ACHIEVING INTERACTIVE INDIRECT ILLUMINATION USING THE GPU

A Thesis

Presented to the

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in Partial Fulfillment

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Master of Science

in

Computer Science

By

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ABSTRACT

With the growth of computers and technology, so to has grown the desire to accurately recreate our world using computer graphics. However, our world is very complex and in many ways beyond our comprehension. Therefore, in order to perform this task, we must consider multiple disciplines and areas of research including physics, mathematics, optics, geology, and many more to at the very least approximate the world around us. The applications of being able to do this are plentiful as well, including the use of graphics in entertainment such as movies and games, science such as in weather forecasts or simulations and medicine with body scans, or use in architecture, design, and many other areas. In order to recreate the world around us, an important task is to accurately recreate the way light travels and affects the objects we see. Rendering lighting has been a heavily researched area since the 1970's and has gotten more sophisticated over the years. Until recent developments in technology, realistic lighting of scenes has only been achievable offline taking seconds to hours or more to create a single image, however, due to advances in graphics technology, realistic lighting can be done in real-time. To achieve real-time rendering, we must make trade offs between scientific accuracy and performance, but as will be discussed later, scientific accuracy may not be necessary after all.

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CHAPTER 1

BACKGROUND

Before discussing current topics of research in the field on realistic light rendering in computer graphics, we first need a basic understanding of optics, a branch of physics that studies the behavior and properties of light. Prior to the nineteenth century, light was regarded as a stream of particles and with this particle theory of light in mind, certain light phenomena such as refraction and reflection could be explained. However in 1801, the first evidence of light acting as a wave was found when light rays were found to interfere with one another. Later, in 1873, light was compared to a form of a high-frequency electromagnetic wave. In 1905, Einstein proposed a theory that explained the photoelectric effect (ejection of electrons from a metal surface when hit by light) that used the concept of quantization and stated that the energy of light waves are present in particles called photons which are quantized. (Serway and Jewett 2004) Therefore, light can have the behavior of a wave and a particle. These findings are important to consider in our research to accurately depict light as it travels throughout a scene. As will be discussed in the related work section, many lighting techniques use the idea that light consists of particles in order to simulate proper lighting conditions. Some even use the terminology of photons such as the lighting technique photon mapping. This paper, however, will try to explore a hybrid approach that considers light as both a particle and as a wave.

Lastly, before jumping into the related work section, we must cover some terminology used in current papers covering lighting in computer graphics. Lighting or illumination, is broken down into different components. Direct illumination is the rendering of a scene as it would appear with the light rays from the light sources terminating at the first surface it hits. Scenes made with this type of illumination are high performance but low realism due to attributes such as hard shadows, no reflection or refraction, and the fact that only surfaces in view of the light source will be illuminated with everything else black. In order to add realism, indirect illumination needs to be added to our prior directly illuminated scene. Indirect illumination considers what happens to the light after it comes into contact with a surface. Indirect illumination takes into account the reflection and refraction of light and produces realistic scenes depending on the extent of the indirect illumination captured. Indirect illumination produces the illumination seen on surfaces that are not directly viewable by the light source such as behind another object. The performance of indirect illumination varies greatly due to the extent of indirect illumination captured. Noting that this indirect illumination can be recursively calculated to infinity, indirect illumination can be expanded to a Neumann series and would be calculated by calculating the sum of light incident on a surface due to the reflection of light n-times all the way up to infinity. This fact will be explored further in the related works section, but the main idea is that the performance of calculating the indirect illumination depends on how large n is along with other factors discussed later, but is typically much slower than calculating direct illumination. Once we have direct illumination and indirect illumination calculated, we can say we have calculated the global illumination of the scene. This paper's technique will try to exploit the high performance of direct illumination by trying to approximate the indirect illumination of a single light source by using the direct illumination of multiple virtual light sources.

CHAPTER 2

PREVIOUS WORK

2.1 REAL-TIME VERSUS OFFLINE RENDERING TECHNIQUES

Rendering techniques can be broken down into two distinct categories: real-time and offline. Offline rendering techniques require anywhere from seconds to many hours to render a single image. Current offline rendering techniques are able to render a ultra-realistic image that take into account many light sources as well as many types of light principles such as reflections, refraction, sub-surface light scatter, and more in very complicated scenes consisting of millions of triangles and at fairly high resolutions. Real-time rendering techniques render images at a fast enough rate to support multiple frames a second and vary greatly. These algorithms can be broken down into dynamic and static. Dynamic scenes allow a user to interact with the scene and actively change the scene such as moving the geometry, the camera, or the light source whereas static scene do not. As opposed to offline algorithms, real-time algorithms often have to make sacrifices when rendering the scene and therefore can't be as scientifically accurate as offline algorithms. Regardless of whether the algorithm is offline or real-time, the algorithm can trace it's roots back to a single equation, *The Rendering Equation*.

2.2 THE RENDERING EQUATION

When discussing light transport in computer graphics, the most significant paper is *The Rendering Equation* (Kajiya 1986). In it Kajiya presents an equation that generalizes most rendering algorithms. Such a statement can be confirmed by the fact that all rendering equations try to recreate the scattering of light off of different types of surfaces and materials. The rendering equation is an integral that is adapted from the study of radiative heat transfer for use in computer graphics with an aim at balancing the energy flow between surfaces. The equation, however, is still an approximation because it does not take into account diffraction and it assumes that the space between objects, such as air, is of homogeneous refractive index meaning that light won't refract due to particles in the air.

$$I(x, x') = g(x, x')[\epsilon(x, x') + \int_S \rho(x, x', x'')I(x', x'')dx''] \tag{2.1}$$

The rendering equation is broken down into 4 parts. First, $I(x, x')$ is the intensity of light or energy of radiation passing from point x' to point x measured in energy of radiation per unit time per unit area. Second, the geometry term, $g(x, x')$, indicates the occlusion of objects by other objects. This term is either 0, if x and x' are not visible from one another, or $1/r^2$ where r is the distance between x and x' if x and x' are visible from one another. Third, the emittance term, $\epsilon(x, x')$, measures the energy emitted from point x' that reaches point x. Lastly, the scattering term, $\rho(x, x', x'')$, is the intensity of energy scattered by a surface point x' that originated from point x'' and then ends at point x. As mentioned in the previous section, illumination can be calculated using the Neumann series. A Neumann series is a mathematical series of the form:

$$\sum_{k=0}^{\infty} T^k \tag{2.2}$$

where T is an operator and therefore T^k is a notation for k consecutive operations of operator T.

Furthermore, the rendering equation can also be approximated using the Neumann series. This is done by rewriting the rendering equation above (2.1) as:

$$I = g\epsilon + gMI \tag{2.3}$$

where M is the linear operator given by the integral in the rendering equation. Next, we rewrite equation 2.3 as:

$$(1 - gM)I = g\epsilon \tag{2.4}$$

so that we can invert it to get:

$$I = g\epsilon + gMg\epsilon + gMgMg\epsilon + g(Mg)^3\epsilon + ... \tag{2.5}$$

Equation 2.5 is a Neumann series of the form:

$$I = g\epsilon \sum_{k=0}^{\infty} (Mg)^k \tag{2.6}$$

Equation 2.6 indicates that the rendering equation (equation 2.1) is the final intensity of radiation transfer as a sum of a direct term, a once scattered term, a twice scattered term, and so on. Therefore, as mentioned in the previous section, indirect illumination can be calculated by summing light incident on a surface due to the reflection of light n-times all the way up to infinity. The more scattered terms we include in the calculation, the better the approximation but worse performance. Therefore, in real-time applications, this needs to be avoided.

Next, we show how the rendering equation can be seen has a generalization of most rendering algorithms, but first we must cover some other rendering equations. A good place to start is with offline rendering techniques.

2.3 OFFLINE RENDERING TECHNIQUES

Key examples of offline rendering techniques are ray tracing, radiosity, and photon maps. We begin with ray tracing.

2.3.1 RAY TRACING

Ray tracing is a technique for rendering an image of a three-dimensional scene by casting rays from a camera positioned somewhere in the scene. These rays are shot into the scene and register the first surface it hits. From this surface point, additional rays goes to each of the light sources to determine occlusion from the light sources as well as to other surfaces to calculate reflections. These rays can also be used to calculate other lighting phenomena such as refractions. The rays from the camera can be cast into the scene using different sampling patterns and techniques such as 1 per pixel or many per pixel. Also, the rays can be cast through the center of each pixel or through the use of stochastic sampling can be cast through non-uniformly spaced location in each pixel to avoid aliasing artifacts or jaggies. Ray tracing is able to recreate ultra-realistic scenes but at a high cost. Examples of ray tracing techniques include (**?**), (Cook 1986), and (Ward, Rubinstein, and Clear 1988). With adaptations to ray tracing techniques and advances in technology, there now exist some interactive ray-tracing techniques mentioned in section **??**.

Ray tracing can also be related to the rendering equation. (**?**) describes a new approximation for ray tracing by rewriting the Phong illumination model in order to improve the quality of specular reflections. The Phong illumination model is a way of calculating lighting on a surface through the combination of three components ambient, diffuse, and specular. Diffuse is the reflection of light from rough surfaces, specular is the reflection of light on shiny surfaces, and the ambient component accounts for the amount of light that is scattered throughout the scene. The ambient term is most similar to indirect lighting, but is a user-specified amount to avoid any actual calculations. The improved model from (**?**) is written:

$$I = I_a + k_d \sum_{j=1}^{j=ls} (\bar{N} \cdot \bar{L}_j) + k_s S + k_t T \tag{2.7}$$

where S is the intensity of light incident from the specular reflection direction, k_t is the transmission coefficient, and T is the intensity of light from the transmitted light direction. k_s and k_t are coefficients that are to be used to try to accurately model the Fresnel reflection law. Equation 2.7 is in the form of equation **??** from (Kajiya 1986) as $I = g\epsilon + gMog + gMogMog +$ with Mo as a scattering model which is the sum of reflection and refraction as well as a cosine term that is the diffuse component. The term g has shadows with point radiators and the ambient term can be interpreted as the ϵ term. Lastly, M is approximated by summing over all the light sources rather than using integration.

2.3.2 RADIOSITY

Radiosity is a type of rendering algorithm that was adapted for use in computer graphics from thermal engineering techniques. The method is based on the fundamental Law of Conservation of Energy within a closed area. It provides a global solution for the intensity of light incident on each surface by solving a system of linear equations that describes the transfer of energy between each surface in the scene. Examples of radiosity are seen in (Immel, Cohen, and Greenberg 1986) and (**?**).

Radiosity is a natural extension from the rendering equation (equation 2.1) since its focus is on the balancing the energy flow. The only difference is that radiosity makes assumptions about the reflectance characteristics of the surface material. Radiosity is found by taking the hemispherical integral of the energy leaving the surface called flux which can be found using the following from (Goral, Torrance, Greenberg, and Battaile 1984):

$$B_j = E_j + \rho_j H_j \tag{2.8}$$

where B_j is the rate of energy leaving the surface j measured in energy per unit time per unit area, E_j is the rate of direct energy emission, $_j$ is the reflectivity of surface j, and $H_j istheincidentradiantenergyarrivingatsurfacejperunittimeperunitarea.Equation$

$$dB(x') = \pi[\epsilon_0 + \rho_0 H(x')]dx' \tag{2.9}$$

where ϵ_0 is the hemispherical emittance of the surface element dx', ρ_0 comes from the reflectance term, and H is the hemispherical incident energy per unit time per unit area. This adaptation of the rendering equation (equation 2.9) is the same as the radiosity equation shown above (equation 2.8).

2.3.3 PHOTON MAPS

Photon maps originally introduced in (**?**) is a two pass global illumination method. As mentioned in the Background section, Einstein coined the term photons as the particles present in the energy of light waves. In the method of photon mapping, the term photon is used in a similar context. The first pass of the method consists of making two photon maps by emitting packets of energy called photons from the light sources and storing where they hit surfaces in the scene. The second pass of the method calls for the use of a distribution ray tracer that is optimized using the data gathered in the photon maps. Photon maps are able to render complex lighting principles such as caustics.

Photon maps are an extension to the rendering equation (equation 2.1) as well. During the second pass of the method, the scene is rendered by calculating the radiance by tracing a ray from the eye through the pixel and into the scene using ray tracing, and the radiance is computed at the first surface that the ray hits. The surface radiance leaving the point of intersection x in some direction is computed using the equation from (**?**):

$$L_s(x, \psi_r = L_e(x, \psi_r + \int_\Omega (f_r(x, \psi_i; \psi_r) L_i(x, \psi_i) \cos(\theta_i) d\omega_i) \tag{2.10}$$

where L_e is the radiance emitted by the surface, L_i is the incoming radiance in the direction ψ_i, and f_r is the BRDF or bidirectional reflectance distribution function, which is a four-dimensional function that describes how light is reflected at a surface point. Lastly, Ω is the sphere of incoming directions. This can be broken down into a sum of four components:

$$L_r = \int_\Omega (f_r L_{i,l} \cos(\theta_i) d\omega_i) + \int (f_{r,s} (L_{i,c} + L_{i,d}) \cos(\theta_i) d\omega_i) + \int_\Omega (f_{r,d} L_{i,c} \cos(\theta_i) d\omega_i) + \int_\Omega (f_{r,d} L_{i,d} \cos(\theta_i) \tag{2.11}$$

where the first term of equation 2.11 is the contribution by direct illumination, the second term is the contribution by specular reflection, the third term is the contribution by caustics, and the fourth term is the contribution by soft indirect illumination. Both equations 2.10 and 2.11 are direct adaptations from the rendering equation in (Kajiya 1986) (equation 2.1).

CHAPTER 3

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The final thesis must be in 12-point size in a standard font such as Times New Roman or Arial or True Type font.

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