

Сходимость интегралов

$$\lim_{b \rightarrow \infty} \int_a^b f(t) dt - \text{попытка}$$

$$\left(\int_a^x f(t) dt \right)' = f(x) \quad \text{7. Барроу}$$

$$1. \int_1^{\infty} \frac{\sin^2 x}{x^4 + 1} dx$$

$$|f(x)| \leq \frac{1}{x^4 + 1} \sim \frac{1}{x^4}$$

$$\int_1^{\infty} \frac{dx}{x^4} \quad Cx - a \Rightarrow \text{исходимость } Cx - a$$

$$2. \int_0^{\infty} \frac{\sin^2 x}{x^2} dx$$

$$\text{Точка } x=0 \text{ не описана т.к. } \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = 1$$

$$\frac{\sin^2 x}{x^2} \leq \frac{1}{x^2} \quad Cx - a$$

$$3. \int_0^{\infty} \sqrt{\frac{16+x^4}{16-x^4}} dx$$

$$\sqrt{\frac{16+x^4}{16-x^4}} = \frac{1}{\sqrt{2-x^2}} \cdot \sqrt{\frac{16+x^4}{(2+x)(4+x^2)}} \sim \frac{C}{\sqrt{2-x^2}}$$

$$\int_0^{\infty} \frac{1}{\sqrt{2-x^2}} dx = - \int_{\frac{\pi}{2}}^0 \frac{dt}{\sqrt{t}} \quad Cx - a$$

$$4. \int_0^{\infty} \frac{dx}{x^2 + \sqrt[3]{x}}$$

$$\frac{1}{x^2 + \sqrt[3]{x}} \sim \frac{1}{\sqrt[3]{x}} \quad Cx - a$$

$$5. \int_1^{\infty} \frac{dx}{x^2 + \sqrt[3]{x}} \quad \frac{1}{x^2 + \sqrt[3]{x}} \sim \frac{1}{x^2} \quad Cx - a$$

$$6. \int_{-1}^1 \frac{dx}{\ln(x+1)} = \int_{-1}^{-1/2} + \int_{-1/2}^0 + \int_0^1 \quad x \rightarrow 0: \frac{1}{\ln(x+1)} \sim \frac{1}{x} \quad \text{расх-а}$$

$$x \rightarrow -1: \quad \frac{1}{\ln t} \xrightarrow[t \rightarrow 0]{t = x+1} 0$$

7. $\int_2^{\infty} \frac{dx}{x^2 (\ln x)^B}$

$L=1: I = \int_2^{\infty} \frac{d \ln x}{(\ln x)^B} = \int_{\ln 2}^{\infty} \frac{dt}{t^B}$ $B > 1$ $Cx-u$
 $B \leq 1$ $pacx-cl$

$L > 1: \frac{1}{x^2 (\ln x)^B} \leq \frac{1}{x^2}$ $Cx-u$

$B \geq 0: \ln x \uparrow \Rightarrow (\ln x)^B \uparrow$

$L > 1$ u $B < 0: \frac{(\ln x)^{\gamma}}{x^2} \leq \frac{x^{2-1-\epsilon}}{x^2} = \frac{1}{x^{1+\epsilon}}$ $\epsilon = \frac{L-1}{2}$ $Cx-u$
 $\gamma = -B > 0$

$\ln x < x^{\delta} \quad \forall \delta > 0$
 non domnuat x

$L < 1: \frac{1}{x^2 (\ln x)^B} > \frac{1}{x^{1+\epsilon}}$ $pacx-cl$
 $B > 0$
 $(\ln x)^B < x^{1-L-\epsilon}$

$B \leq 0: \frac{1}{x^2 (\ln x)^B} \geq \frac{1}{x^2}$ $pacx-cl$

8. $\int_0^{\pi} \frac{\operatorname{sh} x}{\exp x^2 \cos x} dx$

$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$

$x \rightarrow 0: \operatorname{sh} x \sim x$

$\operatorname{sh} x = \frac{1}{2} (e^x - e^{-x}) = \frac{1}{2} (1+x+x^2+o(x) - 1+x+o(1)) = x+o(x)$

$f(x) = \frac{x+o(x)}{1+x^2+o(x) - (1-\frac{x^2}{2}+o(x^2))} = \frac{x+o(x)}{\frac{3}{2}x^2+o(x^2)} \sim \frac{2}{3} \cdot \frac{1}{x}$

9. $\int_1^{\infty} \frac{\cos x}{\ln x} dx$

$x \rightarrow 1: \frac{\cos x}{\ln x} \sim \frac{c}{x-1}$ $pacx-cl$

$x \rightarrow \infty: f(x)$ never $\geq \text{nan}$!



~~$$\frac{|\cos x|}{\ln x} \leq \frac{1}{\ln x} \text{ pacr}$$~~

np. fupax ne: $\int_1^w \cos x dx - \text{op}$

$$\frac{1}{\ln x} \downarrow \quad \text{и} \quad x \rightarrow \infty \rightarrow 0$$

$$\Rightarrow \int \text{cx-ee} \text{ как?}$$

ad c: $\frac{|\cos x|}{\ln x} \geq \frac{\cos^2 x}{\ln x} = \frac{1 + \cos 2x}{2 \ln x} = \underbrace{\frac{1}{2 \ln x}}_{\text{pacr-ee}} + \underbrace{\frac{\cos 2x}{2 \ln x}}_{\text{ct-ee}} \Rightarrow \text{pacr-ee}$

cx-ee $F(w) = \int_1^w \cos 2x dx \leq C$

$$\frac{1}{2 \ln x} \downarrow \quad \text{и} \quad x \rightarrow \infty \rightarrow 0$$

$$\Rightarrow \text{cx-ee y chelno}$$

10. $\int_0^\infty \sin(x^2) dx \odot$

$$x^2 = t$$

$$2x dx = dt$$

$$\odot \int_0^\infty \frac{\sin t}{t} dt \quad \text{ct-ee}$$

$\frac{1}{t}$ uchox и $\rightarrow 0$
 $\int_0^\infty \sin t dt \text{ op.}$

$$\frac{|\sin t|}{t} \rightarrow \frac{\sin^2 t}{t} = \frac{1 - \cos 2t}{2t} = \underbrace{\frac{1}{2t}}_{\text{pacr-ee}} - \underbrace{\frac{\cos 2t}{2t}}_{\text{ct-ee}}$$

$$\Rightarrow \text{cx-ee y chelno}$$

2-oi chod

$$\int_1^\infty \sin x^2 dx = \int_1^\infty \underbrace{\left(\frac{1}{2x}\right)}_{\text{uchox} \rightarrow 0} \underbrace{2x \sin x^2 dx}_{f' = -\cos x^2 / w - \text{op}}$$

11. $\int_0^\infty \frac{\sin x \arctan x}{x} dx \Rightarrow \text{ct-ee}$

np. Abene: $\int_0^\infty f dx \text{ cx-ee и } f \text{ uchox и op}$

$$\frac{\sin x}{x} \cdot \arctan x \underset{x \rightarrow \infty}{\sim} \frac{\pi}{2} \cdot \frac{\ln x}{x} \quad \text{pact-u}$$

$$12 \int_0^1 \frac{\cos^3(\ln x)}{x \ln x} dx = \int_{\infty}^0 \frac{\cos^3 t}{t} dt = - \int_0^{\infty} \frac{\cos^3 t}{t} dt$$

$$t = -\ln x$$

$$t \rightarrow 0: \frac{\cos^3 t}{t} \sim \frac{1}{t} \quad \text{pact-u}$$

$$t \rightarrow \infty: \int \cos^3 t dt = \int (t - \sin^2 t) d \sin t = \sin t - \frac{\sin^3 t}{3} \Big|_{\infty}^{\infty} \quad \text{or}$$

$$\frac{1}{t} \downarrow \rightarrow 0$$