Name: Michael Rogers
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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 4a (11 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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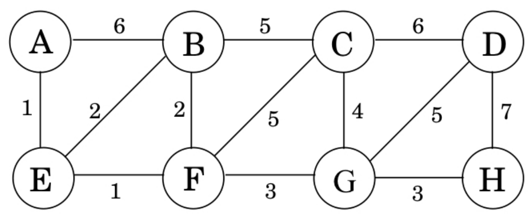
1. (1 pt) What is the definition of a Minimum Spanning Tree (MST)?

   Solution. Given a conncted, undirected graph G=(V,E) where V are verticies and E are edges, a minimum spanning tree is a subgraph that is an undirected tree that contains all verticies in G with minimum weight.

2. (1 pt) Describe in one or two sentences, a greedy rule for constructing an MST.

   Solution. At each step determine an edge (u, v) that can be added to some set of edges A such that A ∪ (u,v) is also a subset of a minimum spanning tree.

3. (3 pts) How many unique MSTs does the following graph have :



   Solution. There are 2 minimum spanning trees.

$$A = [(A, E), (E, B), (E, F), (F, G), (G, C), (G, D), (G, H)] = 19$$

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4. (3 pts) Suppose that you have calculated the MST of an undirected graph $G = (V, E)$ with positive edge weights.
If you increase each edge weight by 2, will the MST change? Prove that it cannot change or give a counterexample if it changes. (Note: Your proof, if there is one, can be a simple logical argument.)

Solution. The MST will not change.

<u>Proof:</u> Spose' some integer k is the weight of a some edge $\in$ G.

If $k = j$ where $j = 2k$ then $\frac{j}{2} = k$ and $\frac{1}{2}\sum_{j=1}^{n} j = \sum_{k=1}^{n} k$

$\Rightarrow \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + ... + \frac{2k}{2} = 1 + 2 + 3 + ... + k$

Since the two series are equal, it holds that if the each weight in the tree is increased by 2 then each weight of the MST will increase by 2 as well. Thus our MST will remain with the same edges in G.

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5. (3 pts) Suppose that you have calculated the shortest paths to all vertices from a fixed vertex $s \in V$ of an undirected graph $G = (V, E)$ with positive edge weights.
   If you increase each edge weight by 2, will the shortest paths from s change? Prove that it cannot change or give a counterexample if it changes.(Note: Just as in Part a, your proof can be a simple logical argument.)

   Solution. The overall weight of the shortest path will change, but the edges comprising the shortest path will not.

   Proof:

   The same logic can be followed as in 4. If all of the edge weights $e_{i,j}$ increase by two, then the graph will simply be the same graph with larger weights. Therefore, the edges comprising the shortest path will not change. However, since every weight in the shortest path will be increased, then the overall weight of the shortest path to each point will become larger.