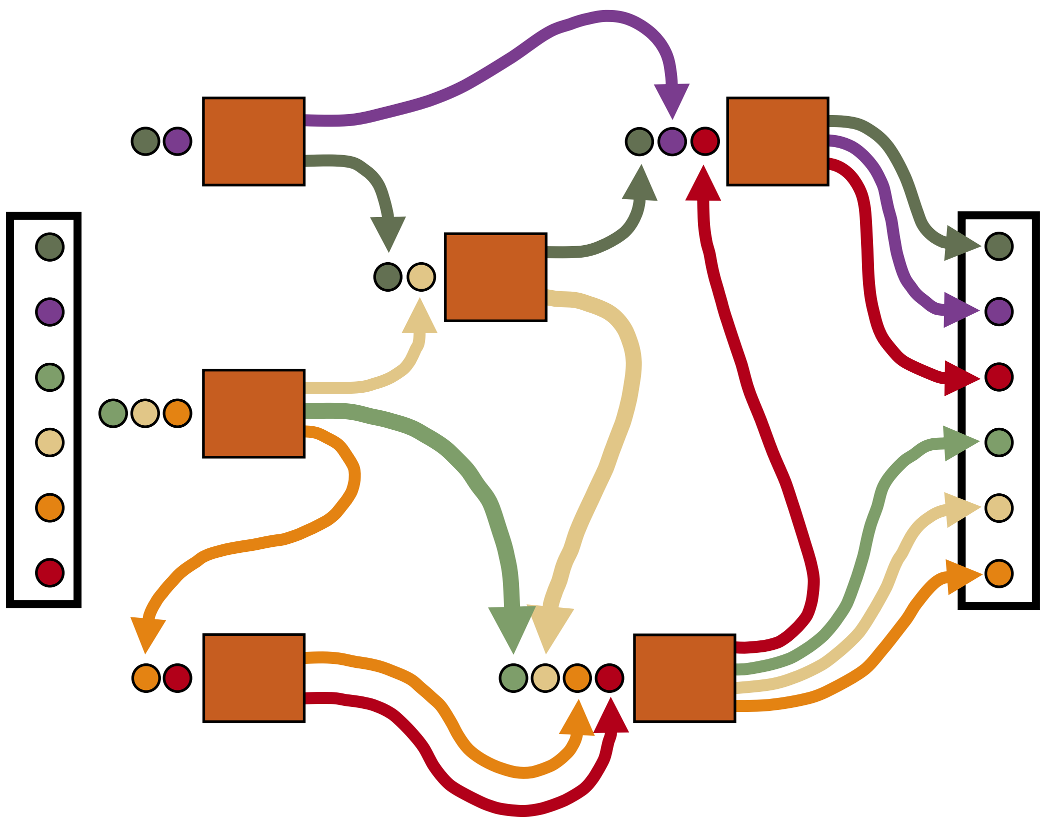
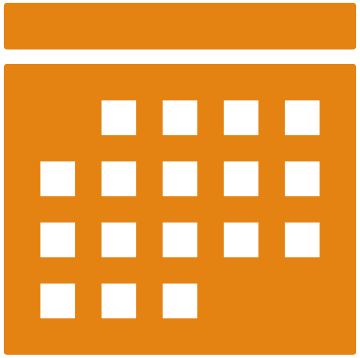
Operations (Machine) Scheduling



Machine Scheduling

- A ***schedule*** is a plan that tells us when certain activities will happen. Schedule often tells us the *sequence* of activities.
- ***Scheduling*** is the process of generating a schedule.
- ***Machine scheduling*** is generating the schedule of activities that needs to take place in the shop floor. It is often called *shop floor control*.
- Machine scheduling is usually the lowest hierarchical level of decision making in an organization. It needs input from several upstream planning decisions.



Definitions

- A ***machine*** is a resource that can perform at most one activity at any time.
- Activities are commonly referred to as ***jobs***, and it is assumed that a job is worked on by at most one machine at any time.
- Jobs are processed on machines for a time period that

  is called ***processing time***.
- In general, a scheduling problem is one in which ***n***

  jobs must be processed through ***m*** **machines**.
- There may be many different ***optimization criteria*** and constraints on ***job sequences*** that may impact the complexity of the problem

Definitions

- Processing time of job j on machine i (tij)
- Start time of job j on machine i is the time that machine i starts processing job j.
- Completion time of job j on machine i is the time that machine i
- finished processing job j. If there is no interruption,
- completion time = start time + processing time
- Completion time (Cj) of a job in the job shop is the time that the job is completed in all machines required for the processing of that job. Completion time of a job is the maximum of completion of that job across all machines.
- Due date (dj) is the time that a job is required (or expected) to be completed in all machines that it is required of.

Broad objectives

Turnaround measures the time required to complete a task.

Timeliness measures the conformance of a particular task's completion to a given deadline.

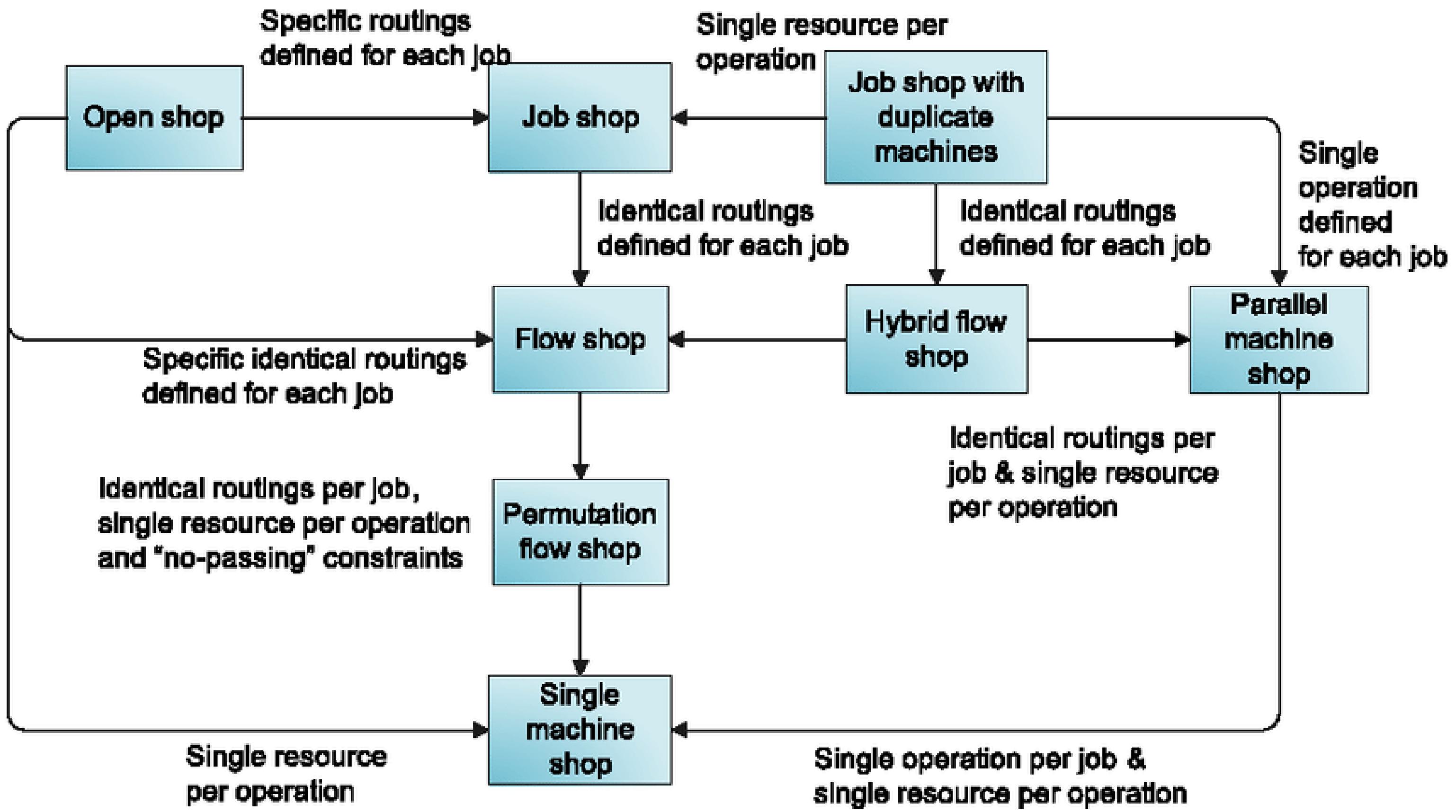
Throughput measures the amount of work completed during a fixed period of time.

More definitions

- *Ready time* (r_j) is the time at which the job is ready (or available) for processing
- *Flow time* (F_j) is the time that the job spends in the system
- *Lateness (L_j)* is the difference between the completion time and the due date *($L_j = C_j - d_j$).*
- *Tardiness* is the positive difference between the completion time and the due date of a job

  ($T_j = max\{C_j - d_j, 0\}$). A tardy job is one that is completed after its due date.
- *Makespan* is the time that all jobs are completed in the job shop. *Makespan=max{C_j}*

A classification of scheduling models



Single machine scheduling

- Basic assumptions
  - Deterministic processing times
  - Machine available at all times
  - No preemption
  - No setup times (or setup times part of processing times)
  - All jobs are ready at time 0.

Example

Job number	Processing Time	Due Date
1	11	61
2	29	45
3	31	31
4	1	33
5	2	32

- Plan using 4 different sequencing rules

Multiple Machines- Parallel machines

- There are m machines that are doing exactly the same thing.
- Each job can be processed in any of the m parallel machines.
- If we allow for pre-emption, minimum makespan is

  $$M^* = \max[\sum p_j/m, \max_j\{p_j\}]$$

- If we do not allow for pre-emption, finding the minimum makespan is an NP-hard problem
- A ***list schedule*** picks up a job from a list and places it to a machine which is available at the earliest time.
- Any list schedule gives a makespan M such that

  $$M/M^* \leq 2 - 1/m$$

- An LPT list schedule gives a makespan M such that

  $$M/M^* \leq 4/3 - 1/3m$$

Multiple machines – Series machines

- Each job needs to be processed on different machines in “some” order
- If the order is same for all jobs, then the shop is called a ***flow shop***
- If the order is fixed a-priori, and can be different for each job, then the shop is called a ***job shop***
- If the order is not fixed a-priori, and can be different for each job, then the shop is called an ***open shop***

Flow shops

- Each job needs to be processed in machines *1,2,3,..,m* in that order
- p_{ij} is the processing time of job *j* on machine *i*
- Consider two jobs to be processes in 4 machines

Job	1	2
p_{1j}	1	4
p_{2j}	4	1
p_{3j}	4	1
p_{4j}	1	4

Description of Job Shop Scheduling

A finite set of n jobs

Each job consists of a chain of operations

A finite set of m machines

Each machine can handle at most one operation at a time

Each operation needs to be processed during an uninterrupted period of a given length on a given machine

Purpose is to find a schedule, that is, an allocation of the operations to time intervals to machines, that has minimal length

Formal Definition of JSS

Job set $J = \{ j_1, j_2, \ldots j_n \}$

Machine set $M = \{m_1, m_2, \ldots m_m\}$

Operations $O = \{o_1, o_2, \ldots o_n\}$ $\quad O_i = \{o_{i1}, o_{i2}, \ldots o_{im_i}\}$

Each operation has processing time $\{\tau_{i1}, \tau_{i2}, \ldots \tau_{im_i}\}$

On O define A, a binary relation represent a precedence between operations. If $(v, w) \in A$ then v has to be performed before w.

A induce the total ordering belonging to the same job; no precedence exist between operations of different jobs.

Job Shop Scheduling

Jobs	Machines		
J1	M2 (3)	M3(4)	M1 (1)
J2	M3 (6)	M1 (2)	M2 (4)
J3	M1 (2)	M2 (5)	M3 (5)

- *Makespan* is the time that all jobs are completed in the job shop. *Makespan=max{C_j}*

- Gantt chart