27-Mavzu. Matematik statistika elementlari

1.1. Elementar hodisalar va ular ustida amallar.
1.2. Tanlanma va bosh to'plam. Tanlash usullari
1.3. Tanlanmaning statistik taqsimoti

Elementar hodisalar va ular ustida amallar.

1. Oilaning uchta A, B, C a'zolari klinikaning 1, 2, 3-statsionerlarida vrachlarga ega. Haftaning ma'lum kunida oilaning har bir a'zosiga tasodifiy ravishda biror statsionarga borish belgilangan. Tajriba oila a'zolarining har biriga statsionar nomerini tanlashdan iborat. Masalan, (1,2,1) A - 1-statsionarga boradi, B – 2-statsionarga boradi, C – 1-statsionarga boradi.

a. Elementar hodisalar fazosining barch 27 hodisasini tuzing.
b. Uchala oila a'zosi ham bitta statsionarga borish elementar hodisalarini yozing.
c. Uchala oila a'zosi boshqa-boshqa satatsionarlarga borish elementar hodisalarini yozing.
d. Oila a'zolarining ikkitasi bitta statsionarga borishi hodisalarini tuzing.

2. Ikkita A va B saylovchi shahar sovetiga 1-,2-,3-nomzodlarni saylashadi. Tajriba A va B saylovchilarning tanlovidan iborat. Masalan, (3,2) hodisa A saylovchi 3-nomzodni, B saylovchi 2-nomzodni saylaydi.

a. Barcha elementar hodisalar fazosini tuzing.
b. A va B saylovchilar bir xil nomzodlarni saylash hodisalarini tuzing.
c. Saylovchilarning hech biri 2-nomzodni saylamaslik hodisalar to'plamini tuzing.

Tajriba uchun ma'lum navdagi 50 ta g'ozaning bo'yi o'lchanib quyidagi tanlanma olingan (sm da):

98	78	68	65	66	87	65	64	82	73
54	72	78	76	75	89	92	94	58	78
69	72	65	76	78	72	79	64	59	65
76	79	85	83	67	65	90	82	76	74
59	68	73	57	87	67	75	92	91	63

a) Tanlanmani o'sib boruvchi variatsion qator ko'rinishida yozing.
b) Tanlanmaning eng kata va eng kichik variantalarini toping.
c) Tanlanmaning chastotalar va nisbiy chastotalar bo'yicha statistic taqsimotini tuzing.
d) Tuzilgan statistic taqsimotnning medianasi (eng kata chastotaga ega variantasini) toping.

Yechish.

a) variatsion qatorni quyidagicha tuzamiz:

54 57 58 59 59 63 64 64 65 65 65 65 65 66 67 67 68 68 69 72 72 72 73 73 74 75 75 76 76 76 76 78 78 78 78 79 79 82 82 83 85 87 87 89 90 91 92 92 94 98

b) variatsion qatordan ko'rinib turibdiki eng kata variant 98 va eng kichik variant 54 ga teng.

c) Tanlanma hajmi katta va variantalar bir xil uzoqlikda bo'lmaganligi uchun intervallar qurishdan foydalahamiz. Buning uchun avvalo

$$k = 1 + 3{,}322\, lgn = 1 + 3{,}322 \cdot lg50 \approx 6{,}67$$

formula orqali intervallar sonini aniqlaymiz, $k \approx 7$ ta. Bunda intervallar uzunligini

$$h = \frac{x_{\max} - x_{\min}}{k} = \frac{98-54}{6{,}67} = 6{,}6$$

bo`ladi. Bu erda $x_{\max}$ va $x_{\min}$ mos ravishda variatsion qatorning eng katta va eng kichik hadlaridir. k- interval sifatida

$$\left[x_{\min} + (k-1)h; x_{\min} + kh\right]$$

interval olinadi.

U holda tanlanma to`plamning intervalli variatsion qatori, yani statistik taqsimoti quyidagi ko`rinishda bo`ladi:

Varianta intervallari	**Interval o'rtasi** $y_i = \frac{x_i + x_{i+1}}{2}$	**Intervalga tegishli variantalar soni (chastotasi)**	**Nisbiy chastotasi**
$[54;54+6{,}6] = [54;60{,}6]$	57,3	5	0,1
$[60{,}6;67{,}2]$	63,9	11	0,22
$[67{,}2;73{,}8]$	70,5	8	0,16
$[73{,}8;80{,}4]$	77,1	13	0,26
$[80{,}4;86{,}0]$	83,7	4	0,08
$[86{,}0;92{,}6]$	90,3	7	0,14
$[92{,}6;99{,}2]$	96,9	2	0,04

d) tanlanmaning eng kata chastotaga ega bo'lgan variantasi 77,1 ga teng (13 ta).

№9. Tajriba uchun ma'lum zotgi 40 ta qo'ylarnng tirik vazni o'lchanib quyidagi tanlanma olingan (*kg* da):

65	68	68	92	74	78	68	79	82	73
64	66	64	69	68	65	73	74	72	69
77	73	78	76	81	84	83	82	86	69
73	54	56	74	78	77	89	81	73	67

a) Tanlanmani o'sib boruvchi variatsion qator ko'rinishida yozing.
b) Tanlanmaning eng kata va eng kichik variantalarini toping.
c) Tanlanmaning chastotalar va nisbiy chastotalar bo'yicha statistic taqsimotini tuzing.
d) Tuzilgan statistic taqsimotnning medianasi (eng kata chastotaga ega variantasini) toping.

Mustaqil ta’lim bo’yicha o’quv materiallari

1. Matritsalar ustida keladigan iqtisodiy masalalar
2. Yuqori tartibli determinantlari
3. Kvadratik forma va uning kanonik ko‘rinishlari
4. Matritsalar ustida amallar va Mathcad dasturida yechish
5. Ikkinchi tartibli egri chiziqlar. Aylana, ellipsning kanonik tenglamalari
6. Ikkinchi tartibli egri chiziq tenglamalari. Giperbola va parabolaning kanonik ko‘rinishi
7. To‘plam tushunchasi va ular ustida amallar
8. Parametr orqali berilgan funksiyalar va ularning hosilalarini hisoblash
9. Limitlarni hisoblashning Lapital qoidasi
10. O‘zgarmas koeffisiyentli chiziqli differensial tenglamalar
11. Tanlanmaning o‘rtacha va tanlanma dispersiyalarini hisoblashning qo‘shish metodi
12. Bernulli sxemasini umumlashtirish
13. Diskret taqsimotning aniqmaslik darajasi
14. Kombinatorika elementlari, o‘rin almashtirish, o‘rinlashtirish va guruhlash
15. Diskret turdagi ikki o‘zgaruvchili tasodifiy miqdorning taqsimot qonunu

28-mavzu. Tanlanmaning yig’ma xarakteristikalarini hisoblash usullari

1. Shartli variantalarni tuzish
2. Tanlanam o’rta qiymat va tanlanma dispersiyani ko’paytmalar usulida hisoblash
3. Normal egri chiziqni tajriba ma’lumotlari bo’yicha yasash

1. Shartli variantalarni tuzish

1-misol. Qiyidagi statistic taqsimotning shartli variantalarini toping:

x_i	23,6	28,6	33,6	38,6	43,6
n_i	5	20	50	15	10

Yechish. Soxta nol sifatida 38,6 variantani olamiz, h=28,6-23,6=5.

Shartli variantani topamiz:

$$u_1 = \frac{x_1 - C}{h} = \frac{23{,}6 - 38{,}6}{5} = -2.$$

Shunga o’xshash qolgan variantalarni topamiz:

$$u_2 = -1,\ u_3 = 0,\ u_4 = 1,\ u_5 = 2.$$

u_i	-2	-1	0	1	2
n_i	5	20	50	15	10

2-misol. Janubiy Irlandiyada 21 ta fermasida 2009-2011 yillar davomida yog'in miqdori o'rganilganda quyidagi natija olingan (mm/yil):

1027 1124 1124 1373 1373 1373 1077 1373 1077 1373 1373 1373 1077 1124 1124 1453 1077 1124 1124 1373 1373[1]

a) Tanlanmaning statistic taqsimotini tuzing;

b) Tuzilgan taqsimot asosida shartli variantalar bo'yicha taqsimot tuzing.

3-misol. Janubiy Irlandiyada 21 ta fermasida 2009-2011 yillar davomida sigirlardan o'rtacha sog'ib oligan sut miqdori o'rganilganda quyidagi natija olingan (litr/bosh):

5319 6010 5688 5309 5149 5672 5080 5671 5431 2507 4229 5613 5290 4415 4671 6038 4928 5549 5500 5174 5522[2]

a) Tanlanmaning statistic taqsimotini tuzing;

b) Tuzilgan taqsimot asosida shartli variantalar bo'yicha taqsimot tuzing.

4-misol. Ma'lum bir hududdagi 4 ta chorvachilik fermasida 30 tadan sigirlarning laktasiya davrida o'rtacha sog'ib olinadigan sut miqdori o'rganilib, quyidagi taqsimot tuzilgan:

x_i	3245	3250	3400	3450	3500	3550
n_i	28	22	45	13	8	4

a) Tanlanmaning statistic taqsimotini tuzing;

b) Tuzilgan taqsimot asosida shartli variantalar bo'yicha taqsimot tuzing.

6.2. Tanlanam o'rta qiymat va tanlanma dispersiyani ko'paytmalar usulida hisoblash

5-misol. Ko'paytmalar usulidan foydalanib quyidagi statistic taqsimotning tanlanma o'rtacha qiymatini va tanlanma dispersiyasini toping:

[1] E.Mihailescu, P.N.C.Murphy, W.Ryan , I.A.Casey and J. Humphreys. Nitrogen balance and use efficiency on twenty-one intensive grass-based dairy farms in the South of Ireland (*Journal of Agricultural Science* (2014), 152, 843–859.)

[2] E.Mihailescu, P.N.C.Murphy, W.Ryan , I.A.Casey and J. Humphreys. Nitrogen balance and use efficiency on twenty-one intensive grass-based dairy farms in the South of Ireland (*Journal of Agricultural Science* (2014), 152, 843–859.)

x_i	10,2	10,4	10,6	10,8	11,0	11,2	11,4	11,6	11,8	12,0
n_i	2	3	8	13	25	20	12	10	6	1

Yechish. tanlanma hajmi n=100, qadam $h=0,2$, soxta nol $C=11,0$. Ko'paytmalar jadvalini tuzamiz:

x_i	n_i	u_i	$u_i n_i$	$n_i u_i^2$	$n_i(u_i+1)^2$
10,2	2	-4	-8	32	18
10,4	3	-3	-9	27	12
10,6	8	-2	-16	32	8
10,8	13	-1	-13	13	0
11,0	25	0	0	0	25
11,2	20	1	20	20	80
11,4	12	2	24	48	108
11,6	10	3	30	90	160
11,8	6	4	24	96	150
12,0	1	5	5	25	36
	n=100		$\sum n_i u_i = 57$	$\sum n_i u_i^2 = 383$	$\sum n_i (u_i+1)^2 = 597$

Tekshirish:

$$\sum n_i u_i^2 + 2\sum n_i u_i + n = 383 + 2\cdot 57 + 100 = 597,$$
$$\sum n_i (u_i + 1)^2 = 597.$$

Demak hisoblash to'g'ri bajarilgan.

Birinchi va ikkinchi tartibli shartli momentlarni hisoblaymiz:

$$M_1^* = \frac{\sum n_i u_i}{n} = \frac{57}{100} = 0,57, \quad M_2^* = \frac{\sum n_i u_i^2}{n} = \frac{383}{100} = 3,83.$$

Bularga asosan, tanlanma o'rtacha qiymat va tanlanma dispersiyani hisoblaymiz:

$$\bar{x}_t = M_1^* h + c = 0,57\cdot 0,2 + 11,0 = 11,1,$$

$$V_t = \left[M_2^* - \left(M_1^*\right)^2\right]\cdot h^2 = \left(3,83 - (0,57)^2\right)\cdot 0,2^2 = 0,14.$$

6-misol. Ko'paytmalar usulidan foydalanib quyidagi statistic taqsimotning tanlanma o'rtacha qiymatini va tanlanma dispersiyasini toping:

x_i	0,05	0,1	0.15	0,2	0,25	0,3	0,35	0,4	0,45	0,5
n_i	1	2	4	6	12	10	6	5	3	1

7-misol. Ko’paytmalar usulidan foydalanib quyidagi statistic taqsimotning tanlanma o’rtacha qiymatini va tanlanma dispersiyasini toping:

x_i	122,4	128,4	132,4	136,4	140,4	144,4	148,4	152,4
n_i	2	5	8	18	25	20	12	10

8-misol. Ko’paytmalar usulidan foydalanib quyidagi statistic taqsimotning tanlanma o’rtacha qiymatini va tanlanma dispersiyasini toping:

x_i	10,2	10,4	10,6	10,8	11,0	11,2	11,4	11,6	11,8
n_i	3	4	6	11	20	26	14	8	8

9-misol. Ko’paytmalar usulidan foydalanib quyidagi statistic taqsimotning tanlanma o’rtacha qiymatini va tanlanma dispersiyasini toping:

x_i	25,38	29,82	34,26	38,7	43,14	47,58	52,02	56,46	60,9	65,34
n_i	6	12	11	14	22	18	15	7	3	2

6.3. Normal egri chiziqni tajriba ma’lumotlari bo’yicha yasash