Неопределенный интеграл

1. $\int\left(\frac{1}{x}+x^2\ln 5-\frac{4}{\sqrt{x}}+\frac{1}{3\sqrt[3]{x^4}}+\frac{7}{\sqrt{1-x^2}}\right)dx=\int\frac{1}{x}dx+\ln 5\int x^2dx+4\cdot\int\frac{1}{\sqrt{x}}dx+$
$+\frac{1}{3}\int\frac{1}{\sqrt[3]{x^4}}dx+7\cdot\int\frac{1}{\sqrt{1-x^2}}dx=\ln x+\ln 5\cdot\frac{x^3}{3}+4\cdot\frac{\sqrt{x}}{1/2}+\frac{1}{3}\cdot\frac{x^{-1/3}}{-1/3}+7\cdot\arcsin x+C$ ✓

2. $\int x(1-2x)^3dx=\int x\left(-8x^3+12x^2-6x+1\right)dx=\int\left(-8x^4+12x^3-6x^2+x\right)dx=$
$=-8\int x^4dx+12\int x^3dx-6\int x^2dx+\int x\,dx=-8\cdot\frac{x^5}{5}+12\cdot\frac{x^4}{4}-6\frac{x^3}{3}+\frac{x^2}{2}+C$ ✓

3. $\int\frac{2x^2+3x^{1/2}-1}{2x}dx=\int\frac{2x^2}{2x}dx+\int\frac{3x^{1/2}}{2x}dx-\int\frac{1}{2x}dx=\int x\,dx+\int\frac{3}{2}\frac{1}{\sqrt{x}}dx-\frac{1}{2}\int\frac{1}{x}dx=$

$=\frac{x^2}{2}+\frac{3}{2}\cdot\frac{\sqrt{x}}{1/2}-\frac{1}{2}\cdot\ln|x|$ ✓

4. $\int\cos\frac{x}{2}dx=\int\cos\frac{x}{2}d\left(\frac{x}{2}\right)=2\cdot\sin\frac{x}{2}+C$ ✓

$\int\cos x\,dx=\sin x+C$

$\left(\frac{x}{2}\right)'=\frac{x'\cdot 2-2'\cdot x}{4}=\frac{1}{2}$

5. $\int 4^{5x}dx=\int 4^{5x}d(5x)=\frac{1}{5}\cdot\frac{4^{5x}}{\ln 4}+C$

$\int a^xdx=\frac{a^x}{\ln a}+C$

6. $\int\sqrt[5]{(1+x)^4}\,dx=\int\sqrt[5]{t^4}\,dt=\sqrt[5]{t^9}\cdot\frac{5}{9}+C=\sqrt[5]{(1+x)^9}\cdot\frac{5}{9}+C$ ✓

$t=1+x$

$dt=d(1+x)=(1+x)'dx=dx$

7. $\int x(1-x)^5dx=\int(1-t)t^5(-dt)=\int -t^5+t^6dt=\int t^5dt+\int t^6dt=-\frac{t^6}{6}+\frac{t^7}{7}+C\ominus$

] $t=1-x$
$\Rightarrow x=1-t$

$\ominus-\frac{(1-x)^6}{6}+\frac{(1-x)^7}{7}+C$ ✓

$dt=d(1-x)=(1-x)'dx=-dx$

8. $\int\frac{x\,dx}{\sqrt{9-8x^2}}\ominus$

] $t=9-8x^2$ $\quad -8x^2=t-9$ $\quad x^2=\frac{t+9}{8}$ $\quad \sqrt{\frac{t+9}{8}}$

$$dt = d(3-8x^2) = (3-8x^2)'dx = -16x\,dx$$

$$dx = \frac{dt}{-16x}$$

$$= \int \frac{dt/_{-16}}{\sqrt{t}} = \int -\frac{1}{16} \cdot \frac{1}{\sqrt{t}}\,dt = -\frac{1}{16} \int \frac{1}{\sqrt{t}}\,dt = -\frac{1}{16} \cdot \frac{\sqrt{t}}{1/2} + C = -\frac{1}{8}\sqrt{(3-8x^2)} + C \checkmark$$

9. $\int e^{2x^3-1} \cdot x^2\,dx = \int e^t \cdot \frac{dt}{6} = \frac{1}{6} e^{2x^3-1} + C \checkmark$

$\exists\, t = 2x^3 - 1$

$dt = d(2x^3-1) = (2x^3-1)'dx = 6x^2 dx$

$dx = \frac{dt}{6}$

10. $\int \frac{\sqrt[3]{\ln(3x+1)}}{3x+1}\,dx = \int \frac{\sqrt[3]{t}}{3x+1} \cdot \frac{3x+1}{3}\,dt = \int \frac{\sqrt[3]{t}}{3}\,dt = \frac{1}{3} \cdot \frac{t^{4/3}}{4/3} + C =$

$\exists\, t = \ln(3x+1)$

$dt = d(\ln(3x+1)) = (\ln(3x+1))'\,dx = \frac{3}{3x+1}\,dx \Rightarrow dx = \frac{3x+1}{3}\,dt$

$= t^{4/3} \cdot \frac{1}{4} + C = \sqrt[3]{\ln^4(3x+1)} \cdot \frac{1}{4} + C \checkmark$