

Определенный интеграл

$$1. \int_{-1}^{-2} \sqrt[3]{x} dx = x^{\frac{4}{3}} \cdot \frac{3}{4} \Big|_{-1}^{-2} = (2)^{\frac{4}{3}} \cdot \frac{3}{4} - (-1)^{\frac{4}{3}} \cdot \frac{3}{4}$$

$$2. \int_0^{\frac{\pi}{2}} \arcsin x dx \quad \ominus$$

$$u = \arcsin x \Rightarrow du = \frac{1}{\sqrt{1-x^2}} dx$$

$$dv = dx \Rightarrow v = x$$

$$\ominus x \arcsin x \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \frac{\pi}{12} + \frac{1}{2} \frac{\sqrt{1-x^2}}{\frac{1}{2}} \Big|_0^{\frac{\pi}{2}} \quad \ominus$$

$$\ominus \frac{\pi}{12} + \sqrt{\frac{3}{4}} - 1$$

$$3. \int_{e^2}^{e^3} \frac{dx}{x \ln x} = \int_{e^2}^{e^3} \frac{d(\ln x)}{\ln x} = \ln(\ln x) \Big|_{e^2}^{e^3} = \ln 3 - \ln 2 = \ln \frac{3}{2}$$

$$3.^* \int \frac{dx}{x \ln x} \quad dt = \frac{1}{x} dx$$
$$\int_{e^2}^{e^3} \frac{dx}{x \ln x} = \int_2^3 \frac{dt}{t} = \ln(t) \Big|_2^3 = \ln 3 - \ln 2 = \ln \frac{3}{2}$$

$$4. \int_0^1 x \sqrt{1+x} dx \quad \ominus$$

$$t = \sqrt{1+x}$$

$$dt = \frac{1}{2\sqrt{1+x}} dx$$

$$dx = 2\sqrt{1+x} dt$$

$$t^2 = 1+x$$

$$x = t^2 - 1$$

$$\ominus 2 \int_1^{\sqrt{2}} (t^2 - 1)(t^2) dt = 2 \left(\frac{t^5}{5} \Big|_1^{\sqrt{2}} - \frac{t^3}{3} \Big|_1^{\sqrt{2}} \right)$$

$$5. \int_{-a}^a \sqrt{a^2 - x^2} dx = \frac{\pi a^2}{2}$$

$$6. \int_1^e \sin(\ln x) dx \quad \textcircled{=}$$

$$\text{let } t = \ln x \quad dt = \frac{dx}{x} \quad d(\ln x) = dx \quad x = e^t$$

$$\textcircled{=} \int_0^1 e^t \sin t dt =$$

$$7. \int_{-a}^a x \sqrt{a^8 - x^8} dx = 0 \quad \text{T.K. } \text{ф-ue heretnal}$$

$$8. \int_0^{\pi} \frac{x \cdot \sin x}{1 + \cos^2 x} dx \quad \textcircled{=}$$

$$\int \frac{\sin x}{1 + \cos^2 x} \textcircled{=} , \quad t = \cos x \quad dt = -\sin x dx \quad dx = -\frac{dt}{\sin x}$$

$$\textcircled{=} \int \frac{dt}{1+t^2} = -\arctg(\cos x)$$

$$\textcircled{=} -\int_0^{\pi} x d(\arctg(\cos x)) = \underbrace{-\arctg(\cos x) \cdot x \Big|_0^{\pi}}_A + \underbrace{\int_0^{\pi} \arctg(\cos x) dx}_B =$$

$$\arctg \cos x = \varphi$$

$$\text{tg } \varphi = \cos x \quad \Leftrightarrow \quad \text{tg}^2 \varphi = \cos^2 x \quad \Leftrightarrow \quad \frac{1}{\cos^2 \varphi} - 1 = \cos^2 x$$

$$\text{let } x = \frac{\pi}{2} + t$$

$$B = \int_{-\pi/2}^{\pi/2} \arctg(\sin t) dt = 0$$

$$9. \int_{-1}^1 \frac{1+x^2}{1+x^4} dx$$

$$\int_{-1}^1 \frac{1+x^2}{1+x^4} dx = \int_{-1}^1 \frac{\frac{1}{x^2} + 1}{\frac{1}{x^2} + x^2} dx \quad \textcircled{=}$$

$$\left(1 + \frac{1}{x^2}\right) dx = d\left(x - \frac{1}{x}\right)$$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$Jt = x - \frac{1}{x}$$

$$\textcircled{=} \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}} \arctg \frac{x^2-1}{x\sqrt{2}} \Big|_{-1}^1 = 0 \quad \text{но ф-ла неопределенного интеграла}$$

невозможно найти дельта на отрезке, а при $x=0$ пропущены \Rightarrow ф-ла Коттона-Лейбница не работает

\Rightarrow разобьем

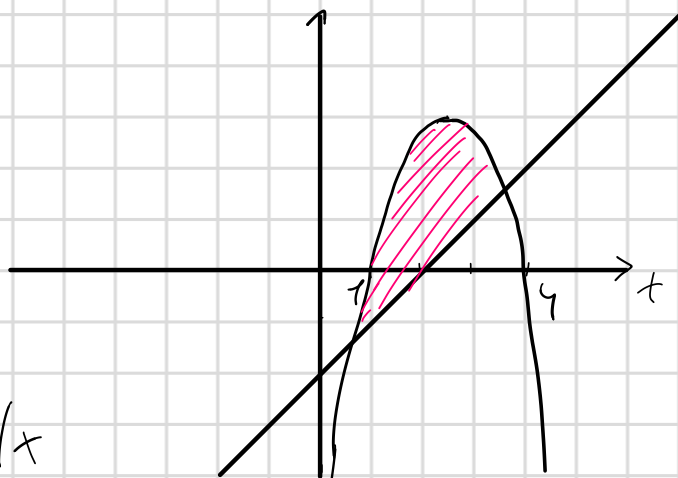
$$\int_{-1}^1 \frac{\frac{x^2+1}{x^2+x^2}}{x^2+x^2} dx = 2 \int_0^1 \frac{\frac{x^2+1}{x^2+x^2}}{x^2+x^2} dx = 2 \cdot \frac{1}{\sqrt{2}} \arctg \frac{x^2-1}{x\sqrt{2}} \Big|_0^1 = \sqrt{2} \left(0 - \left(-\frac{\pi}{2}\right)\right) = \frac{\pi}{\sqrt{2}}$$

10. Площадь?

$$y = -x^2 + 6x - 6$$

$$y = x - 2$$

$$S = \int_1^4 (6x - x^2 - 6 - x + 2) dx$$



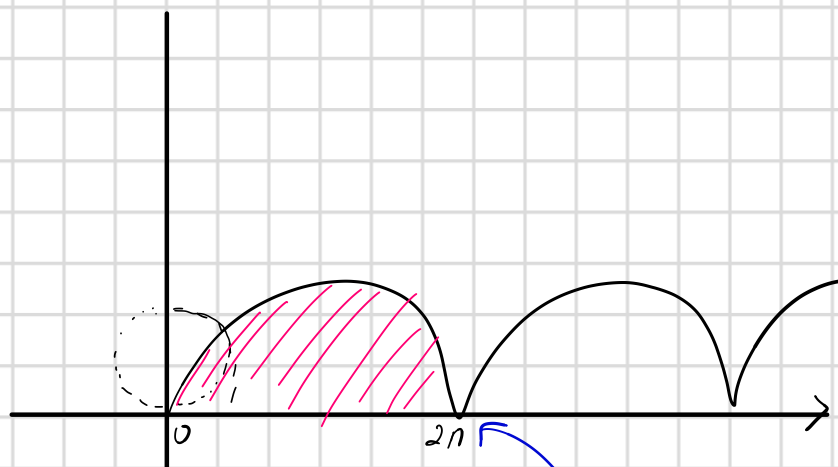
11. Площадь одной арки

$$\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$$

$$y = 0$$

$$S = \int_{x_1}^{x_2} y(t) dx = \int_0^{2\pi} y(t) \cdot x'(t) dt \textcircled{=}$$

$$\textcircled{=} a^2 \int_0^{2\pi} (1 - \cos t)^2 dt = a^2 \int_0^{2\pi} \left(1 - 2\cos t + \frac{1 + \cos 2t}{2}\right) dt \textcircled{=}$$



указания

$t = 2\pi$ т.е. $y = 0$

$$\Rightarrow 3\pi a^2$$

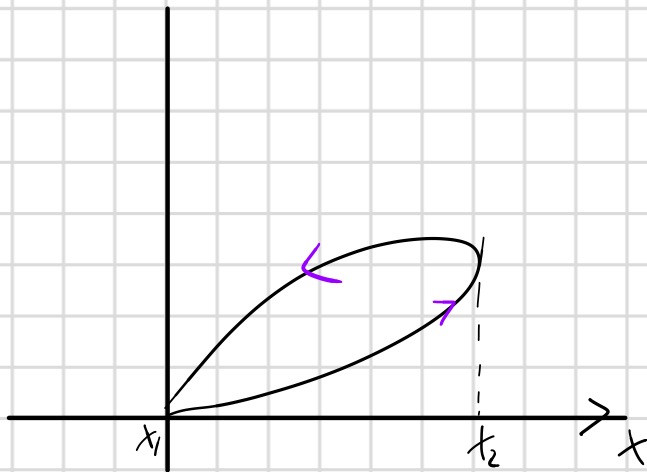
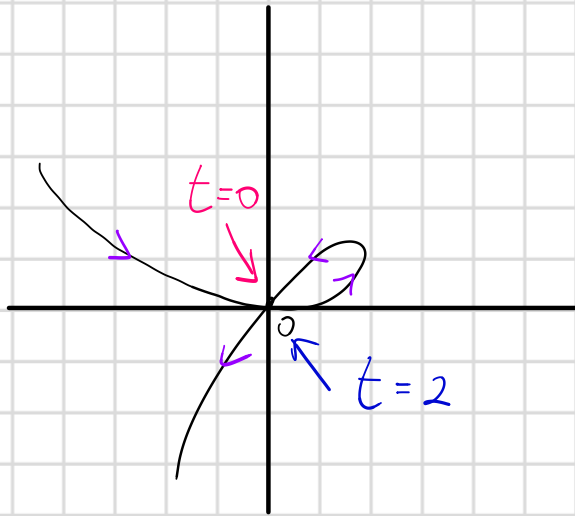
$$12. \begin{cases} x = 2t - t^2 \\ y = 2t^2 - t^3 \end{cases}$$

найдем периметр

$$\begin{cases} x = t(2-t) \\ y = t^2(2-t) \end{cases}$$

$$S = \left| \int_0^2 y \cdot x' dt \right|$$

$$\int_{x_1}^{x_2} y dx$$



$$13. \rho(\varphi) = \cos 3\varphi$$

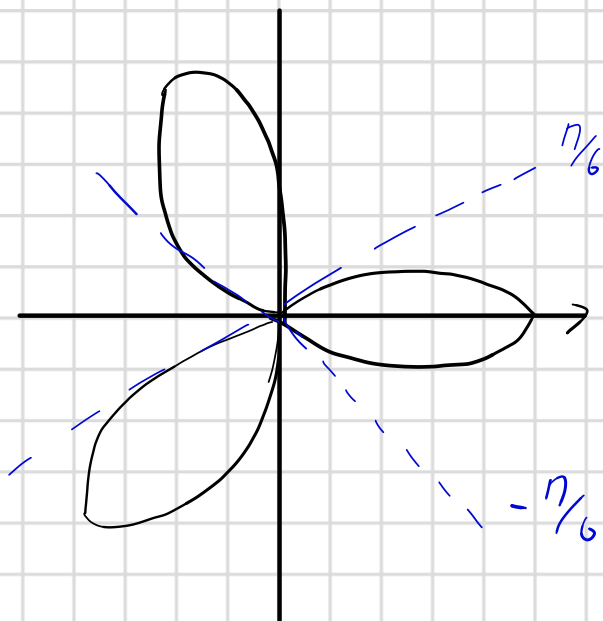
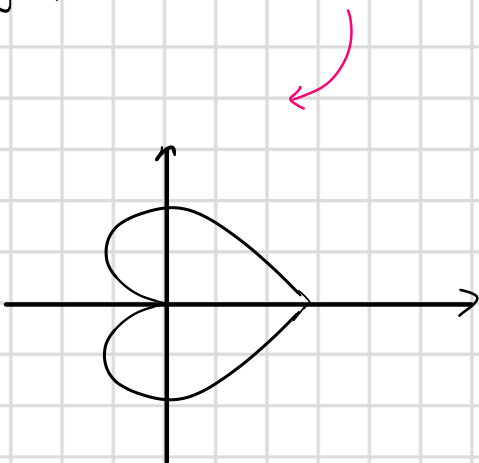
$$\cos 3\varphi = 0$$

$$3\varphi = \frac{\pi}{2}$$

$$\varphi = \frac{\pi}{6}$$

Понесите в Desmos: $r = \cos(\theta) + 1$

$$\rho(\varphi) = 1 + \cos \varphi$$



$$\int = 3 \cdot \frac{1}{2} \int_{-\pi/6}^{\pi/6} \rho^2(\varphi) d\varphi = \frac{3}{2} \int_{-\pi/6}^{\pi/6} \cos^2 3\varphi d\varphi = \frac{3}{4} \int (1 + \cos 6\varphi) d\varphi =$$

$$= \frac{3}{4} \cdot \frac{\pi}{3} = \frac{\pi}{4}$$