Merkle trees

Flag Bits

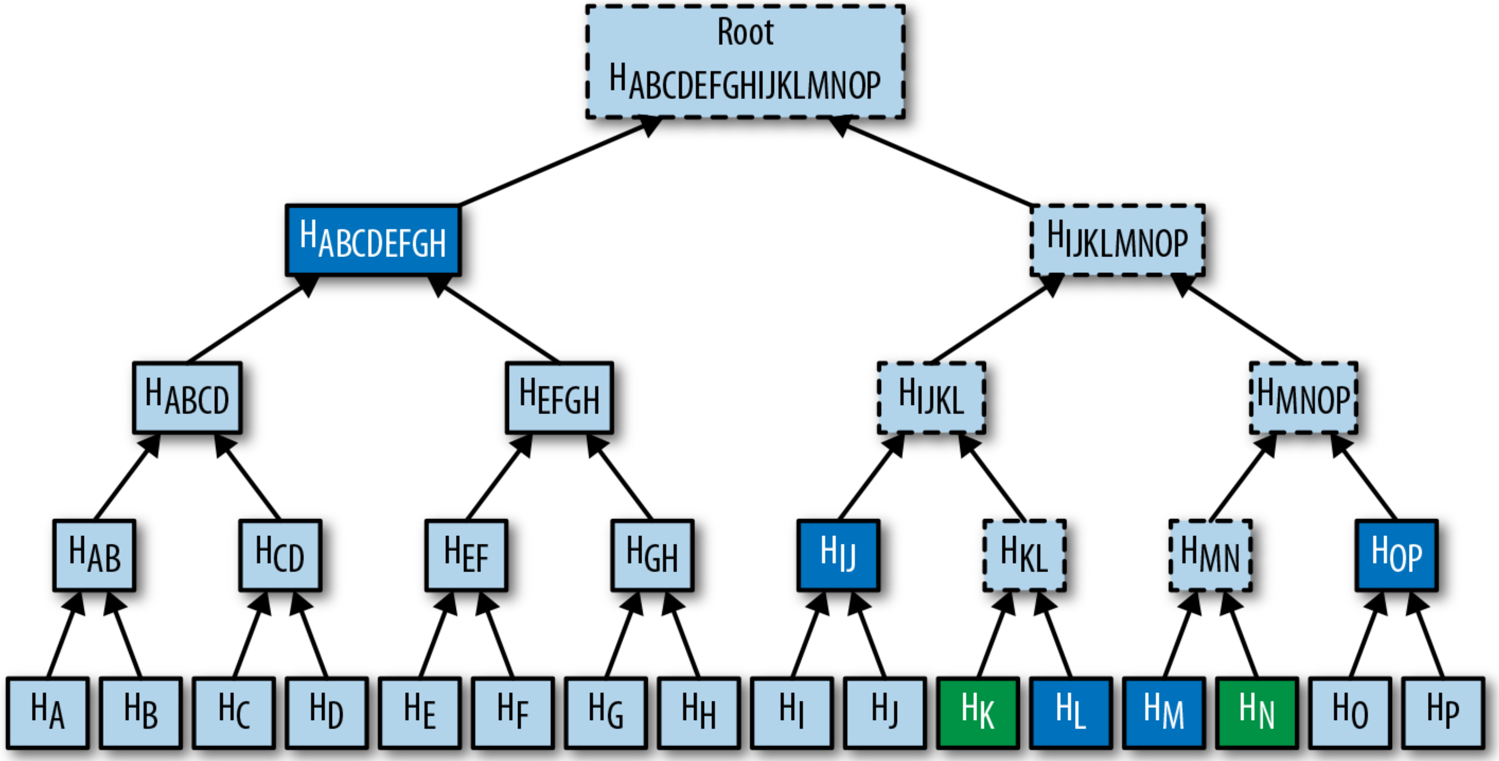
Use of Merkle trees

- Create and verify an inclusion proof
- I.e. I want a compact proof that one or more data belongs to a Merkle tree with a specific Merkle root

- Important: one can get proofs for multiple data points

- Recall: everyting in a Merkle tree is a hash (modulo leaves; sort of)

What does a proof look like?



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I want a Proof that:
- H(K) and H(N) belong
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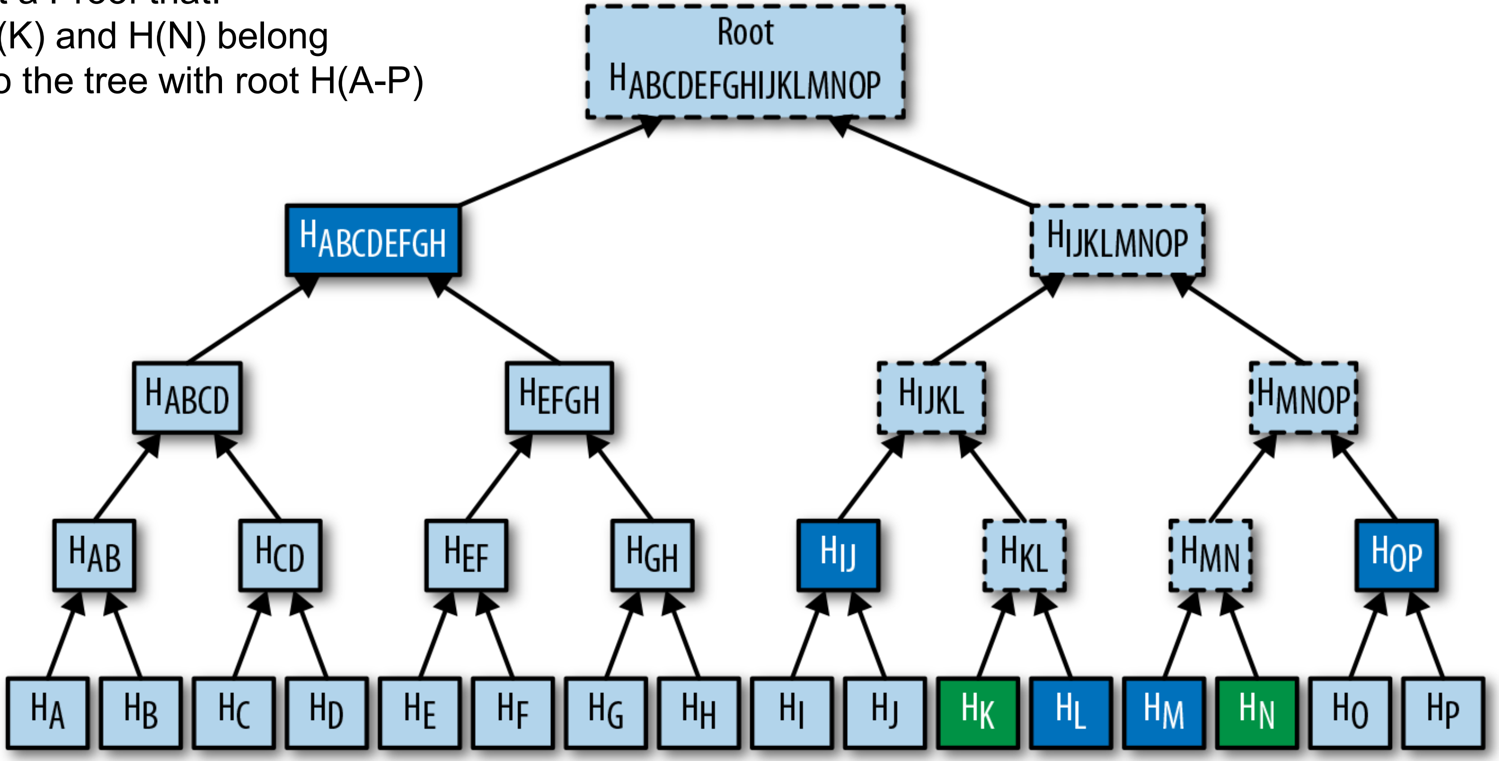

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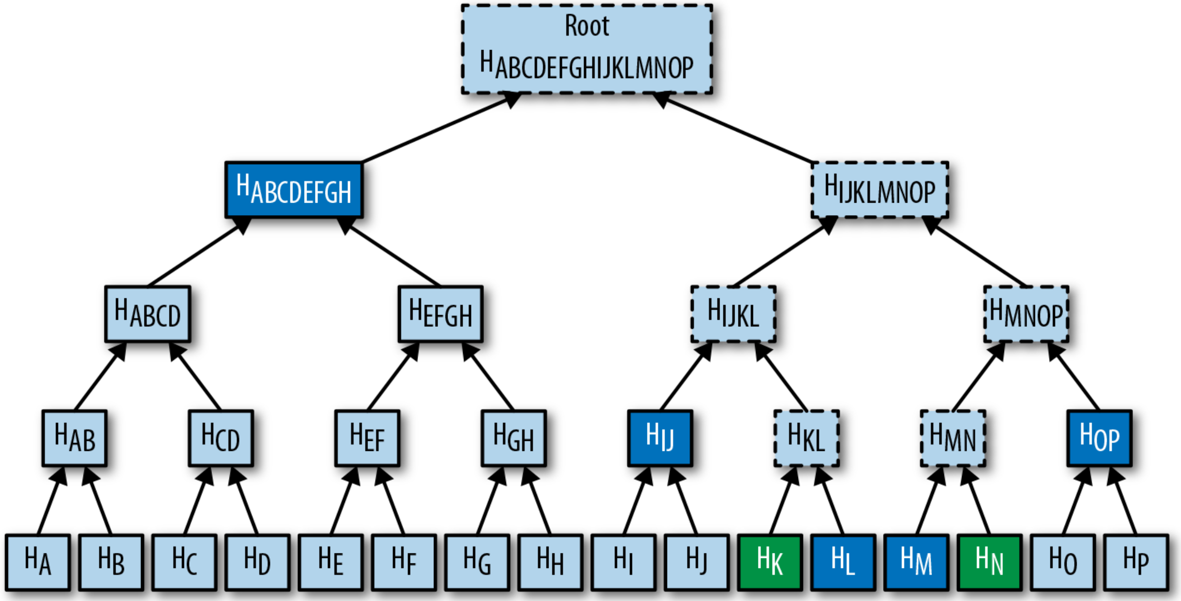

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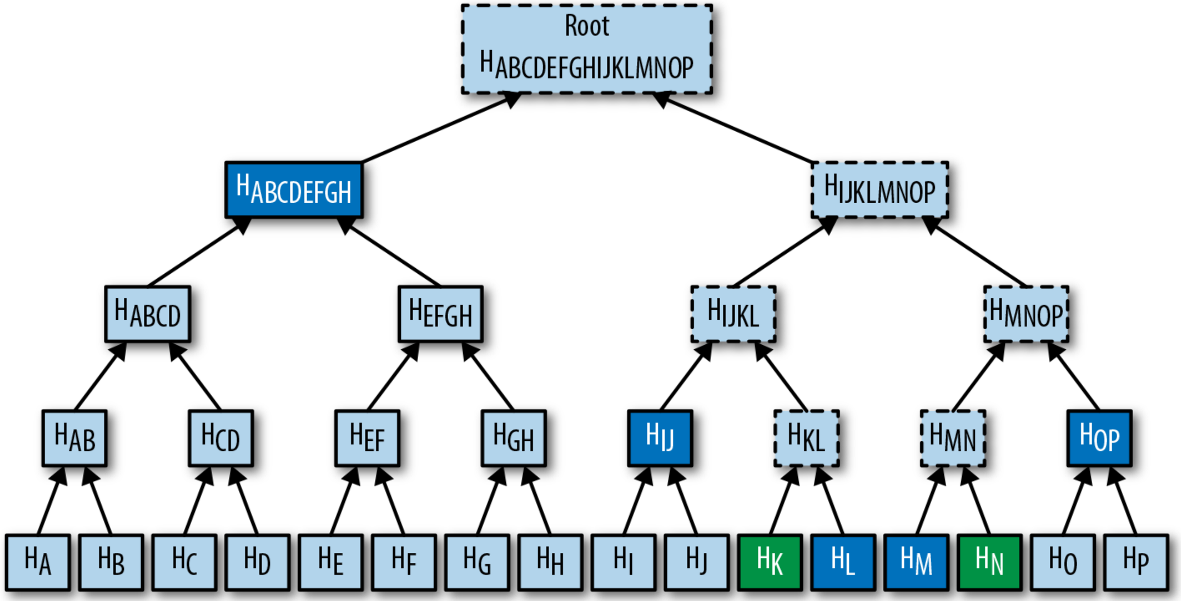

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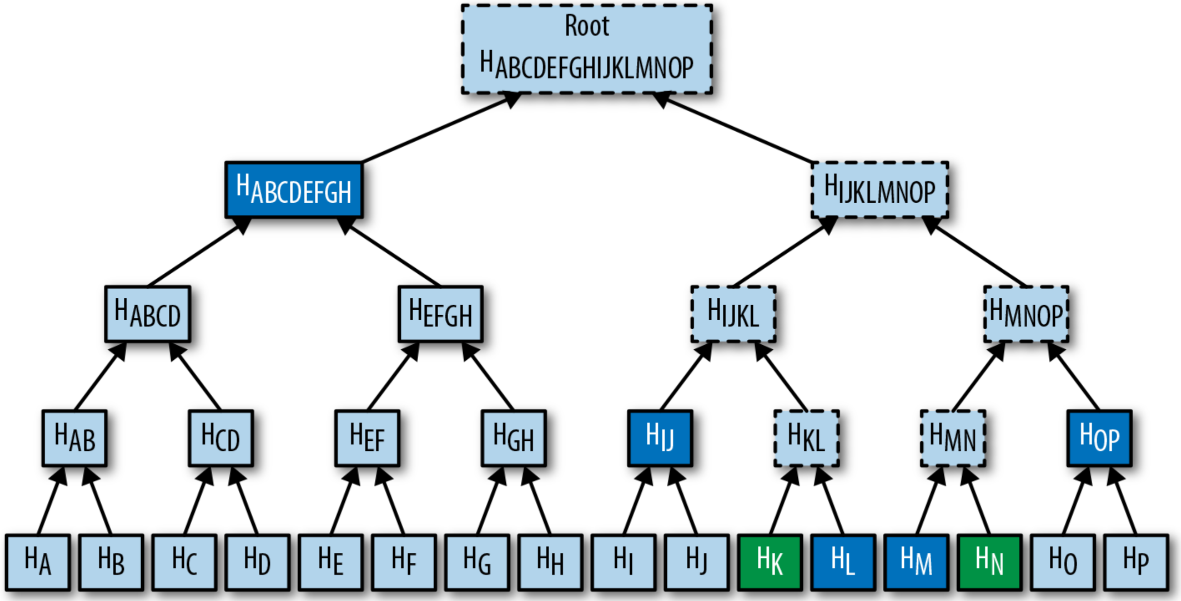

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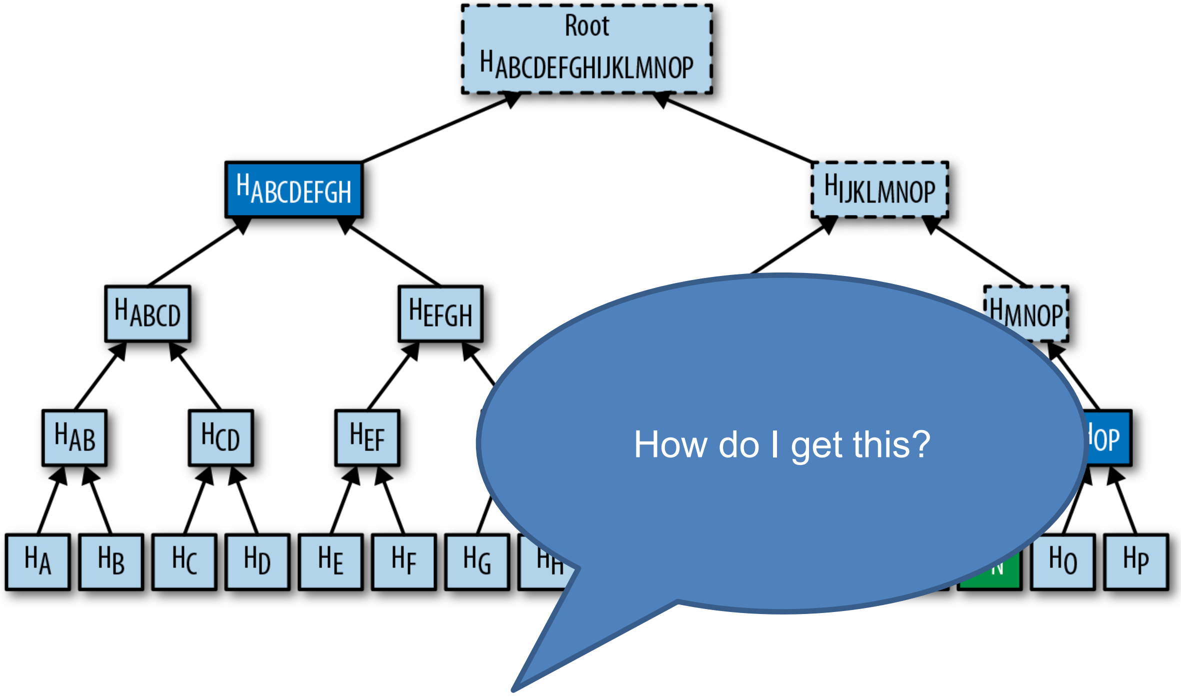

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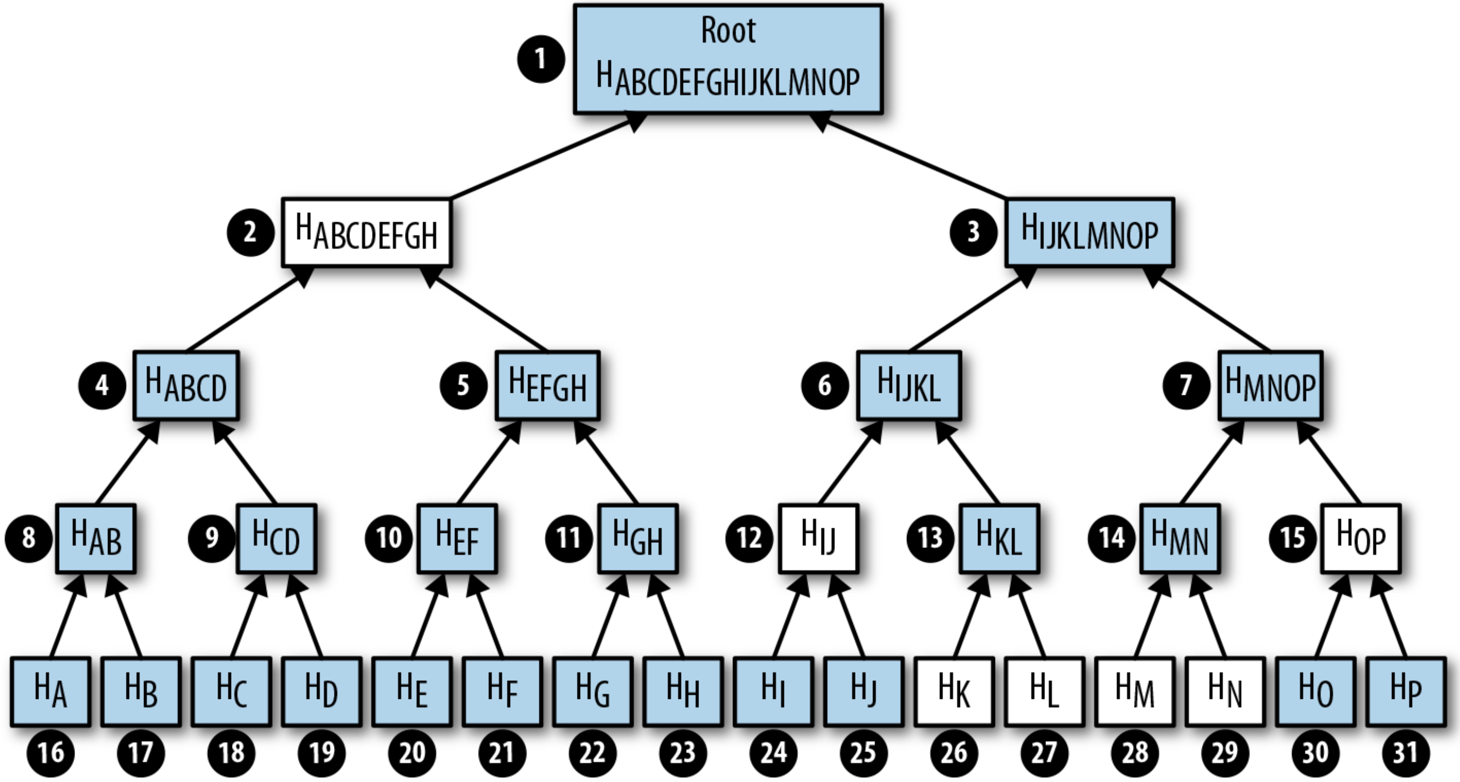
Tree traversal algorithms

BFS



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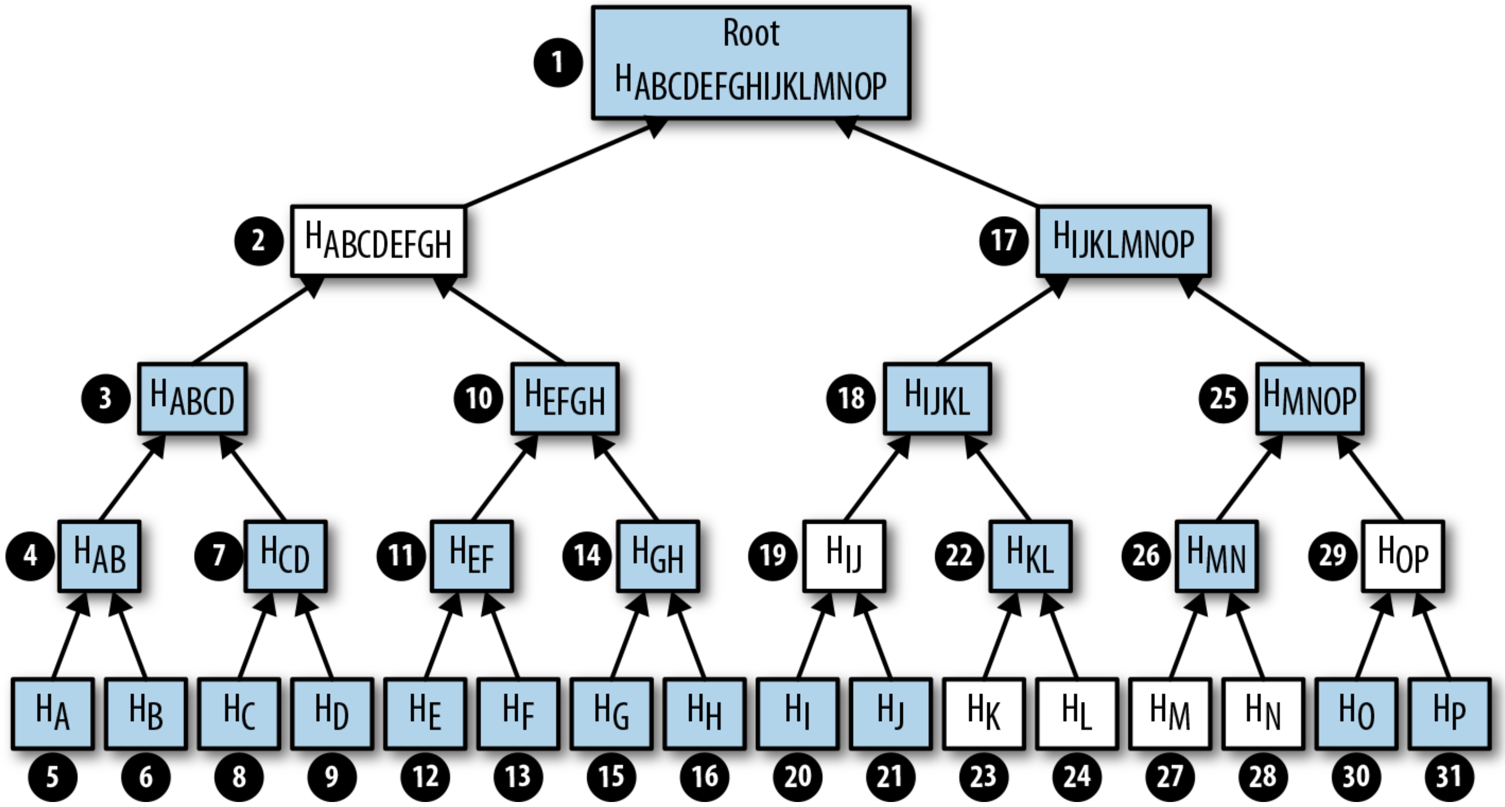
Tree traversal algorithms

DFS



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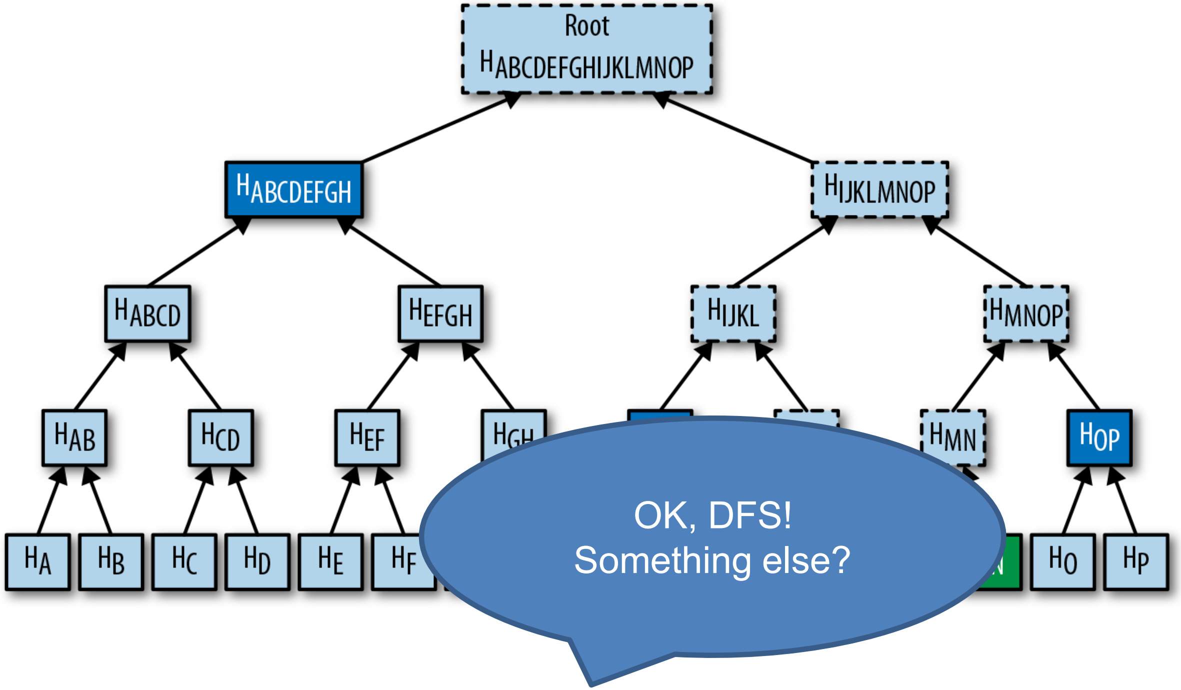

I also need: the position of all blue/green hashes in the tree!!!

One last thing: the number of leaves!

What is done in Bitcoin?

- Proof that hashesOfInterest belong to Merkle tree with the root *Mroot:*

1) Numer of leaves in the tree with root Mroot
2) Hashes needed to reconstruct the root
3) Position of these hashes in the tree (using DFS)

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    - How to represent this information?
    - With a list of bits!!!
    - **Flag Bits**

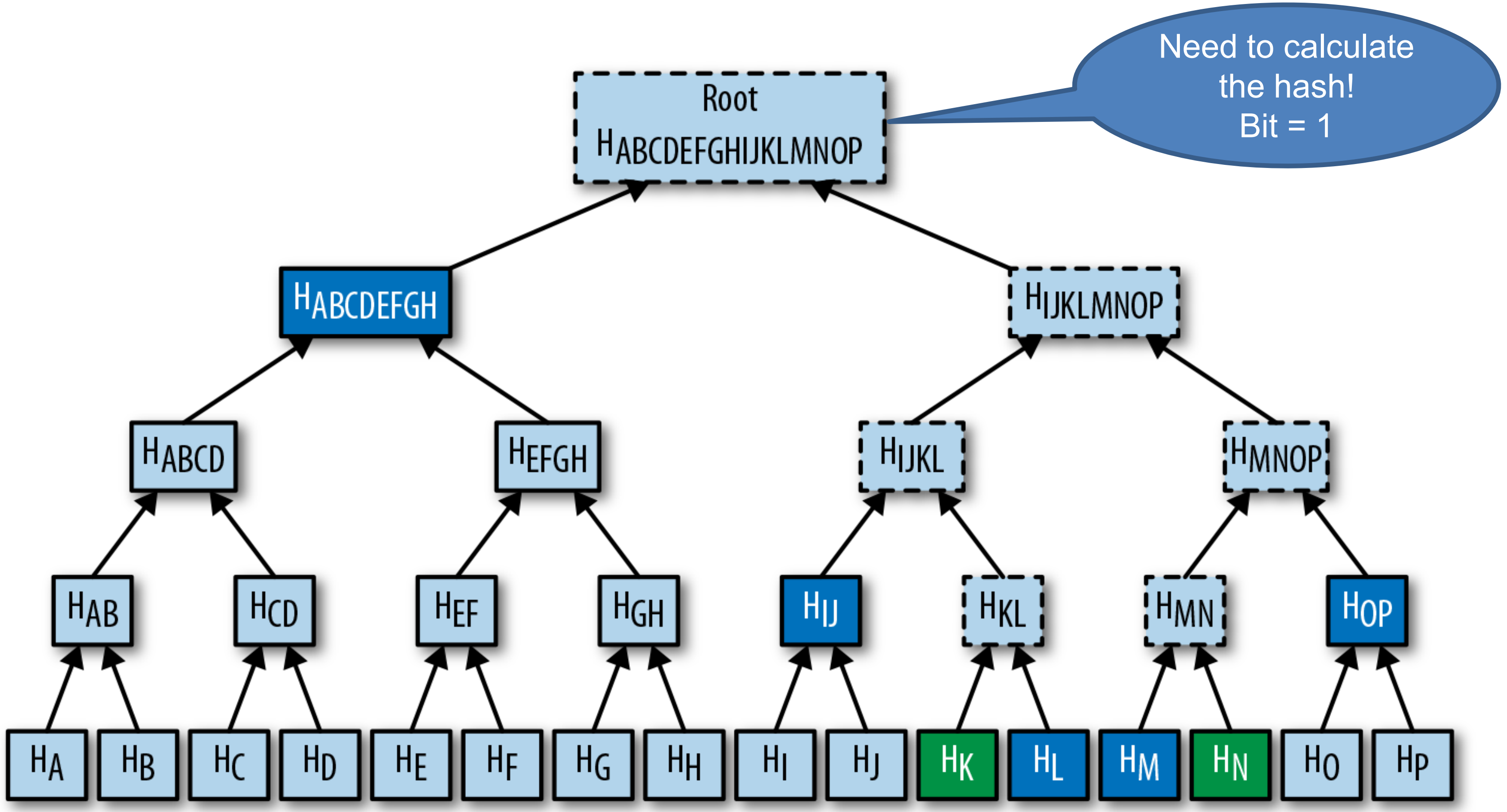
Flag Bits

Proof = (nrLeaves, listOfHashes, flagBits) for hashesOfInterest

- Rules for flagBits:

1. If the hash of the node we are processing appears in listOfHashes, the bit is equal to 0
2. If the node is an internal node (not a leaf), and we need to compute its value, the bit equals 1
3. If the node is a leaf that appears in hashesOfInterest, then the bit equals to 1, and the hash appears in the listOfHashes

Rules for flagBits



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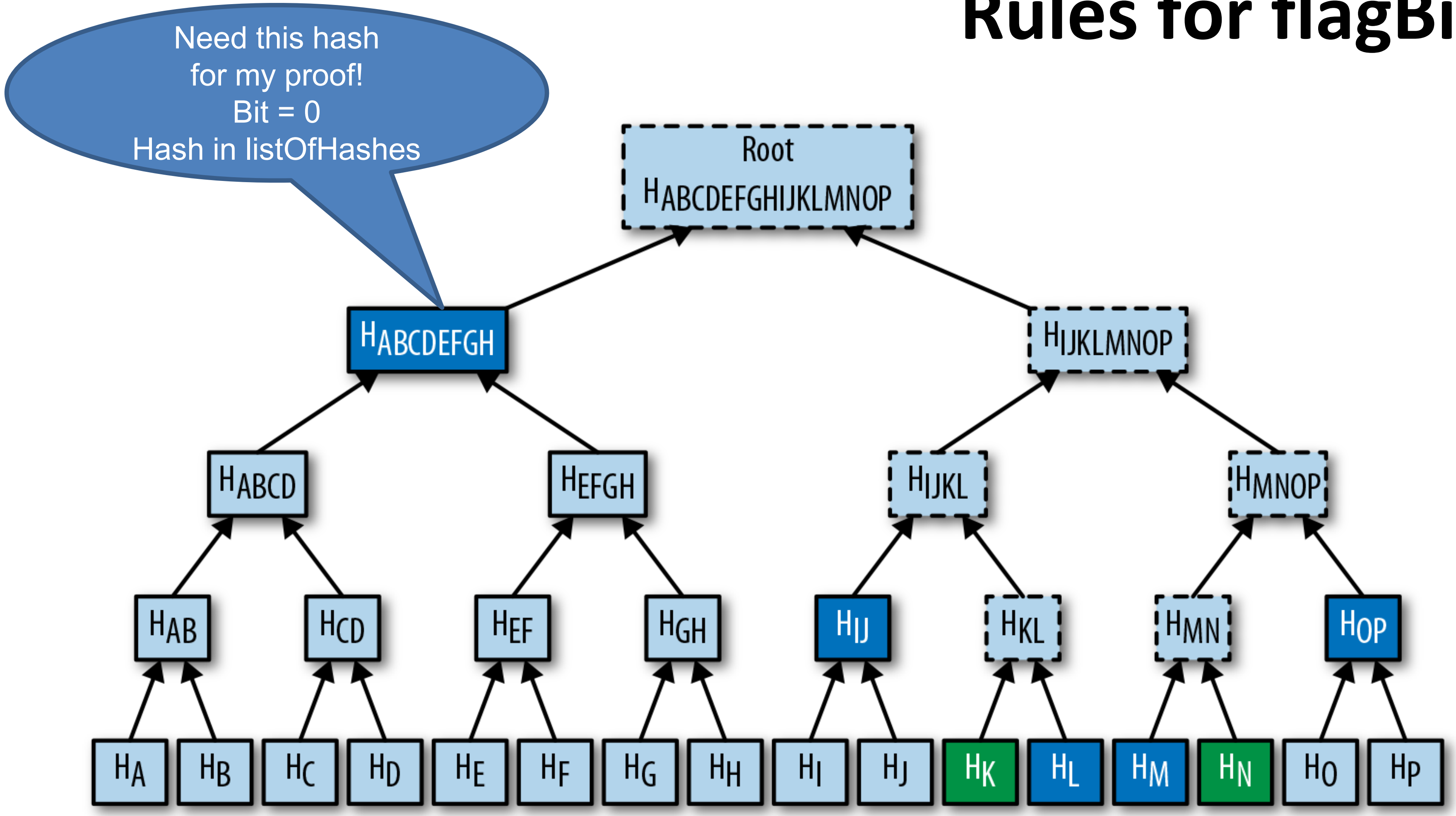

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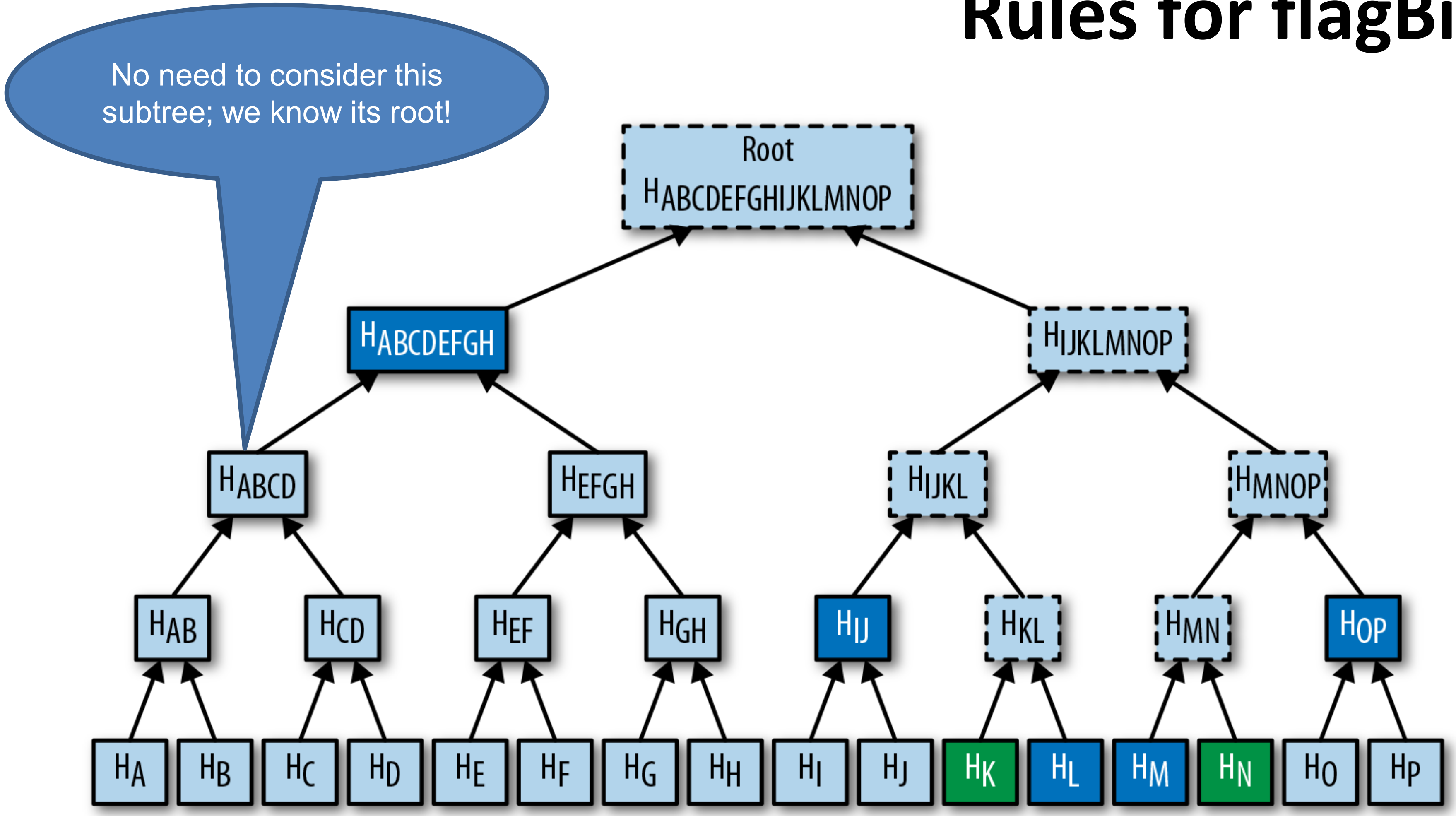

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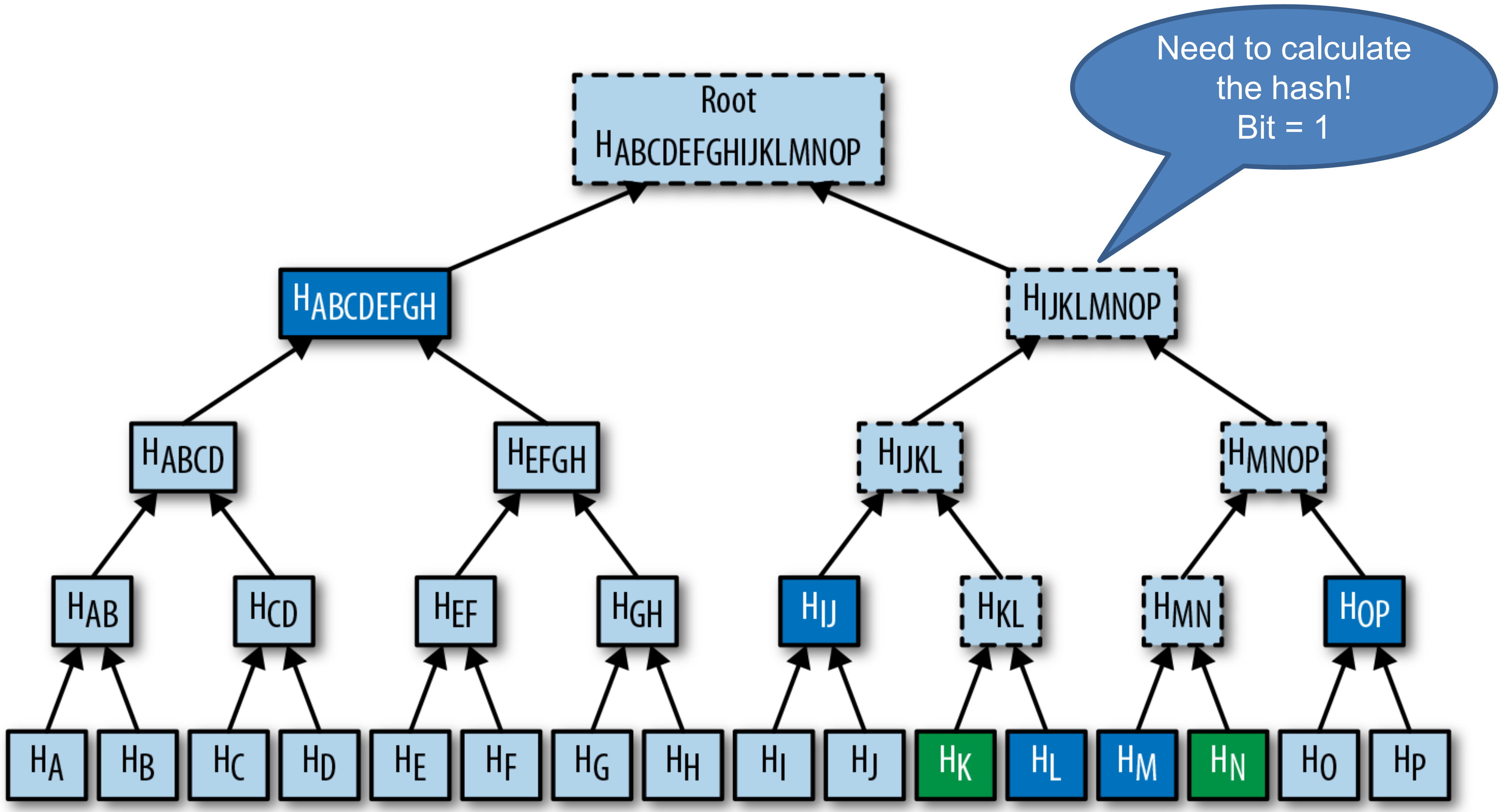

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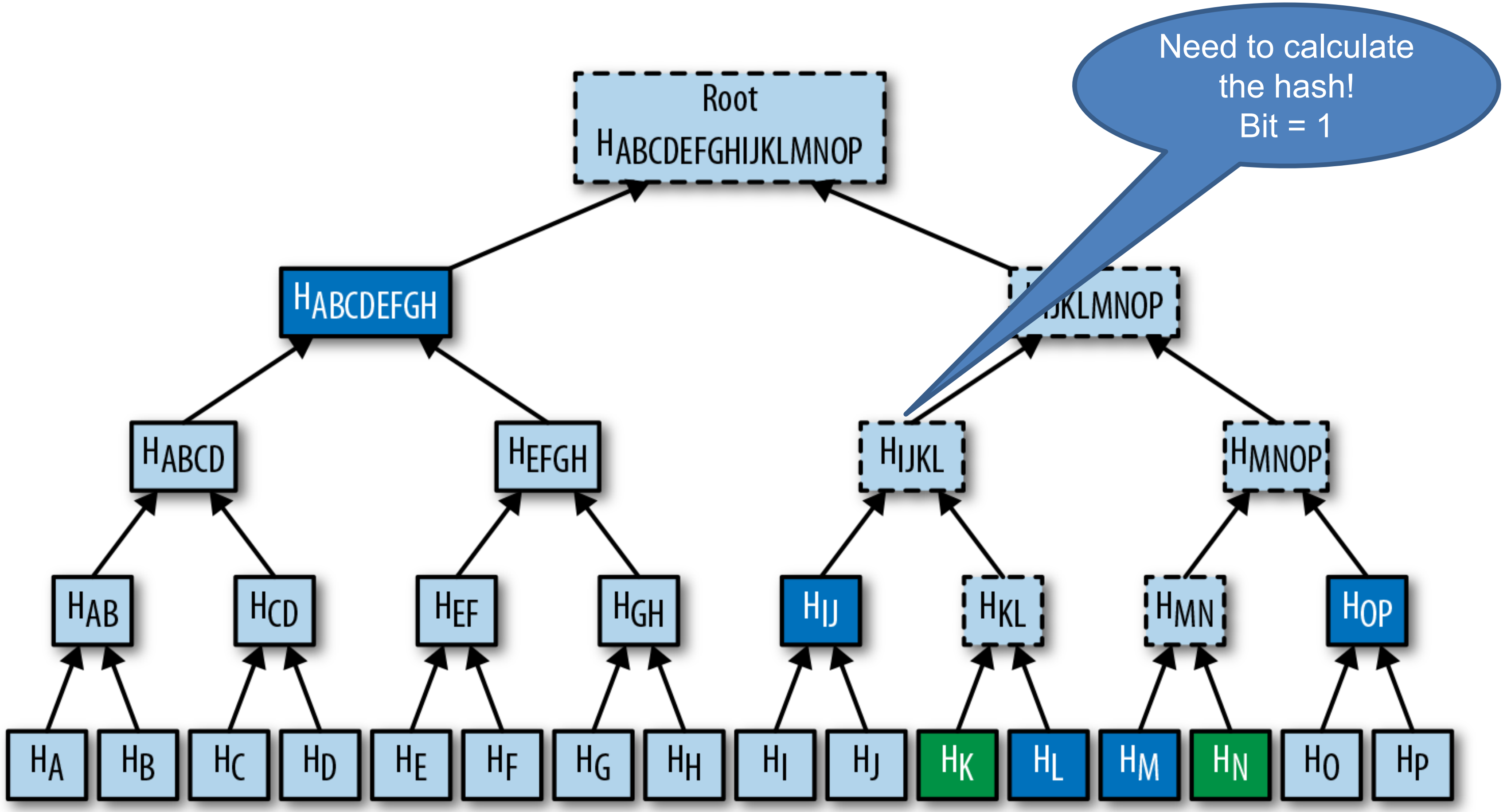

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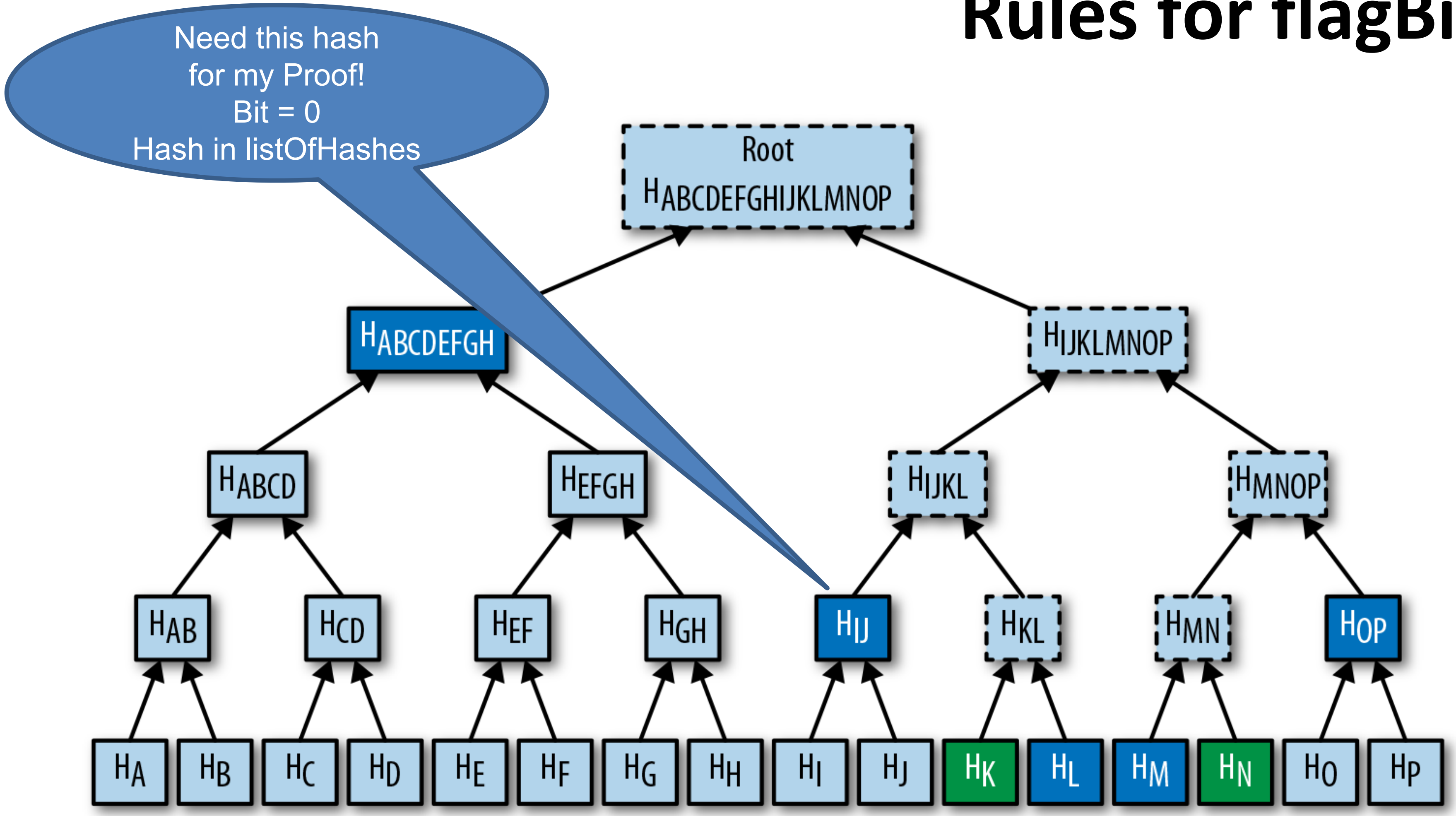

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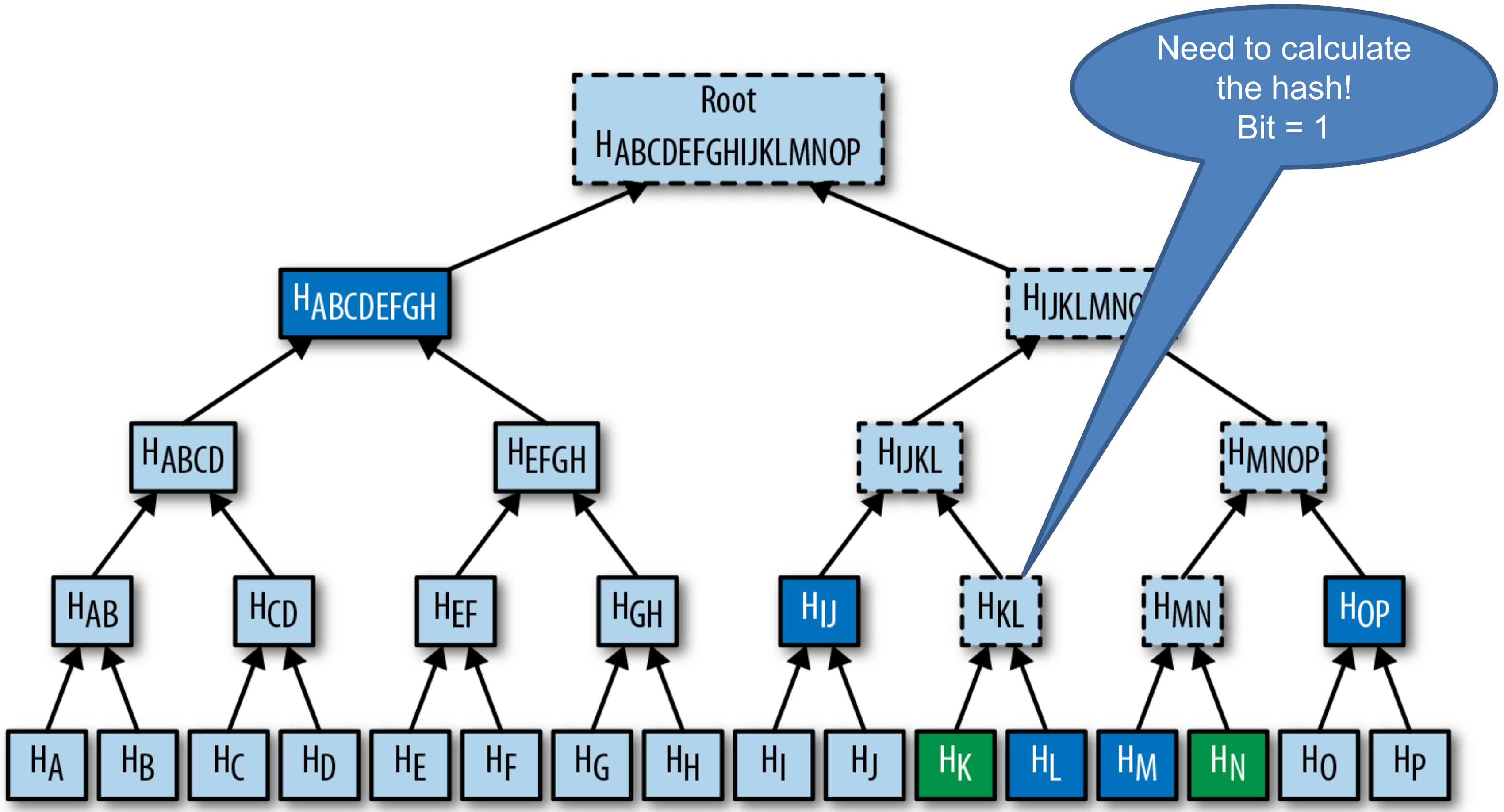

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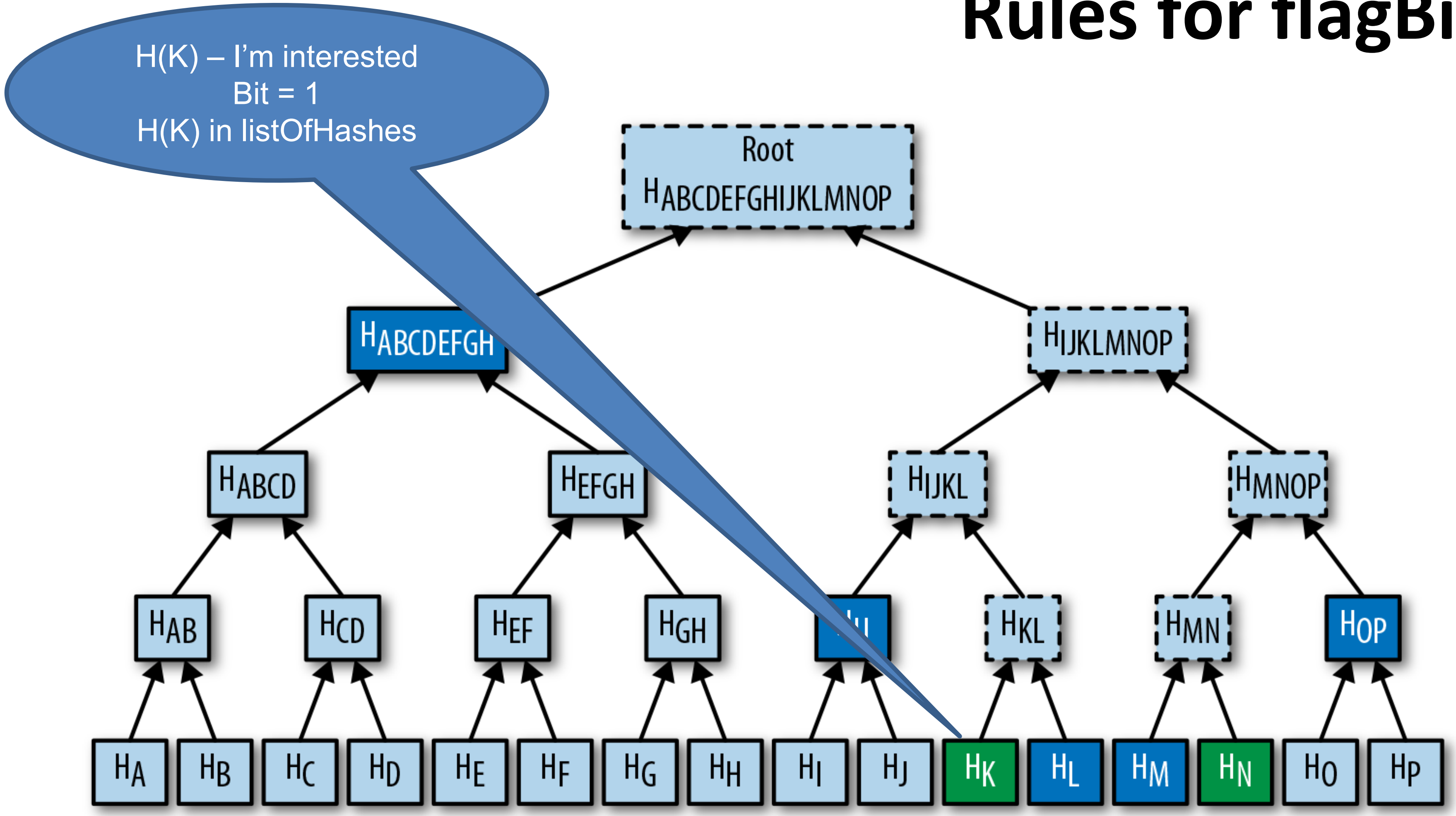

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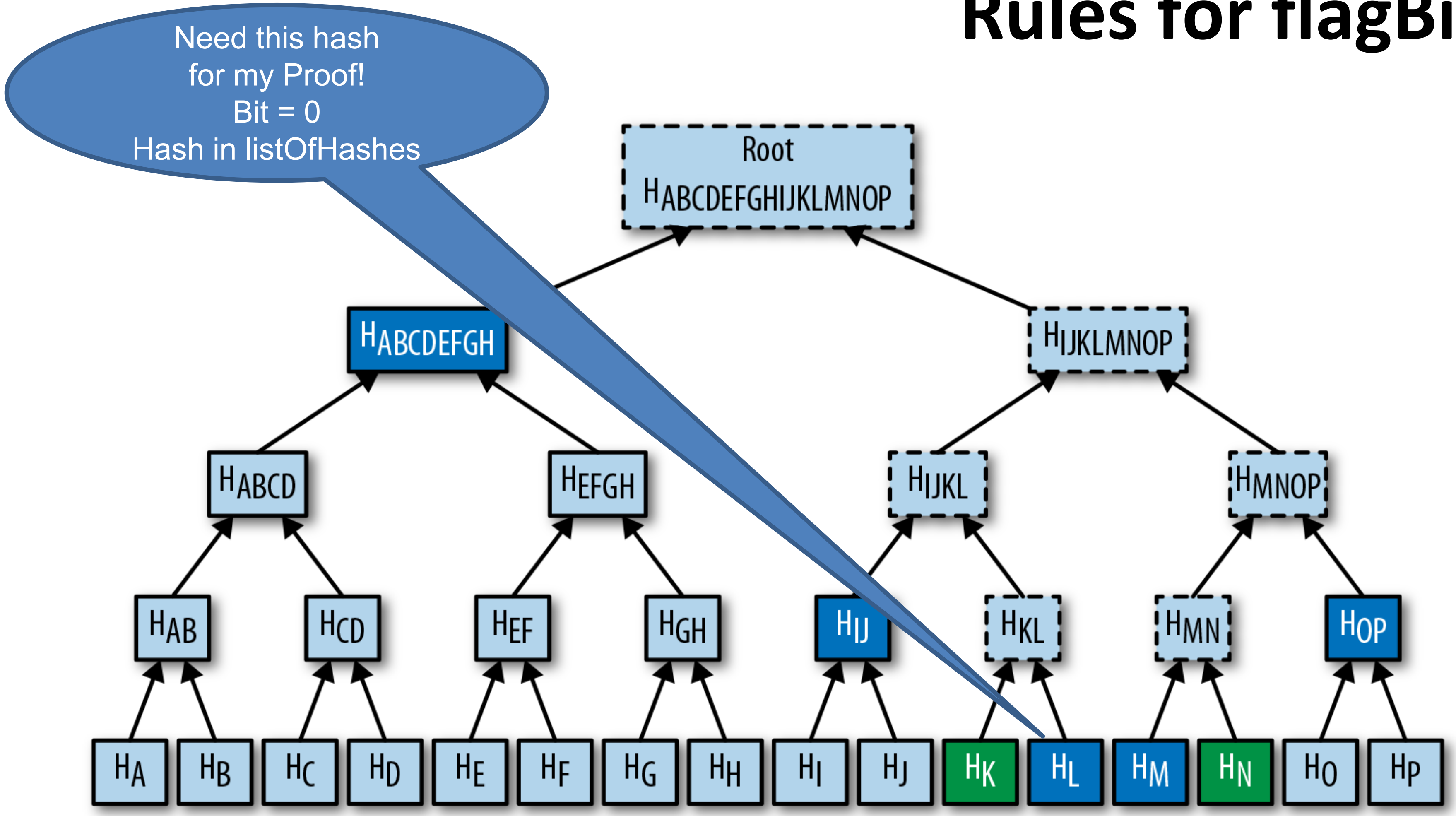

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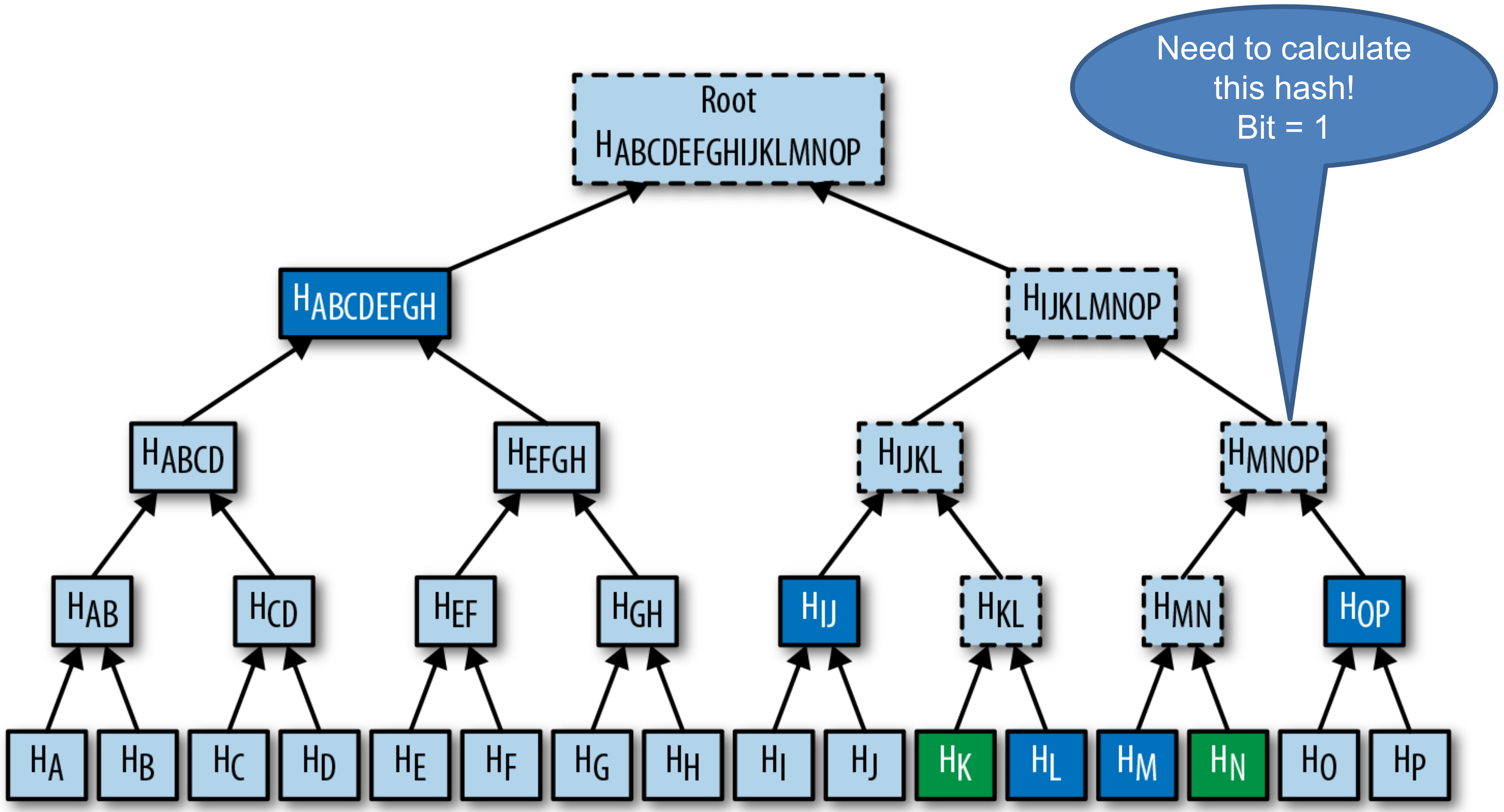


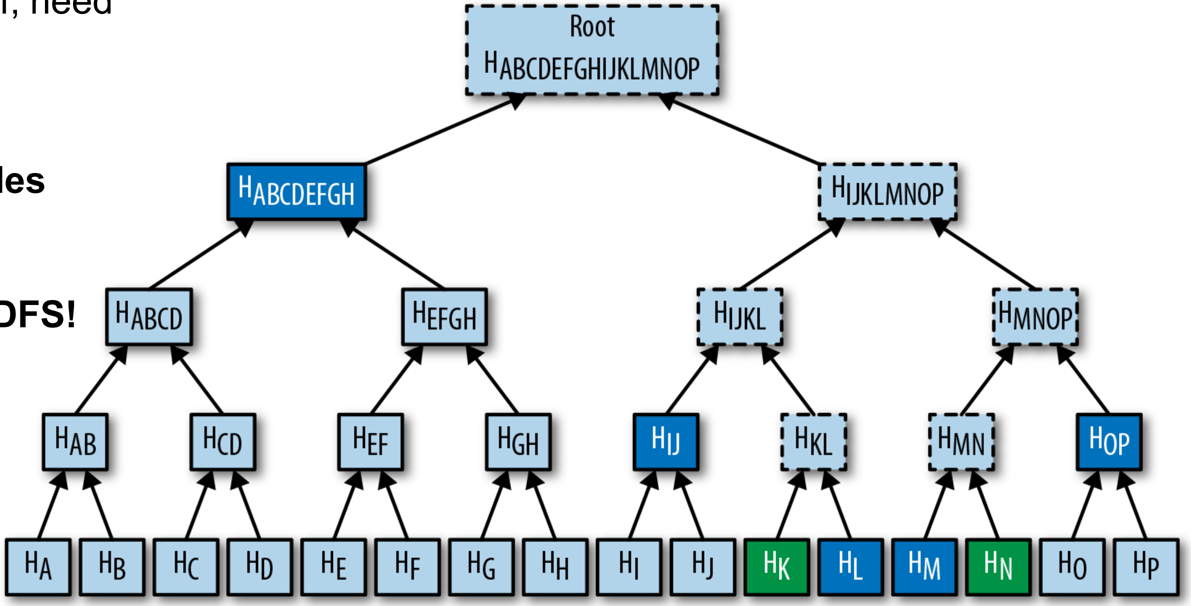
Rules for flagBits

flagBits tell me:

- Using DFS (node, left, right)
- If value is 0 no need to go down this subtree
- If value is 1 and Ip0pm not a leaf; need to parse the subtree
- If I0m a leaf I need the hash

- **flagBits consider only the nodes needed to generate the Proof!**

- **And the rest is processed by DFS!**



Rules for flagBits

flagBits for this tree:

- 1 – root
- 0 – H(ABCDEFGH)
- 1 – H(IJKLMNOP)
- 1 – H(IJKL)
- 0 – H(IJ)
- 1 – H(KL)
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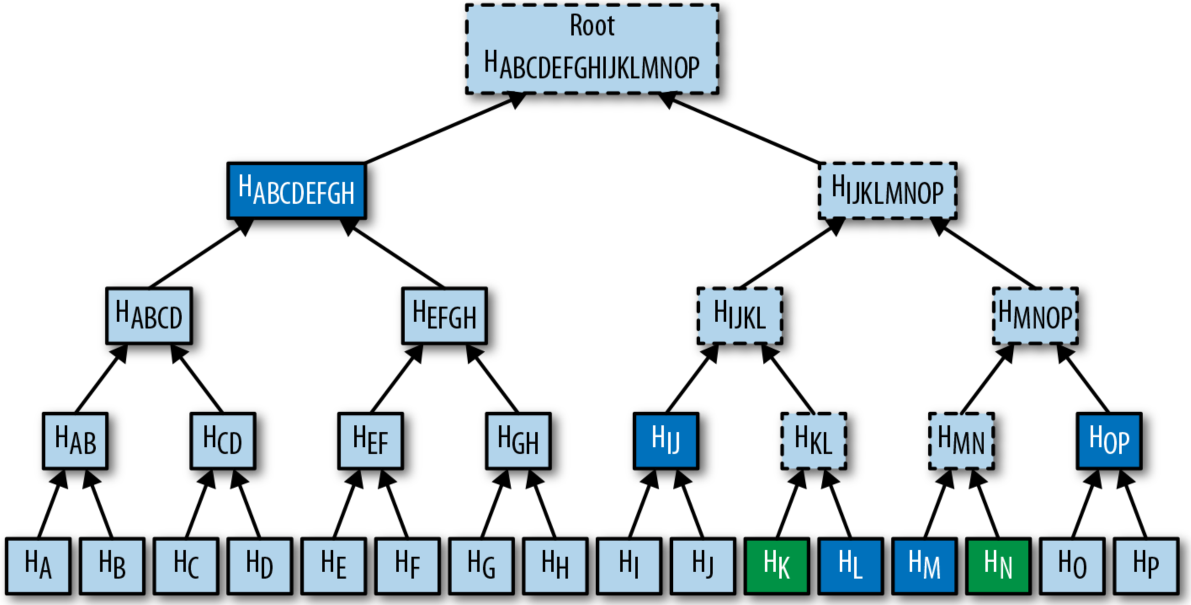
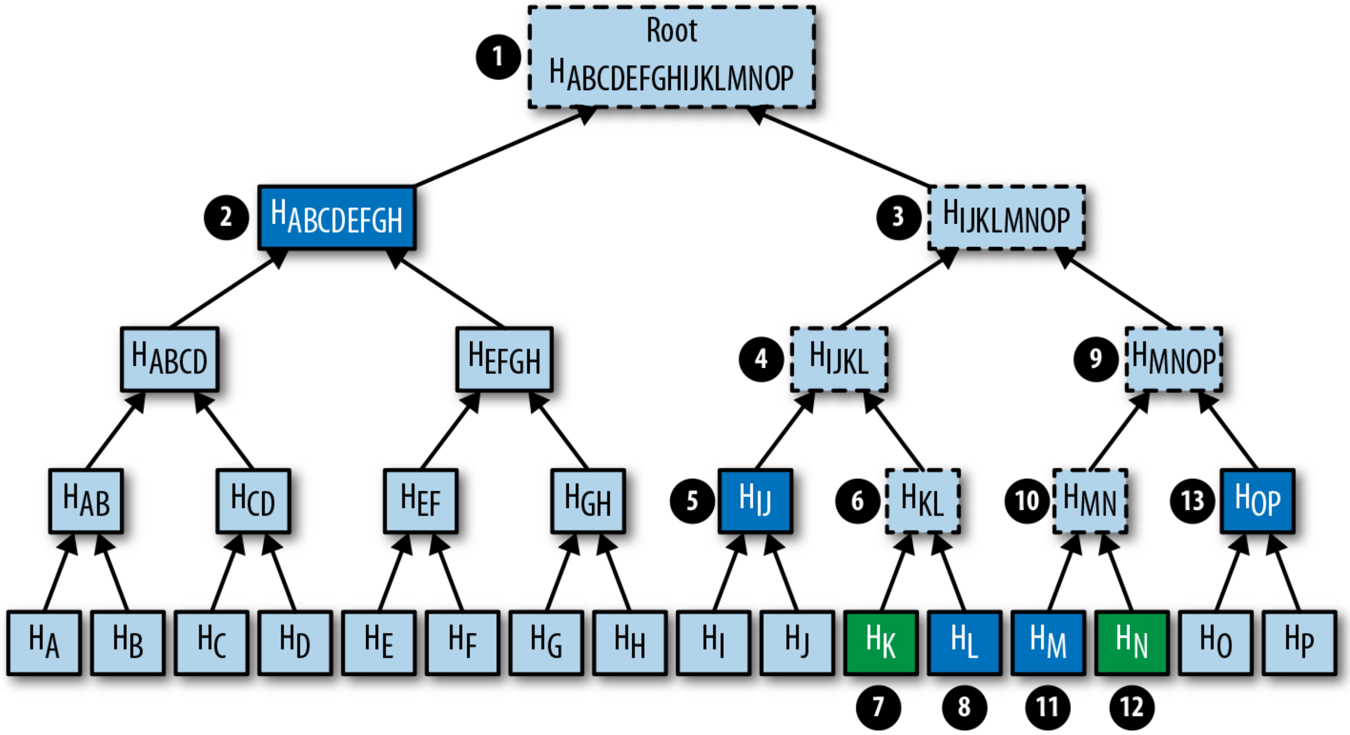


How to do DFS using flagBits?

Proof = (nrLeaves, listOfHashes, flagBits)



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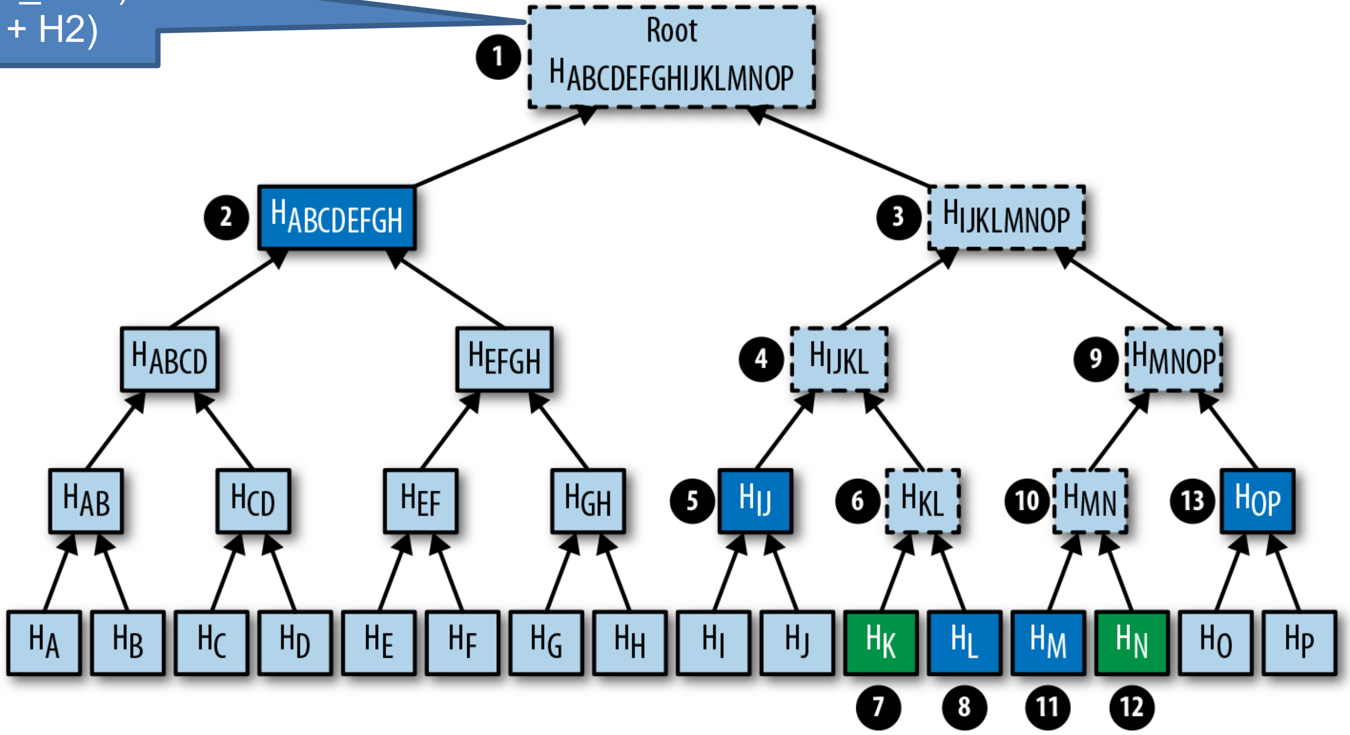

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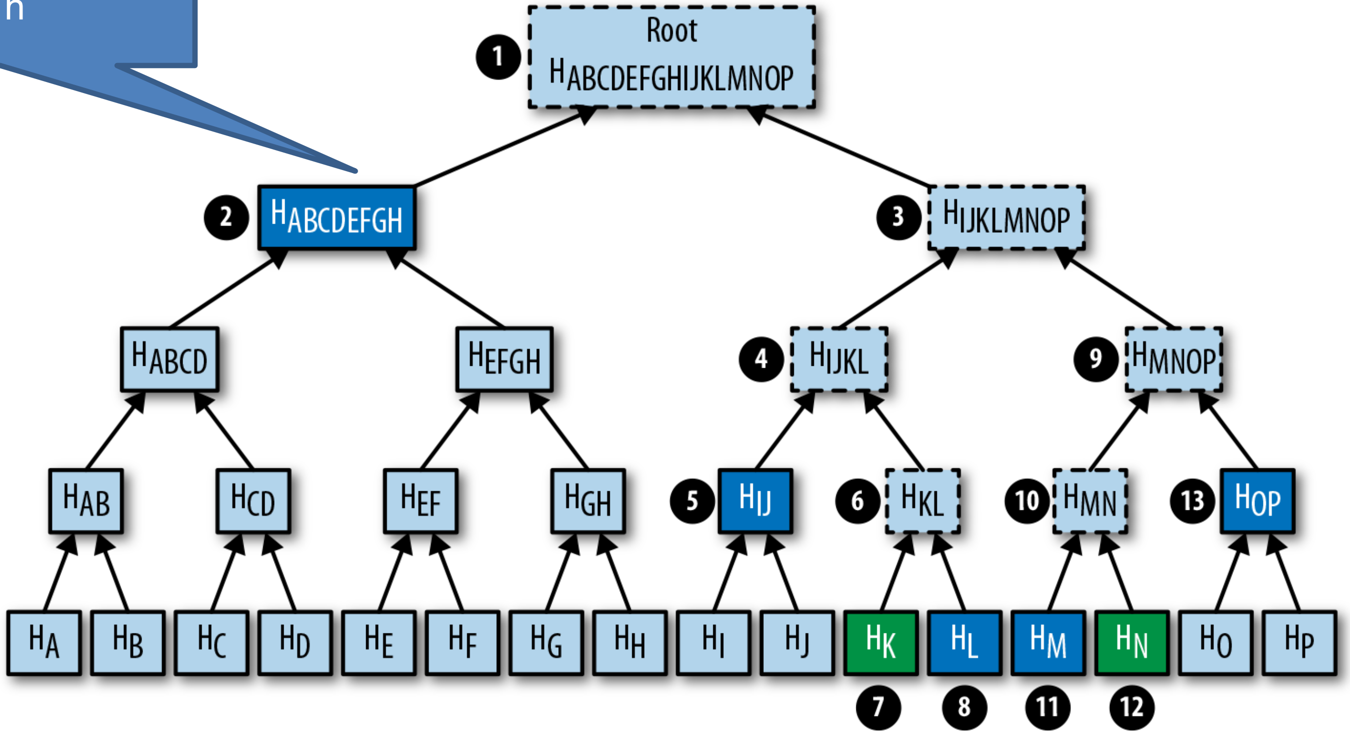

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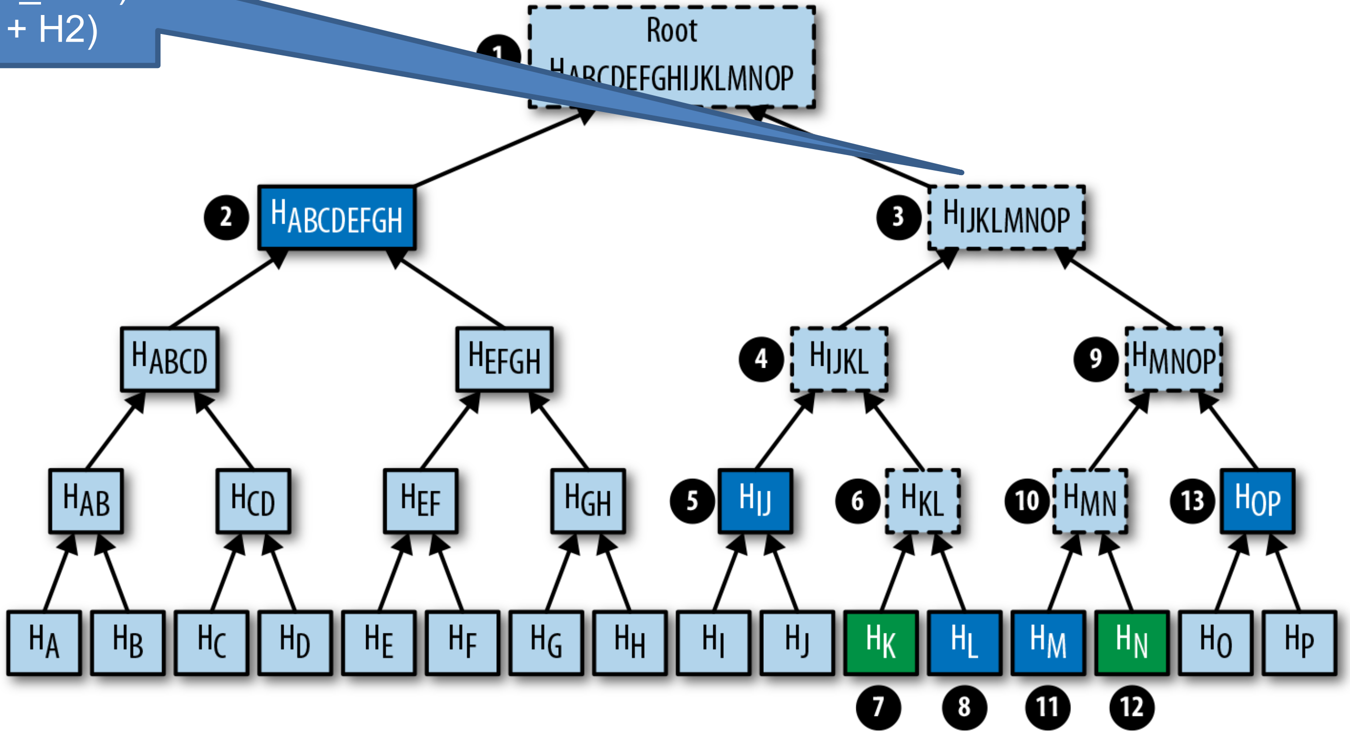

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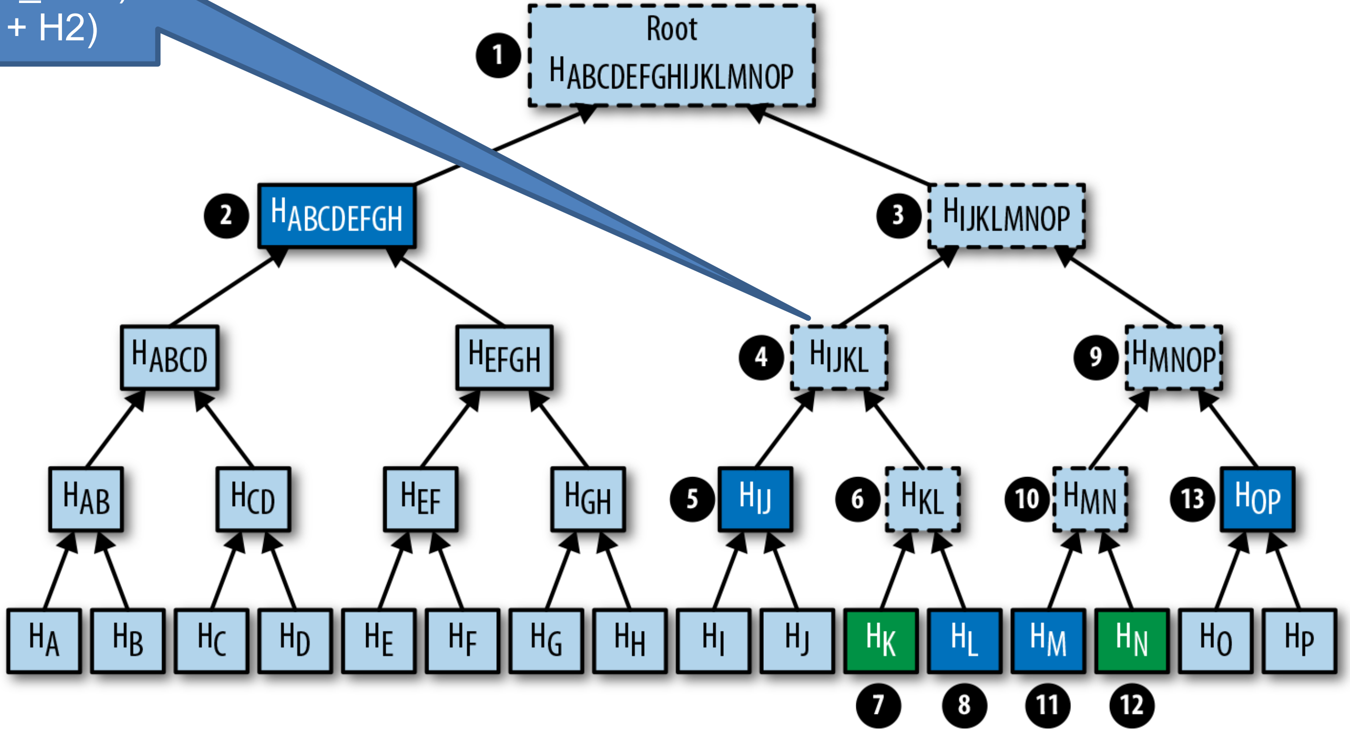

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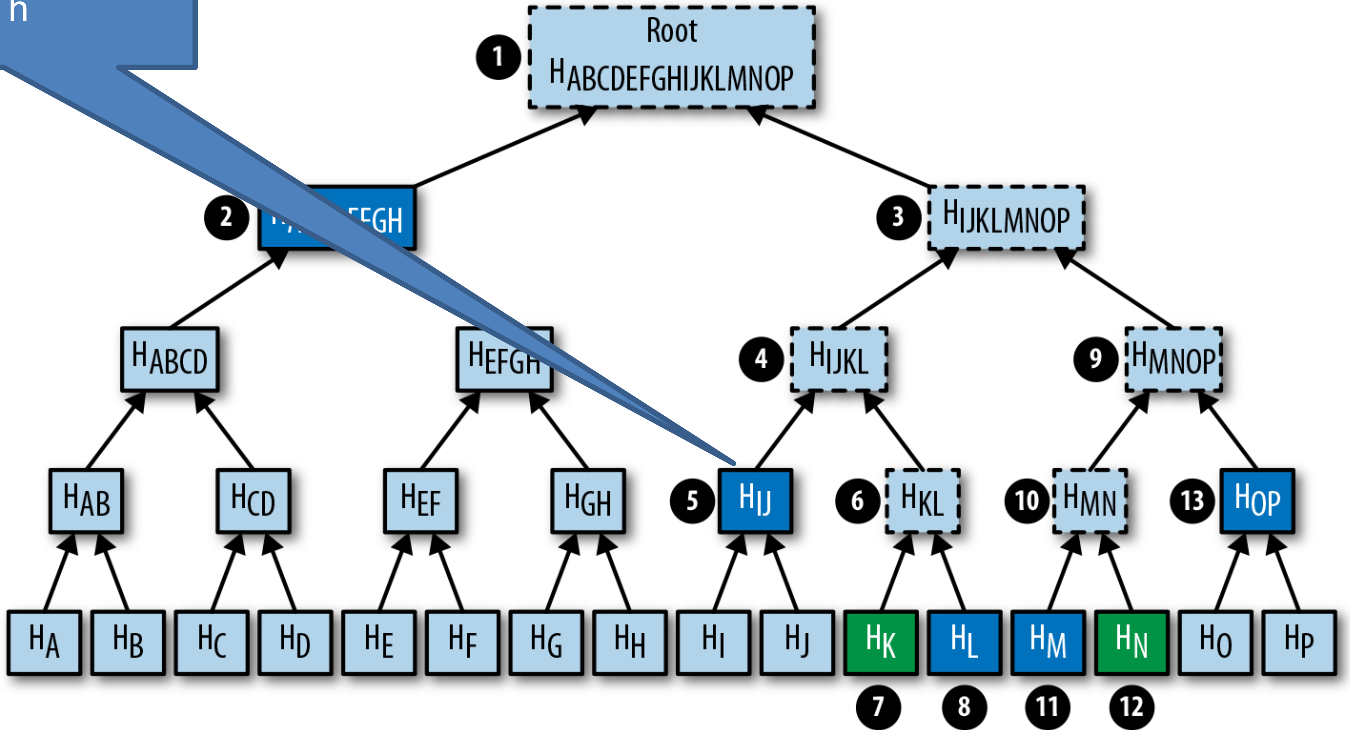

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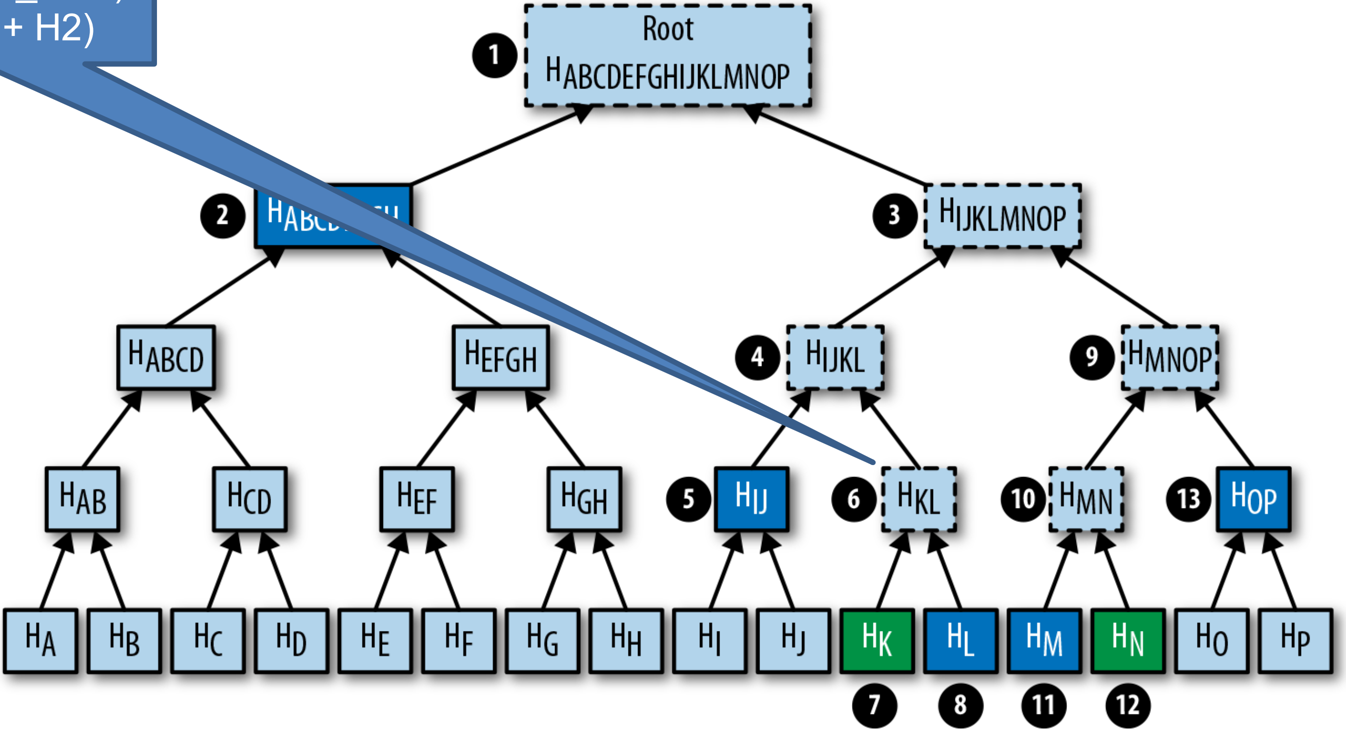

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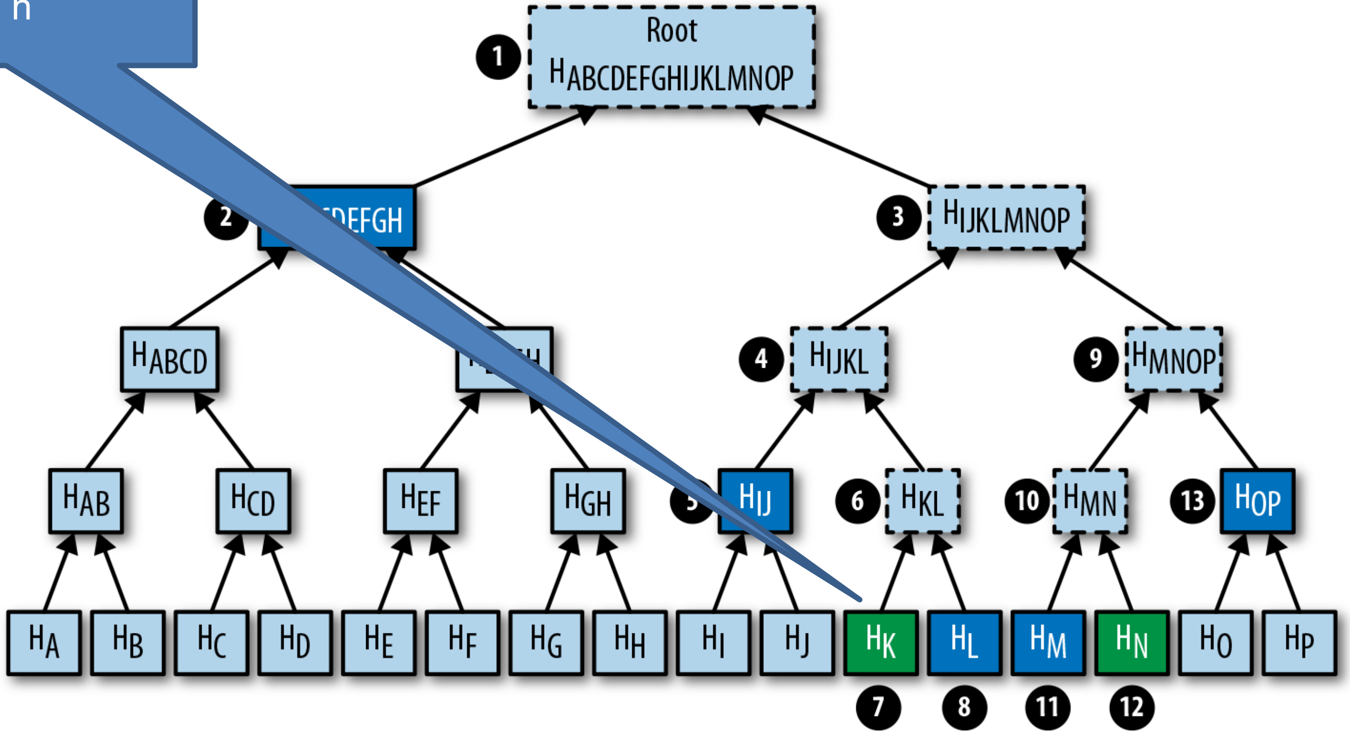

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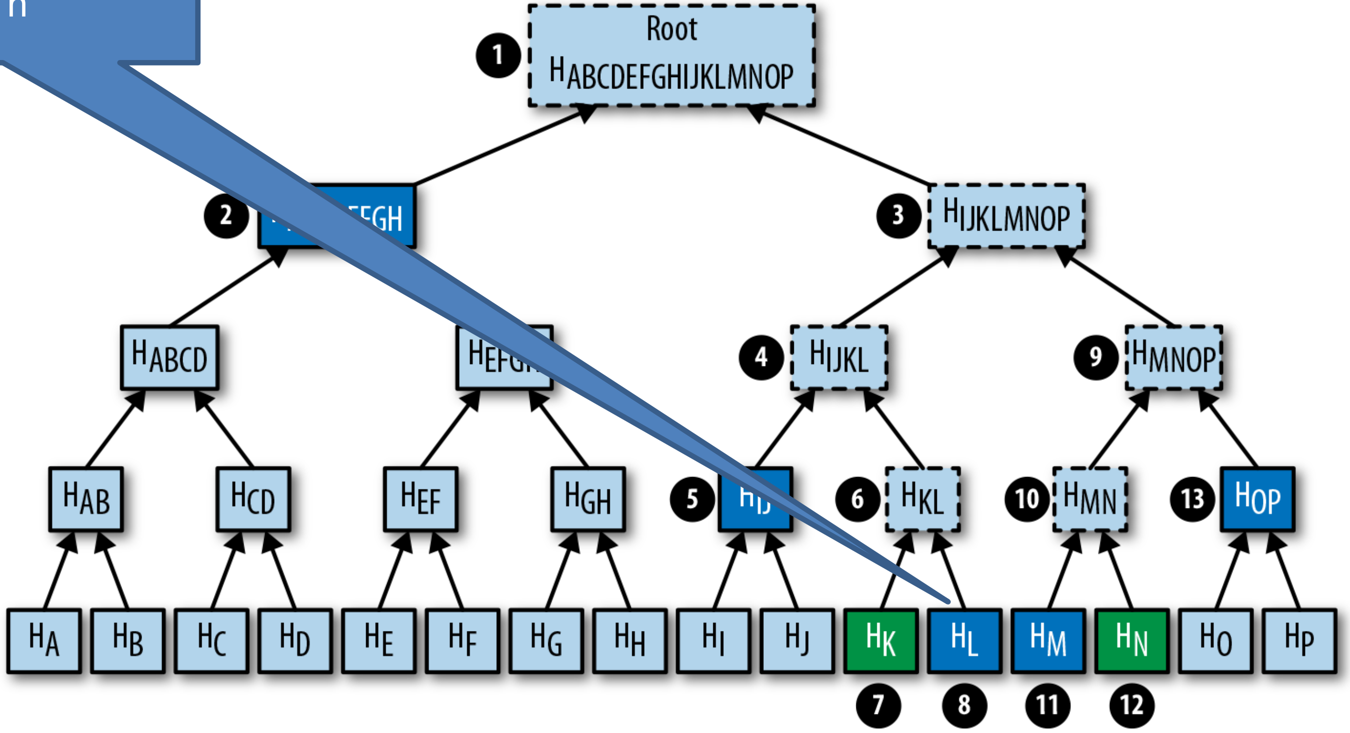

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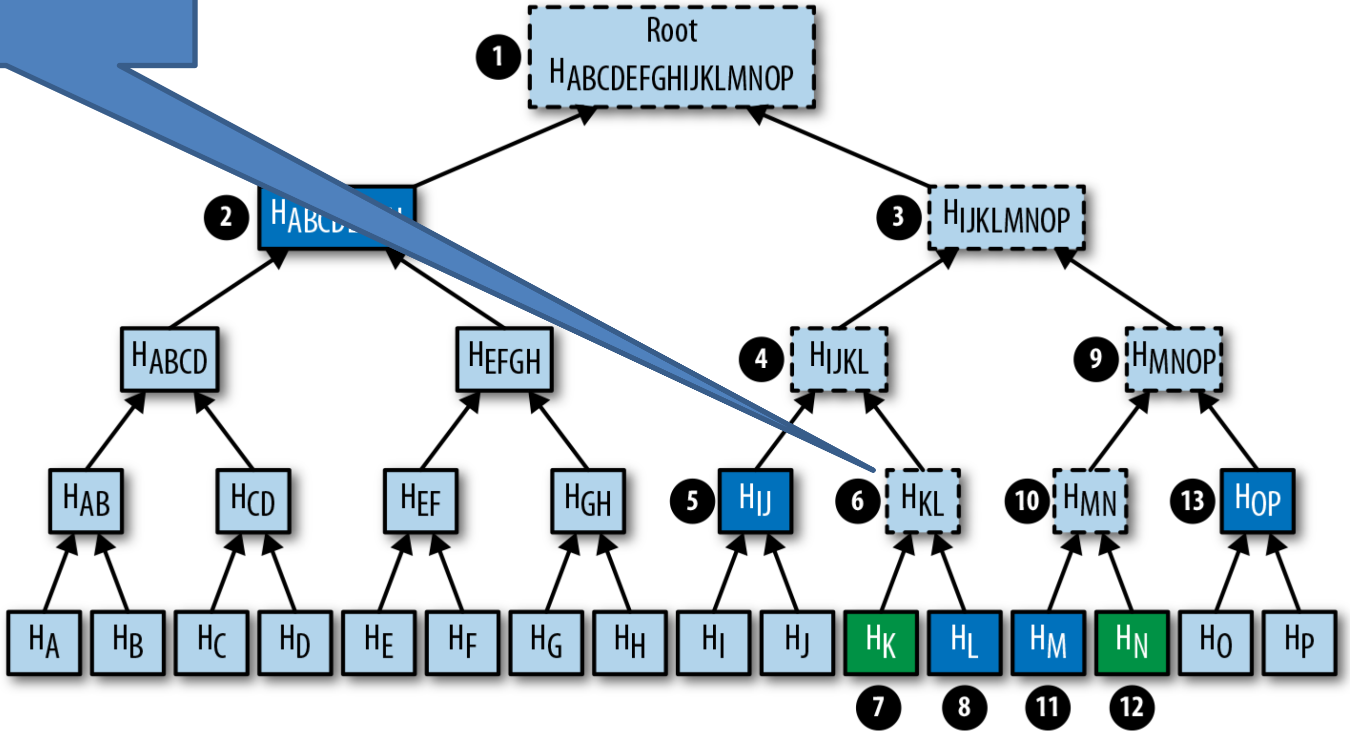
The node can now compute the hash!



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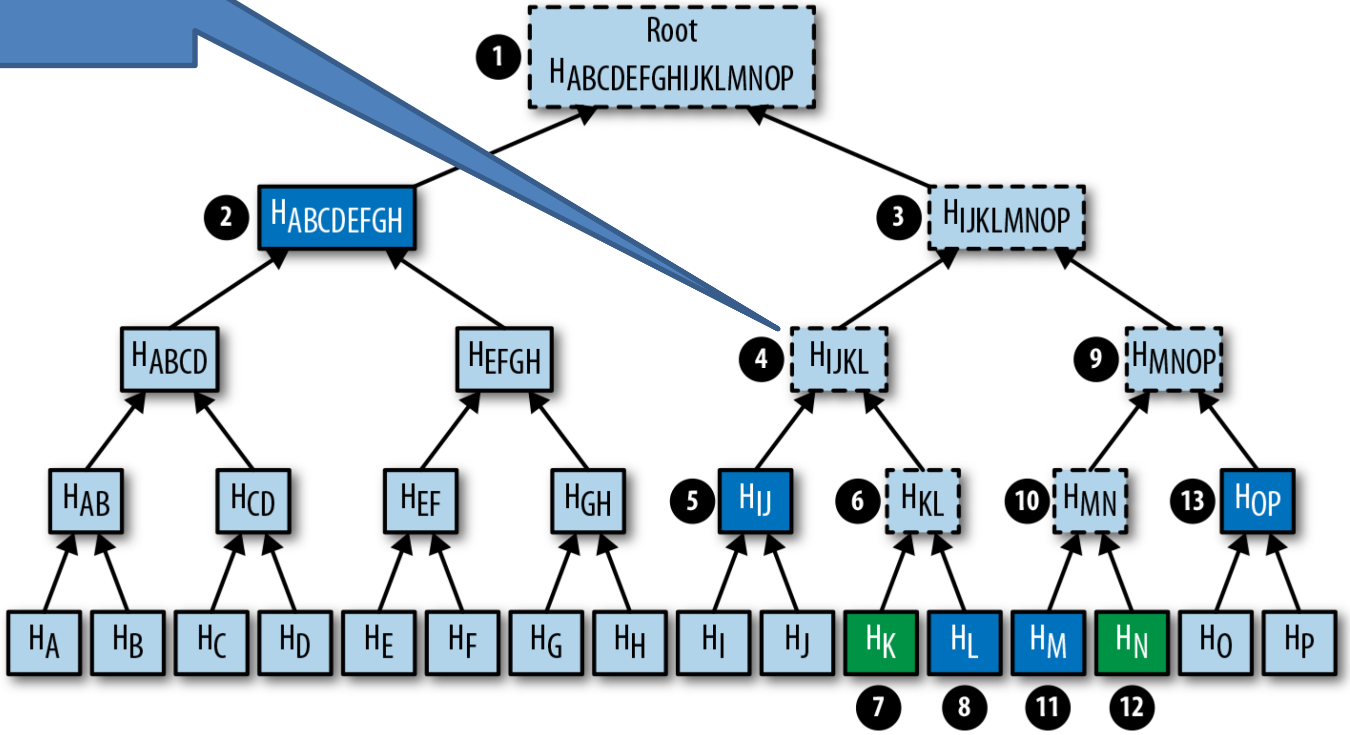

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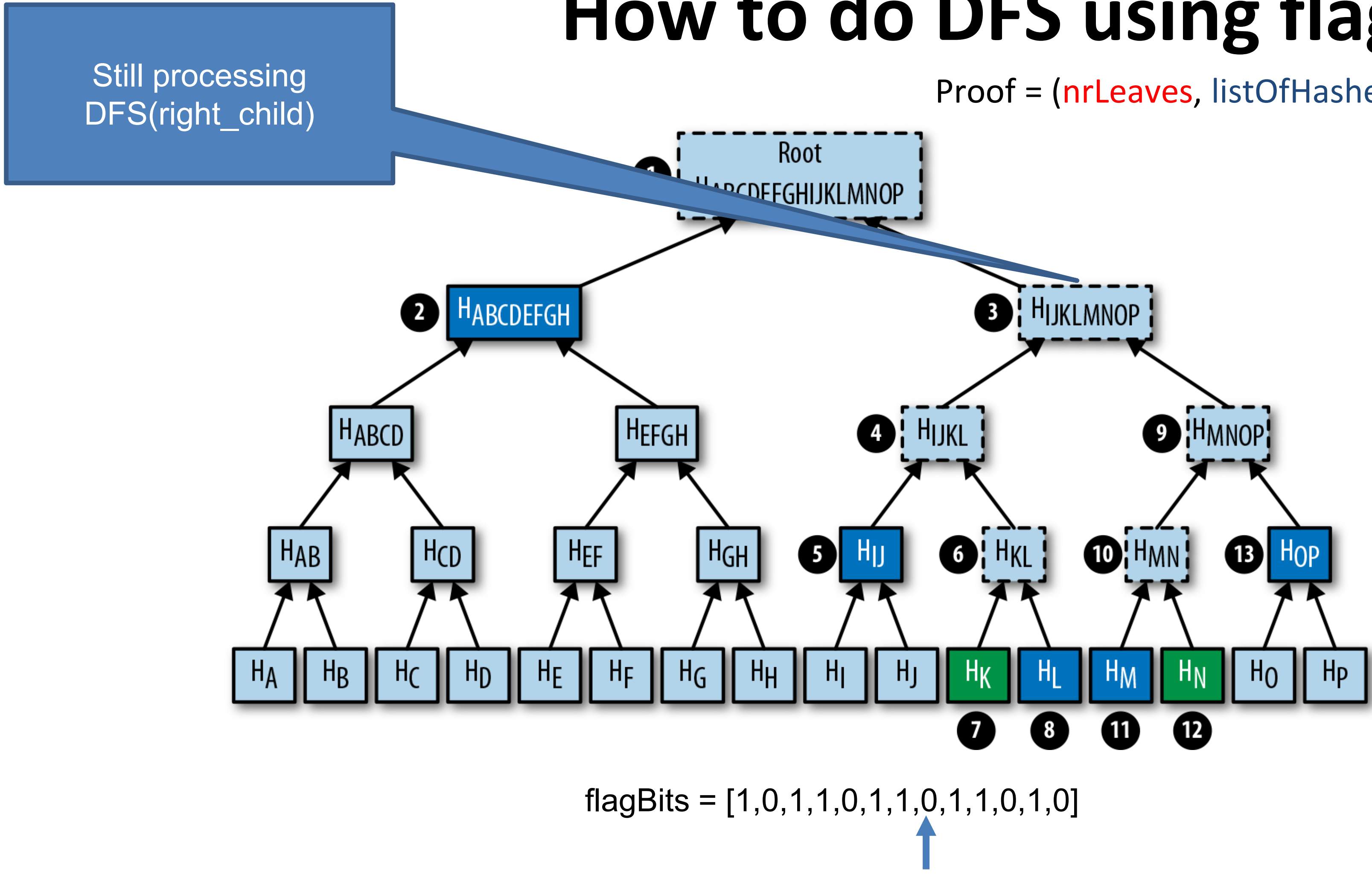

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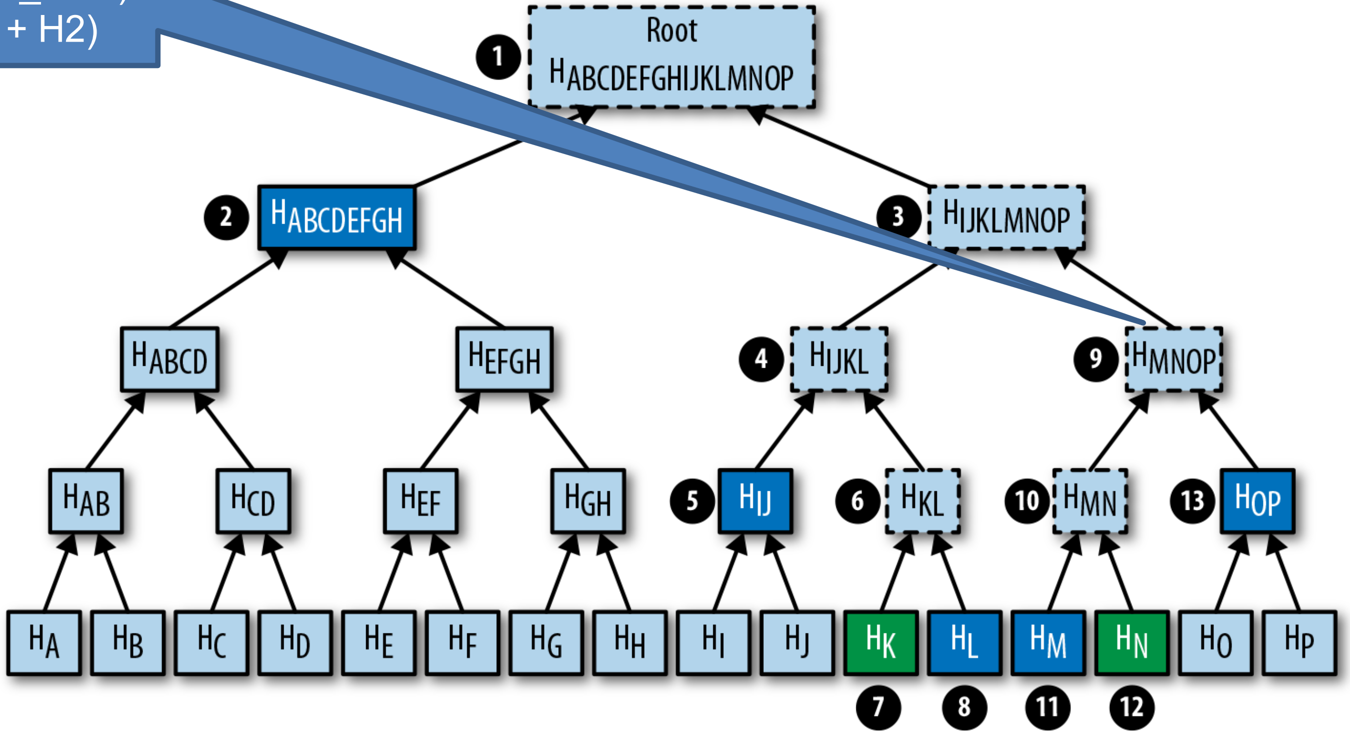

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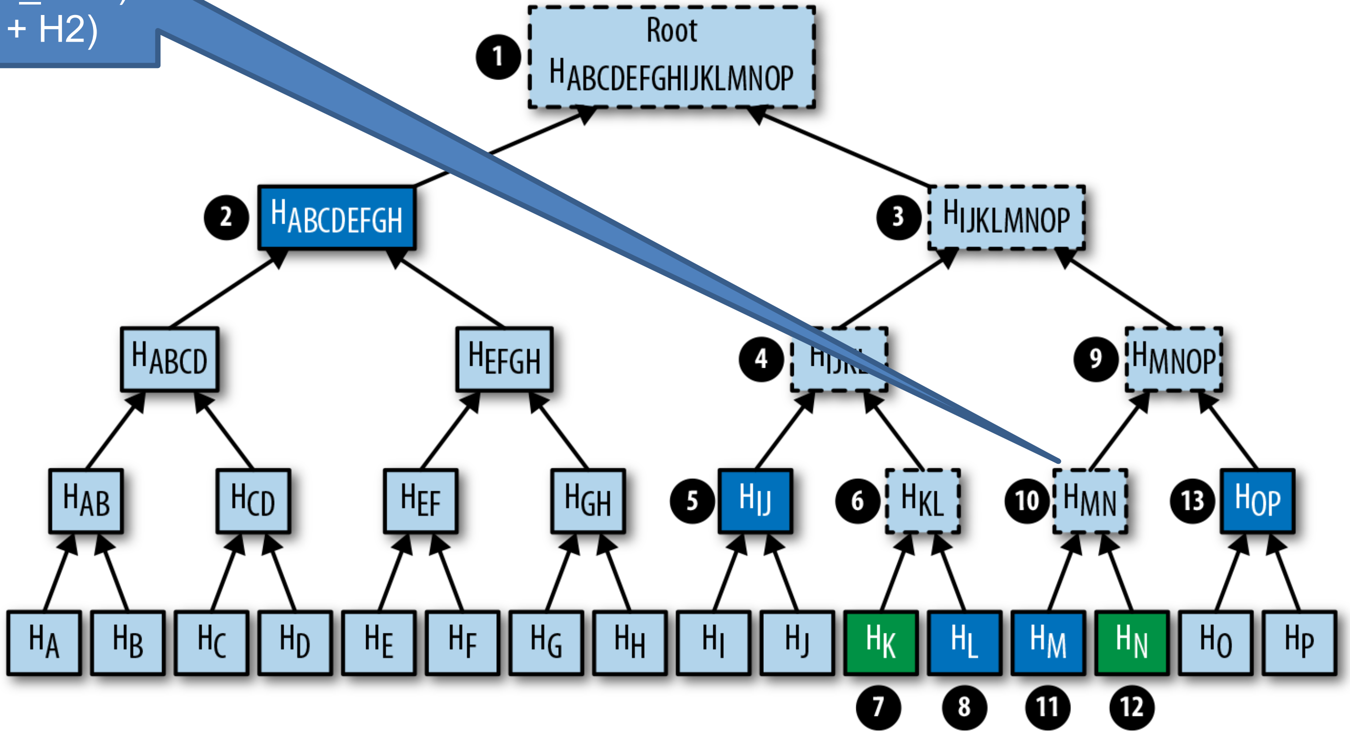

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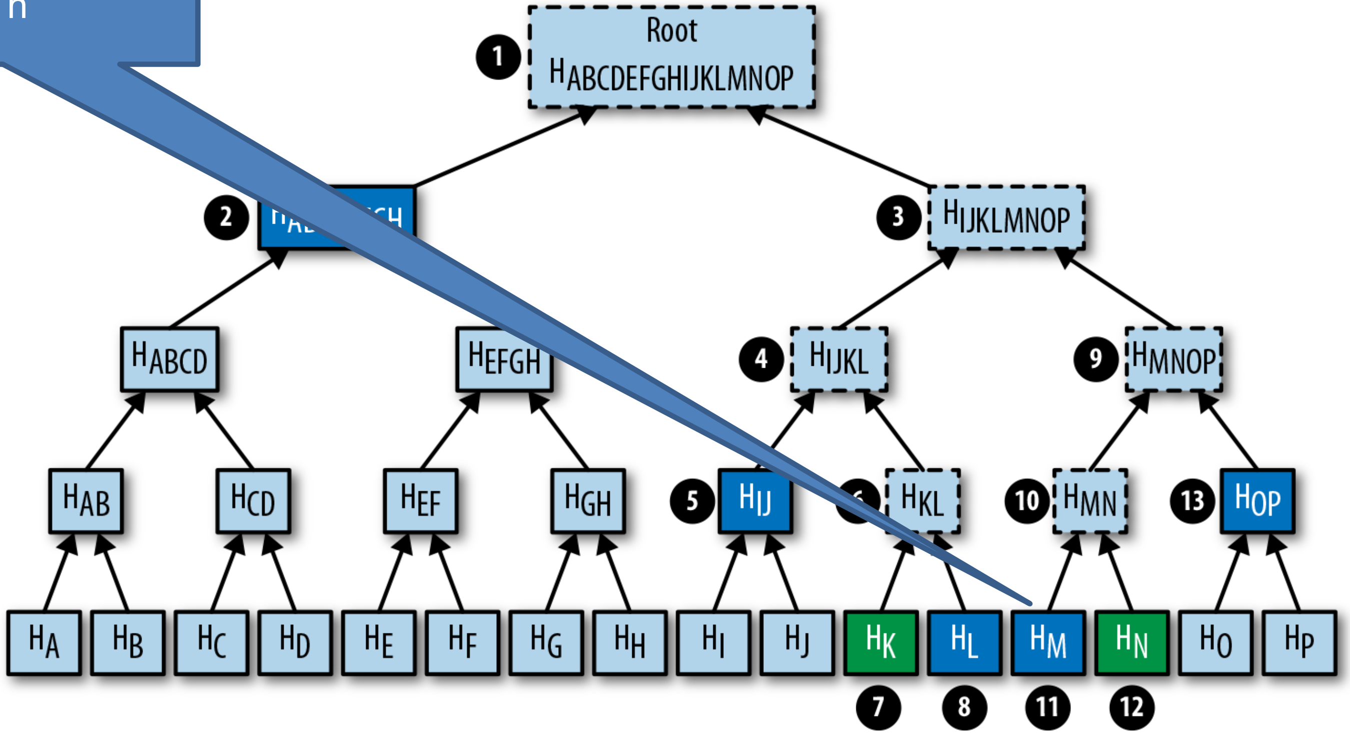

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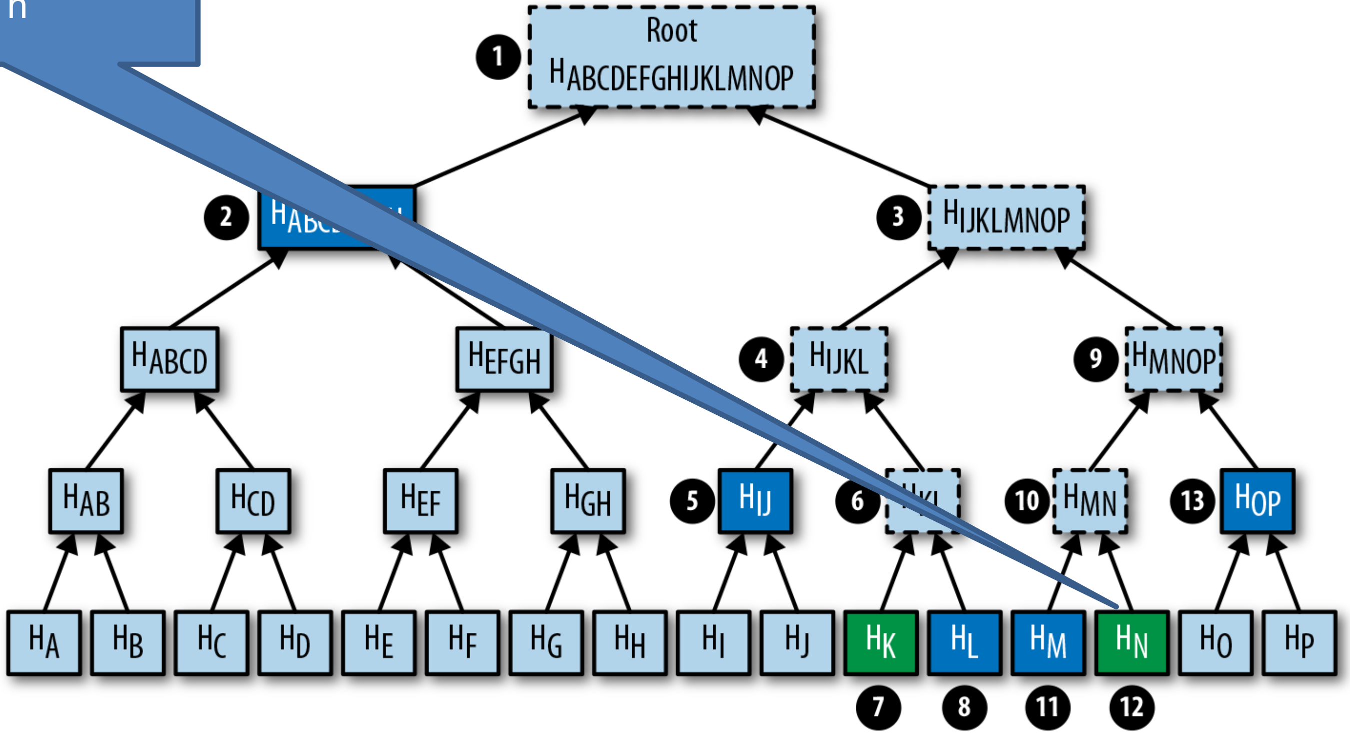

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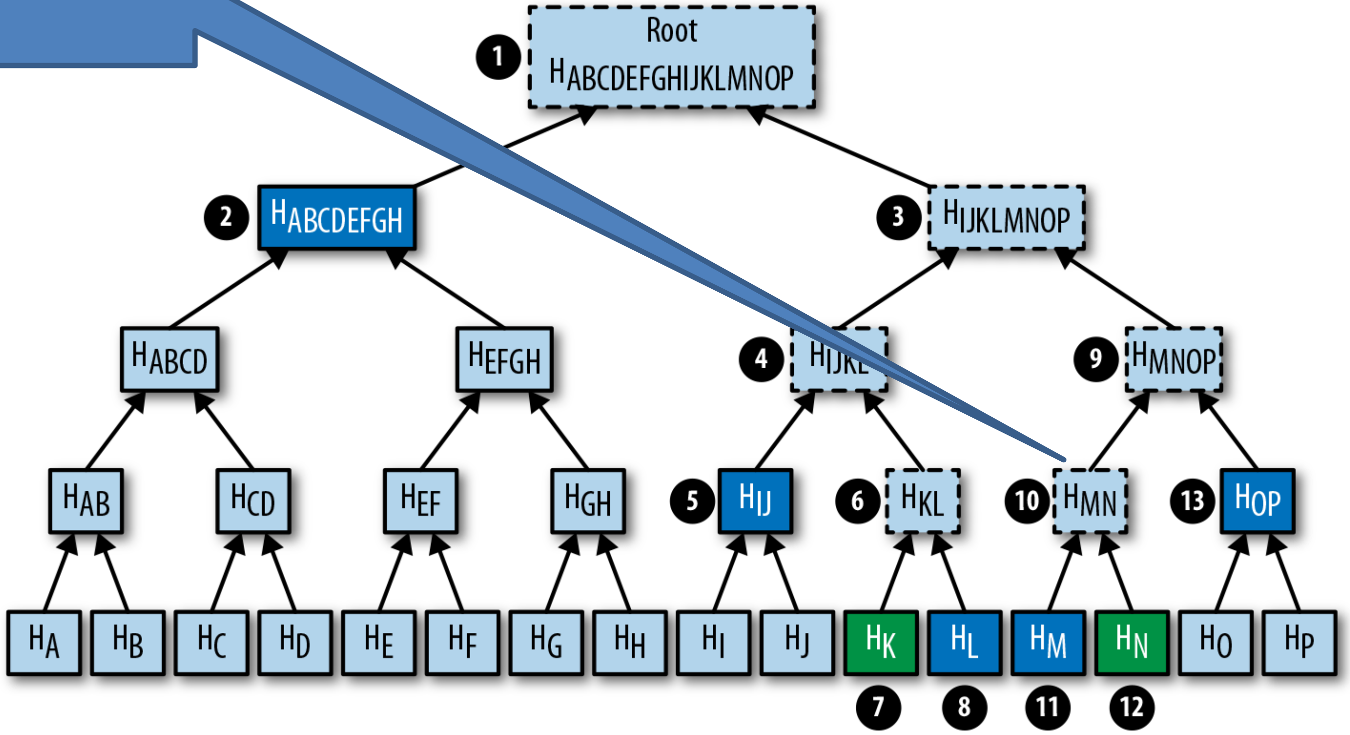

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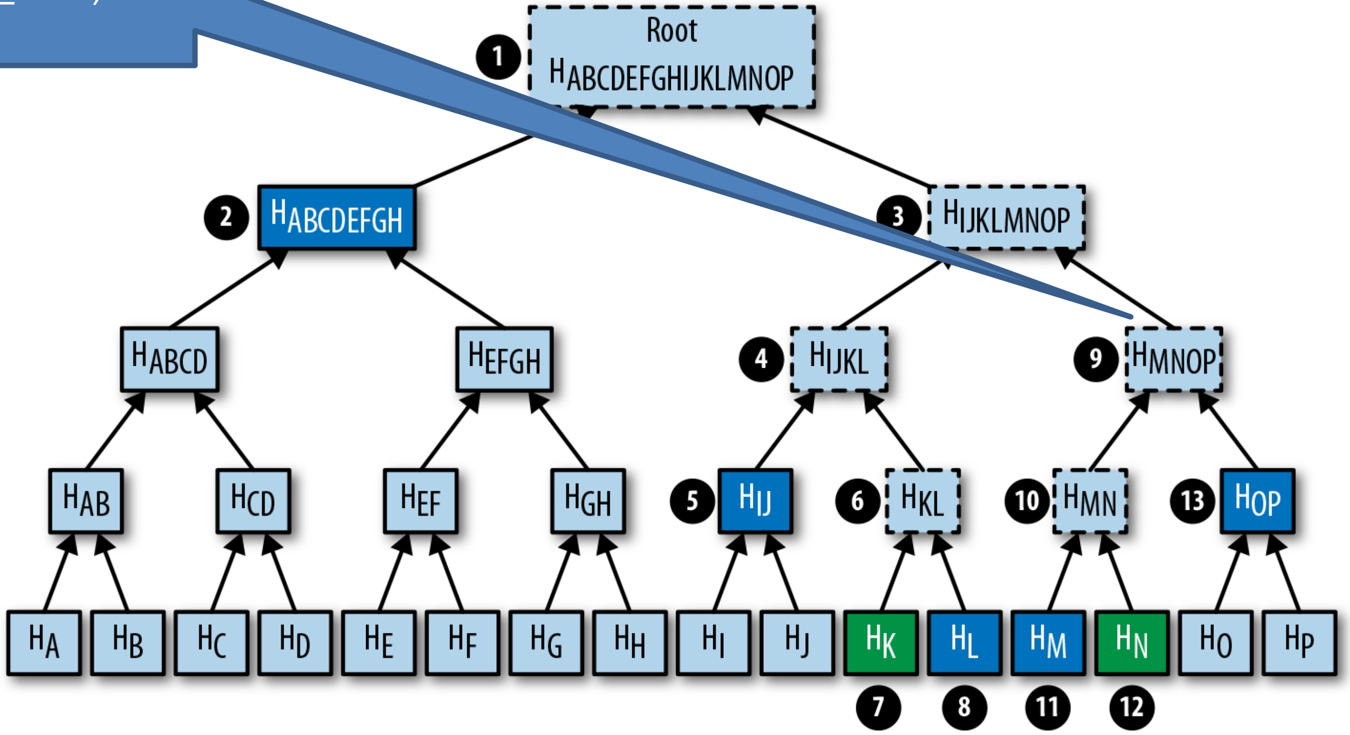
Still processing DFS(right_child)



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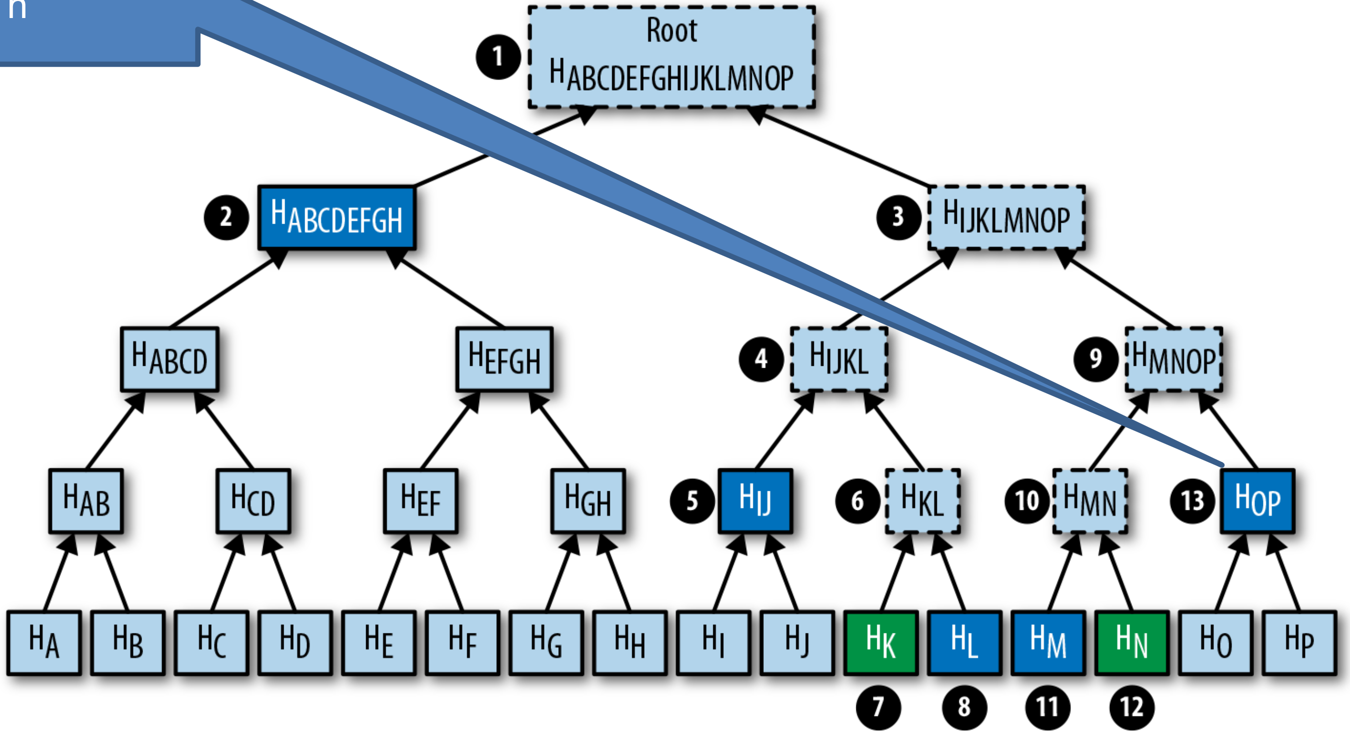

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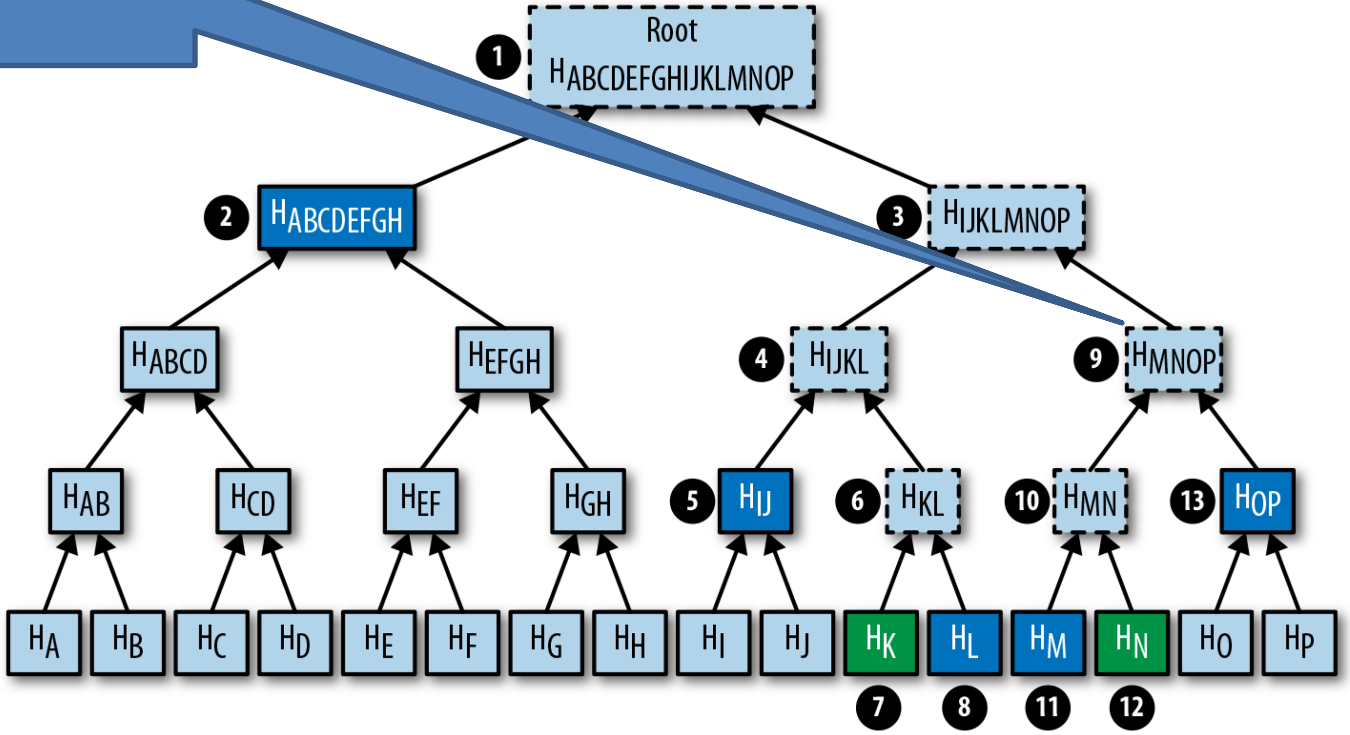

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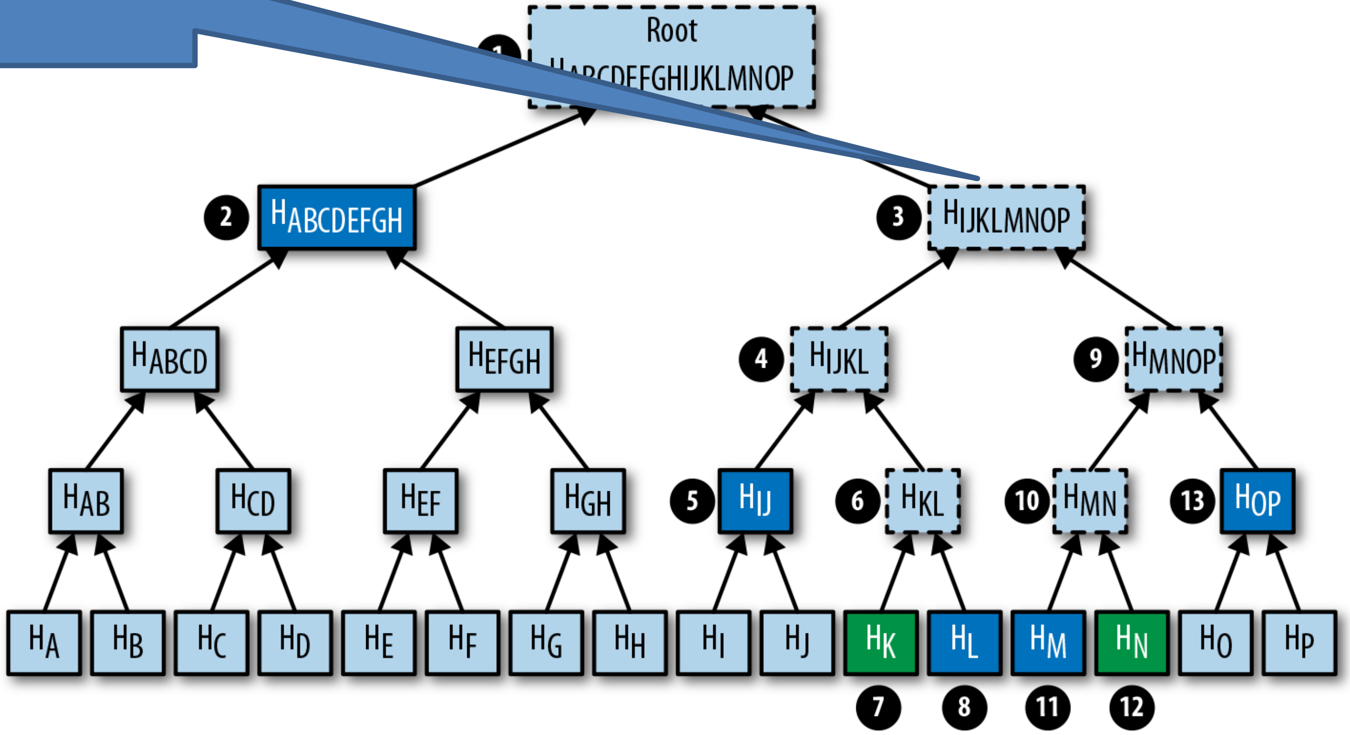

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How to do DFS using flagBits?

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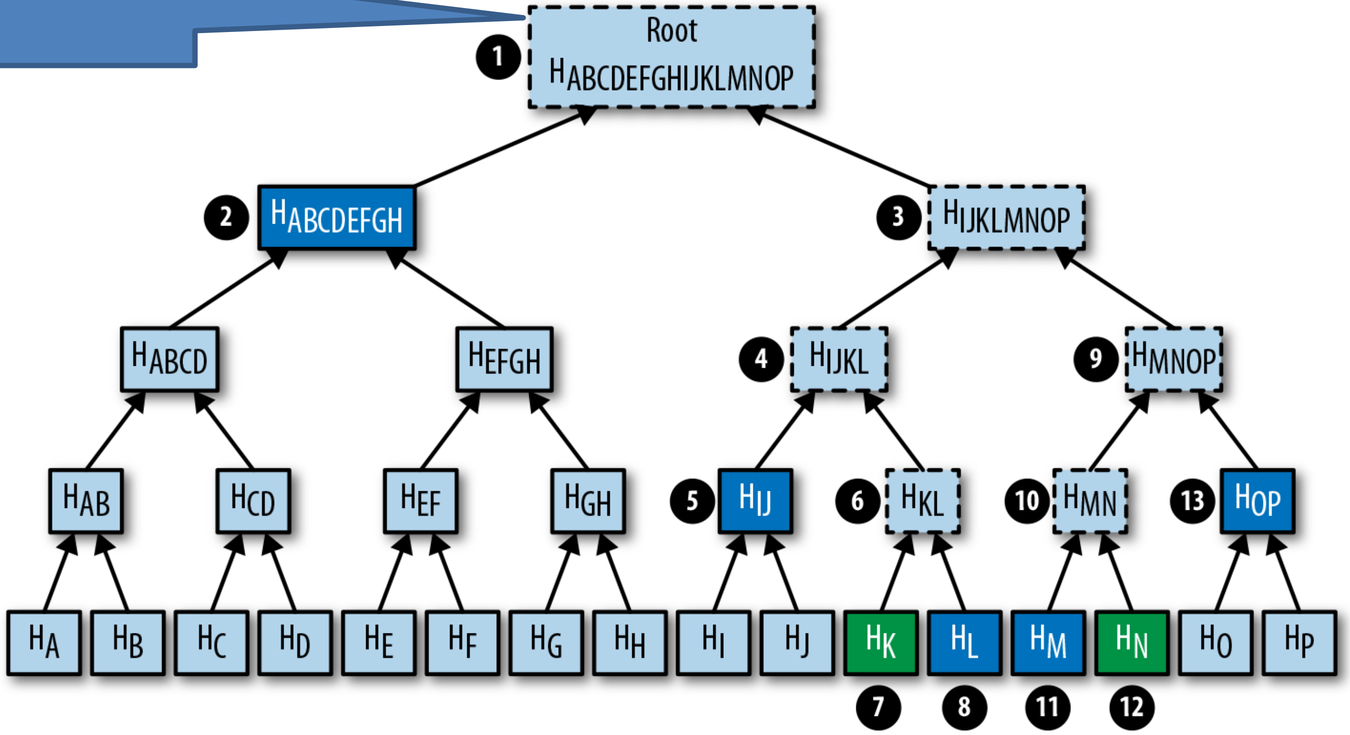
We can compute the root



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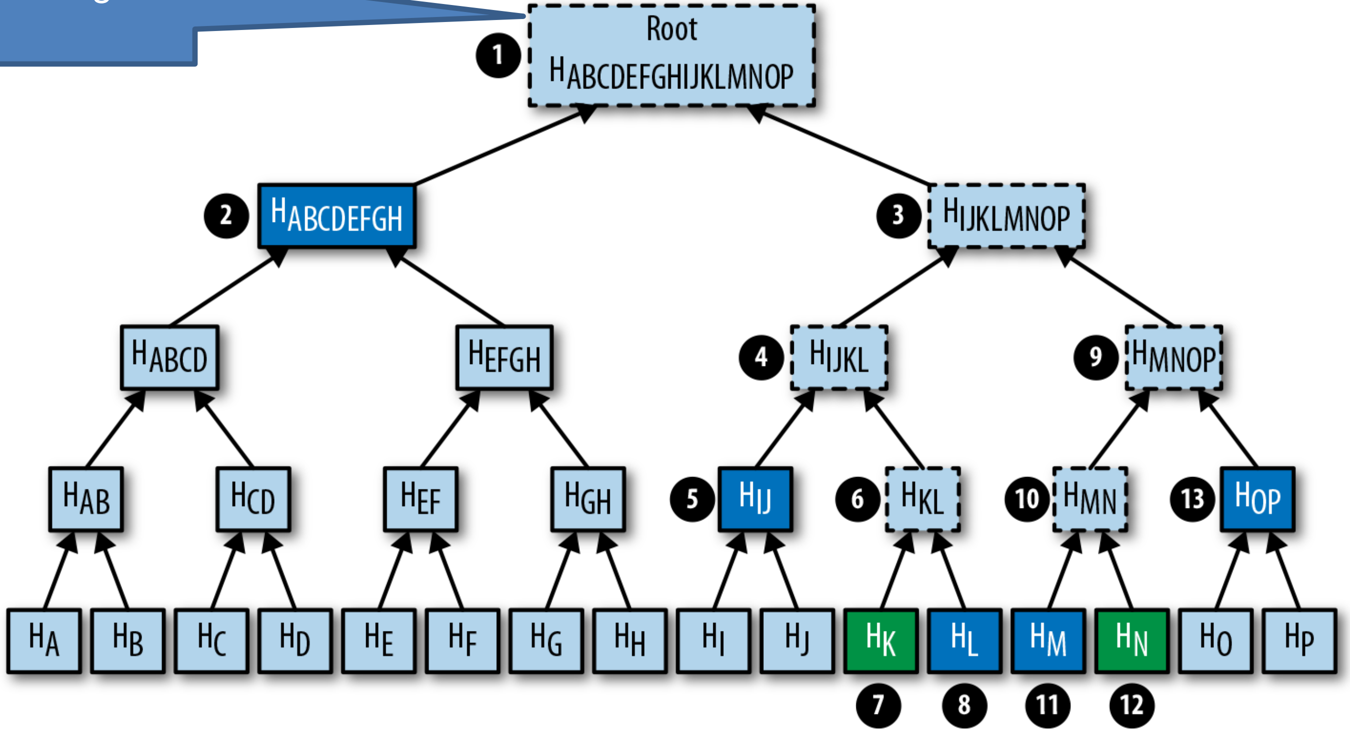
How to do DFS using flagBits?

If the hash is equal to the one we have all good

Proof = (nrLeaves, listOfHashes, flagBits)



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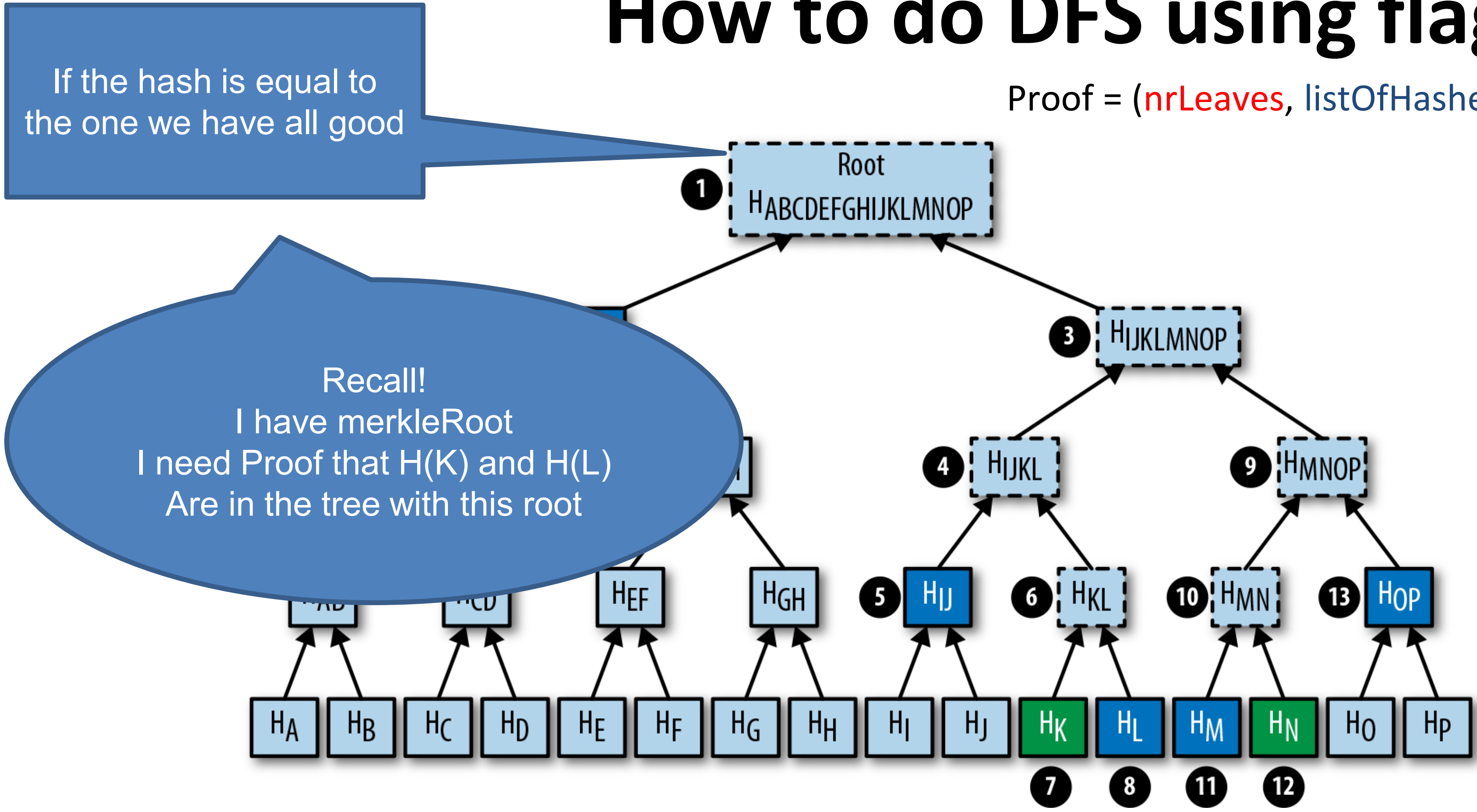
Proof = (nrLeaves, listOfHashes, flagBits)


If the hash is equal to the one we have all good
Recall! I have merkleRoot I need Proof that H(K) and H(L) Are in the tree with this root
1 Root $H_{ABCDEFGHIJKLMNOP}$
3 $H_{IJKLMNOP}$
4 H_{IJKL}
9 H_{MNOP}
H_{AB}
H_{CD}
H_{EF}
H_{GH}
5 H_{IJ}
6 H_{KL}
10 H_{MN}
13 H_{OP}
H_A H_B H_C H_D H_E H_F H_G H_H H_I H_J H_K H_L H_M H_N H_O H_P
7 8 11 12

flagBits = [1,0,1,1,0,1,1,0,1,1,0,1,0]

Practice time!

- Now we need to implement all this!

References

- Jimmy Song, Programming Bitcoin, chapter 11