

30-Chiziqli regressiya tenglanasini tuzishda eng kichik kvadratlar usuli

Misol: Bizga 10 yillik o'rtacha daromad X va o'rtacha iste'mol Y (mln. so'm) berilgan. 1-jadval.

Jadval ma'lumotlariga asoslanib Y ning X faktorga bog'liqligini chiziqli, darajali, ko'rsatkichli funksiya ko'rinishida ifodalang va optimal modelni tanlang (model bahosini tekshirishda o'rtacha aproksimasiya xatosi (A) va F-Fisher kriteriyasi).

1-jadval

Yillar	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
X	10,5	11,6	12,3	13,7	14,5	16,1	17,3	18,7	20,1	21,8
Y	8,12	10	8,41	12,1	12,4	11,4	12,8	13,9	17,3	17,5

Yechish:

a. $y = a + bx$ ko'rinishdagi chiziqli regressiya tenglamasi bo'yicha masalani yechish uchun a va b ga nisbatan quyidagi normal tenglamalar sistemasini yechamiz:

$$\begin{cases} na_0 + a_1 \sum x = \sum y, \\ a_0 \sum x + a_1 \sum x^2 = \sum y \cdot x. \end{cases}$$

Bu tenglamalar sistemasini yechishda bizga quyidagilarni aniqlash kerak bo'ladi.

$$\sum x, \sum y, \sum x^2, \sum y \cdot x$$

2-jadval

N	X	Y	XY	X ²	Y ²	Y _x	Y- Y _x	Ai
1	10,5	8,12	85,26	110,25	65,9344	8,2515	0,1315	1,619458
2	11,6	10	116	134,56	100	9,1348	0,8652	8,652
3	12,3	8,41	103,443	151,29	70,7281	9,6969	1,2869	15,30202
4	13,7	12,1	165,77	187,69	146,41	10,8211	1,2789	10,56942
5	14,5	12,4	179,8	210,25	153,76	11,4635	0,9365	7,552419
6	16,1	11,4	183,54	259,21	129,96	12,7483	1,3483	11,82719
7	17,3	12,8	221,44	299,29	163,84	13,7119	0,9119	7,124219

8	18,7	13,9	259,93	349,6 9	193,21	14,836 1	- 0,93 61	6,7345 32
9	20,1	17,3	347,73	404,0 1	299,29	15,960 3	1,33 97	7,7439 31
10	21,8	17,5	381,5	475,2 4	306,25	17,325 4	0,17 46	0,9977 14
Jami	156,6	123,93	2044,4 13	2581, 48	1629,3 83	123,94 98	0	78,122 91
O'rtac ha	15,66	12,393	204,44 13	258,1 48	162,93 83			7,8122 91
σ	3,5933 83	3,0580 71						
σ^2	12,912 4	9,3518 01						

$$\sigma_x^2 = \overline{x^2} - \bar{x}^2$$

$$\sigma_y^2 = \overline{y^2} - \bar{y}^2$$

$\sigma_x - X$ kvadratik og'ishning o'rtachasi;

$\sigma_y - Y$ kvadratik og'ishning o'rtachasi;

$$b = \frac{cov(x, y)}{\sigma_x^2} = \frac{\overline{yx} - \bar{y} \cdot \bar{x}}{\overline{x^2} - \bar{x}^2} = \frac{204,441 - 12,393 \cdot 15,66}{258,148 - 15,66^2} = 0,803$$

$$a = \bar{y} - b \cdot \bar{x} = 12,393 - 0,803 \cdot 15,66 = -0,180$$

Bundan regressiya tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$y_x = -0,180 + 0,803 \cdot x$$

Regressiya tenglamasidan quyidagi xulosaga kelish mumkin, agar o'rtacha daromadni 1 million so'mga oshirsak, o'rtacha iste'mol hajmi 0,803 mln so'mga oshadi.

Korrelyasiya koeffitsiyentni hisoblaymiz.

$$r_{xy} = b \frac{\sigma_x}{\sigma_y} = 0,803 \cdot \frac{3,59}{3,05} = 0,94517$$

Determinasiya koeffitsiyenti

$$d = r^2 = (0,94517)^2 = 0,893347$$

O'rtacha A aproksimasiya xatosini topamiz:

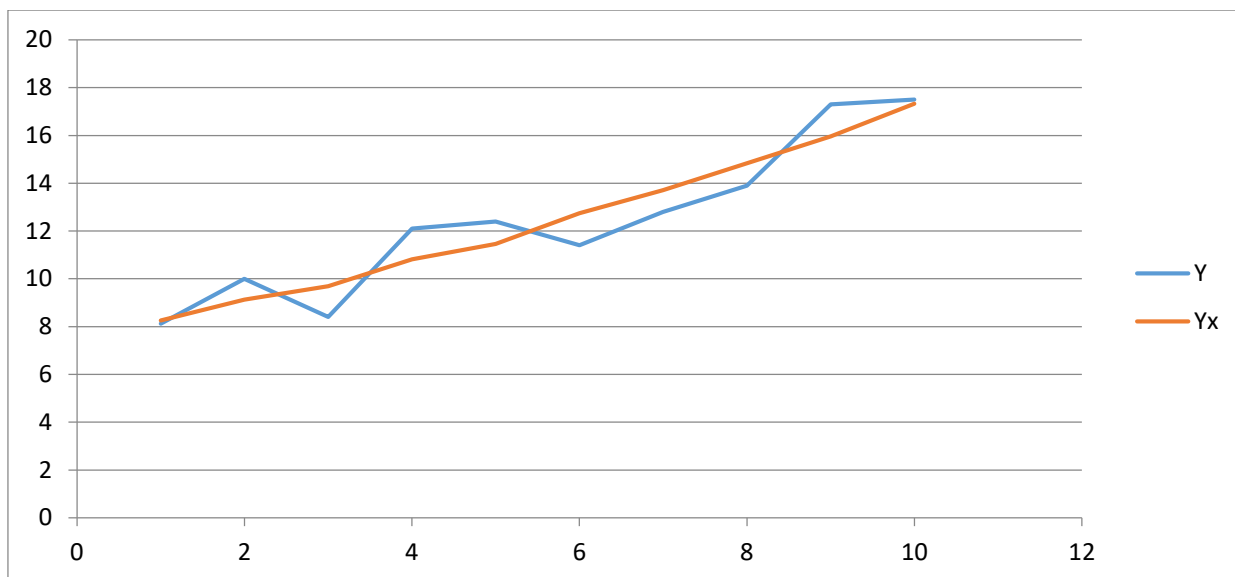
$$A = \frac{1}{n} \sum \left| \frac{y - y_x}{y} \right| \cdot 100\% = \frac{78,123 \cdot 100\%}{10} = 7,8$$

O'rtacha hisob qiymati faktik qiymatdan 7,8% chetlanishini anglatadi.

F-Fisher kriteriyasini hisoblaymiz:

$$F_{fak} = \frac{r_{xy}^2}{1 - r_{xy}^2} \cdot (n - 2) = \frac{0,893347}{0,106653} \cdot 8 = 67,01$$

$$F_{tab} = 3,07 < F_{fak}$$



VAZIFA

Yil lar	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
X	10,5+ 0,n	11,6+ 0,n	12,3+ 0,n	13,7+ 0,n	14,5+ 0,n	16,1+ 0,n	17,3+ 0,n	18,7+ 0,n	20,1+ 0,n	21,8+ 0,n
Y	8,12	10	8,41	12,1	12,4	11,4	12,8	13,9	17,3	17,5

n-talabaniing tartib raqami