

29- Korrelyatsiya koeffitsiyenti. Korrelyatsiya koeffitsiyentini hisoblash usullariga doir masalalar

Misol: Bizga 10 yillik o'rtacha daromad X va o'rtacha iste'mol Y (mln. so'm) berilgan. 1-jadval.

Jadval ma'lumotlariga asosan Y ning X faktorga bog'liqligini chiziqli, darajali, ko'rsatkichli funksiya ko'rinishida ifodalang va optimal modelni tanlang (model bahosini tekshirishda o'rtacha aproksimasiya xatosi (A) va F-Fisher kriteriyasi).

1-jadval

Yillar	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
X	10,6	11,7	12,4	13,8	14,6	16,2	17,4	18,8	20,2	21,9
Y	8,12	10	8,41	12,1	12,4	11,4	12,8	13,9	17,3	17,5

Yechish:

1a. $y = a + bx$ ko'rinishdagi chiziqli regressiya tenglamasi bo'yicha masalani yechish uchun a va b ga nisbatan quyidagi normal tenglamalar sistemasini yechamiz:

$$\begin{cases} na_0 + a_1 \sum x = \sum y, \\ a_0 \sum x + a_1 \sum x^2 = \sum y \cdot x. \end{cases}$$

Bu tenglamalar sistemasini yechishda bizga quyidagilarni aniqlash kerak bo'ladi.

$$\sum x, \sum y, \sum x^2, \sum y \cdot x$$

2-jadval

N	Y	X	XY	X ²	Y ²	Yx	y-yx	Ai
1	8,12	10,6	86,1	112,4	65,9	8,250214	-0,13021	1,60362
2	10	11,7	117,0	136,9	100,0	9,133366	0,866634	8,666338
3	8,41	12,4	104,3	153,8	70,7	9,695372	-1,28537	15,28385
4	12,1	13,8	167,0	190,4	146,4	10,81938	1,280616	10,58361
5	12,4	14,6	181,0	213,2	153,8	11,46168	0,938324	7,567129
6	11,4	16,2	184,7	262,4	130,0	12,74626	-1,34626	11,80931
7	12,8	17,4	222,7	302,8	163,8	13,7097	-0,9097	7,107026
8	13,9	18,8	261,3	353,4	193,2	14,83371	-0,93371	6,717345

9	17,3	20,2	349,5	408,0	299,3	15,9577 2	1,34227 7	7,75882 9
10	17,5	21,9	383,3	479,6	306,3	17,3225 9	0,17740 6	1,01374 9
Jami	123,9	157,6	2056, 8	2612, 9	1629, 4		0,0	74,9035 6
O'rta	12,4	15,8	205,7	261,3	162,9			7,49035 6
□ □ □	9,4	12,9						
□ □	3,05807 1	3,59338 3						

σ^2	12,912 4	9,35180 1						
------------	-------------	--------------	--	--	--	--	--	--

$$\sigma_x^2 = \overline{x^2} - \bar{x}^2$$

$$\sigma_y^2 = \overline{y^2} - \bar{y}^2$$

$\sigma_x - X$ kvadratik og'ishning o'rtachasi;

$\sigma_y - Y$ kvadratik og'ishning o'rtachasi;

$$b = \frac{cov(x, y)}{\sigma_x^2} = \frac{\overline{yx} - \bar{y} \cdot \bar{x}}{\overline{x^2} - \bar{x}^2} = \frac{204,441 - 12,393 \cdot 15,66}{258,148 - 15,66^2} = 0,803$$

$$a = \bar{y} - b \cdot \bar{x} = 12,393 - 0,803 \cdot 15,66 = -0,180$$

Bundan regressiya tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$y_x = -0,180 + 0,803 \cdot x$$

Regressiya tenglamasidan quyidagi xulosaga kelish mumkin, agar o'rtacha daromadni 1 million so'mga oshirsak, o'rtacha iste'mol hajmi 0,803 mln so'mga oshadi.

Korrelyatsiya koeffitsiyentni hisoblaymiz.

$$r_{xy} = b \frac{\sigma_x}{\sigma_y} = 0,803 \cdot \frac{9,4}{12,9} = 0,94341$$

Determinatsiya koeffitsiyenti

$$d = r^2 = (0,94517)^2 = 0,893347$$

O'rtacha A aproksimatsiya xatosini topamiz:

$$A = \frac{1}{n} \sum \left| \frac{y - \bar{y}}{\bar{y}} \right| \cdot 100\% = \frac{78,123 \cdot 100\%}{10} = 7,8$$

O'rtacha hisob qiymati faktik qiymatdan 7,8% chetlanishini anglatadi.

F-Fisher kriteriyasini hisoblaymiz:

$$F_{fak} = \frac{r_{xy}^2}{1 - r_{xy}^2} \cdot (n - 2) = \frac{0,893347}{0,106653} \cdot 8 = 67,01$$

$$F_{tab} = 3,07 < F_{fak}$$

1b. Modelni quyidagi darajali chiziqlashtiriladigan ko'rinishda quramiz.

$$y = a \cdot x^b$$

Masalani chiziqli funksiyaga qo'rinishga keltirish uchun qismlar bo'yicha logarifmik ko'rinishga keltiramiz:

$$\lg y = \lg a + b \cdot \lg x,$$

Bu yerdan agar $y = \lg(y)$, $x = \lg(x)$, $c = \lg(a)$ qo'rinishda belgilash olsak regressiya tenglamamiz quyidagi ko'rinishga ega bo'ladi:

$$y = c + b \cdot x$$

Hisoblash natijalari 3-jadvalda keltirilgan.

3-jadval

N	Y	X	XY	X ²	Y ²	Yx	y-yx	
1	0,909556	1,025306	0,932573	1,051252	0,827292	10,59109	2,47109	30
2	1	1,068186	1,068186	1,141021	1	11,70246	1,70246	17
3	0,924796	1,093422	1,011192	1,195571	0,855248	12,4097	-3,9997	47
4	1,082785	1,139879	1,234244	1,299324	1,172424	13,82417	1,72417	14
5	1,093422	1,164353	1,273129	1,355718	1,195571	14,63244	2,23244	18
6	1,056905	1,209515	1,278342	1,462927	1,117048	16,24899	4,84899	42
7	1,10721	1,240549	1,373548	1,538962	1,225914	17,46139	4,66139	36
8	1,143015	1,274158	1,456381	1,623478	1,306483	18,87587	4,97587	3
9	1,238046	1,305351	1,616085	1,703942	1,532758	20,29034	2,99034	17
10	1,243038	1,340444	1,666223	1,79679	1,545144	22,00792	4,50792	25
Jami	10,79877	11,86116	12,9099	14,16899	11,77788	0		28
O'rta	1,079877	1,186116	1,29099	1,416899	1,177788	0		28
□□□	0,011653	0,010027						
□□	0,10795	0,100134						

$$b = \frac{cov(x, y)}{\sigma_x^2} = \frac{\overline{yx} - \bar{y} \cdot \bar{x}}{\overline{x^2} - \bar{x}^2} = \frac{1,28791 - 1,079877 \cdot 1,183201}{1,416899 - 1,183201^2} = 1,004$$

$$s = \bar{y} - b \cdot \bar{x} = 1,079877 - 1,004 \cdot 1,183201 = -0,10806$$

Bundan regressiya tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$y_x = -0,10806 + 1,004 \cdot x$$

Potensiallab quyidagiga ega bo'lamiz:

$$y_x = 10^{-0,10806} \cdot x^{1,004} = 0,779722 \cdot x^{1,004}$$

$$r_{xy} = \sqrt{1 - \frac{\sum(y - y_x)^2}{\sum(y - \bar{y})^2}} = \sqrt{1 - \frac{10,31902}{93,51801}} = 0,943217$$

$$A = \frac{1}{n} \sum \left| \frac{y - \bar{y}}{y} \right| \cdot 100\% = 7,8$$

1c. Tenglamani ko'rsatkichli funksiya ko'rinishida olamiz.

$$y = a \cdot b^x$$

$$\lg y = \lg a + x \cdot \lg b,$$

$$y = c + b \cdot x$$

4-jadval

N	X	Y	XY	X2	Y2	Yx	Y-Yx	(Y-Yx)^2	Ai
1	10,5	0,909 556	9,550 338	110,2 5	0,827 292	8,603 496	0,483 5	0,233 769	5,954 388
2	11,6	1	11,6	134,5 6	1	9,239 085	0,760 915	0,578 991	7,609 148
3	12,3	0,924 796	11,37 499	151,2 9	0,855 248	9,667 785	1,257 78	1,582 022	14,95 582
4	13,7	1,082 785	14,83 416	187,6 9	1,172 424	10,58 578	1,514 218	2,292 855	12,51 419
5	14,5	1,093 422	15,85 461	210,2 5	1,195 571	11,14 898	1,251 024	1,565 062	10,08 891
6	16,1	1,056 905	17,01 617	259,2 1	1,117 048	12,36 685	0,966 85	0,934 792	8,481 107
7	17,3	1,107 21	19,15 473	299,2 9	1,225 914	13,36 678	0,566 78	0,321 245	4,428 006
8	18,7	1,143 015	21,37 438	349,6 9	1,306 483	14,63 602	0,736 02	0,541 723	5,295 098
9	20,1	1,238 046	24,88 473	404,0 1	1,532 758	16,02 577	1,274 228	1,623 658	7,365 482
10	21,8	1,243 038	27,09 823	475,2 4	1,545 144	17,89 192	0,391 92	0,153 601	2,239 542
Jami	156,6	10,79 877	172,7 423	2581, 48	11,77 788	123,5 325		9,827 718	78,93 169
O'rta cha	15,66	1,079 877	17,27 423	258,1 48	1,177 788			0,982 772	7,893 169

σ	3,593 383	0,107 95							
σ^2	12,91 24	0,011 653							

$$b = \frac{cov(x, y)}{\sigma_x^2} = \frac{\overline{yx} - \bar{y} \cdot \bar{x}}{\overline{x^2} - \bar{x}^2} = \frac{17,27423 - 15,66 \cdot 1,079877}{258,148 - 15,66^2} = 0,02814$$

$$s = \bar{y} - b \cdot \bar{x} = 1,079877 - 0,02814 \cdot 15,66 = 0,639205$$

Bundan regressiya tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$y_x = 0,639205 + 0,02814 \cdot x$$

Potensiallab quyidagiga ega bo'lamiz:

$$y_x = 10^{0,639205} \cdot 10^{0,2814} = 4,357175 \cdot 1,06694^x$$

$$r_{xy} = \sqrt{1 - \frac{\sum(y - y_x)^2}{\sum(y - \bar{y})^2}} = \sqrt{1 - \frac{9,827718}{93,51801}} = 0,945997$$

$$A = \frac{1}{n} \sum \left| \frac{y - \bar{y}}{y} \right| \cdot 100\% = 7,9$$

1d. Tenglamani giperbola ko'rinishida olamiz.

$$y = a + b \cdot \frac{1}{x}$$

Bu regressiya tenglamasini chiziqli tenglama ko'rinishiga keltirish uchun quydagicha bulgilash olamiz:

$$z = \frac{1}{x}$$

U holda tenglama $y = a + b \cdot z$ ko'rinishga keladi.

N	Z	Y	ZY	Z2	Y2	Yz	Y-Yz	(Y-Yz)^2	Ai
1	0,095 238	8,12		0,009 07	65,93 44	7,627 674	0,492 326	0,242 385	6,063 13
2	0,086 207	10	0,862 069	0,007 432	100	9,172 081	0,827 919	0,685 45	8,279 19
3	0,081 301	8,41	0,683 74	0,006 61	70,72 81	10,01 106	- 06	2,563 395	19,03 758
4	0,072 993	12,1	0,883 212	0,005 328	146,4 1	11,43 181	0,668 186	0,446 472	5,522 196
5	0,068 966	12,4	0,855 172	0,004 756	153,7 6	12,12 049	0,279 505	0,078 123	2,254 074
6	0,062 112	11,4	0,708 075	0,003 858	129,9 6	13,29 254	- 54	3,581 689	16,60 119

7	0,057 803	12,8	0,739 884	0,003 341	163,8 4	14,02 929	- 1,229 29	1,511 165	9,603 863
8	0,053 476	13,9	0,743 316	0,002 86	193,2 1	14,76 934	- 0,869 34	0,755 747	6,254 224
9	0,049 751	17,3	0,860 697	0,002 475	299,2 9	15,40 629	1,893 711	3,586 14	10,94 63
10	0,045 872	17,5	0,802 752	0,002 104	306,2 5	16,06 975	1,430 254	2,045 626	8,172 878
Jami	0,673 718	123,9 3	7,138 916	0,047 834	1629, 383	123,9 303	0	15,49 619	92,73 463
O'rta cha	0,067 372	12,39 3	0,793 213	0,004 783	162,9 383			1,549 619	9,273 463
σ	0,015 635	3,058 071							
σ^2	0,000 244	9,351 801							

$$b = \frac{cov(z, y)}{\sigma_z^2} = \frac{\bar{y}\bar{z} - \bar{y} \cdot \bar{z}}{\bar{z}^2 - z^2} = \frac{0,793213 - 12,393 \cdot 0,067372}{0,004783 - 0,067372^2} = -171,008$$

$$a = \bar{y} - b \cdot \bar{z} = 12,393 + 171,008 \cdot 0,067372 = 23,91415$$

Bundan regressiya tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$y_x = 23,91415 - 171,008 \cdot \frac{1}{x}$$

$$r_{xy} = \sqrt{1 - \frac{\sum(y - y_z)^2}{\sum(y - \bar{y})^2}} = \sqrt{1 - \frac{15,49619}{93,51801}} = 0,913399$$

$$A = \frac{1}{n} \sum \left| \frac{y - \bar{y}}{y} \right| \cdot 100\% = 9,3$$

№	Regressiya tenglamasi	Korrelyasiya koeffitsiyenti	O'rtacha aproksimasiya xatosini
1a	$y_x = -0,180 + 0,803 \cdot x$	$r_{xy} = 0,94517$	$A = 7,8$
1b	$y_x = 0,779722 \cdot x^{1,004}$	$r_{xy} = 0,943217$	$A = 7,8$
1c	$y_x = 4,357175 \cdot 1,06694^x$	$r_{xy} = 0,945997$	$A = 7,9$
1d	$y_x = 23,91415 - 171,008 \cdot \frac{1}{x}$	$r_{xy} = 0,913399$	$A = 9,3$

