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SARAH GETS A DIAMOND

Greg Mills proposed to Sarah Staggers in the summer of 2006. This is the brief story of their relationship, the proposal, and Greg's search for the perfect diamond.

Background

After graduating with an engineering degree from a southeastern university, Greg accepted a position as a structural engineer with a national defense contractor in eastern Virginia. It was there that he met an attractive engineering colleague by the name of Sarah Staggers. Though significant professional rivalry and tension initially existed between the two, Greg and Sarah ultimately became friends. Before long, both realized a relationship was developing and that these two closely matched, highly competitive colleagues actually complemented each other quite well. Greg and Sarah started dating in the summer of 2003.

After several years of tolerating corporate and federal bureaucracy, they had both concluded they were ill suited for government defense contracting and left together to pursue MBAs at a well-known southeastern business school. After a successful first year for the couple, Greg's career interest in real estate development led him to take a summer internship with a large national homebuilder in Naples, Florida. Sarah chose a consulting internship in Cleveland, Ohio—effectively separating the two for the first time in nearly three years. The separation was difficult for each to bear, and in May 2006, Greg made the decision to ask Sarah for her hand in marriage.

After Sarah's internship ended in mid-July, she flew to Naples intent on spending a week drinking margaritas and lounging on the beach while her boyfriend toiled away at work. Greg saw the visit as the ideal opportunity to propose.

Sarah Describes the Proposal

After three years, one month, and 16 days, Greg decided that I was the one. It was Sunday, July 16, 2006, and Greg was in Naples, Florida, working at a real estate development firm for the summer. Greg was living in this incredible oceanfront condo owned by the CFO of his firm's southeast division. Since the beginning of the summer, I planned to visit him for a week following the conclusion of my summer internship in Cleveland with a management consulting firm.

On that wonderful Sunday morning, Greg picked me up from the airport, and we had a great day in Naples on the beach. That evening, Greg suggested that we grab an early dinner at a Japanese steakhouse in order to make it to the beach to watch the sun set over the Gulf. During my prior visits to Naples, we routinely went down to the beach to watch the sun set, so I did not suspect anything extraordinary. On this particular evening, the sky was so clear because it had not rained all day.

As we arrived on the beach, we decided to walk as we watched the sun sink into the ocean. Surprisingly, there were very few people out that night. Just as the sun was minutes away from disappearing, Greg suggested that we sit down in the sand. We were sitting there side-by-side, making small talk, when Greg asked me if we could continue our earlier conversation about our relationship. I turned to look at him, and it appeared that he was having trouble finding his words. After a few seconds that seemed like minutes, he told me that he had no doubts that I was the one for him. He positioned himself so we were facing each other and started pulling a box out of his cargo shorts. Greg looked at me and said that he wanted to share a lifetime of sunsets with me. Then, he asked, "Will you marry me?"

As he was uttering those glorious life-changing words, I was in total shock. I gasped and put my hands over my mouth. My first reply was to ask him if he was joking because this was the same Greg who would never even utter the word "commitment." I kept saying, "Oh my gosh!" in total disbelief, and he quietly said, "You haven't answered me yet."

The Perfect Engagement Ring

Will Sarah say yes? *Should* Sarah say yes? Will Sarah open the box before answering? When she opens the box, will she be thrilled by the ring inside? And if she is not thrilled by the ring, will she pretend to be anyway?

If this were an organizational behavior case, we might address these important questions. But this is a data-analysis case. This case takes you back in time to examine Greg's quest for the perfect engagement ring. Ever the engineer, Greg firmly believed his engineering skills and newfound business acumen would allow him to optimize his purchase decision.

The setting

Since the moment he decided to propose, Greg had been looking for the perfect engagement ring. Remembering comments that Sarah had made in the past about friends' rings, he was sure she would prefer a unique platinum setting for her diamond, preferably with an antique flair, supporting a round brilliant-cut diamond. Greg spoke with multiple jewelers, looked at hundreds of designs, and even went so far as to create a design of his own. With time running out and frustratingly few suitable options, a friend suggested the ideal solution. Greg explained:

> I had very little success finding a ring and setting that I felt would be perfect for Sarah. I knew she wanted something unique, preferably antique in style, but I had serious problems locating *the one*. Each design was wrong; it was either too ornate, or too traditional, or too something else. I wasn't sure exactly what I was looking for, but I was sure that I'd know it when I saw it. I really was getting frustrated. This is a once-in-a-lifetime purchase and I wanted it to be perfect—no compromises.
>
> I only had about a month left before I planned to propose, when a close friend of mine suggested a great alternative. He said, "Why not buy the diamond and build the setting together with Sarah, exactly like she wants?" Brilliant idea. Sarah would get the ring she always dreamed about and I would handle the diamond purchase.
>
> That settled it. I would purchase a diamond stone and a basic setting, propose, and later work with Sarah and my jeweler to create a ring exactly as she wanted. It was the perfect solution in just the right amount of time.

Diamond grading

With the ring-setting design issue out of the way, Greg focused his attention on selecting a diamond. A few simple online queries unearthed a wealth of knowledge on diamond selection, grading, and pricing.

Diamonds are primarily based on four criteria known as the “four Cs”: cut, color, clarity, and carat weight. In addition to these factors, other less-known qualifiers (e.g., fluorescence, polish, and symmetry) marginally influence the value of the stone.

Cut: In 1919, Marcel Tolkowsky, a young mathematician from a Belgian family of diamond cutters, proposed a series of diamond proportions that mathematically maximized the light refraction characteristics of round-cut diamonds. Known as the Tolkowsky cut, it remains as a North American industry benchmark for ideal-cut round diamonds. The geometric dimensions and proportions in which a diamond is cut drastically influence a stone’s ability to reflect and refract light. For this reason, a diamond’s cut grade is typically considered the most important value driver in the quality of the diamond. These are the diamond cut ratings, in ascending order: poor, fair, good, very good, ideal, and signature ideal. **Exhibit 1** shows the approximate distribution of the number of cut diamonds in each category.

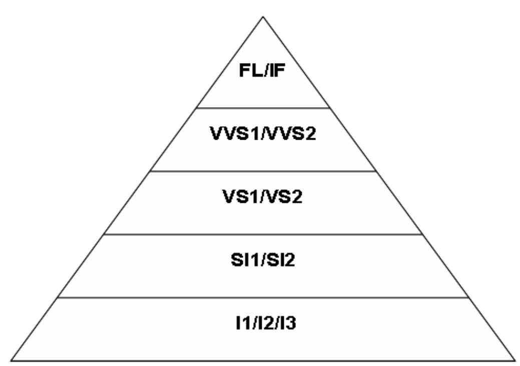
Color: Color in a diamond acts like a filter to block refracted colors from incoming light; as a result, the sparkle of the diamond is diminished. Diamonds are graded on a letter scale (see **Table 1**), from Z (noticeable color) to D (absolutely colorless). Diamonds with color grades of J or better are considered gem-quality, and most reputable dealers refrain from selling diamonds below this grade.

Table 1. Diamond color scale.

Grade		**Description**
D	Completely colorless	Extremely rare, absolutely undetectable color
E	Colorless	Miniscule traces of color detectable only by expert gemologists
F		Very little color traces detectable only by expert gemologists
G–H	Near-colorless	Color noticeable when compared to higher-quality diamonds
I–J		Slight color noticeable by untrained eye
K–Z	Noticeable color	Easily detectable by untrained eye

Clarity: Mineral deposits and other imperfections reduce the quality and light reflection techniques of diamonds; the result is a decrease in value. Gems that are absolutely free of such blemishes are known to be FL (flawless), followed by IF (internally flawless), VVS1 and VVS2 (very, very slight inclusions), VS1 and VS2 (very slight inclusions), SI1 and SI2 (slight inclusions), and I1, I2, and I3 (see **Figure 1**).

Figure 1. Clarity quality pyramid.



Carat weight: Finally, size is the last prime driver of value in diamond pricing. A diamond's rarity—and therefore its price—increases exponentially with size. For example, a two-carat diamond is worth more than twice the value of two one-carat stones of similar cut, clarity, and color.

Miscellaneous factors: Most gem-quality diamonds are accompanied by a grading report from a major gemological laboratory. The American Gemological Society Laboratory (AGSL) and Gemological Institute of America (GIA) serve as North America's industry leaders. A report accompanying a diamond from either of these two labs ensures the quality of the diamond and potentially increases the value of the stone.

The cut symmetry and surface polish—both of which affect light reflection/refraction characteristics—marginally affect the quality and price of a diamond because both of these characteristics are indirectly considered in cut and clarity. Both symmetry and polish are graded using G (good), VG (very good), and EX (excellent) categories. The AGSL reports also use an ID (ideal) category.

Diamond pricing standards

The Rapaport Diamond Report (RDR) is published monthly as a diamond wholesale price list, based on current-market diamond purchases and availability. Used by diamond buyers and sellers worldwide, it is the industry standard and the definitive guide for wholesale gem prices. The values in the RDR are generated through market activity and are general in nature, dealing primarily in one-quarter carat increments. Unfortunately, due to the broad nature of the report, and with tens of thousands of unique diamonds available for purchase, no practical method exists to specifically predict the wholesale market of a unique stone.

Modeling Diamond Prices

With extensive market research and a wealth of information at his fingertips, it was almost time for Greg to purchase a diamond. After choosing an online dealer with a long history of superior customer service and exceptional pricing, Greg downloaded information on 6,000 diamonds available for purchase. **Exhibit 2** is a glimpse at what the data look like. **Exhibit 3** gives descriptive statistics for each of the variables and a scatterplot of price versus carat weight. Greg's plan was to use these data to build a predictive model of diamond prices so he could then find a high-quality diamond that he believed was priced below its inherent value.

Additive model

Greg's first attempt was to build a simple linear model for the relationship between price and carat weight. The resulting model was

$$\text{Price} = -\$12{,}739 + \$18{,}381 \times \text{Carat Weight}.$$

Although this simple model had an adjusted R-square of 74% and a t-stat of 130, Greg was concerned that it reflected some unrealistic assumptions about the way prices behaved. In particular, the model indicated that each additional carat increased prices by $18,381 across the entire range of carat weights. The model forecasts are given in **Table 2**.

Table 2. Simple linear model forecasts.

Carat Weight	Price
0	−$12,738.58
1	$5,642.68
2	$24,023.94
3	$42,405.20

In addition, if other terms were added to this model, the terms would simply add or subtract a fixed-dollar amount to the price of the diamond. Thus, this model assumed diamond prices worked the same way new car prices worked in that optional features *add* a fixed-dollar amount to the price of the car. This would mean, for example, that improving the color of a one-carat diamond would increase the price by the same dollar amount as improving the color of a two-carat diamond. Greg did not think that made sense.

Multiplicative models

From his engineering background, Greg knew that when several positive factors multiply together to produce an overall result, the sum of the natural logs of each factor would equal the natural log of the result. In short, one can accomplish multiplication by adding logs. This is the principle behind the slide rule.

For diamond prices, this meant that if Greg believed that the price of a diamond resulted from the multiplication of several factors (e.g., improved color increasing the price by 10%, flaws decreasing price by 20%), then he should use l*n*(price) as his dependent variable.

His second modeling attempt was to regress the natural log of price against carat weight. The resulting model was the following:

$$\ln(\text{Price}) = 7.26 + 1.38 \times \text{Carat Weight}$$

This is sometimes called a log-linear model because the natural log of Y is a linear function of X. The resulting t-stat jumped to 183.

This model showed that each unit increase in carat weight increased l*n*(price) by 1.38. Because of the way the natural log function works, this is equivalent to saying that each unit increase in carat weight *multiplies* the price by exp(1.38) = 3.96. **Table 3** illustrates how this model works. A linear relationship between l*n*(price) and carat weight is equivalent to a multiplicative relationship between price and carat weight. Notice that forecast prices increase 396% with each additional carat.

Table 3. Natural log model forecasts.

Carat Weight	ln(price)	Price
0	7.265	$1,428.78
1	8.640	$5,652.65
2	10.015	$22,363.51
3	11.390	$88,476.46

If other terms were added to this model, these other terms would multiply the forecast diamond price by a fixed multiplicative factor. Improving the color of a one-carat diamond, for example, would increase the price by the same percentage as improving the color of a two-carat diamond.

Finally, Greg fit a log-log model by regressing the natural log of price on the natural log of carat weight:

$$\ln(\text{Price}) = 8.639 + 1.995 \times \ln(\text{Carat Weight}).$$

The t-stat for the l*n*(carat weight) coefficient was 199. Because this model uses l*n*(price) as the dependent variable, it too is a multiplicative model. Adding other terms to this model means these other terms will change price by a fixed multiplicative factor.

The difference between the log-log model and the log-linear model is that in the log-log model, a small *percentage* change in carat weight results in a fixed percentage change in price. For that reason, the log-log model is also known as the constant elasticity model. For the diamond data, the estimated elasticity of price to carat weight is 1.995. In other words, the ratio of percentage change in price to percentage change in carat weight is 1.995 for very small changes in carat weight. The following table shows the forecasts from the log-log model.

Table 4. Log-log model forecasts.

Carat Weight	ln(Carat Weight)	ln(Price)	Price
0.5	−0.693	7.256	$1,416.89
1.0	0.000	8.639	$5,648.22
1.5	0.405	9.448	$12,683.09
2.0	0.693	10.022	$22,515.73
2.5	0.916	10.467	$35,142.10
3.0	1.099	10.831	$50,559.11

Another way to interpret the 1.995 coefficient in this model is to realize that multiplying carat weight by λ results in the price being multiplied by $\lambda^{1.995}$. Doubling the carat weight, for example, means the forecast price will go up approximately four times. (To see this, compare the forecast price of a 2-carat-weight diamond with that of a 1-carat-weight diamond. Also, compare the price of a 3-carat-weight diamond with that of a 1.5-carat-weight diamond. In both of those cases, the forecast price of the larger diamond is about four times the forecast price of the smaller diamond.)

The forecasts from the three models are charted in **Exhibit 3**. As expected, the two multiplicative models show an accelerating relationship between the price and the carat weight. For the multiplicative models, the forecast price accelerates at an increasing rate as diamonds get bigger.

Romancing the Stone

Once Greg had selected from among the three models, his plan was to include the other drivers of price in the model. With a full-blown model that accounted for all value drivers, he should be able to price each diamond under consideration and find one that was underpriced. As with almost every important purchase, Greg thought being an informed shopper was essential. Greg knew that Sarah felt the same way. Why, she might even find the fact that he was using regression to help select her diamond to be … well … romantic.

Exhibit 1

SARAH GETS A DIAMOND

Distribution of Diamond Cuts

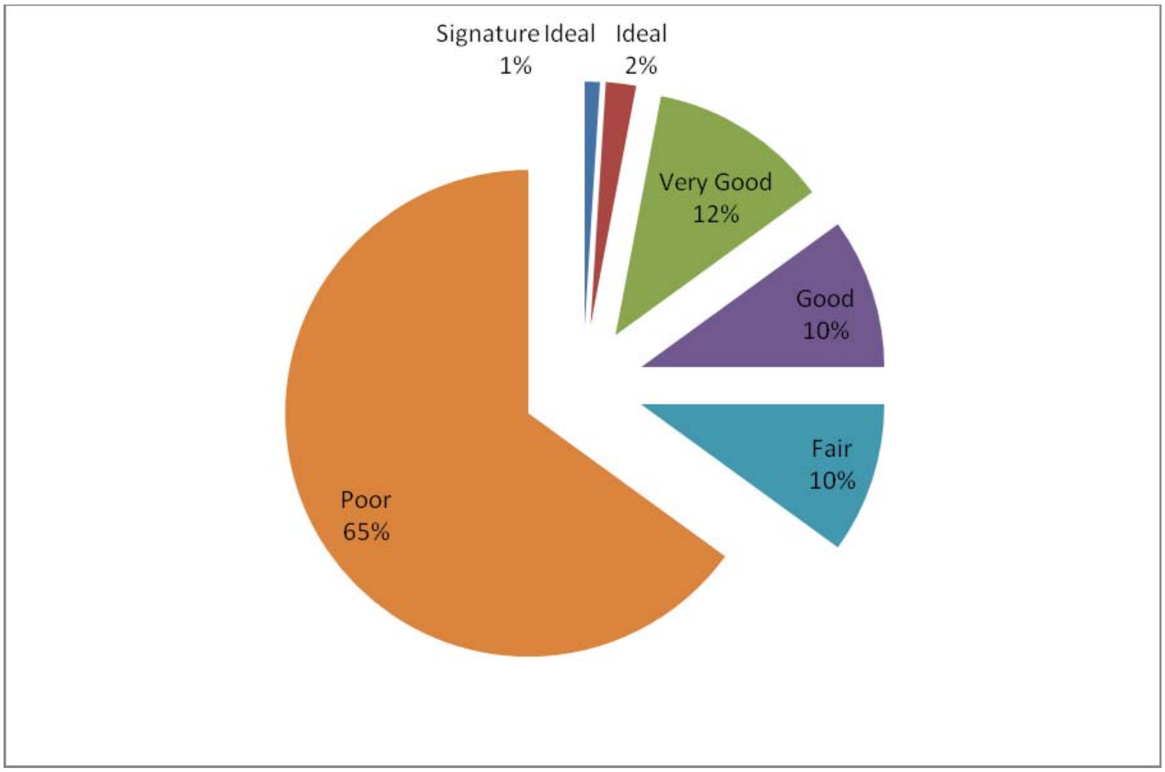


Exhibit 2

SARAH GETS A DIAMOND

The 6,000 Diamond Data Set

ID	Carat Weight	Cut	Color	Clarity	Polish	Symmetry	Report	Price
1	1.10	Ideal	H	SI1	VG	EX	GIA	$5,169
2	0.83	Ideal	H	VS1	ID	ID	AGSL	$3,470
3	0.85	Ideal	H	SI1	EX	EX	GIA	$3,183
4	0.91	Ideal	E	SI1	VG	VG	GIA	$4,370
5	0.83	Ideal	G	SI1	EX	EX	GIA	$3,171
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5996	1.03	Ideal	D	SI1	EX	EX	GIA	$6,250
5997	1.00	Very Good	D	SI1	VG	VG	GIA	$5,328
5998	1.02	Ideal	D	SI1	EX	EX	GIA	$6,157
5999	1.27	Signature-Ideal	G	VS1	EX	EX	GIA	$11,206
6000	2.19	Ideal	E	VS1	EX	EX	GIA	$30,507

Exhibit 3

SARAH GETS A DIAMOND

Summary Statistics for the 6,000 Diamonds

	Average Price	Count
CUT		
Signature-Ideal	$11,541.53	253
Ideal	$13,127.33	2,482
Very Good	$11,484.70	2,428
Good	$9,326.66	708
Fair	$5,886.18	129
COLOR		
D	$15,255.78	661
E	$11,539.19	778
F	$12,712.24	1,013
G	$12,520.05	1,501
H	$10,487.35	1,079
I	$8,989.64	968
CLARITY		
FL	$63,776.00	4
IF	$22,105.84	219
VVS1	$16,845.68	285
VVS2	$14,142.18	666
VS1	$13,694.11	1,192
VS2	$11,809.05	1,575
SI1	$8,018.86	2,059
TOTAL	$11,791.58	6,000

Exhibit 3 (continued)

	Carat Weight	Price
Mean	1.33452	$11,791.6
Median	1.13	$7,857.0
Mode	1.01	$4,466.0
Standard Deviation	0.47569628	$10,184.4
Sample Variance	0.226286951	$103,720,986.0
Kurtosis	−0.416759	$7.9
Skewness	0.8912170	$2.3
Range	2.16	$99,377.0
Minimum	0.75	$2,184.0
Maximum	2.91	$101,561.0
Sum	8,007.12	$70,749,476.0
Count	6,000	6,000

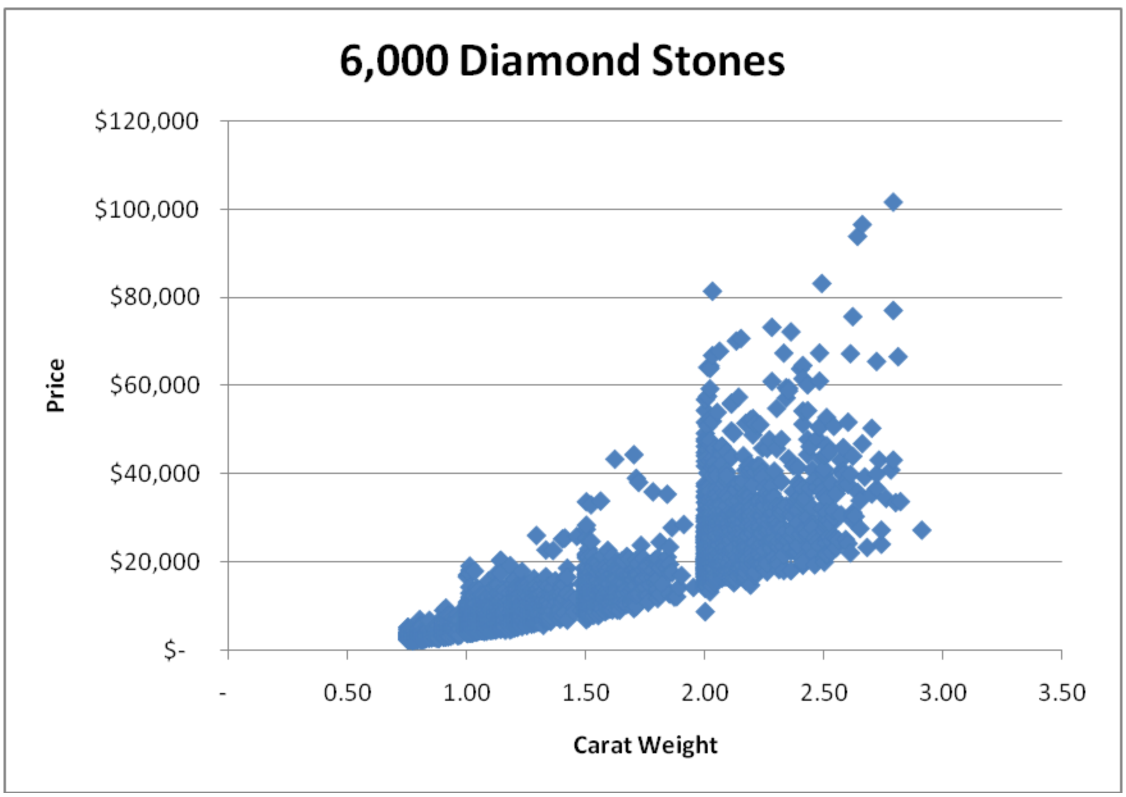

Exhibit 4

SARAH GETS A DIAMOND

Chart of Model Forecasts