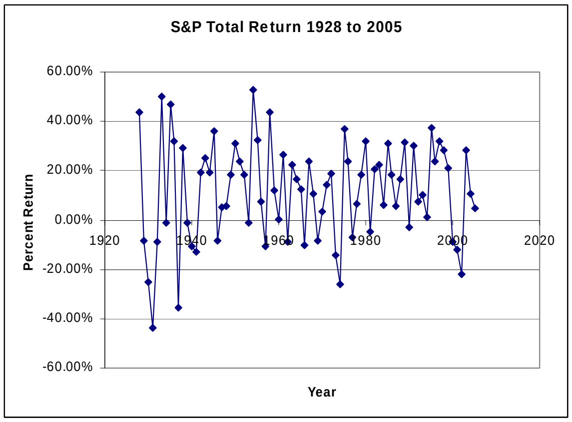
UV1301

TIME SERIES

What Is a Time Series?

A time series is a sequence of observations in time order. The observations are usually measured using an interval (numerical) scale with equal time gaps between successive observations. The chart to the right shows an example of a time series: annual total returns from the S&P 500 from 1928 to 2005.



Time Series Analysis

Time series analysis attempts to identify patterns in the time series, build a model that explains the past patterns, and then use that model to forecast subsequent observations of the time series. Because time series analysis only attempts to extrapolate past time patterns into the future, some have suggested that it is like trying to drive an automobile while looking in the rearview mirror. Obviously this will be a challenge, and we can not expect time series analysis to predict the future with pinpoint accuracy.

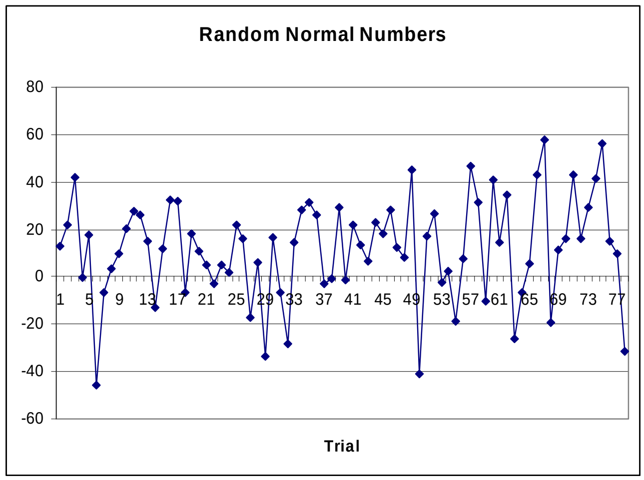
So why do we use time series analysis? Why try to drive our automobile while looking in the rearview mirror? We use time series analysis when we have no better alternative. In many business situations, we have no windshield into the future. And without a windshield, it is better to take advantage of past patterns than to ignore them. If the time series has been relatively high in the recent past, it may make sense to predict that it will be high in the future. If the time series regularly peaked in winter months and bottomed out in the summer, it makes sense to expect the same thing in the future.

This note was prepared by Professor Phillip E. Pfeifer.

Time Series Patterns

Before we begin to talk about the different kinds of patterns one finds in time series, let us address what it means if a time series has no pattern. A time series completely lacking in pattern is one in which each observation is independent of all others. Such a time series is sometimes called white noise.

White noise: no pattern



One way to see what white noise looks like is to use Crystal Ball to generate a column of numbers from a distribution and then to graph them in a line chart. For reasons that will become apparent, enter =CB.NORMAL(12,20) into a cell and copy it down to create a column of 78 observations.[1] Next, create the line chart. You should see something similar to the chart to the right. What you are looking at is an example of a white noise sequence. What's cool (to me at least) is that every time you hit F9, you get another example of a white noise sequence. In one sense, each sequence is different. But it another sense, all sequences are the same in that they were all generated by the same process.

Because we know how the sequence was generated, we know exactly how to forecast a 79th observation. The best point forecast for the 79th observation is 12, the mean of the normal distribution used to generate the sequence. If we want a complete forecast, we would say the 79th observation will be normally distributed with a mean of 12 and a standard deviation of 20. We know this to be true, because we know how the sequence was generated.

Notice that in this very special time series, there is nothing in the sequence of observations (no real pattern) we can make use of to extrapolate into the future. Each observation comes out independently from all others. If observation 78 was unusually high, our forecast of observation 79 will be 12. If observation 78 was unusually low, our forecast for 79 is still 12. When there is no pattern in the time series, there is nothing one can do except forecast the past average … and keep forecasting that average.

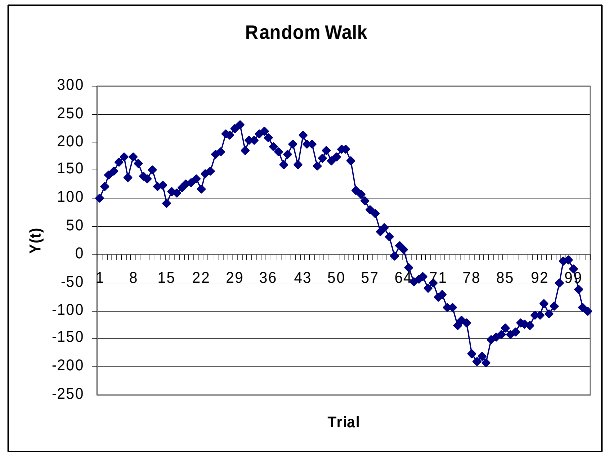
Have you figured out why we asked you to use a mean of 12 and a standard deviation of 20 to generate a white noise time series of length 78? We chose those numbers to match the average, sample standard deviation, and length of the S&P return time series charted at the

[1] Crystal Ball® is a registered trademark of Decisioneering, Inc. If you do not want to use Crystal Ball, you can simply use =NORMINV(RAND(),12,20) to accomplish the same thing.

beginning of the note. If you believe in an efficient market, there should be no time series pattern in stock returns. Consequently, the time series of S&P returns should look like an example of white noise.

Random walk: the ultimate cycle

We now turn to a particular kind of time series that exhibits a very strong cyclical pattern. Enter the number 100 in cell A1, and then below it enter =A1+CB.NORMAL(0,20). Copy this new cell down so that you end up with a time series with 100 observations in column A. Use Excel's charting capabilities to put the resulting time series into a line chart. The chart to the right is what I got.



What you will see is an example of a random walk. It is called a random walk because in each period, the series adds a random amount to the previous value of the series. If that random amount is normally distributed with a mean of zero, the series is just as likely to take a step up as to take a step down each period. But what is key about a random walk is that once the series takes a step, its journey starts anew from where it ended up. If it happens to end up at a high value, that high value is the starting point for the future.

Now the fun begins. Hit F9 a few times. Although one might think that the series will not stray very far from the starting value of 100, it does. Sometimes the series goes up. Sometimes down. But in comparison to white noise, the random walk series is smooth and meandering. The patterns you see in the random walk series are called *cycles*. The series moves up and down but with turning points that are not predictable.

Once again, since we generated these random walks, we know how to forecast a 101st observation. The best point forecast of Y(101) is Y(100). If we want a complete forecast of the 101st observation, we say that Y(101) is normally distributed with a mean of Y(100) and a standard deviation of 20.

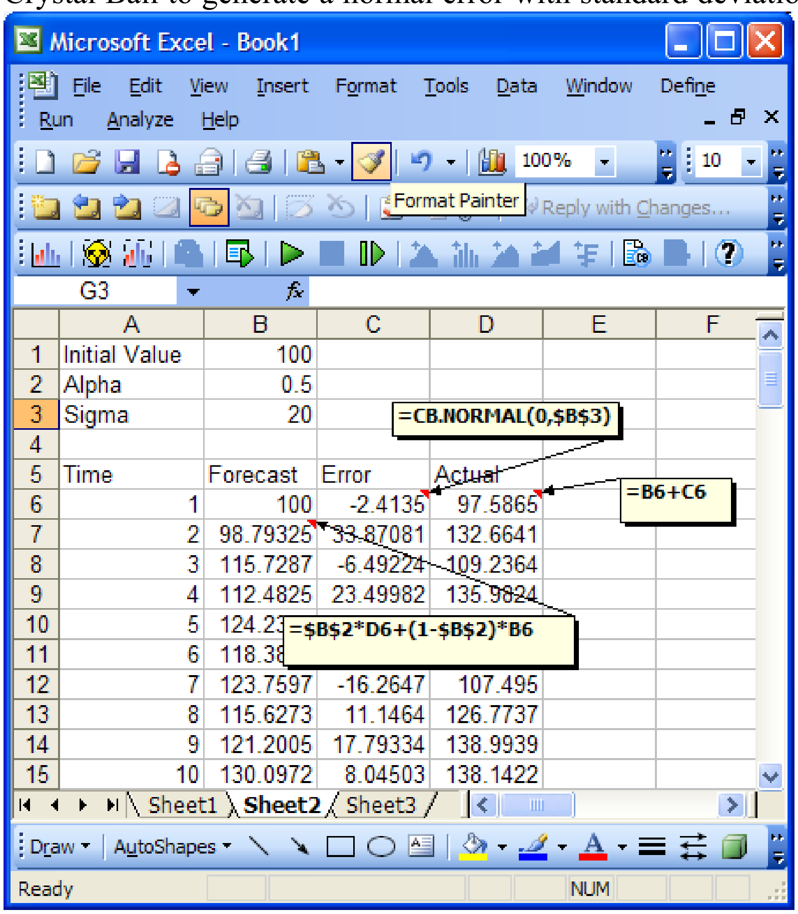
In many ways, random walks are the opposite of white noise. With white noise, the previous observation is ignored when forecasting the future. With random walks, the previous observation *is* the forecast of the future. With white noise, our forecast stays the course and the series varies randomly about the forecast. With a random walk, our forecast tracks the series and the series varies randomly about the forecast.

Please notice that although random walks are the opposite of white noise in many ways, they are identical in that *the series varies randomly about the forecast*. In fact, this will be true of all good time-series forecasting models. If the forecasting model is “right”, then the series will vary randomly about the forecast. In other words, the forecast errors will be white noise if we use the right forecasting model.

Exponential smoothing: anywhere in between

Our exploration of cyclic patterns has identified two extremes: white noise that exhibits no pattern and random walks that exhibit very strong cyclical patterns. As you might expect, the behaviors of actual time series often fall in between these two extremes. The *exponential smoothing model* is a simple model that can produce cyclical behavior between these extremes. The model does this using a smoothing parameter α, which controls the strength of the cyclical behavior. When α = 0 the exponential smoothing model is equivalent to white noise. When α = 1 the exponential smoothing model is equivalent to a random walk.

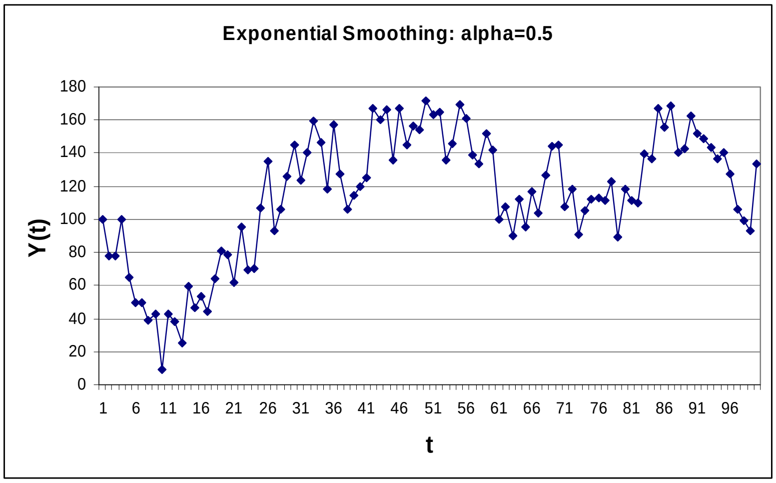
To see exponential smoothing in action, set the initial forecast to 100 (cell B6), use Crystal Ball to generate a normal error with standard deviation 20 (cell C6), and add the error to the forecast to get the first observation of the time series (cell D6). The magic of exponential smoothing happens in cell B7. The forecast for period 2 is a weighted combination of the actual for period 1 and the forecast for period 1. The immediate past actual receives a weight of α, and the immediate past forecast receives a weight of (1 − α). Enter this relationship in cell B7 and copy it down. Also, copy down the normal error and the relationship in cell D6. What you now have in column D is a time series generated by the exponential smoothing model with parameter α (cell B2). You also have the ability to have some fun hitting F9. And you can also change α as you see fit.



For α = 0.5, I got the series shown in the Exponential Smoothing chart. Clearly, it exhibits a cyclical pattern—but one that is not as smooth as those produced by a random walk. What this means is that the series meanders, but with irregularity. When we forecast such a series, we want to pay some, but not total, attention to the previous observation. The compromise inherent in exponential smoothing is seen clearly in the equation used for forecasting:

$$\text{New Forecast} = \alpha \times \text{ Actual} + (1 - \alpha) \times \text{ Old Forecast.}$$

For my particular series, the 100th observation was 133.6 and the forecast of that observation was 101.2. (The 101.2 should make some sense. At period 99, 101.2 looked like a very reasonable forecast of the 100th observation.) The fact that observation 100 was much higher than expected coupled with a history that shows cyclical behavior suggests we want to adjust the new forecast upward from 101.2. Since we know that $\alpha = 0.5$, we know exactly what the new forecast should be.[2]



$$\begin{aligned}\text{New Forecast} &= \alpha \times \text{ Actual} + (1 - \alpha) \times \text{ Old Forecast} \\ &= 0.5(133.6) + 0.5(101.2) \\ &= 117.5.\end{aligned}$$

In addition to modeling time series exhibiting cyclical behavior, exponential smoothing has the attractive feature that its forecasts are very easy to update. With each period, we combine the new observation with the old forecast (using α) to calculate the new forecast. Once we have the new forecast, we can "throw away" the old forecast and the recent observation. We'll never need them again for forecasting purposes.

Before we move on, I want to let you in on a secret about exponential smoothing. The secret is that *exponential smoothing can only forecast one period into the future*. Let me explain what I mean. In period 100, our forecast for period 101 is 117.5. If we wait until the end of period 101 to forecast period 102, we will use our forecasting equation to update the 117.5 forecast based on what happens in period 101:

$$\text{New Forecast(102)} = \alpha \times \text{ Actual(101)} + (1 - \alpha) \times \text{ 117.5.}$$

[2] The calculation of the 117.5 was conducted with greater precision (more decimal places for the input numbers) than is illustrated here.

But if we have to forecast 102 now without benefit of knowing the actual for 101, we simply do the next best thing and substitute our forecast of 101 into the forecasting equation. Of course, the forecast of 101 is 117.5, which means our forecast of 102 will also be 117.5. Similar logic says that our forecast of every future period is 117.5. It is in that sense that I say that exponential smoothing can only forecast one period into the future. At the end of period 100, the forecast of period 101 is also the forecast for all subsequent periods. While this is as it should be (because that's the nature of the unpredictability of cycles), it highlights the difficulty of driving based on what's in the rearview mirror when the road is full of cycles.

Trend

We will now attempt to make a distinction between cycles and *trends*. We attempt this with full recognition that in other contexts and with other audiences this distinction is impossible. From the simulations of cyclic behavior you just conducted, however, you might have an appreciation for how a cycle is different from a trend.

If we look carefully at periods 11 through 36 of the exponential smoothing series in the chart on the previous page, we see that the series is (for the most part) increasing during this period. You and I know that this is an up cycle. We know this because we generated the series using exponential smoothing. We also know from our experience with exponential smoothing that what goes up does not necessarily continue to go up. That is the nature of cycles, and that is what distinguishes cycles from trends. Even though most of us would be willing to say the series *trended* up during the period, we know that it was simply an up cycle. We will reserve the word "trend" to refer to general increases (decreases) that are expected to continue into the future. All of the upward or downward trends in the exponential smoothing series are examples of cycles, not trends.

The distinction we make between cycles and trends is consistent with the Deardorff's Glossary of International Economics definition of an economic trend:

> The long-term movement of an economic variable, such as its average rate of increase or decrease, over enough years to encompass several business cycles.

This definition captures the idea that trends persist whereas cycles do not.

We admit from the outset that it is often difficult to distinguish trends from cycles based solely on the data. The debates about global warming are a case in point. The distinction matters a great deal, however, when one attempts to forecast the future. Models that account for trend produce forecasts that increase (decrease) throughout the forecast period. Models (such as exponential smoothing) that do not account for trend produce forecasts that are flat.

Holts: adding trend to exponential smoothing

Exponential smoothing does not do very well if there is a trend. If the series increases for an extended period of time, exponential smoothing consistently under-forecasts each new observation. Although this under-forecasting is entirely appropriate (and temporary) for a cycle, it is a mistake in the presence of trend. C. C. Holt (who invented exponential smoothing in 1957) recognized the opportunity to modify his model so that it would account for trend. In 1958 he introduced a revised model, which today is known as Holt's method.

The idea behind Holt's method is to include a trend component in each forecast:

$$\text{New Forecast} = \text{New Level} + \text{New Trend}.$$

We use the term "level" to refer to the starting value each period before we add the anticipated trend. Trend is the increase we expect next period. Holt decided to apply exponential smoothing separately to update each component. The smoothing parameter for the level component is the familiar α. The smoothing parameter for the trend component is … you may have already guessed it … β. The updating equations are

$$\text{New Level} = \alpha \times \text{Actual} + (1-\alpha) \times \text{ Old Forecast}$$

$$\text{New Trend} = \beta \times (\text{New Level} - \text{Old Level}) + (1-\beta) \times \text{ Old Trend}.$$

Because Holt includes a smoothed trend in his model, it can accommodate series for which the trend changes slowly over time. In practice, one does not expect economic trends (not to be confused with cycles) to change very rapidly. As a consequence, most applications of Holt's will require a small value of β. Do not, however, confuse a small β with a lack of trend. A small β means an unchanging trend.

As mentioned earlier, models that account for trend produce forecasts that increase (decrease) throughout the forecast period. The Holts forecast of the observation p periods into the future is

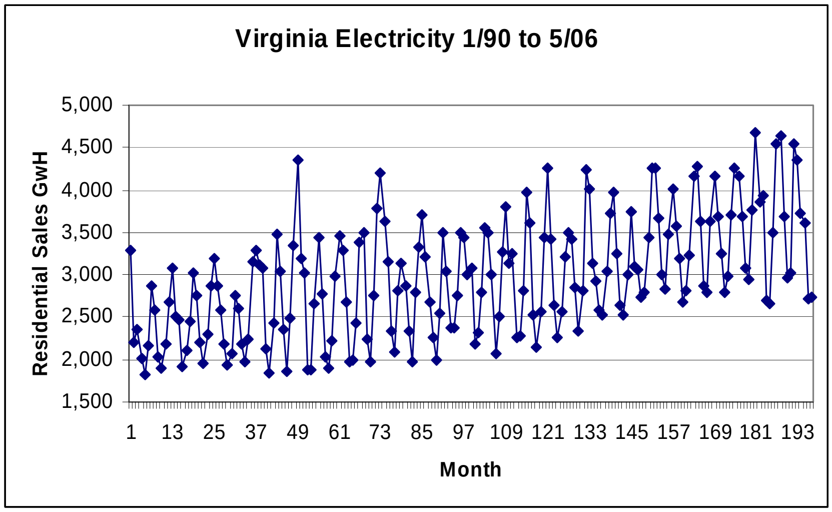
$$\text{Forecast } p \text{ Periods into Future} = \text{New Level} + p \times \text{New Trend}.$$

Even though Holts can produce forecasts p periods into the future, we must recognize that the uncertainty surrounding these point forecasts increases the further we extrapolate into the future.

Seasonality

In addition to cycles and trends, the third important kind of time series pattern is seasonality. Seasonality is a pattern of highs and lows that repeats every s periods. Parameter s is called the seasonal period. If the time series is monthly data, we would look for s to be 12. If the time series is daily data, we might expect *s* to be 7. The quick way to check for seasonality is to see if the peaks (valleys) in the series occur at regular intervals. If they do, that interval is the seasonal period.

Consider the time series of monthly residential sales of electricity in the Commonwealth of Virginia from January 1990 through May 2006.[3] The series exhibits a clear trend. (We call this a trend rather than an up cycle under the assumption that electricity sales will continue to increase into the future.) As expected, we also notice seasonality of period 12. If you look closely, the valleys and the peaks are exactly 12 months apart. The valleys are at months April/May and the peaks at months July/August and December/January.



Winter's: adding seasonality to Holts

Neither exponential smoothing nor Holt's can handle seasonality. C.C. Holt apparently ran out of steam, because he left it to P.R. Winters to figure out how to modify the Holt's method to account for seasonality. In 1960, Winters published the obvious solution. Use α to smooth the level component, use β to smooth a trend component, and use γ to smooth the seasonality components. For seasonality of period *s*, the model uses *s* seasonal indices, which are subject to exponential smoothing with parameter γ. As with the β parameter, we must be careful not to conclude that a low value of γ means a small seasonality component. Whereas a Holt's model forecast is the sum of the level and trend components, a Winters forecast starts with the sum of level and trend and then multiplies that sum by the seasonal index. Thus if the seasonal index for month 12 is 1.2, that means our December forecasts will be 20% above "average." Winters' method produces the most interesting forecasts in that they account for cycle, trend, and seasonality. With trend and seasonality present, the forecasts rise (or fall) into the future with a seasonal pattern reflecting the seasonality in the past time series.

[3] Source: Energy Information Administration, Department of Energy, U.S. Government, www.eia.doe.gov/cneaf/.

Exhibit 1 shows the results of applying Winters' method to the Virginia electricity time series. The software reports the best values of the three smoothing parameters,[4] the seasonal indices, and the initial trend. As expected, we find pronounced seasonality (the January index is 1.3 whereas both April and May have indices of 0.8) and a trend (ranging from 5 to 15 GwH per month). The trend and seasonality are relatively stable (both β and γ are close to 0.000) whereas the level has a smoothing constant of 0.6.

The resulting forecasts of the subsequent 24 months are shown in the first chart and in the table. Each forecast starts with the most recent smoothed level (3,506.11), increases at a rate of 13.34 GwH per month (the most recent, smoothed trend), and is multiplied by the appropriate seasonal index. For example, the forecast for January 2007 is[5]

$$\text{Forecast } p \text{ Periods into Future} = (\text{New Level} + p \times \text{New Trend}) \times \text{Seasonal Index}$$
$$\text{Forecast of January 2007} = (3{,}506.11 + 8 \times 13.34) \times 1.31$$
$$= 4742.89.$$

As is always the case with time series forecasts, the uncertainty surrounding point forecasts gets larger the farther into the future we forecast.

Deseasonalized data

A slightly simpler approach for dealing with data exhibiting seasonal patterns is to deseasonalize the data as a first step. Deseasonalizing the data means calculating a series of *s* seasonal indices and dividing each observation by the appropriate index. The resulting time series is said to have been deseasonalized. It may exhibit trends and cycles, but it should no longer show seasonality. A technique such as Holt's might then be applied to the deseasonalized time series with the resulting forecasts reseasonalized by multiplying by the appropriate seasonal index.

Even if one does not use a formal time series model to forecast the deseasonalized series, managers often find it useful to examine the deseasonalized data. The deseasonalized data show how past performances compare to the average for that month (or quarter or week).

Fitting Time Series Models

As always, there is an important distinction between fit and forecast. Fit refers to how well the model mimics the past data, and forecast refers to how well the model will forecast future data. Unfortunately, the model that fits best usually does not forecast the best if the past fit was achieved using an overly complicated model.

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[4] In actuality, the software did not optimize the three parameter values due to a flaw in the software. We used Excel's solver to accomplish the optimization (minimizing root mean squared error), and Palisades Corporation is trying to fix its software.

[5] The calculation of the 4,742.89 was accomplished using more precision than is illustrated here.

In regression analysis with normal errors, the standard error is the accepted measure of fit. The measure of time series fit that is closest to standard error is root mean squared error (RMSE). The RMSE is calculated as the square root of the average squared error. For the Winters model fit to the Virginia electricity data, the RMSE is calculated as

$$\begin{aligned}\text{RMSE} &= [(-590.85^2 + 108.18^2 + \ldots + -15.18^2)/196]^{1/2}\\ &= 249.20.\end{aligned}$$

The smoothing parameters were optimized so as to minimize RMSE. An approximate probability forecast for June 2006 would be that electricity sales are normally distributed with mean 3,377.10 GwH and standard deviation 249.20.[6]

If enough data are available, the modeler can use a holdout sample to guard against over-fitting. The trailing observations of the time series are set aside as a holdout sample. The remaining observations are called the training set. A time series model is selected and fit to the training set and used to forecast the observations in the holdout sample. (When applied to the forecast holdout sample, the model forecasts should be updated period by period using each new actual. But parameters should not be re-estimated.) If the RMSE for the model when used to forecast the holdout sample is larger than the RMSE of the model fit to the training set, the model has been over-fit and should not be used.

Diagnostic checking

Recall that when building regression models we pay attention to four assumptions. *Linearity* means we selected the correct model form. With time series, we also want to select the model that best describes the patterns evident in the time series. In this note we described three time series models appropriate for certain kinds of time series patterns.

Patterns in the Time Series	***Time Series Model***
None	Simple Average
Cycles	Exponential Smoothing
Cycles and Trend	Holt's
Cycles, Trend, and Seasonality	Winters'

Please know that there are time series models other than the three described in this note. For example, there is an entire family of time series models called ARIMA or Box-Jenkins models.[7] Exponential smoothing is one model in the ARIMA family.

[6] The RMSE understates the actual standard deviation of the prediction error because it does not account for uncertainty in parameter estimates, initial level and trend, or seasonal indices. Also, the RMSE applies only to one-step-ahead forecasts.

[7] The author wrote his doctoral dissertation on STARIMA models—an extension of ARIMA models into the spatial domain.

The assumptions of *normality* and *homoskedasticity* also apply to time series models. These two assumptions are checked in much the same way as they are checked for a regression model.

The last of the four assumptions is probably the most important in the context of time series analysis. *Independence* or lack of temporal autocorrelation means that the forecast errors from a time series model must contain no time series patterns. In other words, the time series model is acceptable only if its errors behave as if they came from a white noise process. This should make a lot of sense. Our job of modeling is complete only if what is left over after modeling is nothing but white noise.

In addition to examining the times series plot of the errors (the third chart in **Exhibit1**) to check the independence assumption, there are a couple of other tests we can apply. The first is called a *runs test*. It compares the actual number of runs (the number of times the time series of errors goes from negative to positive or vice versa) to the distribution of the number of runs expected from a white noise process. If the series is too smooth, it will have fewer than expected runs. If the series is too jagged (bouncing back and forth across the axes), it will have more runs than expected. The runs test applied to the Winters errors in **Exhibit 1** shows some evidence that the series is too smooth. We saw 89 runs and expected to see 99. The difference has a p-value of 0.15.

Runs Test for Randomness	**Errors**
Observations	196
Below Mean	98
Above Mean	98
Number of Runs	89
Mean	-8.81
E(R)	99.0000
StdDev(R)	6.9820
Z-Value	-1.4322
P-Value (two-tailed)	0.1521

A more sophisticated approach to checking independence is to examine the sample *autocorrelations* of the errors. As the name suggest, an autocorrelation is the correlation of a variable with itself. For time series, autocorrelations are the correlation coefficients of observations 1, 2, 3, … lags apart. One way to see what this means is to think about a scatter plot of Y(t+1) versus Y(t). If the Y's came from a white noise process, a regression line through this scatter plot should be flat. A flat regression line is equivalent to a first lag autocorrelation of zero. If the Y's show some kind of smooth cycle, the scatter plot will show a positive relationship reflecting the fact that Y(t+1) is correlated with Y(t). If there is a positive relationship between Y(t+1) to Y(t), the first lag autocorrelation will be positive.

If the errors came from a white noise process, we expect all autocorrelations (regardless of the lag) to be zero. If the sample autocorrelations are different from zero, we have evidence that the errors did not come from a white noise process. This would mean that the independence assumption is violated, and our time series model has not explained fully the time series patterns in the original time series.

Exhibit 2 shows the autocorrelations of the errors from the Winters model in Exhibit 1. Given the number of data points, the standard error of these sample autocorrelations is 0.0714. Our software listed in bold the autocorrelations more than two standard errors away from zero.

After examining these autocorrelations, we conclude that the model errors are not independent across time. In particular, they exhibit a seasonal pattern—perhaps with a seasonality of three months. (I will resist the temptation to call attention to the four seasons.[8]) What this means is that our series is more complicated (and interesting?) than the Winters method can handle. The Winters method did not explain adequately the seasonal pattern(s) in the original series. Our choices are to go ahead and use the imperfect Winters forecasts or search for a better time series model.

[8] In other words, I will refer to neither Vivaldi nor Valli.

Exhibit 1

TIME SERIES

Winters-Method Forecast of Virginia Electricity Sales

StatTools	(Core Analysis Pack)
Analysis:	Forecast
Performed By:	Darden Graduate Business School
Date:	Monday, September 11, 2006
Updating:	Live/Unlinked

Forecasting Constants (Optimized)	
Level (Alpha)	0.594
Trend (Beta)	-0.011
Season (Gamma)	-0.092

Winters Exponential	
Mean Abs Err	196.63
Root Mean Sq Err	249.20
Mean Abs Per% Err	6.77%

Forecast and Original Observations

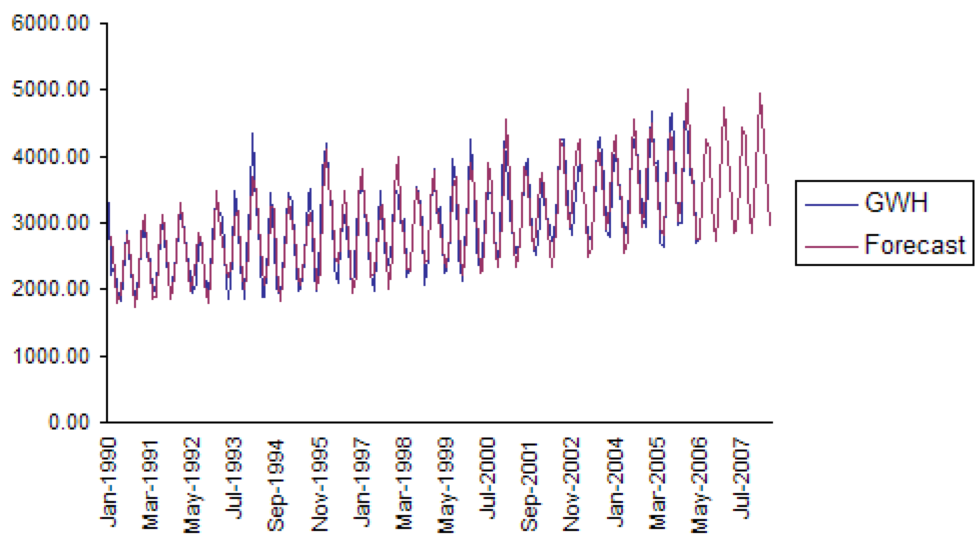
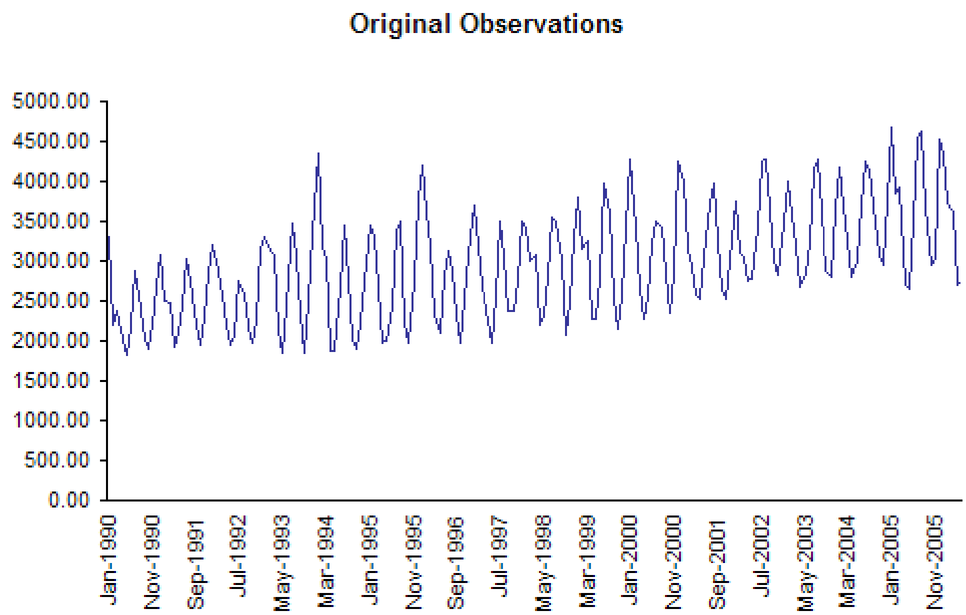


Exhibit 1 (continued)



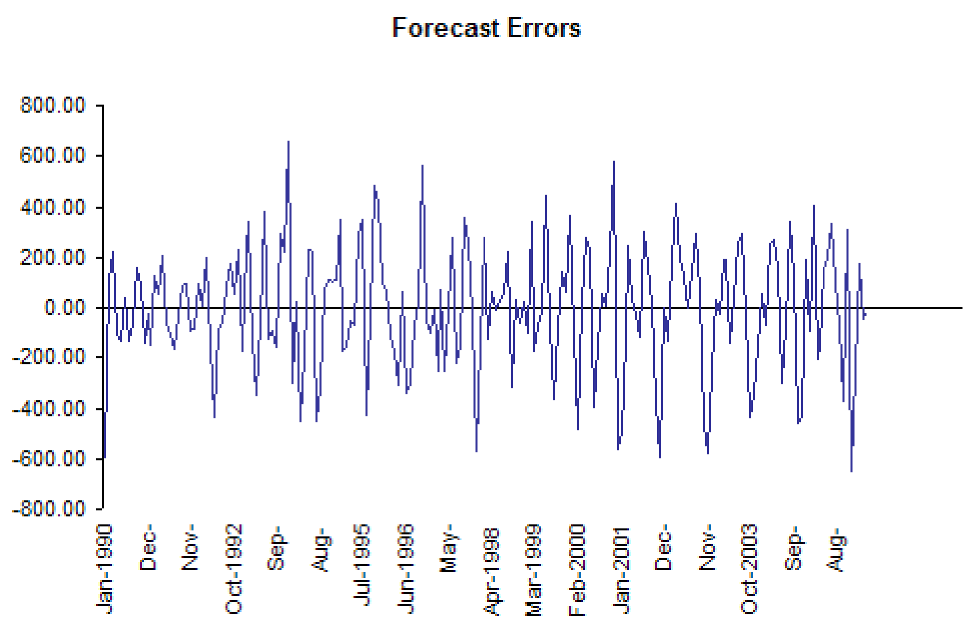


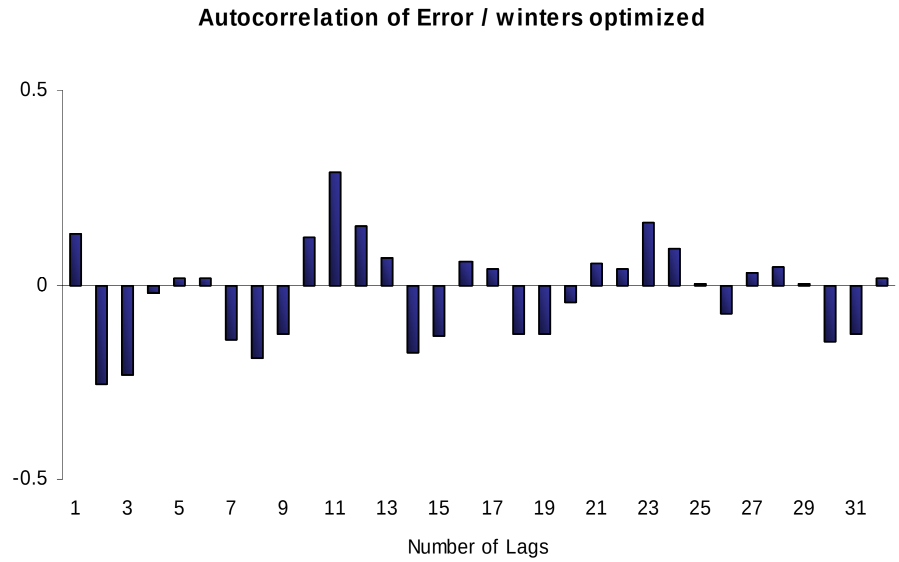
Exhibit 1 (continued)

Forecasting Data	GwH	Level	Trend	Season	Forecast	Error
Jan-1990	3,294.2730	2,536.55	5.08	1.30		
Feb-1990	2,208.6290	2,222.87	8.44	1.11	2,799.48	−590.85
Mar-1990	2,365.3900	2,294.86	7.77	1.01	2,257.21	108.18
Apr-1990	2,022.4180	2,472.96	5.98	0.78	1,798.54	223.87
May-1990	1,819.6310	2,400.77	6.80	0.78	1,921.59	−101.96
Jun-1990	2,169.3260	2,324.45	7.68	0.96	2,303.15	−133.82
Jul-1990	2,866.2060	2,350.84	7.48	1.21	2,828.02	38.18
Aug-1990	2,576.4990	2,287.93	8.22	1.15	2,712.76	−136.26
Sep-1990	2,036.9150	2,254.66	8.66	0.92	2,100.79	−63.88
Oct-1990	1,906.8450	2,384.61	7.38	0.77	1,749.11	157.74
Nov-1990	2,189.7420	2,451.88	6.75	0.88	2,101.22	88.52
Dec-1990	2,673.6800	2,383.69	7.54	1.15	2,818.22	−144.54
Jan-2006	4,354.5430	3,535.07	12.48	1.31	5,000.48	−645.94
Feb-2006	3,721.3600	3,413.57	13.89	1.12	3,973.92	−252.56
Mar-2006	3,619.3450	3,530.73	12.80	1.00	3,444.68	174.67
Apr-2006	2,709.2060	3,504.41	13.21	0.78	2,760.50	−51.30
May-2006	2,742.1200	3,506.11	13.34	0.78	2,757.30	−15.18
Jun-2006					3,377.10	
Jul-2006					4,257.55	
Aug-2006					4,111.70	
Sep-2006					3,277.20	
Oct-2006					2,741.43	
Nov-2006					3,122.34	
Dec-2006					4,147.85	
Jan-2007					4,742.89	
Feb-2007					4,072.00	
Mar-2007					3,651.01	
Apr-2007					2,847.63	
May-2007					2,874.32	
Jun-2007					3,530.66	
Jul-2007					4,450.41	
Aug-2007					4,297.25	
Sep-2007					3,424.54	
Oct-2007					2,864.22	
Nov-2007					3,261.68	
Dec-2007					4,332.27	
Jan-2008					4,952.98	
Feb-2008					4,251.71	
Mar-2008					3,811.55	
Apr-2008					2,972.39	
May-2008					2,999.79	

Exhibit 2

TIME SERIES

A Winters-Method Forecast of Virginia Electricity Sales



Autocorrelation Table	*Winters Errors*
Number of Values	196
Standard Error	0.0714
Lag #1	0.1318
Lag #2	**−0.2557**
Lag #3	**−0.2333**
Lag #4	−0.0238
Lag #5	0.0169
Lag #6	0.0157
Lag #7	−0.1410
Lag #8	**−0.1903**
Lag #9	−0.1253
Lag #10	0.1241
Lag #11	**0.2882**
Lag #12	**0.1517**
Lag #13	0.0687
Lag #14	**−0.1760**
Lag #15	−0.1339
Lag #16	0.0600
Lag #17	0.0388
Lag #18	−0.1275
Lag #19	−0.1290
Lag #20	−0.0437
Lag #21	0.0555
Lag #22	0.0386
Lag #23	**0.1612**
Lag #24	0.0917