

Неопределенный интеграл

$$1. \int \left(\frac{1}{x} + x^2 \ln 5 - \frac{4}{\sqrt{x}} + \frac{1}{3\sqrt[3]{x^4}} + \frac{7}{\sqrt{1-x^2}} \right) dx = \int \frac{1}{x} dx + \ln 5 \int x^2 dx + 4 \int \frac{1}{\sqrt{x}} dx + \frac{1}{3} \int \frac{1}{\sqrt[3]{x^4}} dx + 7 \int \frac{1}{\sqrt{1-x^2}} dx = \ln x + \ln 5 \cdot \frac{x^3}{3} + 4 \cdot \frac{\sqrt{x}}{1/2} + \frac{1}{3} \cdot \frac{x^{-1/3}}{-1/3} + 7 \cdot \arcsin x + C \quad \checkmark$$

$$2. \int x(1-2x)^3 dx = \int x(-8x^3 + 12x^2 - 6x + 1) dx = \int (-8x^4 + 12x^3 - 6x^2 + x) dx = -8 \int x^4 dx + 12 \int x^3 dx - 6 \int x^2 dx + \int x dx = -8 \cdot \frac{x^5}{5} + 12 \cdot \frac{x^4}{4} - 6 \cdot \frac{x^3}{3} + \frac{x^2}{2} + C \quad \checkmark$$

$$3. \int \frac{2x^2 + 3x^{1/2} - 1}{2x} dx = \int \frac{2x^2}{2x} dx + \int \frac{3x^{1/2}}{2x} dx - \int \frac{1}{2x} dx = \int x dx + \int \frac{3}{2} \frac{1}{\sqrt{x}} dx - \frac{1}{2} \int \frac{1}{x} dx = \frac{x^2}{2} + \frac{3}{2} \cdot \frac{\sqrt{x}}{1/2} - \frac{1}{2} \cdot \ln|x| \quad \checkmark$$

$$4. \int \cos \frac{x}{2} dx = \int \cos \frac{x}{2} d\left(\frac{x}{2}\right) = 2 \cdot \sin \frac{x}{2} + C \quad \checkmark$$

$$\int \cos x dx = \sin x + C$$

$$\left(\frac{x}{2}\right)' = \frac{x' \cdot 2 - 2' \cdot x}{4} = \frac{1}{2}$$

$$5. \int 4^{5x} dx = \int 4^{5x} d(5x) = \frac{1}{5} \cdot \frac{4^{5x}}{\ln 4} + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$6. \int \sqrt[5]{(1+x)^4} dx = \int \sqrt[5]{t^4} dt = \sqrt[5]{t^3} \cdot \frac{5}{9} + C = \sqrt[5]{(1+x)^3} \cdot \frac{5}{9} + C \quad \checkmark$$

$$t = 1+x$$

$$dt = d(1+x) = (1+x)' dx = dx$$

$$7. \int x(1-x)^5 dx = \int (1-t)t^5 (-dt) = \int -t^5 + t^6 dt = -\int t^5 dt + \int t^6 dt = -\frac{t^6}{6} + \frac{t^7}{7} + C \quad \ominus$$

$$\int t = 1-x$$

$$\Rightarrow x = 1-t$$

$$\ominus -\frac{(1-x)^6}{6} + \frac{(1-x)^7}{7} + C \quad \checkmark$$

$$dt = d(1-x) = (1-x)' dx = -dx$$

$$8. \int \frac{x dx}{\sqrt{9-8x^2}} \quad \ominus$$

$$\int t = 9-8x^2 \quad -8x^2 = t-9 \quad x^2 = \frac{t-9}{-8} \quad \sqrt{\frac{t-9}{-8}}$$

$$dt = d(8-8x^2) = (8-8x^2)' dx = -16x dx$$

$$dx = \frac{dt}{-16x}$$

$$\Leftrightarrow \int \frac{\frac{dt}{-16}}{\sqrt{t}} = \int -\frac{1}{16} \cdot \frac{1}{\sqrt{t}} dt = -\frac{1}{16} \int \frac{1}{\sqrt{t}} dt = -\frac{1}{16} \cdot \frac{\sqrt{t}}{\frac{1}{2}} + C = -\frac{1}{8} \cdot \sqrt{8-8x^2} + C \quad \checkmark$$

$$9. \int e^{2x^3-1} \cdot x^2 dx = \int e^t \cdot \frac{dt}{6} = \frac{1}{6} e^{2x^3-1} + C \quad \checkmark$$

$$t = 2x^3 - 1$$

$$dt = d(2x^3-1) = (2x^3-1)' dx = 6x^2 dx$$

$$dx = \frac{dt}{6}$$

$$10. \int \frac{\sqrt[3]{\ln(3x+1)}}{3x+1} dx = \int \frac{\sqrt[3]{t}}{3x+1} \cdot \frac{3x+1}{3} dt = \int \frac{\sqrt[3]{t}}{3} dt = \frac{1}{3} \cdot \frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C \Leftrightarrow$$

$$t = \ln(3x+1)$$

$$dt = d(\ln(3x+1)) = (\ln(3x+1))' dx = \frac{3}{3x+1} dx \Rightarrow dx = \frac{3x+1}{3} dt$$

$$\Leftrightarrow t^{\frac{4}{3}} \cdot \frac{1}{4} + C = \sqrt[3]{\ln^4(3x+1)} \cdot \frac{1}{4} + C \quad \checkmark$$