version)
https://doi.org/10.48550/arXiv.2309.15600 1



#### References I



- Proust-Lima, C., Amieva, H., & Jacqmin-Gadda, H. (2013). Analysis of multivariate mixed longitudinal data: A flexible latent process approach. *British Journal of Mathematical and Statistical Psychology*, 66(3), 470–487.
- Signorelli, M. (2023). pencal: an R Package for the Dynamic Prediction of Survival with Many Longitudinal Predictors. *arXiv Preprint arXiv:2309.15600*.
- Signorelli, M., Ayoglu, B., Johansson, C., Lochmüller, H., Straub, V., Muntoni, F., Niks, E., Tsonaka, R., Person, A., Aartsma-Rus, A., Nilsson, P., Al-Khalili Szigyarto, C., & Spitali, P. (2020). Longitudinal serum biomarker screening identifies MDH2 as candidate prognostic biomarker for Duchenne muscular dystrophy. *Journal of Cachexia, Sarcopenia and Muscle*, 11(2), 505–517.
- Signorelli, M., Spitali, P., Al-Khalili Sgyziarto, C., The Mark-MD Consortium, & Tsonaka, R. (2021). Penalized regression calibration: A method for the prediction of survival outcomes using complex longitudinal and high-dimensional data. *Statistics in Medicine*.

vnamic prediction of survival

pencal

The method: Penalized Regression Calibration

The R package: pencal

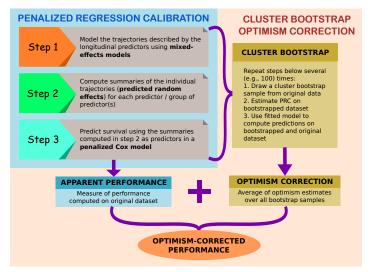
Prediction of survival

Evaluation of predictive performance

Package overview

### Cluster-bootstrap optimism correction





 $\rightarrow$  go back

Vignette: bit.ly/pencal-CMS

