Statistical methods for the dynamic prediction of survival in settings with numerous longitudinal predictors

Mirko Signorelli

★: mirkosignorelli.github.io

✓: @signormirko

Mathematical Institute Leiden University

April 3, 2024 Dipartimento di Scienze Statistiche, UniPD







- Assistant prof. in Statistics at the Mathematical Institute of Leiden University (NL)
- ▶ PhD between University of Padova (IT) and of Groningen (NL)
- Research interests:

- Assistant prof. in Statistics at the Mathematical Institute of Leiden University (NL)
- ▶ PhD between University of Padova (IT) and of Groningen (NL)
- Research interests:
 - statistical modelling of networks (graphs & hypergraphs)

- Assistant prof. in Statistics at the Mathematical Institute of Leiden University (NL)
- ▶ PhD between University of Padova (IT) and of Groningen (NL)
- Research interests:
 - statistical modelling of networks (graphs & hypergraphs)

 VENERDI' 5 APRILE 2024 ORE 9.00 IN AULA SC140

 COMPLESSO SANTA CATERINA VIA CESARE BATTISTI. 241

Orario	rio N. Corso di Laurea Matricola COGNOME		COGNOME	TITOLO DELLA TESI	
		Lauree Magistrali			
09:00	1	SS1736	2053005	CALORE ALBERTO	Metodi di bilanciamento in presenza di trattamenti multipil: MARMOT e Template Matching a confronto
09:30	2	SS1736	2058300	DRIUSSO EUGENIA	Statistical modelling of time-stamped hypergraphs: a model-based clustering approach
10:00	3	SS1736	2053208	FRAULINI ENRICO	Effetto delle differenze di reddito sulle elezioni presidenziali statunitensi

- Assistant prof. in Statistics at the Mathematical Institute of Leiden University (NL)
- ▶ PhD between University of Padova (IT) and of Groningen (NL)
- Research interests:
 - statistical modelling of networks (graphs & hypergraphs)
 - longitudinal data analysis
 - survival analysis
 - biomedical applications

The dynamic prediction problem

Dynamic prediction in high dimensions

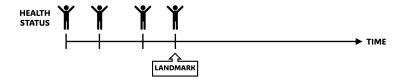
Benchmarking

The R package pencal

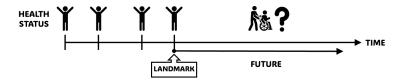
Prediction of survival

Conclusion

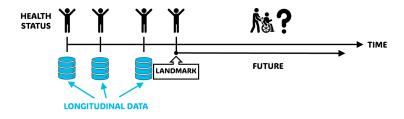
Appendix



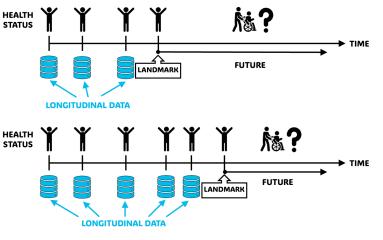
 $\rightarrow \mathsf{example}$



 \rightarrow example



 $\rightarrow \mathsf{example}$



ightarrow example

The goal(s)

Goals of dynamic prediction:

- ▶ predict future survival $S(t|\ell_1) = P(T > t|T > \ell_1)$, $t > \ell_1$, using repeated measurements over $[0, \ell_1]$
- update predictions when newer information becomes available, i.e. update $S(t|\ell_2)$ given repeated measurements over $[0,\ell_2],$ $t>\ell_2>\ell_1$

The problem

- ► Traditional methods for dynamic prediction:
 - joint models: very computationally-intensive. Can't usually be estimated with more than 3-5 longitudinal predictors!
 - ► landmarking with LOCF¹: no modelling of the longitudinal trajectories + no measurement error correction (important for biomarkers)
- Problem: nowadays, longitudinal studies can comprise tens, hundreds, or even thousands longitudinal predictors ("biomarkers")
- ► How to do dynamic prediction with "many" longitudinal predictors?

¹LOCF = Last Observation Carried Forward

The dynamic prediction problem

Dynamic prediction in high dimensions

Benchmarking

The R package pencal

Prediction of survival

Conclusion

Appendix

Recent methodological solutions

Several solutions proposed over the last 5 years:

- 1. Li & Luo (2019): MFPCCox
- 2. Signorelli et al. (2021)²: Penalized Regression Calibration (PRC)
- 3. Lin et al. (2021): Functional Random Survival Forest (FunRSF)
- 4. Devaux et al. (2023): DynForest

 $^{^{2}}$ but see Signorelli (2023) for a better explanation of the dynamic prediction setting

Modelling approaches

- ► Two-step modelling:
 - 1. model trajectories of longitudinal predictors over $[0,\ell]$
 - 2. use summaries of longitudinal predictors to predict $S(t|\ell),\ t\geq \ell$

		Longitudinal covariates		
		Multivariate Functional PCA	Mixed Effects Models	
Survival	Cox PH model	MFPCCox (Li and Luo, 2019)	Penalized Regression Calibration (Signorelli et al., 2021)	
outcome	Random survival forest	Functional Random Survival Forest (Lin et al., 2021)	DynForest (Devaux et al., 2023)	

Notation

Covariates:

- ▶ P baseline covariates $x_i = (x_{1i}, ..., x_{Pi})$ measured at $t_{i1} = 0$
- ▶ Q longitudinal covariates measured at $t_{i1}, t_{i2}, ..., t_{im_i}$
- $y_{qij} = y_{qi}(t_{ij})$

Survival outcome:

- $ightharpoonup T_i$ time-to-event outcome for subject i
- \triangleright δ_i event indicator:
 - $\delta_i = 1$: event observed at $T_i = t_i$
 - $\delta_i = 0$: right-censoring at $T_i = t_i$

The 4 bricks

		Longitudinal covariates		
		Multivariate Functional PCA	Mixed Effects Models	
Survival	Cox PH model	MFPCCox (Li and Luo, 2019)	Penalized Regression Calibration (Signorelli et al., 2021)	
outcome	Random survival forest	Functional Random Survival Forest (Lin et al., 2021)	DynForest (Devaux et al., 2023)	

- ▶ We need 4 "bricks":
 - 1. MFPCA / mixed-effects models
 - 2. Cox model / random survival forest

Building block 1: MFPCA

▶ MFPCA (Happ & Greven, 2018) decomposition:

$$y_{qij} = y_{qi}(t_{ij}) \approx \mu_q(t_{ij}) + \sum_{k=1}^K \rho_{ki} \psi_{kq}(t_{ij}), \ i \in \mathcal{I}(\ell), \ t_{ij} \leq \ell,$$
 (1)

where:

- $\psi_k(t) = (\psi_{k1}(t), ..., \psi_{kQ}(t))$ are Q-variate orthonormal eigenfunctions
- ho_{ki} are subject-specific MFPCA scores (shared across $Y_1, ..., Y_Q$)
- eigenfunctions are ordered by decreasing percentage of variance explained (PVE)
- ightharpoonup K chosen so that eigenfunctions explain a certain PVE of the original variables (usually: 90 / 95%)

Building block 2: linear mixed models (LMMs)

Linear mixed model (LMM):

$$y_{qij} = y_{qi}(t_{ij}) = W_{qi}(t_{ij})\beta_q + Z_{qi}(t_{ij})u_{qi} + \varepsilon_{qi}, \ i \in \mathcal{I}(\ell), \ t_{ij} \leq \ell, \ (2)$$

where:

- $ightharpoonup W_{qi}(t_{ij}), Z_{qi}(t_{ij})$ are design matrices
- ▶ $u_{qi} \sim N(0, \Sigma_q) \rightarrow \text{random effects}$
- $ightharpoonup arepsilon_{qi} \sim N(0, \sigma_q^2)$
- ▶ $\beta_q, \Sigma_q, \sigma_q^2 \rightarrow \text{fixed effects}$

Building block 3: Cox proportional-hazards model

Cox model (Cox, 1972):

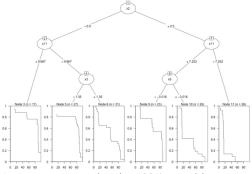
$$h_i(t) = h_0(t) \exp\left(\sum_j \gamma_j z_j\right),$$
 (3)

where:

- ▶ $h_i(t) = \lim_{h\to 0^+} P(T_i \in [t, t+h))$ is the hazard function
- \blacktriangleright $h_0(t)$ is the baseline hazard
- $ightharpoonup z_j$ covariate with regression coefficient γ_j

Building block 4: random survival forest

- ▶ Draw B bootstrap samples from data
- ▶ For b = 1, ..., B:
 - specify tuning parameters, e.g. number of candidate covariates for node splitting, minimum node size, minimum number of events...
 - build a survival tree

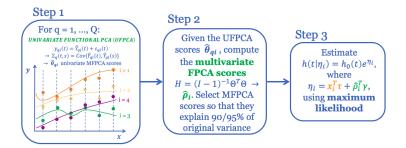


Building block 4: random survival forest

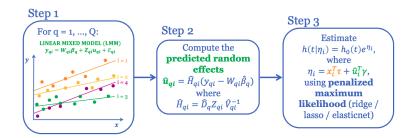
- Draw B bootstrap samples from data
- ▶ For b = 1, ..., B:
 - specify tuning parameters, e.g. number of candidate covariates for node splitting, minimum node size, minimum number of events...
 - build a survival tree
 - predicted survival for subject i with covariates z_i given by cumulative hazard function (CHF) of the terminal node where i ends, $\hat{H}_b(t|x_i)$ $(\rightarrow \hat{S}(t) = \exp \hat{H}(t))$
- Average CHF predictions across trees:

$$\hat{H}(t|z_i) = \frac{1}{B} \sum_{b=1}^{B} \hat{H}_b(t|x_i)$$
 (4)

MFPCCox (Li & Luo, 2019)

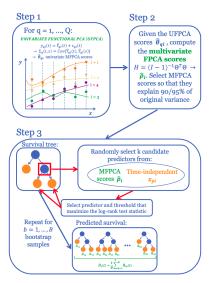


Penalized Regression Calibration (PRC, Signorelli et al. (2021))

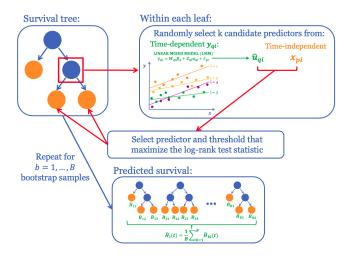


▶ Steps 1-2: multivariate version with MLPMM possible

Functional Survival Random Forest (FunRSF, Lin et al. (2021))



DynForest (Devaux et al., 2023)



The dynamic prediction problem

Dynamic prediction in high dimensions

Benchmarking

The R package pencal

Prediction of survival

Conclusion

Appendix

Motivation

- ▶ Methods proposed quite recently: between 2019 and 2023
- Very little knowledge about their performance with real data from longitudinal studies
- ► Let's compare them!

Preprint (Signorelli & Retif, 2024)



An empirical appraisal of methods for the dynamic prediction of survival with numerous longitudinal predictors

Signorelli Mirko, Sophie Retif

Recently, the increasing availability of repeated measurements in biomedical studies has motivated the development of several statistical methods for the dynamic prediction of survival in settings where a large (potentially high-dimensional) number of longitudinal covariates is available. These methods differ in both how they model the longitudinal covariates trajectories, and how they specify the relationship between the longitudinal covariates and the survival outcome. Because these methods are still quite new, little is known about their applicability, limitations and performance when applied to real-world data. To investigate these questions, we present a comparison of the predictive performance of the aforementioned methods and two simpler prediction approaches to three datasets that differ in terms of outcome type, sample size, number of longitudinal covariates and length of follow-up. We discuss how different modelling choices can have an impact on the possibility to accommodate unbalanced study designs and on computing time, and compare the predictive performance of the different approaches using a range of performance measures and landmark times.

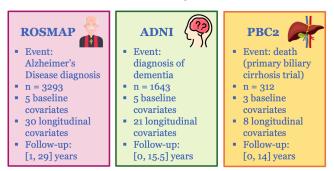
Subjects: Methodology (stat.ME); Applications (stat.AP)

Cite as: arXiv:2403.14336 [stat.ME]

(or arXiv:2403.14336v1 [stat.ME] for this version)

Longitudinal studies

We considered data from three longitudinal studies:

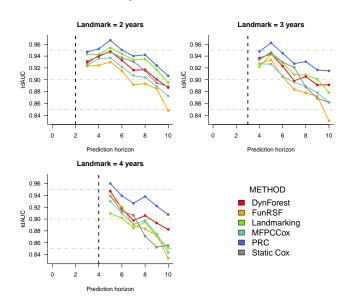


- Methods included: MFPCCox, PRC, FunRSF, DynForest + static Cox + LOCF landmarking
- Performance evaluated at multiple landmark times
- ▶ Performance measures: C index, tdAUC, Brier score
- ▶ 10-fold cross-validation, repeated 10 times

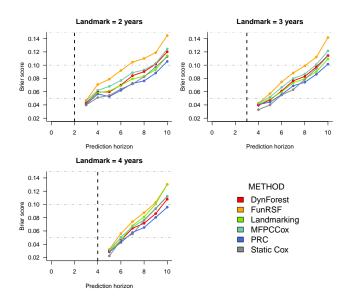
ADNI dataset: C index

		Landmark	
Method	2	3	4
Static Cox	0.901 (0.002)	0.885 (0.006)	0.856 (0.008)
Landmarking	0.906 (0.001)	0.89 (0.004)	0.855 (0.009)
MFPCCox	0.889 (0.003)	0.872 (0.009)	0.859 (0.008)
PRC	0.913 (0.001)	0.908 (0.003)	0.904 (0.003)
FunRSF	0.873 (0.004)	0.858 (0.011)	0.845 (0.012)
DynForest	0.891 (0.003)	0.883 (0.005)	0.871 (0.011)

ADNI dataset: time-dependent AUC



ADNI dataset: Brier score



Results overview

- ▶ PRC, landmarking, DynForest > MFPCCox, static Cox, FunRSF
- Methods that use LMMs > methods using MFPCA
- Conditionally on method used to model longitudinal predictors (MFPCA / LMMs), methods that use Cox model > methods that use RSF
- ► LOCF landmarking often second / third best model
- ▶ Relative performance of landmarking and static Cox worsens with higher landmark & horizon times

Limitations

- lacktriangle MFPCA-based methods: regular measurement grid required ightarrow unrealistic & unflexible
- LMM-based methods: using GLMMs would allow for more modelling flexibility
- Methods using RSF: need to choose value of multiple tuning parameters
- Competing risks: only in DynForest
- Interval censoring: none of the methods
- Software
 - MFPCCox, FunRSF: no software implementation!
 - ightharpoonup PRC ightarrow pencal, DynForest

The dynamic prediction problem

Dynamic prediction in high dimensions

Benchmarking

The R package pencal

Prediction of survival

Conclusion

Appendix

Where to find the package

- PRC implemented in the R package pencal
- Available on CRAN:

```
pencal: Penalized Regression Calibration (PRC) for the Dynamic Prediction of Survival
```

Computes penalized regression calibration (PRC), a statistical method for the dynamic prediction of survival when many longitudinal predictors are available. PRC is described in Signorelli et al. (2021) doi:10.1082/sim.9178> and Signorelli (2023) doi:10.1085/s0/arXiv.2309.1560/05

Version: 2.2.1 Depends: R (≥ 4.1.0)

Imports: doParallel, dplyr, foreach, glmnet, lcmm, magic, MASS, Matrix, methods, nlme, purrr, riskRegression, stats, survival, survival,

knitr, ptmixed, rmarkdown, survminer

Published: 2024-03-31

Author: Mirko Signorelli 👵 [aut, cre, cph], Pietro Spitali [ctb], Roula Tsonaka [ctb], Barbara Vreede [ctb]

Maintainer: Mirko Signorelli <msignorelli.rpackages at gmail.com>

License: $\underline{GPL} (\geq 3)$

URL: https://mirkosignorelli.github.io/r

NeedsCompilation: no

Suggests:

Citation: pencal citation info
Materials: NEWS

CRAN checks: pencal results

Documentation:

Reference manual: pencal.pdf

Vignettes: pencal: an R Package for the Dynamic Prediction of Survival with Many Longitudinal Predictors

Example dataset

- ▶ Data from the PBC2 clinical trial (1974-1984)
 - ightharpoonup n = 312, P = 3 baseline and Q = 7 longitudinal predictors
 - Outcome: time to death
 - Follow-up up to 14.3 years

```
library(pencal)
data(pbc2data)
sdata = pbc2data$baselineInfo
ldata = pbc2data$longitudinalInfo
```

Data preparation

Let's choose $\ell = 2$ as landmark:

```
# remove subjects with event / censoring before landmark
lmark = 2
sdata = subset(sdata, time > lmark)
ldata = subset(ldata, id %in% sdata$id)

# remove repeated measurements taken after landmark
ldata = subset(ldata, fuptime <= lmark)</pre>
```

We log-transform some highly-skewed predictors:

```
ldata$logSerBil = log(ldata$serBilir)
ldata$logSerChol = log(ldata$serChol)
ldata$logAlk = log(ldata$alkaline)
ldata$logSGOT = log(ldata$SGOT)
ldata$logProthr = log(ldata$prothrombin)
```

Inputs

 A dataset (ldata) with the longitudinal covariates measured up to the landmark time:

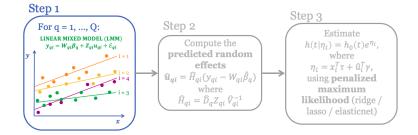
```
##
      id
           age fuptime logSerBil logSerChol albumin logAlk
## 3
       2 56.45
                  0.00
                            0.10
                                        5.71
                                                4.14
                                                       8.91
## 4
       2 56.95
                  0.50
                           -0.22
                                          NA
                                                3.60
                                                       7.65
## 5
       2 57.45
                1.00
                            0.00
                                          NA
                                                3.55
                                                       7.44
## 16
      4 54.74
                0.00
                            0.59
                                        5.50
                                                2.54
                                                       8.72
## 17
       4 55.26
                0.51
                            0.47
                                          NA
                                                       7.07
                                                2.88
## 18
       4 55.76
                1.02
                            0.53
                                          NA
                                                2.80
                                                       7.05
## 19
       4 56.74
                  2.00
                            1.16
                                          NΑ
                                                2.92
                                                       7.07
      logSGOT platelets logProthr
##
## 3
         4.73
                    221
                             2.36
## 4
         4.94
                    188
                             2.40
## 5
         4.97
                    161
                             2.45
## 16
        4.10
                    183
                             2.33
## 17
        5.13
                             2.94
                    240
## 18
         5.11
                    251
                             2.45
## 19
         5.12
                    220
                             2.38
```

Inputs

2. A dataset (sdata) with the survival outcome, and baseline covariates:

```
id
            time event baselineAge sex treatment
##
                           56.44782 female D-penicil
## 3
      2 14.152338
## 12 3 2.770781
                           70.07447
                                     male D-penicil
## 16
        5.270507
                           54.74209 female D-penicil
## 23 5 4.120578
                           38.10645 female
                                            placebo
## 29
      6 6.853028
                          66.26054 female placebo
## 35 7
         6.847552
                           55.53609 female
                                            placebo
```

Step 1: estimating the LMMs

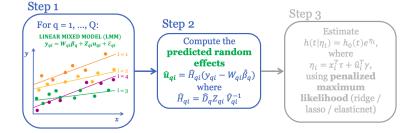


Step 1: estimating the LMMs

Extracting output from the fitted LMMs

```
summary(step1, vname = 'logSerBil', what = 'betas') |> round(6)
## (Intercept) age
     0.518320 -0.001045
##
summary(step1, yname = 'logSerBil', what = 'tTable') |> round(4)
##
              Value Std.Error DF t-value p-value
## (Intercept) 0.5183 0.2788 566 1.8590 0.0636
## age -0.0010 0.0055 566 -0.1884 0.8506
summary(step1, yname = 'logSerBil', what = 'variances')
## id = pdLogChol(age)
             Variance StdDev Corr
##
## (Intercept) 7.332118e-01 0.856277849 (Intr)
## age 4.731627e-05 0.006878682 0.103
## Residual 1.437622e-01 0.379159888
```

Step 2: computing the predicted random effects



Step 2: computing the predicted random effects

```
step2 = summarize_lmms(step1)
```

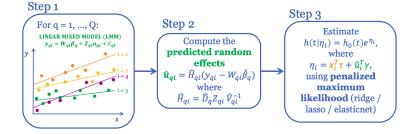
► Handy: summarize_lmms automatically inherits relevant arguments from fit_lmms ⊕

Step 2: sample output

round(step2\$ranef.orig[1:5, 1:6], 6)

```
##
    logSerBil_b_int logSerBil_b_age logSerChol_b_int
## 2
          -0.382988
                       -0.001661
                                          -0.071154
          -0.117107
## 3
                        -0.000584
                                         -0.598453
## 4
           0.168600
                         0.000922
                                         -0.370434
## 5
                          0.001170
                                       -0.291031
           0.380035
## 6
          -0.473763
                        -0.002305
                                         -0.248214
##
    logSerChol_b_age albumin_b_int albumin_b_age
## 2
            0.000660
                         0.179725
                                        3.0e-06
## 3
            0.004916
                         0.018124
                                        1.0e-06
## 4
            0.003468
                     -0.529776
                                       -7.0e-06
## 5
            0.002886
                         -0.148329
                                        8.0e-06
## 6
            0.002136
                         0.292353
                                        1.7e-05
```

Step 3: estimate the penalized Cox model



Step 3: estimate the penalized Cox model

```
step3 = fit_prclmm(step2, surv.data = sdata,
    baseline.covs = ~ baselineAge + sex + treatment,
    penalty = 'ridge', standardize = T)
```

Step 3: fitted model

summary(step3)

```
## Fitted model: PRC-LMM
## Penalty function used: ridge
## Tuning parameters:
       lambda alpha
##
## 1 0.2126761
## Sample size: 278
## Number of events: 107
## Bootstrap optimism correction: not computed
## Penalized likelihood estimates (rounded to 4 digits):
     baselineAge sexfemale treatmentD-penicil logSerBil_b_int
##
## 1
          0.0476 - 0.2872
                                      -0.0157
                                                       0.4341
     logSerBil_b_age logSerChol_b_int logSerChol_b_age
##
## 1
            111.3935
                               0.0986
                                              -10.5311
##
     albumin b int albumin b age logAlk b int logAlk b age
                        23070.92
                                       0.0874
                                                  -12.5617
## 1
           -1.1361
##
     logSGOT_b_int logSGOT_b_age platelets_b_int
## 1
             0.238
                         272,246
                                         -0.0011
##
    platelets_b_age logProthr_b_int logProthr_b_age
## 1
            -0.2046
                              2.8114
                                           -573.3093
```

The dynamic prediction problem

Dynamic prediction in high dimensions

Benchmarking

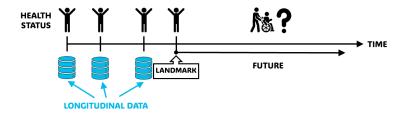
The R package pencal

Prediction of survival

Conclusion

Appendix

Back to our goal: predicting survival



Prediction of survival

```
Shat = survpred_prclmm(step1, step2, step3, times = 3:5)
```

► This will compute $\hat{S}(t|2)$, t = 3, 4, 5:

```
head(Shat$predicted_survival, 4) |> dfround(3)
```

```
## id S(3) S(4) S(5)
## 2 2 0.940 0.887 0.833
## 3 3 0.856 0.739 0.632
## 4 4 0.814 0.671 0.545
## 5 5 0.946 0.898 0.849
```

- Prediction for new subjects? Possible through additional arguments new.longdata and new.basecovs
- ► Evaluation of predictive performance: see → Appendix

The dynamic prediction problem

Dynamic prediction in high dimensions

Benchmarking

The R package pencal

Prediction of survival

Conclusion

Appendix

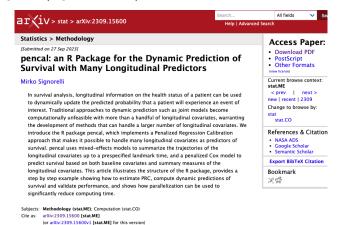
Package overview table

Table 1: Overview of the pencal functions that implement the different modelling steps for the PRC LMM and PRC MLPMM approaches.

Task	PRC LMM	PRC MLPMM
Step 1: estimate the mixed-effects models	fit_lmms	fit_mlpmms
Step 2: compute the predicted random effects	summarize_lmms	summarize_mlpmms
Step 3: estimate the penalized Cox model	fit_prclmm	fit_prcmlpmm
Computation of predicted survival probabilities	survpred_prclmm	survpred_prcmlpmm
Evaluation of predictive performance	performance_prc	performance_prc

More about pencal

▶ Vignette (Signorelli, 2023) available at 🗹 arXiv:2309.15600:



https://doi.org/10.48550/arXiv.2309.15600

44

Limitations & future work

Still plenty of work to do:

- ► GLMMs
- Competing risks
- ► Interval censoring



Preprints:

▶ pencal vignette: ✓arXiv:2309.15600

benchmarking study:

ZarXiv:2403.14336

: mirkosignorelli.github.io

🔰: @signormirko

References I

- Cox, D. R. (1972). Regression Models and Life-Tables. *Journal of the Royal Statistical Society: Series B (Methodological)*, 34(2), 187–202.
- Devaux, A., Proust-Lima, C., & Genuer, R. (2023). Random Forests for time-fixed and time-dependent predictors: The DynForest R package. arXiv:2302.02670.
- Happ, C., & Greven, S. (2018). Multivariate Functional Principal Component Analysis for Data Observed on Different (Dimensional) Domains. *Journal of the American Statistical Association*, 113(522), 649–659.
- Li, K., & Luo, S. (2019). Dynamic prediction of Alzheimer's disease progression using features of multiple longitudinal outcomes and time-to-event data. *Statistics in Medicine*, *38*(24), 4804–4818.
- Lin, J., Li, K., & Luo, S. (2021). Functional survival forests for multivariate longitudinal outcomes: Dynamic prediction of Alzheimer's disease progression. *Statistical Methods in Medical Research*, 30(1), 99–111.
- Signorelli, M. (2023). Pencal: An R Package for the Dynamic Prediction of Survival with Many Longitudinal Predictors. arXiv:2309.15600.

References II

- Signorelli, M., & Retif, S. (2024). An empirical appraisal of methods for the dynamic prediction of survival with numerous longitudinal predictors. arXiv:2403.14336.
- Signorelli, M., Spitali, P., Szigyarto, C. A., The MARK-MD Consortium, & Tsonaka, R. (2021). Penalized regression calibration: A method for the prediction of survival outcomes using complex longitudinal and high-dimensional data. *Statistics in Medicine*, 40(27), 6178–6196.

The dynamic prediction problem

Dynamic prediction in high dimensions

Benchmarking

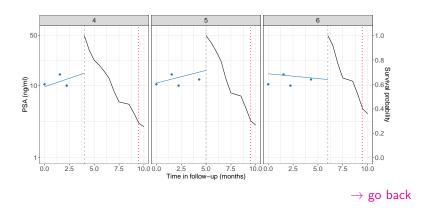
The R package pencal

Prediction of survival

Conclusion

Appendix

Dynamic prediction example





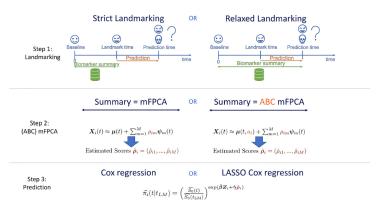


Figure 1. Graphical summary of the methods proposed in Section 2. See also Section 2.6.

Strict vs relaxed landmarking with two-step methods

Original Research Article



Dynamic prediction of survival using multivariate functional principal component analysis: A strict landmarking approach

Daniel Gomon¹ D, Hein Putter² D, Marta Fiocco^{1,2} and Mirko Signorelli¹

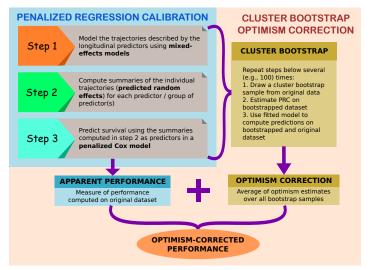
Statistical Methods in Medical Research 2024, Vol. 33(2) 256–272 The Author(s) 2024

Article reuse guidelines:

sagepub.com/journals-permissions DOI: 10.1177/09622802231224631 journals.sagepub.com/home/smm



Cluster-bootstrap optimism correction



ightarrow go back

Internal validation

- ▶ Performance measures: time-dependent AUC, C index, Brier score
- ▶ Internal validation of predictive performance:
 - Cluster bootstrap optimism correction procedure (Signorelli et al. (2021))
 → appendix
 - repeated cross-validation also possible as an alternative

Computing the CBOCP

➤ To compute the cluster bootstrap optimism correction procedure, rerun steps 1, 2 and 3 specifying nboots = B > 0 inside fit_lmms:

- ▶ NB: n.boots needs to be specified just in step 1, but it is used also in steps 2 and 3
- n.cores allows you to parallelize computations within each step!

Computing the performance measures

Predictive performance

predPerf

```
## $call
## performance_prc(step2 = step2b, step3 = step3b, metric = c("tdauc",
       "brier"), times = 3:5, n.cores = 8)
##
##
## $tdAUC
##
     pred.time tdAUC.naive optimism.correction tdAUC.adjusted
## 1
             3
                    0.9439
                                       -0.0056
                                                        0.9383
## 2
             4
                    0.9351
                                       -0.0143
                                                        0.9208
## 3
                    0.9266
                                       -0.0125
                                                        0.9141
##
## $Brier
     pred.time Brier.naive optimism.correction Brier.adjusted
##
## 1
             3
                    0.0571
                                        0.0142
                                                        0.0713
## 2
             4
                    0.0699
                                        0.0266
                                                       0.0965
## 3
                    0.0844
                                        0.0324
                                                        0.1168
```