Определенный интеграл

1. $\int_{-1}^{-2} \sqrt[3]{x}\,dx = x^{4/3} \cdot \frac{3}{4} \Big|_{-1}^{2} = (2)^{4/3} \cdot \frac{3}{4} - (-1)^{4/3} \cdot \frac{3}{4}$

2. $\int_{0}^{1/2} \arcsin x\,dx \ominus$

$u = \arcsin x \Rightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx$

$dv = dx \Rightarrow v = x$

$\ominus\ x\arcsin x \Big|_0^{1/2} + \frac{1}{2}\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,d(1-x^2) = \frac{\pi}{12} + \frac{1}{2}\,\frac{\sqrt{1-x^2}}{1/2}\Big|_0^{1/2} \ominus$

$\ominus\ \frac{\pi}{12} + \sqrt{\frac{3}{4}} - 1$

3. $\int_{e^2}^{e^3} \frac{dx}{x\ln x} = \int_{e^2}^{e^3} \frac{d(\ln x)}{\ln x} = \ln(\ln x)\Big|_{e^2}^{e^3} = \ln 3 - \ln 2 = \ln\frac{3}{2}$

3.* $\exists\ t = \ln x \qquad dt = \frac{1}{x}\,dx$

$\int_{e^2 = x}^{e^3} \frac{dx}{x\ln x} = \int_2^3 \frac{dt}{t} = \ln(t)\Big|_2^3 = \ln 3 - \ln 2 = \ln\frac{3}{2}$

4. $\int_0^1 x\sqrt{1+x}\,dx \ominus$

$t = \sqrt{1+x}$

$dt = \frac{1}{2\sqrt{1+x}}\,dx$

$dx = 2\sqrt{1+x}\,dt$

$t^2 = 1+x$

$x = t^2 - 1$

$\ominus\ 2\int_1^{\sqrt{2}} (t^2-1)(t^2)\,dt = 2\left(\frac{t^5}{5}\Big|_1^{\sqrt{2}} - \frac{t^3}{3}\Big|_1^{\sqrt{2}}\right)$

5. $\int_{-a}^{a} \sqrt{a^2-x^2}\,dx = \frac{\pi a^2}{2}$

6. $\int_{1}^{e} \sin(\ln x)\,dx \;\textcircled{=}$

$\exists\, t = \ln x \quad dt = \frac{dx}{x} \quad dt \cdot x = dx \quad x = e^t$

$\textcircled{=} \int_0^1 e^t \sin t\,dt =$

7. $\int_{-a}^{a} x\sqrt{a^8-x^8}\,dx = 0$ Т.К. ф-ия нечетная

8. $\int_0^{\pi} \frac{x\cdot\sin x}{1+\cos^2 x}\,dx \;\textcircled{=}$

$\int \frac{\sin x}{1+\cos^2 x} \textcircled{=}, \quad t = \cos x \quad dt = -\sin x\,dx \quad dx = -\frac{dt}{\sin x}$

$\textcircled{=} \int \frac{dt}{1+t^2} = -\operatorname{arctg}(\cos x)$

$\textcircled{=} -\int_0^{\pi} x\,d(\operatorname{arctg}(\cos x)) = \underbrace{-\operatorname{arctg}(\cos x)\cdot x\Big|_0^{\pi}}_{A} + \underbrace{\int_0^{\pi} \operatorname{arctg}(\cos x)\,dx}_{= B} =$

$\operatorname{arctg} \cos x = \varphi$

$\operatorname{tg}\varphi = \cos x \iff \operatorname{tg}^2\varphi = \cos^2 x \iff \frac{1}{\cos^2\varphi} - 1 = \cos^2 x$

$\exists\, x = \pi/2 + t$

$B = \int_{-\pi/2}^{\pi/2} \operatorname{arctg}(\sin t)\,dt = 0$

9. $\int_{-1}^{1} \frac{1+x^2}{1+x^4}\,dx$

$\int_{-1}^{1} \frac{1+x^2}{1+x^4}\,dx = \int_{-1}^{1} \frac{1/x^2+1}{1/x^2+x^2}\,dx \;\textcircled{=}$

$(1+1/x^2)\,dx = d(x - 1/x)$

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$$

Э $t = x-\frac{1}{x}$

$$\ominus \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x^2-1}{x\sqrt{2}}\Bigg|_{-1}^{1} = 0$$ но ф-ия положительна везде

первообразная должна быть на отрезке, а при $x=0$ проиграли $\Rightarrow$

ф-ла Ньютона-Лейбница не работает

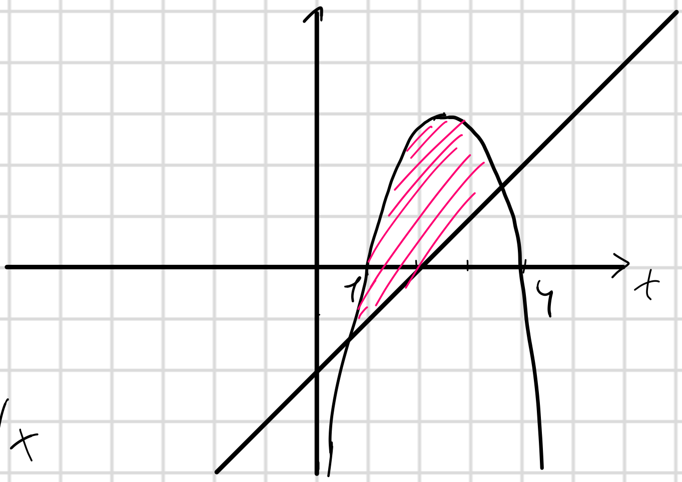
$\Rightarrow$ разобьем

$$=\int_{-1}^{1}\frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+x^2}\,dx = 2\int_{0}^{1}\frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+x^2}\,dx = 2\cdot\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x^2-1}{x\sqrt{2}}\Bigg|_{0^+}^{1} = \sqrt{2}\left(0-\left(-\frac{\pi}{2}\right)\right) = \frac{\pi}{\sqrt{2}}$$

10. Площадь - ?

$y=-x^2+6x-6$

$y = x-2$

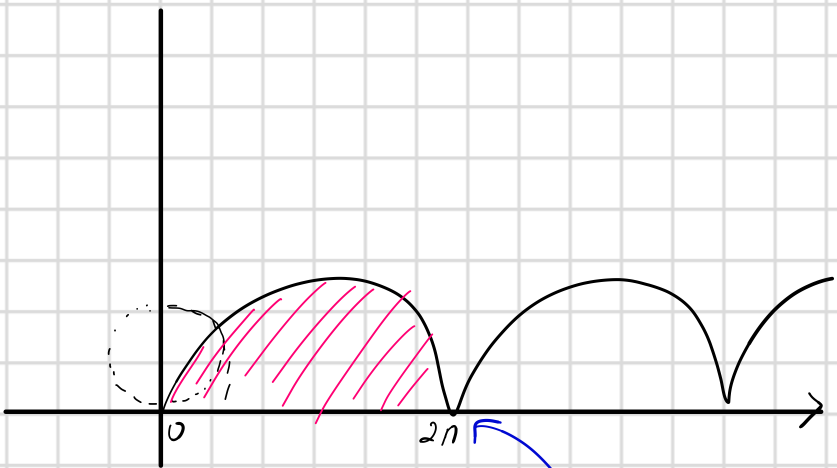


$$S = \int_{1}^{4}(6x-x^2-6-x+2)\,dx$$

11. Площадь одной арки

$$\begin{cases} x = a(t-\sin t) \\ y = a(1-\cos t) \end{cases}$$

$y=0$



$$S = \int_{x_1}^{x_2} y(x)\,dx = \int_{0}^{2\pi} y(t)\cdot x'(t)\,dt \ominus$$

$t = 2\pi$ т.к. $y=0$

$$\ominus\ a^2\int_{0}^{2\pi}(1-\cos t)^2\,dt = a^2\int_{0}^{2\pi}\left(1-\underbrace{2\cos t}_{0}+\frac{1+\overbrace{\cos 2t}^{0}}{2}\right)dt \ominus$$

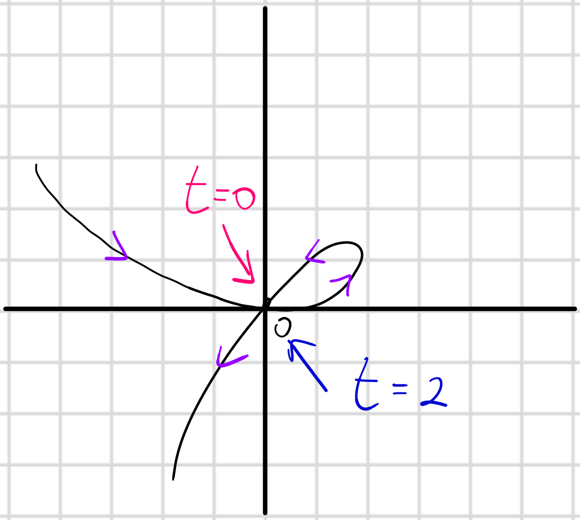
= $3\pi a^2$

12. $\begin{cases} x = 2t - t^2 \\ y = 2t^2 - t^3 \end{cases}$

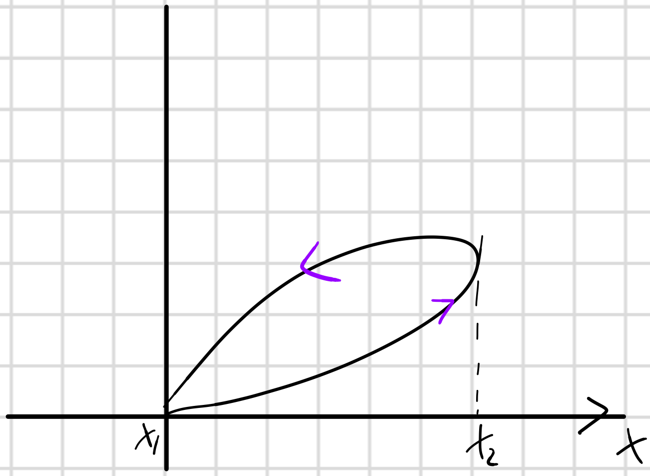
площадь петли

$\begin{cases} x = t(2-t) \\ y = t^2(2-t) \end{cases}$

$S = \left| \int_0^2 y \cdot x' dt \right|$



$\int_{x_1}^{x_2} y\,dx$



13. $\rho(\varphi) = \cos 3\varphi$

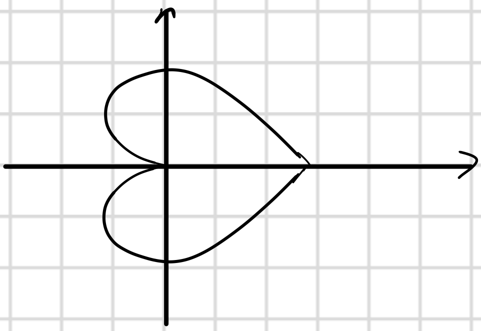
$\cos 3\varphi = 0$

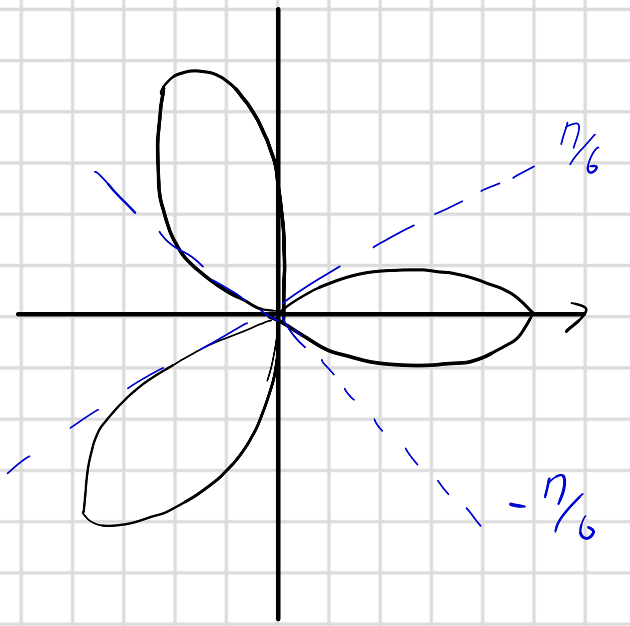
$3\varphi = \frac{\pi}{2}$

$\varphi = \frac{\pi}{6}$

Полярные в Desmos: r=cos(theta)+1

$\rho(\varphi) = 1 + \cos\varphi$





$$S = 3 \cdot \frac{1}{2} \int_{-\pi/6}^{\pi/6} \rho^2(\varphi)\, d\varphi = \frac{3}{2} \int_{-\pi/6}^{\pi/6} \cos^2 3\varphi \, d\varphi = \frac{3}{4} \int (1 + \cos 6\varphi)\, d\varphi =$$

$$= \frac{3}{4} \cdot \frac{\pi}{3} = \frac{\pi}{4}$$