31- Mavzu: Kompyuter dasturlari yordamida Korrelyatsion-regrission tahlillarni o‘tkazish.

1. Chiziqli va chiziqsiz regression bog’lanishlarda qanday turdagi funksiyalardan foydalaniladi
2. Korrelyatsion bog’lanishlarning geometric ko’rinishi
3. Korrelyasion-regression tahlilda eng kichik kvadratlar usulining qo’llanilishi
4. Juft, xususiy va ko’plikdagi korrelyasiya koyeffisiyentlarining farqi

Ijtimoiy-iqtisodiy jarayonlar o’rtasida bog’lanishlarni o’rganishda quyidagi funksiyalar bilan foydalaniladi:

Chiziqli – $y = a_0 + a_1 x$

Ikkinchi darajali parabola – $y = a_0 + a_1 x + a_2 x^2$

Uchinchi darajali parabola – $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

***n*-darajali parabola** – $y = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$

Giperbola – $y = a_0 + \frac{a_1}{x}$

***b*- darajali giperbola** – $y = a_0 + \frac{a_1}{x^b}$

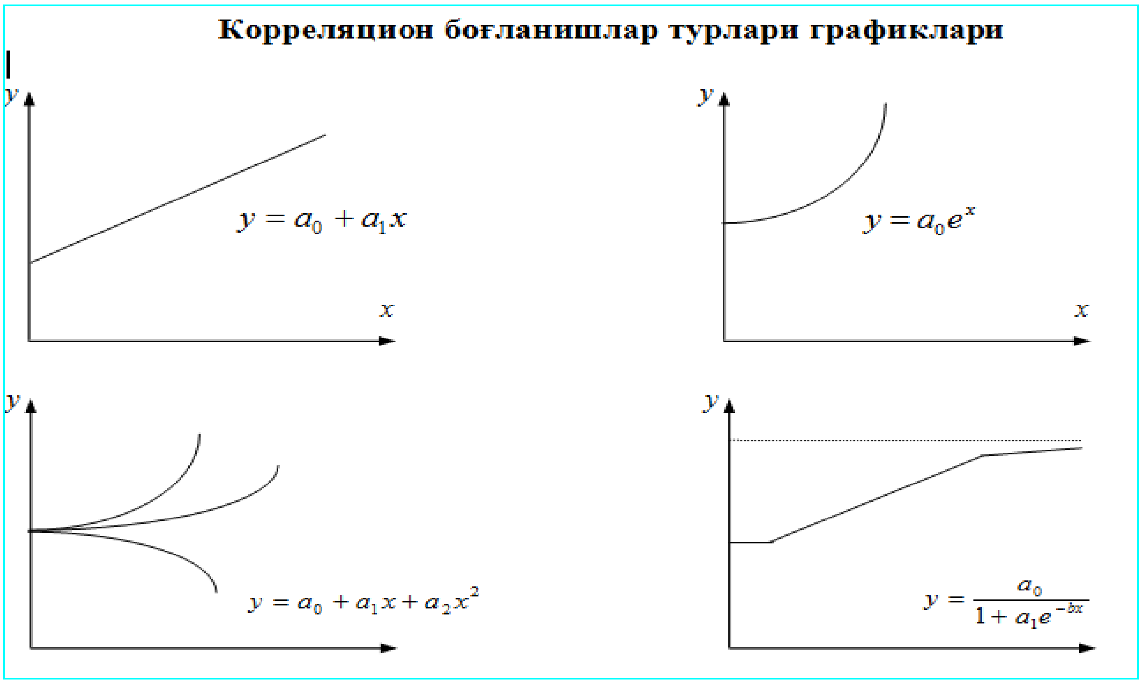
Logarifmik – $\log y = a_0 + a_1 x$

Yarim logarifmik – $y = a_0 + a_1 \ln x$

Ko’rsatkichli funksiya – $y = a_0 a_1^x$

Darajali funksiya – $y = a_0 x_1^{a_1}$

Logistik funksiya – $y = \frac{a_0}{1 + a_1 e^{-bx}}$



3.-rasm.Chiziqli va chiziqsiz regression bog’lanishlar

Bog’lanishlar chiziqli bo’lsa, u holda bog’lanish zichligi baholashda korrelyasiya koeffisiyentidan foydalanish mumkin:

$$r = \frac{\overline{x \cdot y} - \bar{x} \cdot \bar{y}}{\sigma_x \cdot \sigma_y}, \qquad (4.4)$$

bu yerda, σ_x va σ_y mos ravishda x va y o’zgaruvchilarning o’rtacha kvadratik chetlanishidir va ular quyidagi formulalar yordamida hisoblanadi:

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}, \quad \sigma_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}} \qquad (4.5)$$

Shuningdek, korrelyasiya koeffisiyentini hisoblashning quyidagi modifikasiyalangan formulalaridan ham foydalanish mumkin:

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n \cdot \sigma_x \cdot \sigma_y} \text{ yoki } r = \frac{n\sum_{i=1}^{n} xy - \sum_{i=1}^{n} x \sum_{i=1}^{n} y}{\sqrt{\left[n\sum_{i=1}^{n} x^2 - \left(\sum_{i=1}^{n} x\right)^2\right] \cdot \left[n\sum_{i=1}^{n} y^2 - \left(\sum_{i=1}^{n} y\right)^2\right]}} \qquad (4.6)$$

Regression tahlil natijaviy belgiga ta’sir etuvchi omillarning samaradorligini aniqlab beradi.

Regressiya so’zi lotincha ***regressio*** so’zidan olingan bo’lib, orqaga harakatlanish degan ma’noga ega. Bu atama korrelyasion tahlil asoschilari *F.Galton* va *K.Pirson* nomlari bilan bog’liqdir.

Regression tahlil natijaviy belgiga ta’sir etuvchi belgilarning samaradorligini amaliy jihatdan yetarli darajada aniqlik bilan baholash imkonini beradi. Regression tahlil yordamida ijtimoiy-iqtisodiy jarayonlarning kelgusi davrlar uchun bashorat qiymatlarini baholash va ularning ehtimol chegaralarini aniqlash mumkin.

Regression va korrelyasion tahlilda bog’lanishning regressiya tenglamasi aniqlanadi va u ma’lum ehtimol (ishonchlilik darajasi) bilan baholanadi, so’ngra iqtisodiy-statistik tahlil qilinadi.

Korrelyasion-regression tahlilda eng kichik kvadratlar usulining qo’llanilishi.

Funksiyalar parametrlari odatda **“eng kichik kvadratlar”** usuli bilan aniklanadi. Eng kichik kvadratlar usulini mazmuni quyidagicha: xaqiqiy miqdorlarning tekislangan miqdorlardan farqining kvadratlari yigindisi eng kam bo’lishi zarur:

$$S = \sum\left(Y - \bar{Y}_t\right)^2 \to \min \qquad (4.7)$$

Bir omilli chiziqli bog’lanishni olaylik:

$$Y_t = a_0 + a_1 t \qquad (4.8)$$

Qiymat $\sum\left(Y - \bar{Y}_t\right)^2$ eng kam bo’lishi uchun birinchi darajali xosilalar nolga teng bo’lishi kerak:

$$S = \sum\left(Y - \bar{Y}_t\right)^2 = \sum\left(Y - a_0 - a_1 t\right)^2 \to \min \qquad (4.9)$$

$$\frac{\partial S}{\partial a_0}=0 \qquad \frac{\partial S}{\partial a_0}=0 \qquad \rightarrow\begin{cases} n\cdot a_0+a_1\sum t=\sum y \\ a_0\sum t+a_1\sum t^2=\sum y\cdot t \end{cases} \qquad (4.10)$$

Bu normal tenglamalar tizimi.

Regression modelning parametrlarini baholash bog'liq o'zgaruvchi *Y* ning taqsimlanish ehtimolini topishdir. Modelda Y_i normal taqsimlangan va variasiyasi:

$var\,(Y)=\sigma^2$ ga teng

Eng kichik kvadratlar usulida hisoblash tamoyili Y_i larning xaqiqiy qiymatlarining o'rtacha qiymatidan farqining kvadrati summasini topishdan iborat. Demak:

$$S=\sum_{i=1}^{n}\left[Y_i-E(Y_i)\right]^2$$

Yoki (4.11)

$$S=\sum_{i=1}^{n}\left[Y_i-\alpha-\beta\cdot X_i\right]^2$$

bu yerda, S - farqlar kvadratlari summasi.

α va β , qiymatlarini topish uchun S ning α va β bo'yicha birinchi xosilasini topamiz:

$$\frac{\partial S}{\partial \alpha}=\sum_i\frac{\partial\left(Y_i-\alpha-\beta\cdot X_i\right)^2}{\partial\alpha}=-\sum_i 2\left(Y_i-\alpha-\beta\cdot X_i\right)=-2\sum_i Y_i-\alpha-\beta\cdot X_i,$$

(4.12)

$$\frac{\partial S}{\partial \beta}=\sum_i\frac{\partial\left(Y_i-\alpha-\beta\cdot X_i\right)^2}{\partial\beta}=-\sum_i 2\left(Y_i-\alpha-\beta\cdot X_i\right)\cdot(-X_i)=-2\sum_i X_i(Y_i-\alpha-\beta\cdot X_i)$$

Har bir xosilani nolga tenglashtirib hisoblab topilgan $\hat{\alpha}$ ва $\hat{\beta}$ larning qiymatini hisoblaymiz.

$$-2\sum_i\left(Y_i-\hat{\alpha}-\hat{\beta}\cdot X_i\right)=0$$

$$-2\sum_i X_i\left(Y_i-\hat{\alpha}-\hat{\beta}\cdot X_i\right)=0 \qquad (4.13)$$

yoki bunga ekvivalent ravishda

$$\sum Y_i=\hat{\alpha}\cdot n+\hat{\beta}\left(\sum_i X_i\right),$$

$$\sum X_i\cdot Y_i=\hat{\alpha}\left(\sum X_i\right)+\hat{\beta}\left(\sum_i X_i^2\right) \qquad (4.14)$$

Bu tenglamalar eng kichik kvadratlar usulida normal tenglamalar deb ataladi. Bunda *ye* eng kichik kvadratlar qoldig'i:

$$\sum e_i=0$$

$$\sum X_i\cdot e_i=0 \qquad (4.15)$$

tenglama $\hat{\alpha}$ ва $\hat{\beta}$ larga nisbatan yechiladi.

$$\hat{\beta} = \frac{n\left(\sum X_i \cdot Y_i\right) - \left(\sum X_i\right)\cdot\left(\sum Y_i\right)}{n\left(\sum X_i^2\right) - \left(\sum X_i\right)^2} \qquad (4.16)$$

Bu tenglikni boshqacha ko'rinishda ham yozish mumkin:

$$n\cdot\sum\left(X_i - \overline{X}\right)\cdot\left(Y_i - \overline{Y}\right) = n\cdot\sum\left(X_i\cdot Y_i\right) - n\cdot\overline{X}\cdot\left(\sum Y_i\right) - n\cdot\overline{Y}\cdot\left(\sum X_i\right) + n^2\cdot\overline{X}\cdot\overline{Y} =$$
$$= n\cdot\left(\sum X_i\cdot Y_i\right) - \left(\sum X_i\right)\cdot\left(\sum Y_i\right) - \left(\sum X_i\right)\cdot\left(\sum Y_i\right) + \left(\sum X_i\right)\cdot\left(\sum Y_i\right) =$$
$$= n\cdot\left(\sum X_i\cdot Y_i\right) - \left(\sum X_i\right)\cdot\left(\sum Y_i\right)$$

Demak

$$\hat{\beta} = \frac{\sum\left(X_i - \overline{X}\right)\cdot\left(Y_i - \overline{Y}\right)}{\sum\left(X_i - \overline{X}\right)^2} \qquad (4.17)$$

$\hat{\beta}$ larning qiymati topilgandan so'ng α' larni birinchi tenglamadan topamiz. Demak,

$$\hat{\alpha} = \left(\frac{1}{n}\right)\cdot\left(\sum Y_i\right) - \beta\cdot\left(\frac{1}{2}\right)\cdot\left(\sum X_i\right) = \overline{Y} - \hat{\beta}\cdot\overline{X} \qquad (4.18)$$

Nazorat uchun savollar

1. Korrelyasion-regressiontahlilning maqsadlari nimalardan iborat?
2. Juft, xususiy va ko'plikdagi korrelyasiya koyeffisiyentlarining farqi nimadan iborat?
3. Qaysi hollarda korrelyasiya indeksi qo'llaniladi?
4. Regressiya koyeffisiyentlarining iqtisodiy mohiyati nimadan iborat?
5. "Eng kichik kvadratlar usuli" ning mohiyatini tushuntirib bering.
6. Normal tenglamasini yechish usullarini tushuntirib bering.

Real iqtisodiy jarayonlar bo'yicha turli xildagi bog'lanishlarga 10 ta misol tuzing.