

Было:

$$F_{\mathcal{G}}(a) = P(\mathcal{G} \leq a)$$

$$f_{\mathcal{G}}(a) = P(\mathcal{G} = a)$$

$$F_{\mathcal{G}}(a) = \sum_{\mathcal{G} \leq a} f_{\mathcal{G}}(a)$$

$$E_{\mathcal{G}} = \sum_{\omega \in \Omega} p(\omega) \mathcal{G}(\omega) = \sum_a a \cdot P(\mathcal{G} = a)$$

$$E(\mathcal{G} + \mathcal{H}) = E(\mathcal{G}) + E(\mathcal{H})$$

$$E(\mathcal{G} - E_{\mathcal{G}}) = E_{\mathcal{G}} - \overbrace{E E_{\mathcal{G}}}^{E_{\mathcal{G}}} = 0$$

$$\mathcal{G} \mapsto \mathcal{H} = \mathcal{G} - E_{\mathcal{G}}$$

$$D_{\mathcal{G}} = E(\mathcal{G} - E_{\mathcal{G}})^2 = E_{\mathcal{G}^2} - (E_{\mathcal{G}})^2$$

$$\begin{aligned} D(\mathcal{G} + \mathcal{H}) &= E(\mathcal{G} + \mathcal{H})^2 - (E(\mathcal{G} + \mathcal{H}))^2 = \underline{E_{\mathcal{G}^2}} + \underline{E_{\mathcal{H}^2}} + 2E_{\mathcal{G} \cdot \mathcal{H}} - \underline{(E_{\mathcal{G}})^2} - \underline{(E_{\mathcal{H}})^2} - 2E_{\mathcal{G}}E_{\mathcal{H}} = \\ &= D_{\mathcal{G}} + D_{\mathcal{H}} + 2(E_{\mathcal{G}\mathcal{H}} - E_{\mathcal{G}}E_{\mathcal{H}}) \end{aligned}$$

$$\text{Cov}(\mathcal{G}, \mathcal{H}) = E_{\mathcal{G}\mathcal{H}} - E_{\mathcal{G}}E_{\mathcal{H}} \leftarrow \text{ковариация}$$

$$D_{\mathcal{G}} = \text{Cov}(\mathcal{G}, \mathcal{G})$$

$$\text{Corr}(\mathcal{G}, \mathcal{H}) = \frac{\text{Cov}(\mathcal{G}, \mathcal{H})}{\sqrt{D_{\mathcal{G}} D_{\mathcal{H}}}} \leftarrow \text{корреляция}$$

Теорема:

$$-1 \leq \text{Corr}(\mathcal{G}, \mathcal{H}) \leq 1$$

$$\triangleright \triangleleft \lambda = \mathcal{G} - \lambda \mathcal{H}$$

$$D_{\lambda} = E_{\mathcal{G}^2} - 2\lambda E_{\mathcal{G}\mathcal{H}} + \lambda^2 E_{\mathcal{H}^2} - (E_{\mathcal{G}})^2 + 2\lambda E_{\mathcal{G}}E_{\mathcal{H}} - \lambda^2 (E_{\mathcal{H}})^2 \geq 0$$

$$D_\lambda = E_\eta + 2\lambda \text{Cov}(\eta, \epsilon) + \lambda^2 D_\epsilon \geq 0$$

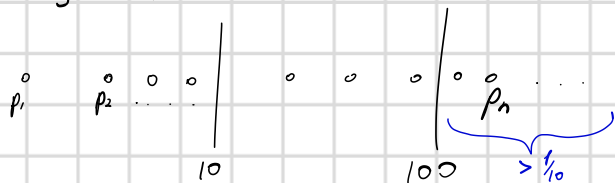
$$4 \text{Cov}(\eta, \epsilon)^2 - 4 D_\eta D_\epsilon \leq 0 \quad \eta = \lambda \epsilon + C \quad \blacktriangleleft$$

Хвостовые неравенства

Пр:

$$\exists \eta, E_\eta = 10, \eta \geq 0$$

$$P(\eta \geq 100) < \frac{1}{2} \quad ?$$



Теорема (Чер-во Маркова)

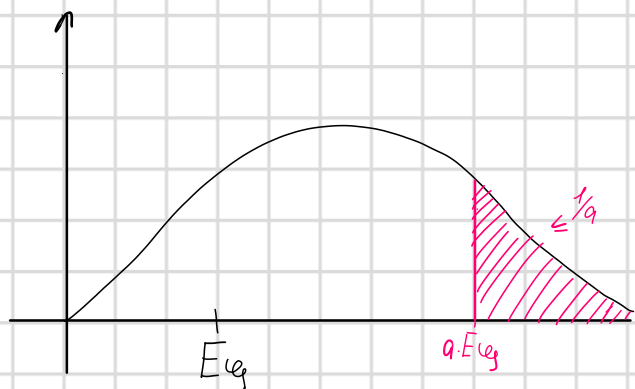
$$\exists \eta \geq 0, \eta \neq 0$$

$$P(\eta \geq a \cdot E_\eta) \leq \frac{1}{a}$$

$$\triangleright E_\eta = \sum_v v \cdot P(\eta = v) = \sum_{v < a \cdot E_\eta} v \cdot P(\eta = v) + \sum_{v \geq a \cdot E_\eta} v \cdot P(\eta = v) \quad \textcircled{\geq}$$

$$\geq a \cdot E_\eta \cdot \sum_{v \geq a \cdot E_\eta} P(\eta = v)$$

$$\textcircled{\geq} a \cdot E_\eta \cdot P(\eta \geq a \cdot E_\eta) \quad \blacktriangleleft$$



$$\text{Есть } a = \frac{c}{E_\eta} \rightarrow P(\eta \geq c) \leq \frac{E_\eta}{c}$$

$$D_\eta = E(\eta - E_\eta)^2$$

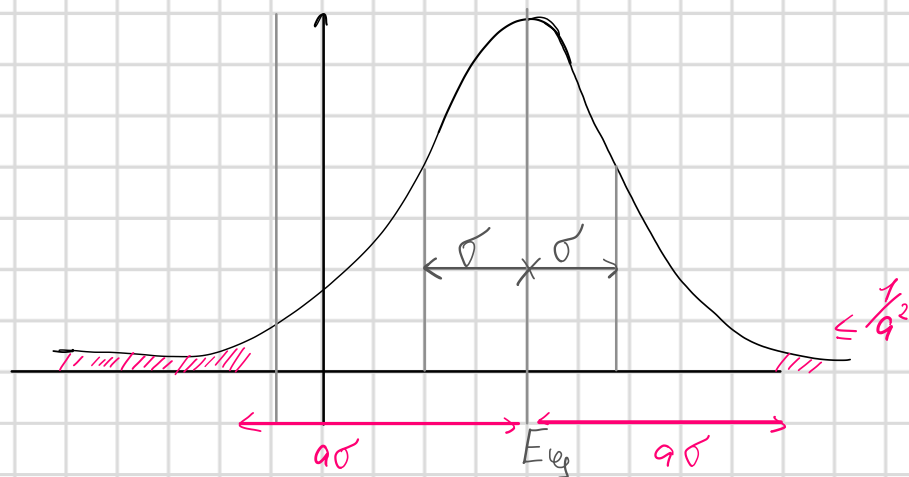
$$\eta = (\eta - E_\eta)^2$$

$$P((\xi - E\xi)^2 \geq a \cdot D\xi) \leq \frac{1}{a^2}$$

Теорема (Пер-во Чебышева)

$$P(|\xi - E\xi| \geq a\sigma) \leq \frac{1}{a^2} \leftarrow \text{среднеквадратичное отклонение}$$

$$P(|\xi - E\xi| \geq c) \leq \frac{D\xi}{c^2}$$



Пр:

10 монет, # единиц ξ

$$E\xi = 5$$

$$D\xi = 2,5$$

$$P(\xi \leq 0) \leq P(|\xi - E\xi| \geq 5) \leq \frac{2,5}{25} = \frac{1}{10}$$

Пр:

небросная монета, $p \neq \frac{1}{2}$

Вопрос: $p > \frac{1}{2}$ или $p < \frac{1}{2}$?

С единиц, и n -с нулей

$$\exists c < \frac{n}{2}, p > \frac{1}{2} \Rightarrow pn > \frac{n}{2}$$

$$P(\xi = c) \leq P(\xi \leq c) \leq P(|\xi - pn| \geq pn - c) \leq P(|\xi - pn| \geq \frac{n}{2} - c) \leq \frac{h}{4(\frac{n}{2} - c)^2}$$

$$\exists \xi_i, P(\xi_i=1)=p, P(\xi_i=0)=q$$

$$\text{и } \xi = \sum_{i=1}^n \xi_i, E\xi = n \cdot p = \mu, \text{ тогда}$$

Теорема (Лемма Чернова)

$$P(|\xi - \mu| \geq \delta \mu) \leq e^{-\mu \frac{\delta^2}{3}}$$