Introduction to Operation Research (Operation Scheduling)

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INTRODUCTION

… where we see the ORigins and an outline of the OR methodology

What is Operations Research (OR)?

- Operations Research (OR)
  - scientific approach to decision making
  - seeks to determine how best to design and operate a system
  - under conditions requiring the allocation of scarce resources
- Operations Research (OR) is a discipline that applies advanced analytical methods to help make better decisions.
- Provides a set of algorithms that act as tools for effective problem solving
  and decision making

What is Operations Research (OR)?

- Objectives
  - Optimize performance.
  - Improve efficiency.
  - Allocate resources effectively.
- Extensive applications
  - Engineering
  - Business
  - Public systems
  - Industry 4.0
- Used extensively by manufacturing and service industries in decision making

Operations Research vs Machine Learning

Definitions and Objectives:

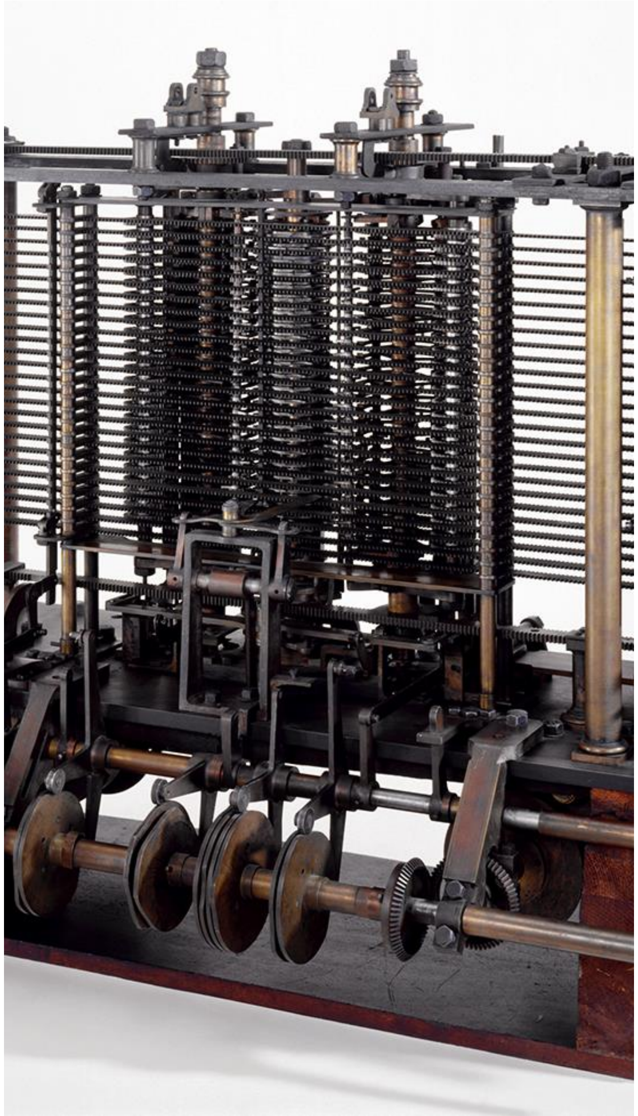
- **Machine Learning**: Focus on developing algorithms that enable computers to learn from and make decisions based on data.
- **Operations Research**: Focus on applying mathematical methods to decision-making, resource allocation, and optimization problems.

History of OR

‘OR’igin

Based on the **history of Operations Research**, it is believed that Charles Babbage (1791-1871) is the father of Operational Research due to the fact that his research into the cost of transportation and sorting of mail resulted in England’s universal Penny Post in 1840.

(Fact: Penny post- every post weighing less than 1 pound was charged 1 Penny)



Father of the Computer

In 1837, Charles Babbage proposed the first general mechanical computer, the Analytical Engine.

The Analytical Engine contained an ALU (Arithmetic Logic Unit), basic flow control, punch cards (inspired by the Jacquard Loom), and integrated memory.

It is the first general-purpose computer concept. Unfortunately, because of funding issues, this computer was also never built while Charles Babbage was alive.

In 1910, Henry Babbage, Charles Babbage's youngest son, was able to complete a portion of this machine and was able to perform basic calculations.

1840
Charles Babbage research into the cost of transportation and sorting of mail

1890
Frederick Taylor Scientific Management [Industrial Engineering]

1900
- Henry Gannt [Project Scheduling]
- Andrey A. Markov [Markov Processes]
- Assignment [Networks]

1910
- F. W. Harris [Inventory Theory]
- E. K. Erlang [Queuing Theory]

1920
- William Shewart [Control Charts]
- H.Dodge – H.Roming [Quality Theory]

1930
Jon Von Neuman – Oscar Morgenstern [Game Theory]

1940
- World War 2
- George Dantzig [Linear Programming]
- First Computer

1950
- H.Kuhn - A.Tucker [Non-Linear Prog.]
- Ralph Gomory [Integer Prog.]
- PERT/CPM
- Richard Bellman [Dynamic Prog.

1960
- John D.C. Litle [Queuing Theory]
- Simscript [Simulation]

1970
- Microcomputer

1980
- H. Karmarkar [Linear Prog.]
- Personal computer
- OR/MS Softwares

1990
- Spreadsheet Packages
- INFORMS

2021-
- You are here
- Age of Big Data, Machine Learning, Artifical Intelliegence

The roots of OR can be traced back many decades, when early attempts were made to use a scientific approach in the management of organizations.

However, the beginning of the activity called operations research has generally been attributed to the military services early in **World War II**.

The name *operations research evolved in the year 1940*.

During World War II, a team of scientists (Blackett's Circus) in UK applied scientific techniques to research military operations to win the war and the techniques thus developed was named as OR.

As a formal discipline, operations research originated from the efforts of army advisors at the time of World War II.

Because of the war effort, there was an urgent need to allocate scarce resources to the various military operations and to the activities within each operation in an effective manner.

Therefore, the British and then the U.S. military management called upon a large number of scientists to apply a scientific approach to dealing with this and other strategic and tactical problems. In effect, they were asked to do research on (military) operations. These teams of scientists were the first OR teams.

By developing effective methods of using the new tool of radar, these teams were instrumental in winning the Air Battle of Britain.

Patrick Blackett worked for several different organizations during the war. Early in the war while working for the Royal Aircraft Establishment (RAE) he set up a team known as the "Circus" which helped to reduce the number of anti-aircraft artillery rounds needed to shoot down an enemy aircraft from an average of over 20,000 at the start of the Battle of Britain to 4,000 in 1941.

Through their research on how to better manage convoy and antisubmarine operations, they also played a major role in winning the Battle of the North Atlantic.

Similar efforts assisted the Island Campaign in the Pacific. When the war ended, the success of OR in the war effort spurred interest in applying OR outside the military as well.

As the industrial boom following the war was running its course, the problems caused by the increasing complexity and specialization in organizations were again coming to the forefront.

It was becoming apparent to a growing number of people, including business consultants who had served on or with the OR teams during the war, that these were basically the same problems that had been faced by the military but in a different context. By the early 1950s, these individuals had introduced the use of OR to a variety of organizations in business, industry, and government. The rapid spread of OR soon followed.

Factors that played a key role Post World War II

One was the substantial progress that was made early in improving the techniques of OR. After the war, many of the scientists who had participated on OR teams or who had heard about this work were motivated to pursue research relevant to the field; important advancements in the state of the art resulted.

A prime example is the simplex method for solving linear programming problems, developed by George Dantzig in 1947. Many of the standard tools of OR, such as linear programming, dynamic programming, queueing theory, and inventory theory, were relatively well developed before the end of the 1950s.

A second factor that gave great impetus to the growth of the field was the onslaught of the computer revolution.

A large amount of computation is usually required to deal most effectively with the complex problems typically considered by OR. Doing this by hand would often be out of the question.

Therefore, the development of electronic digital computers, with their ability to perform arithmetic calculations thousands or even millions of times faster than a human being can, was a tremendous boon to OR.

A further boost came in the 1980s with the development of increasingly powerful personal computers accompanied by good software packages for doing OR. This brought the use of OR within the easy reach of much larger numbers of people.

Today, literally millions of individuals have ready access to OR software. Consequently, a whole range of computers from mainframes to laptops now are being routinely used to solve OR problems.

Operations Research Today

- Large professional society (INFORMS, https://www.informs.org)
- Many publications
  Management Science, Operations Research, Annals of Operations Research, European Journal of Operational Research, Mathematics of Operations Research, OR Today, Interface, IIE Transactions....
- Applications
  - Transportation - How can an airline quickly and efficiently get its flight crews in place to fly following a major disruption to operations?
    Continental airlines: http://www.orms-today.org/orms-6-02/edelman.html
  - Telecommunication – How to reroute traffic in case of link failures? How to design networks with sufficient restoration capacity for rerouting?
    AT&T Network: https://www.informs.org/Sites/Getting-Started-With-Analytics/Analytics-Success-Stories/Case- Studies/AT-T-Network
  - Services – Logistical nightmares of package delivery: Fast, Limited by crew, weight, dimensions, packing capacity Federal Express: http://fedexlegends.info/earlyit/absolutely.pdf
  - Finance – How to invest to maximize return?
    Operations Research and Bank Planning:
    http://www.sciencedirect.com/science/article/pii/0024630175900928
  - Health Care
    Operations Research for Surgical Services:
    http://www.franklindexter.net/PDF%20Files/SurgicalServicesCourse.pdf
  - Sports – Scheduling baseball games: want good games in primetime, want primetime to be equally distributed, …
    http://mat.gsia.cmu.edu/trick/tourn_final.pdf

Key Concepts

- **Optimization**
  - Finding the best solution from a set of feasible solutions.
  - Types: Linear programming, nonlinear programming, integer programming.
- **Modeling**
  - Creating mathematical representations of real-world systems.
  - Example: Queueing models, inventory models.
- **Simulation**
  - Using models to simulate complex systems and analyze their behavior.
  - Example: Monte Carlo simulation

Techniques in Operations Research

- **Linear Programming (LP)**
  - Solving optimization problems where the objective function and constraints are linear.
  - Example: Maximizing profit, minimizing cost.
- **Integer Programming (IP)**
  - Optimization where some or all variables are constrained to be integers.
  - Example: Scheduling, resource allocation.
- **Network Models**
  - Analyzing and optimizing network flows.
  - Example: Transportation, supply chain networks.
- **Decision Analysis**
  - Evaluating and comparing different decision options.
  - Example: Decision trees, utility theory.

Operations … what??

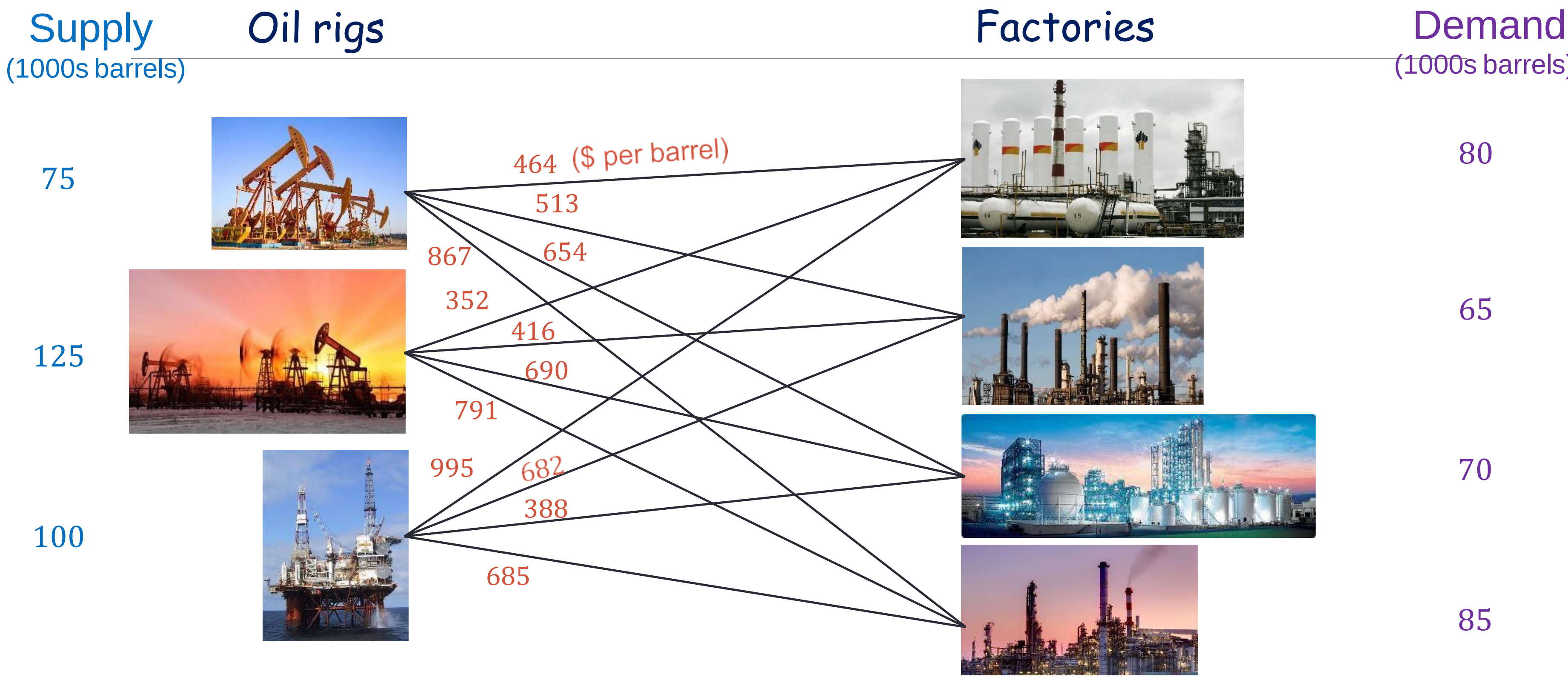
- Cancer treatment machine sends ionized radiation beam through a patient's body
- Use two beams

	Fraction of Entry Dose Absorbed by area	
Area	Beam 1	Beam 2
Healthy Tissues	0.4	0.5
Critical Tissues	0.3	0.1
Tumor Tissues	0.5	0.5
Tumor Center Tissues	0.6	0.4

Question: What is the dosage of beams 1 and 2 that minimizes the damage to the healthy tissues, does not over-damage the critical tissues, kills the tumor and the tumor center tissues?

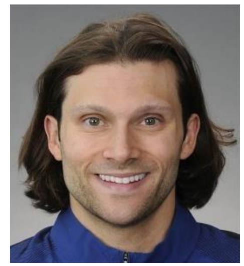
Example 2: Transportation

Operations … what??



Question: How much to ship from each supply point to each demand point to meet the demands while minimizing shipping cost?

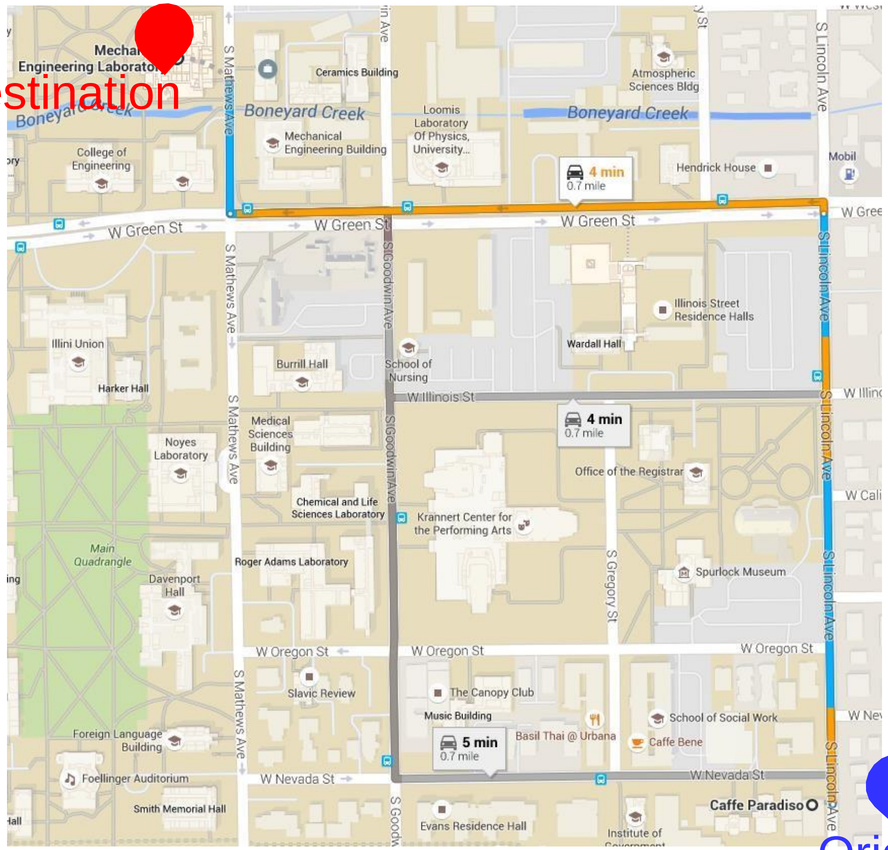
Operations … what??

	Adrian	Miller	Phelps	Murphy
Backstroke	$51.77s$	$51.99s$	$52.33s$	$51.85s$
Breaststroke	$58.86s$	$58.87s$	$58.91s$	$58.95s$
Butterfly	$51.59s$	$51.17s$	$51.14s$	$51.83s$
Freestyle	$47.85s$	$48.93s$	$48.01s$	$49.31s$

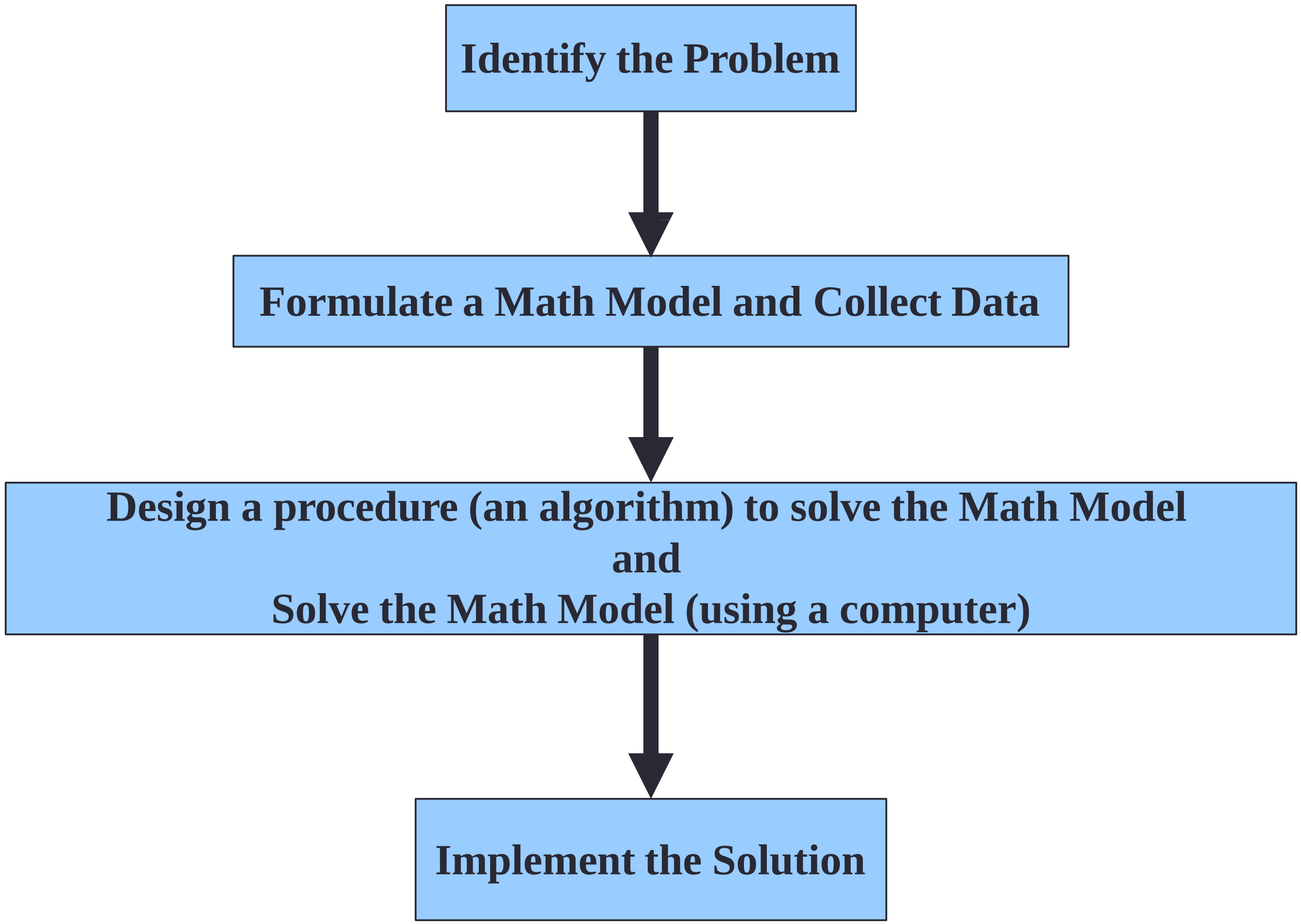
Question: Who swims which leg of the relay to minimize total time?

Operations … what??



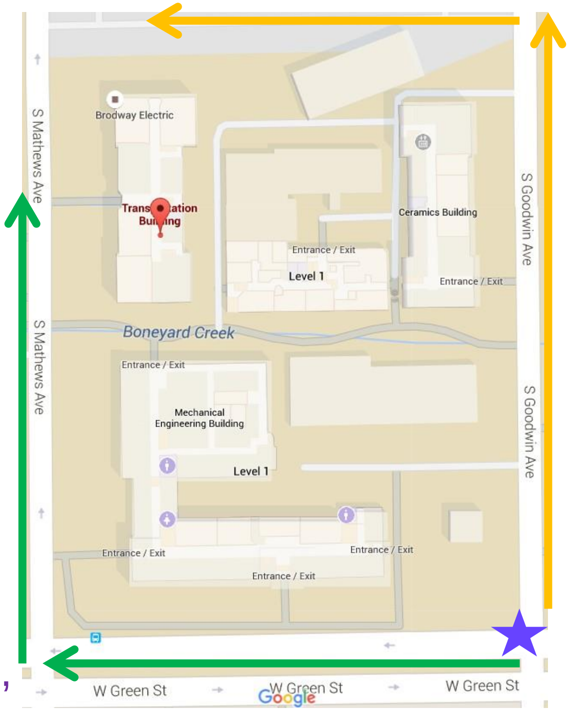
Question: What is the shortest route to the destination?

The OR Methodology



Example

Problem:
choose the faster route to get to TB from where you are



Data:

route 1: Goodwin-Springfield, travel time t_1
route 2: Green-Mathew, travel time t_2

Model/Formulation:

Decisions: $x_1 = 1$ if route 1 is chosen, otherwise $x_1 = 0$,
$x_2 = 1$ if route 2 is chosen, otherwise $x_2 = 0$

Restrictions: x_1, x_2 is 1 or 0, $x_1 + x_2 = 1$ (choose exactly one route)

Objective: minimize $t_1 x_1 + t_2 x_2$

Solution Procedure: set $x_1 = 1$ if $t_1 < t_2$, otherwise set $x_2 = 1$

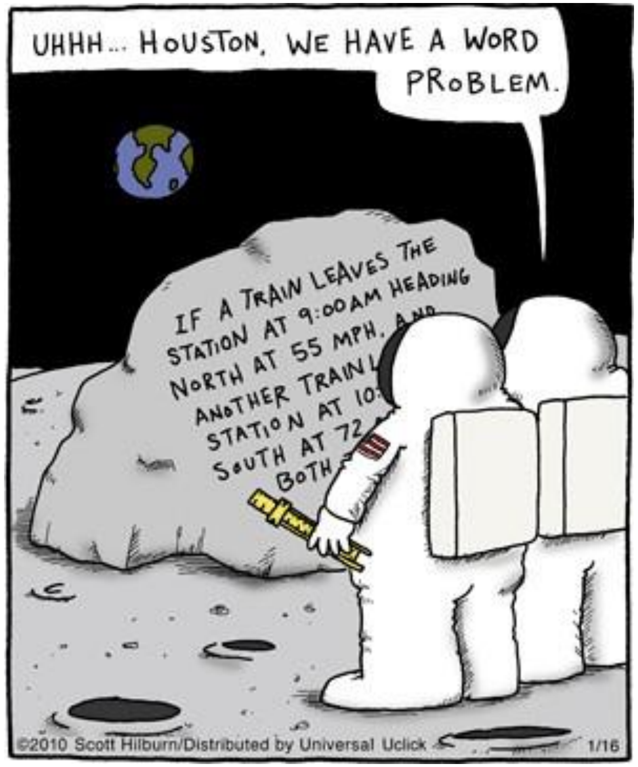
Implementation: start ~~your hoverboard~~ biking!

Elements of Formulating Optimization Models

- **Decision variables:** what we need to decide
  - example:
    - which route to take
  - represent numerical value(s)

- **Objective function:** goal to achieve
  - example:
    - minimize travel time
  - quantifiable, and can be expressed as a function of decision variables

- **Constraints:** conditions to satisfy
  - example:
    - only one route can be chosen
  - quantifiable, and can be expressed as functions of decision variables



FORMULATIONS

… where we model problems mathematically

- So that we can use standard solution techniques

Example 1

- Consider a small manufacturer making two commodities A and B
- Two resources R_1 and R_2 are required to make these commodities
- Each batch of commodity A requires 1 batch of R_1 and 3 batches of R_2
- Each batch of commodity B requires 1 batch of R_1 and 2 batches of R_2
- Manufacturer has 5 batches of R_1 and 12 batches of R_2
- Manufacturer makes a profit of
  - 6$ per batch of commodity A sold
  - 5$ per batch of commodity B sold

Question: What is the best production mix?

Example 1: Summarize data

- Consider a small manufacturer making two commodities A and B
- Two resources R_1 and R_2 are required to make these commodities
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Question: What is the best production mix?

	A	B	Availability
R_1	1	1	5
R_2	3	2	12
Profit	6	5	

Example 1: Formulation

	A	B	Availability
R_1	1	1	5
R_2	3	2	12
Profit	6	5	

Step 1: identify decision variables

x_1: number of batches of commodity A to make
x_2: number of batches of commodity B to make

Step 2: determine the objective function

$$\max Z(x_1, x_2) = 6x_1 + 5x_2$$

Step 3: identify constraints

- Resource constraints

$$x_1 + x_2 \leq 5 \qquad \text{(Resource } R_1\text{)}$$
$$3x_1 + 2x_2 \leq 12 \qquad \text{(Resource } R_2\text{)}$$

- “Common Sense” (non-negativity) constraints

$$x_1 \geq 0, x_2 \geq 0$$

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Question: What is the best production mix?

Maximize $Z = 6x_1 + 5\,x_2$
subject to

$$x_1 + x_2 \leq 5$$
$$3x_1 + 2x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

Example 2

- Manufacturer makes two commodities
  - Glass-door with an aluminum frame: sells for $\$3{,}000$ per batch
  - Wood-window with wood frame: sells for $\$5{,}000$ per batch
- The manufacturer has three plants with limited working hours
  - Plant 1: Aluminum frames
    - at most 4 working hours per week
    - 1 hour to prepare a batch of aluminum frame
  - Plant 2: Wood frames
    - at most 12 working hours per week
    - 2 hours to prepare a batch of wood frame
  - Plant 3: Assembly
    - at most 18 working hours per week
    - 3 hours to assemble a batch of glass-doors with aluminum frames and 2 hours to assemble a batch of wood windows with wood frames

Question: What is the best product mix?

Example 2: Summarize data

- Manufacturer makes two commodities
  - Glass-door with an aluminum frame: sells for $\$3{,}000$ per batch
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	C_1	C_2	Availability
P_1	1		4
P_2		2	12
P_3	3	2	18
Profit	3	5	

Question: What is the best product mix?

Example 2: Formulation

	C_1	C_2	Availability
P_1	1		4
P_2		2	12
P_3	3	2	18
Profit	3	5	

Step 1: identify decision variables

x_1: number of batches of doors to make per week
x_2: number of batches of windows to make per week

Step 2: determine the objective function

$$\max Z(x_1, x_2) = 3x_1 + 5x_2$$

Step 3: identify constraints

- Hour constraints at each plant

$$x_1 \leq 4 \qquad \text{(plant 1)}$$
$$2x_2 \leq 12 \qquad \text{(plant 2)}$$
$$3x_1 + 2x_2 \leq 18 \qquad \text{(plant 3)}$$

- "Common Sense" (non-negativity) constraints

$$x_1 \geq 0, x_2 \geq 0$$

Example 2: Formulation

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	C_1	C_2	Availability
P_1	1		4
P_2		2	12
P_3	3	2	18
Profit	3	5	

Question: What is the best product mix?

$\max\ Z = 3x_1 + 5x_2$ (profit)

$x_1 \leq 4$ (hour constraint for plant 1)

$2x_2 \leq 12$ (hour constraint for plant 2)

$3x_1 + 2x_2 \leq 18$ (hour constraint for plant 3)

$x_1 \geq 0$ (non-negative amount of commodity 1)

$x_2 \geq 0$ (non-negative amount of commodity 2)

Example 2: Formulation

	C_1	C_2	Availability
P_1	1		4
P_2		2	12
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Step 1: identify decision variables

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- "Common Sense" (non-negativity) constraints

$$x_1 \ge 0, x_2 \ge 0$$

Formulating a Mathematical Model for a Problem

Step 1: identify decision variables

Step 2: determine the objective function

Step 3: identify constraints

Example 2: Formulation

$$\max\ Z = 3x_1 + 5x_2 \quad \text{(profit)}$$

$$x_1 \leq 4 \quad \text{(hour constraint for plant 1)}$$

$$2x_2 \leq 12 \quad \text{(hour constraint for plant 2)}$$

$$3x_1 + 2x_2 \leq 18 \quad \text{(hour constraint for plant 3)}$$

$$x_1 \geq 0 \quad \text{(non-negative amount of commodity 1)}$$

$$x_2 \geq 0 \quad \text{(non-negative amount of commodity 2)}$$

The objective function and the constraints are linear

Therefore, the model that we formulated is a linear programming formulation

Benefits of Operations Research

- **Enhanced Decision-Making**
  - Provides quantitative basis for decisions.
- **Efficiency Improvements**
  - Identifies optimal solutions, reduces waste.
- **Cost Savings**
  - Optimizes resource allocation, minimizes costs.
- **Strategic Planning**
  - Supports long-term planning and policy-making.

Application Areas

- Strategic planning
- Supply chain management
- Pricing and revenue management
- Logistics and site location
- Optimization
- Marketing research

Applications Areas (cont.)

- Scheduling
- Portfolio management
- Inventory analysis
- Forecasting
- Sales analysis
- Auctioning
- Risk analysis

Challenges in Operations Research

- **Data Quality**
  - Reliable, accurate data is crucial.
- **Model Complexity**
  - Balancing model detail with computational feasibility.
- **Interdisciplinary Knowledge**
  - Requires understanding of mathematics, computer science, and the specific application domain.
- **Implementation**
  - Translating OR solutions into practical, real-world actions.

Future Directions

- **Integration with AI and Machine Learning**
  - Combining OR with advanced AI techniques for more powerful solutions.
- **Big Data Analytics**
  - Leveraging large datasets for more accurate models and predictions.
- **Sustainability**
  - Applying OR to address environmental and sustainability challenges.
- **Real-Time Decision Making**
  - Developing models for real-time optimization and decision support.

Conclusion

- **Summary**
  - Operations Research is a powerful tool for optimizing decision-making and resource allocation.
  - It has broad applications across various industries.
  - Despite challenges, its integration with modern technologies promises exciting future developments.
- **Final Thoughts**
  - Encourage continued learning and exploration in the field of OR.