21-mavzu: Hodisa ehtimolini hisoblash.

Ta'rif. m elementni n tadan o`rinlashtirish deb, shunday birlashmalarga aytiladiki, ularning har birida berilgan m elementdan olingan n ta element bo`lib, ular bir- birlaridan yo elementlari bilan, yoki elementlarining tartibi bilan farq qiladi(n $\leq$ m bo`lishi shart).

m ta elementdan n tadan tuzulgan o`rinlashtirishlar soni A_m^n simvol bilan belgilanadi.

(A fransuzcha "arrangement", ya'ni orinlashtirish degan so`zning bosh harfidir.)

$A_m^n = m(m-1)(m-2)....(m-n+1)$. Bu o`rinlashtirishlar sonini topish formulasi hisoblanadi.

1-misol. $A_7^4 = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

2-misol. Sinfda 10 fandan dars bo`lib har kuni 5 xil usul bilan dars o`tiladi. Bir kunlik dars necha xil usul bilan taqsimlanishi mumkin?

Yechish: Masala o`rinlashtirishlar sonini aniqlash bilan yechiladi.

$A^5{}_{10} = 10*9*8*7*6 = 30240$ usul bilan taqsimlab qo`yish mumkin.

O`rin almashtirish.

Ta'rif. Faqat elementlarning tartibi bilangina farq qilgan(yani n=m) o`rinlashtirishlar o`rin almashtirishlar deyiladi.

M elementdan tuzulgan o`rin almashtirishlar soni P_m simvol bilan belgilanadi.

(P fransuzcha "Permution", ya'ni o`rin almashtirish so`zining bosh harfidir.)

$P_m = 1 \cdot 2 \cdot 3 \cdot ... \cdot (m-2) \cdot (m-1) \cdot m$.

Bu formula o`rin almashtirishlar sonini topish formulasi hisoblanadi.

Misollar: $P_3 = 1 \cdot 2 \cdot 3 = 6; P_6 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720; P_5 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$ va hokazo.

3 Masala. 8 ta stol qo`yilgan; unga 8 kishini necha xil usul bilan o`tqazish mumkin.

Yechish. Bu masala o`rin almashtirishlar sonini aniqlash bilan yechiladi.

$P_8 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 40320$ xil usul bilan.

Gruppalash.

*Ta`rif.* m ta elementdan n tadan tuzulgan gruppalash deb, m elementdan n tadan tuzulgan o`rinlashtirishlardan bir- biridan eng kamida bitta elementi bilan farq qiladigan o`rinlashtirishlarga aytiladi.

M elementdan n tadan gruppalash soni C_m^n simvol bilan belgilanadi.

(C-fransuzcha "Combinasion" ,ya'ni gruppalash degan so`zning bosh harfi).

$C_m^n = \dfrac{A_m^n}{P_n} = \dfrac{m(m-1)(m-2)...(m-n+1)}{1 \cdot 2 \cdot 4 \cdot .... \cdot n}$.

Bu formula gruppalashlar sonini topish formulasi hisoblanadi.

4. Misolar: $\quad 1) C_9^5 = \dfrac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 126.$

$\quad 2) C_{25}^4 = \dfrac{25 \cdot 24 \cdot 23 \cdot 22}{1 \cdot 2 \cdot 3 \cdot 4} = 12650.$

3) $C_{2x}^2 = 1$ tenglamani yeching.

Yechish:

$$C_{2x}^2 = \frac{2x \cdot (2x-1)}{1 \cdot 2} = 1 \, yoki \, 2x^2 - x - 1 = 0, bundan:$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4}; \, x_1 = 1, x_2 = -\frac{1}{2}.$$

Bulardan yolg`iz $x_1 = 1$ berilgan tenglamani qanoatlantiradi, $x = -\frac{1}{2}$ berilgan tenglamaning chet ildizidir.

4) $C_{m+1}^{n+1} \div C_{m+1}^n \div C_{m+1}^{n-1} = 5 \div 5 \div 3$ berilgan. n va m sonlar topilsin.

Yechish:

$$1 = \frac{5}{5} = \frac{C_{m+1}^{n+1}}{C_{m+1}^n} = \frac{\dfrac{(m+1) \cdot m \cdot (m-1) \cdot ... \cdot (m+1-n+1) \cdot (m+1-n)}{1 \cdot 2 \cdot 3 \cdot ... n \cdot (n+1)}}{\dfrac{(m+1) \cdot m \cdot (m-1) \cdot ... \cdot (m+1-n+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot n}} = \frac{m-n+1}{n+1} \, yoki. \, n+1 = m-n+1,$$

Bundan: $m = 2 \cdot n$.

Yana:

$$\frac{5}{3} = \frac{C_{m+1}^n}{C_{m+1}^{n-1}} = \frac{\dfrac{(m+1) \cdot m \cdot (m-1) \cdot ... \cdot (m+1-n+2) \cdot (m+1-n+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot (n-1) \cdot n}}{\dfrac{(m+1) \cdot m \cdot (m-1) \cdot ... \cdot (m+1-n+2)}{1 \cdot 2 \cdot 3 \cdot ... \cdot (n-1)}} = \frac{m-n+2}{n} \, yoki. \, 5n = 3m - 3n + 6 \, bundan$$

$n = \frac{3m+6}{8}$. Endi bundagi m ning o`rniga topilgan m=2n ni qo`ysak:

$n = \frac{3 \cdot 2 + 6}{8} \, yoki \, 2n = 6, . \, n = 3 \, bo`ladi.$ Bu holda $m = 2 \cdot 3 = 6$.

5) $12C_{x+3}^{x-1} = 55A_{x+1}^2$ tenglamani qanoatlantiruvchi x ning qiymatini toping.(Bu misolda $C_m^n = C_m^{m-n}$ formuladan foydalandik bu formulaning isboti keyinchalik beriladi.)

Yechish:

$$12 \cdot C_{x+3}^{x-1} = 12 C_{x+3}^{x+3-x+1} = 12 C_{x+3}^4 = 12 \cdot \frac{(x+3) \cdot (x+2) \cdot (x+1) \cdot x}{1 \cdot 2 \cdot 3 \cdot 4} = \frac{(x+3) \cdot (x+2) \cdot (x+1) \cdot x}{2};$$

$$A_{x+1}^2 = (x+1) \cdot x; \frac{(x+3) \cdot (x+2) \cdot (x+1) \cdot x}{2} = 55(x+1) \cdot x \text{ yoki } (x+3) \cdot (x+2) = 110,$$

$$x^2 + 5x - 104 = 0, x_{1,2} = \frac{-5 \pm \sqrt{25+416}}{2} = \frac{-5 \pm \sqrt{441}}{2} = \frac{-5 \pm 21}{2} \quad ; \quad x_1 = 8 \quad ; x_2 = -13$$

(chet ildiz).

6). $\begin{cases} C_x^y = C_x^{y+2} \\ C_x^2 = 153 \end{cases}$ ni yeching.

Echish. $C_x^y = \frac{x \cdot (x-1) \cdot ... \cdot (x-y+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot y}$,

$C_x^{y+2} = \frac{x \cdot (x-1) \cdot ... (x-y+1) \cdot (x-y) \cdot (x-y-1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot y \cdot (y+1) \cdot (y+2)}.$

$C_x^2 = \frac{x \cdot (x-1)}{1 \cdot 2}.$

Bularni o`rinlariga qo`yib soddalashtirsak:

$$\begin{cases} (x-y) \cdot (x-y-1) = (y+1) \cdot (y+2), \\ x^2 - x - 306 = 0 \end{cases}$$

tenglamalar sistemasi hosil bo`ladi. Ikkinchi tenglamadan : x=18 bo`lib,buni birinchi tenglamaga qo`ysak: $(18-y)(17-y) = y^2 + 3y + 2$ yoki $38y = 304; y = \dfrac{304}{39} = 8.$

$C_m^n = C_m^{m-n}$ tenglikning isboti .Buning uchun quyidagi ishlarni bajaramiz .

$$C_m^n = \frac{m \cdot (m-1) \cdot (m-2) \cdot \ldots \cdot (m-n+1)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n} = \frac{m \cdot (m-1) \cdot \ldots \cdot (m-n+1) \cdot 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (m-n)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \cdot 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (m-n)} =$$

$$\frac{1 \cdot 2 \cdot 3 \ldots (m-n) \cdot (m-n+1) \cdot \ldots \cdot (m-1) \cdot m}{P_n \cdot P_{m-n}} = \frac{P_m}{_n P_n \cdot P_{m-n.}} \quad .$$

Bu gruppalashlar soni formulasining boshqacha ko`rinishi. Endi chiqarilgan formulada n ni (m-n) bilan almashtirsak:

$$C_m^{m-n} = \frac{P_m}{P_{m-n} \cdot P_n} = C_m^n$$

hosil   bo`ladi. Demak, $C_m^n = C_m^{m-n}$.

Misol. $C_{25}^{23} = C_{25}^{25-23} = C_{25}^2 = \dfrac{25 \cdot 24}{1 \cdot 2} = 300$.