

Task 1

Research object:

Bullet with projectile motion

constraints:

$$\text{mass} = 13.6 \text{ g}$$

$$v_0 = 870 \text{ m s}^{-1}$$

$$L = 1500 \text{ m}$$

H
air friction coefficient $k = 1.3 \times 10^{-5}$

Solution:

Basic projectile motion

• without friction

$$v_{x0} = v \cos \alpha$$

$$v_{y0} = v \sin \alpha$$

$$v_x = v \cos \alpha$$

$$v_y = v \sin \alpha - gt$$

$$a_x = 0$$

$$a_y = -g$$

$$H = \frac{v^2 \sin^2 \alpha}{2g}$$

$$R = L = \frac{v^2 \sin 2\alpha}{2g}$$

we can find α from L equation

$$\alpha = \frac{\arcsin(Lg/v^2)}{2} = 0.556$$

we can find the cargo ship maximum height from L equation

$$H_{\max} = 10741.825354 \text{ m}$$

• without air drag

$$F_c = -kv^2$$

$$v_{x0} = v \cos \alpha$$

$$v_{y0} = v \sin \alpha$$

$$a_y = -\frac{g}{m} - \frac{kv_y^2}{m}$$

$$a_x = \frac{kv_x^2}{m}$$

$$v_x = v \cos \alpha - \frac{F_c}{m} t = v \cos \alpha - \frac{kv_x^2 \cdot t}{m}$$

$$v_y = v \sin \alpha - \frac{gt}{m} - \frac{kv_y^2}{m}$$

we need to use numerical integration to find the velocity
then integrate in order to find the position in x-axis
Then we can substitute initial conditions to get α

By numerical integration for the differential equation:

$$\frac{dv_x}{dt} = -\frac{k v_x^2}{m}$$

Task 2

Research object:

Particle M - translatory motion in tube
~~rotational motion~~ (planar motion)

Object A - rotational motion

Constraints:

$$m = 0.02 \text{ kg}$$

$$\omega = \pi \text{ rad.s}^{-1}$$

$$r = 0.5 \text{ m}$$

initial conditions:

$$t_0 = 0$$

$$x_0 = 0$$

$$\dot{x}_0 = 0.4 \text{ m.s}^{-1}$$

Force analysis:

G (particle weight)

N

Solution: when the particle is at the origin (starting point)

$$m\ddot{x} = \sum_i F_i + F_c$$

$$F_c = m\omega^2 x$$

$$m\ddot{x} = mg \sin \omega t + m\omega^2 x$$

$$\ddot{x} = g \sin \omega t + \omega^2 x$$

$$\ddot{x} - \omega^2 x = g \sin \omega t$$

$$x_1 = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$x_2 = A \sin \omega t$$

till the time the point won't leave the channel $t = 0.3$

To find pressure: $N + F_c - mg \cos \omega t = 0$

$$F_c = 2m\omega v_r \rightarrow \ddot{x}$$

$$\Rightarrow N = mg \cos(\omega t) - 2m\omega v_r$$

Task 3

Research object:

- System of
- 3 masses
 - 2 pulleys
 - strings
 - Big block

Force analysis:

T_1, T_2

R_1, R_2

G_1, G_2, G_3

N_2, N_3

F_{r3}, F_{r2}

Solution:

$$(M_1 + M_2 + M_3) a = F_{r1} + F_{r2} \cos(60)$$

$$N_2 = M_2 g$$

$$N_3 = M_3 g \cos(60)$$

$$F_{r2} = \mu N_2 \quad F_{r3} = \mu N_3$$

we can integrate acceleration with respect to time to get velocity, then integrate to get position

Another solution: using center of mass theorem of motion

$$\begin{aligned} x_c &= \frac{\sum m_i x_{ci}}{\sum m_i} = \frac{m_1 x_{c1} + m_2 x_{c2} + m_3 x_{c3} + m x_c}{\sum m_i} \\ &= \frac{m_1 (x_{c1} - \Delta) + m_2 (x_{c2} + 1 - \Delta) + m_3 (x_{c3} + \frac{1}{2} - \Delta) + m (x_c - \Delta)}{\sum m_i} \end{aligned}$$

$$-m_1 \Delta + m_2 - m_2 \Delta + \frac{m_3}{2} - m_3 \Delta = 0$$

$$\Delta = \frac{m_2 + m_3/2}{m + m_1 + m_2 + m_3} = \frac{15 + \frac{10}{2}}{20 + 15 + 10 + 100} = 0.14 \text{ m}$$

$$\Delta > 0$$

Task 4

Research object:

- Pulley
- rope
- Load

Force analysis:

R_m R_y

G_{man}

G_{pulley}

G_{load}

By intuition, the man climbs the rope then the load on the other side will go up too.

Solution:

$$J = \frac{mr^2}{4}$$

$$v_B = \omega r$$

$$m v_B \cdot r + m (v_B - v) \cdot r + \frac{m}{4} r^2 \omega = 0$$

$$\Rightarrow v_B \cdot r + (v_B - v) + \frac{1}{4} v_B = 0 \quad \Rightarrow \quad v_B = \frac{4}{9} v$$

the load will go up with velocity $\frac{4}{9} v$