Name: Michael Rogers

ID: 105667404

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**

Problem Set 3a (9 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

Name: Michael Rogers
ID: 105667404

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 3a (9 points) **Fall 2019, CU-Boulder**

1. Suppose we have a number of events m_i. Each event starts at time s_i and finishes at time e_i, where $0 \leq s_i < e_i$. We represent the event m_i with the closed interval $[s_i, e_i]$. Our goal is to construct a maximum size set of events, where no two events in the set overlap.

   Suppose the following intervals are provided.

Event Index	Interval
1	$[1, 2]$
2	$[3, 4]$
3	$[5, 6]$
4	$[7, 8]$
5	$[0, 20]$.

   (a) (1 pt) What is the maximum size set of events that can be selected such that no two events in the set overlap? Include the list of the events selected in your answer.

   Solution. Reordered by smallest e_i:

Event Index	Interval
1	$[1, 2] \rightarrow$ Select
2	$[3, 4] \rightarrow$ Select
3	$[5, 6] \rightarrow$ Select
4	$[7, 8] \rightarrow$ Select
5	$[0, 20] \rightarrow$ Delete.

   This is essentially the same problem as the interval scheduling problem. If we prioritize the events by smallest end time, we will have an optimal solution. That is:

$$e_i < s_{i+1}$$

   By this logic, the maximum size set is size 4. This set would be $[1, 2, 3, 4]$. Event 5 would be excluded as $e_4 = 8$ and $s_5 = 0$ thus breaking our greedy rule above.

(b) (2 pt) Suppose we sort the intervals in ascending order by start time. Consider a greedy algorithm that selects the next event based on earliest start time, so long as the interval selected does not conflict with any previously selected interval. Using the intervals provided, show that this greedy algorithm fails to provide a maximum size set of events, where no two events in the set overlap. That is, the solution returned by this greedy algorithm is not optimal.

Solution. Reordered Table:

Event Index	Interval
5	$[0, 20] \rightarrow$ Select
1	$[1, 2] \rightarrow$ Delete
2	$[3, 4] \rightarrow$ Delete
3	$[5, 6] \rightarrow$ Delete
4	$[7, 8] \rightarrow$ Delete.

Final Set:

$$[5]$$

This solution would fail because event 5 has the earliest start time. $s_5 = 0 < s_{1:4}$ but, $e_5 = 20 > s_{1:4}$ and thus $m_{1:4} \in m_5$. Since this algorithm selects the event based on the smallest start time, it would select 5 first, and then all of the other events would conflict, causing a size set of 1, instead of the optimal 4. All the others conflict because $s_{1:4} > s_5$ and $e_{1:4} < e_5$

2. Using the same definition in Problem 1, suppose the following intervals are provided.

Event Index	Interval
1	$[1, 10]$
2	$[11, 20]$
3	$[21, 30]$
4	$[9, 12]$
5	$[19, 22]$.

(a) (1 pt) What is the maximum size set of events that can be selected such that no two events in the set overlap? Include the list of the events selected in your answer.

Solution. Reordered by smallest e_i:

Event Index	Interval
1	$[1, 10] \rightarrow$ Select
4	$[9, 12] \rightarrow$ Delete
2	$[11, 20] \rightarrow$ Select
5	$[19, 22] \rightarrow$ Delete
3	$[21, 30] \rightarrow$ Select.

The optimal solution is one such that $e_i < s_{i+1}$ sorted by end time.
$\Rightarrow$ The algorithm will select $[1, 2, 3]$. Therefore the maximum size of events is 3.

(b) (2 pt) Suppose we sort the intervals in ascending order by interval length. For events with the same length, order by start time. Consider a greedy algorithm that selects the next interval based on the smallest interval length, so long as the interval selected does not conflict with any previously selected interval. Using the intervals provided, show that this greedy algorithm fails to provide a maximum size set of events, where no two events in the set overlap. That is, the solution returned by this greedy algorithm is not optimal.

Solution. Reordered Table:

Event Index	Interval
4	$[9, 12] \rightarrow$ Select
5	$[19, 22] \rightarrow$ Select
1	$[1, 10] \rightarrow$ Delete
2	$[11, 20] \rightarrow$ Delete
3	$[21, 30] \rightarrow$ Delete.

Final Set:

$$[4, 5]$$

As illistrated above, index 4 and 5 are selected, resulting in a 2 size list. As shown in part a, the maximum size list is 3, and since this list is 2 then there's no way it can be the optimal solution.

Name: Michael Rogers
ID: 105667404

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 3a (9 points) **Fall 2019, CU-Boulder**

3. Consider again the same scenario as in Problems 1 and 2, and suppose the following intervals are provided.

Event Index	Interval
1	$[1, 3]$
2	$[4, 6]$
3	$[7, 9]$
4	$[10, 12]$
5	$[2, 5]$
6	$[2, 5]$
7	$[2, 5]$
8	$[5.5, 7.5]$
9	$[8, 11]$
10	$[8, 11]$
11	$[8, 11]$

(a) (1 pt) What is the maximum size set of events that can be selected such that no two events in the set overlap? Include the list of the events selected in your answer.

Solution. Reordered by Smallest e_i:

Event Index	Interval
1	$[1, 3] \rightarrow$ Select
5	$[2, 5] \rightarrow$ Delete
6	$[2, 5] \rightarrow$ Delete
7	$[2, 5] \rightarrow$ Delete
2	$[4, 6] \rightarrow$ Select
8	$[5.5, 7.5] \rightarrow$ Delete
3	$[7, 9] \rightarrow$ Select
9	$[8, 11] \rightarrow$ Select
10	$[8, 11] \rightarrow$ Delete
11	$[8, 11] \rightarrow$ Delete
4	$[10, 12] \rightarrow$ Delete

Final Optimal Set of 4 Elements:

$$[1, 2, 3, 9]$$

(b) (2 pts) Let c_i denote the number of intervals on our list in which interval i conflicts. For example, interval 1 participates in 3 conflicts: with intervals $5, 6$, and 7. So $c_1 = 3$.

Suppose we sort the intervals in ascending order based on the number of conflicts. So if $c_i < c_j$, then interval i comes before interval j. Consider a greedy algorithm that selects the next interval based on the smallest number of conflicts, so long as the interval selected does not conflict with any previously selected interval. Using the intervals provided, show that this greedy algorithm fails to provide a maximum size set of events, where no two events in the set overlap. That is, the solution returned by this greedy algorithm is not optimal.

Solution. Reordered based on c_i:

Event Index	Interval
8	$[5.5, 7.5] \rightarrow$ Select
1	$[1, 3] \rightarrow$ Select
2	$[4, 6] \rightarrow$ Delete
3	$[7, 9] \rightarrow$ Delete
4	$[10, 12] \rightarrow$ Select
5	$[2, 5] \rightarrow$ Delete
6	$[2, 5] \rightarrow$ Delete
7	$[2, 5] \rightarrow$ Delete
9	$[8, 11] \rightarrow$ Delete
10	$[8, 11] \rightarrow$ Delete
11	$[8, 11] \rightarrow$ Delete

Final Set:

$$[8, 1, 4]$$

Since the size of this set is less than the optimal size, this greedy algorithm cannot be optimal.