1. $\lim\limits_{x\to x} \sin(\sqrt{x^2+1} - \sin\sqrt{x^2-1}) =$

$$= \lim_{x\to\infty}\Big(\sin\underbrace{\frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{2}}_{\text{Б.М.П.}}\cdot\underbrace{\cos\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{2}}_{\text{огранич}}\Big) = 0$$

$$\frac{2}{2\sqrt{x^2+1}+\sqrt{x^2-1}} \longrightarrow 0$$

Б.М.П. $\cdot$ огранич = Б.М.П

при $x \longrightarrow 0$

$\sin x \sim x$

при $\alpha(x)\longrightarrow 0$

$\sin(\alpha(x)) \underset{x\to x_0}{\sim} \alpha(x)$ если $\alpha(x) \underset{x\to x_0}{\to} 0 \Leftrightarrow \lim\limits_{x\to x_0}\frac{\sin(\alpha(x))}{\alpha(x)} = 1$

2. $\lim\limits_{x\to 0}\frac{\sin 5x}{x} \overset{(2)}{=} \lim\limits_{x\to 0}\underbrace{\frac{\sin 5x}{5x}}_{\to 1}\cdot 5 = 5$

$\|^{(1)}$

$\lim\limits_{x\to 0}\frac{5x}{x} = 5$

3. $\lim\limits_{x\to \pi}\frac{\sin 3x}{x} = \lim\limits_{x\to\pi}\frac{\sin 3\pi}{\pi} = \frac{0}{\pi} = 0$

4. $\lim\limits_{x\to \pi}\frac{\operatorname{tg} 3x}{\operatorname{tg} 8x} = \lim\limits_{x\to\pi}\frac{3x}{8x} = \frac{3}{8}$

$\boxed{\operatorname{tg} x \sim x}$

5. $\lim\limits_{x\to 0} \sin 2x \cdot ctg\, 3x \cdot \underset{\searrow 1}{2^x} = \lim\limits_{x\to 0} \frac{\sin 2x}{2x} \cdot 2 \cdot \frac{3x}{tg\, 3x} \cdot \frac{1}{3} =$

$= \frac{2}{3}$

убрали период, заменили на эквивалент

6. $\lim\limits_{x\to \pi} \frac{\sin 3x}{\sin 4x} = \lim\limits_{y\to 0} \frac{\sin 3(y+\pi)}{\sin 4(y+\pi)} = \lim\limits_{y\to 0} \frac{-\sin 3y}{\sin 4y} = -\frac{3}{4}$

] $y = x - \pi$
$x = y + \pi$

7. $\lim\limits_{x\to 0} \frac{\cos x - 1}{x \operatorname{arctg}^3 x} \circleddash$

При $x \to 0$	
$\sin x \sim x$	$e^x - 1 \sim x$
$tg\, x \sim x$	$a^x - 1 \sim x \cdot \ln a$
$\arcsin x \sim x$	$\ln(1+x) \sim x$
$\operatorname{arctg} x \sim x$	$\log_a(1+x) \sim \frac{x}{\ln a}$
$1 - \cos x \sim \frac{x^2}{2}$	$(1+x)^\alpha - 1 \sim \alpha x$

$\circleddash \lim\limits_{x\to 0} \frac{-\cos x + 1}{-x \cdot 3x} = \frac{x^2}{2} \cdot \frac{1}{-3x^2} = -\frac{1}{6}$

8. $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+x} - 1}{\ln(1+2x)} = \lim\limits_{x\to 0} \frac{\frac{1}{3}x}{2x} = \frac{1}{6}$

9. $\lim\limits_{x\to 0} \frac{\ln \cos x}{\sqrt[n]{2+x} - \sqrt[n]{2}} = \lim\limits_{x\to 0} \frac{\ln(1 + \overset{\nearrow 0}{(\cos x - 1)})}{\sqrt[n]{2}\left(\sqrt[n]{1 + \underset{\searrow 0}{\frac{x}{2}}} - 1\right)} = \lim\limits_{x\to 0} \frac{\cos x - 1}{\sqrt[n]{2} \cdot \frac{1}{n} \cdot \frac{x}{2}} \circleddash$

$$= \lim_{x\to 0} \frac{-\frac{x^2}{2}}{\sqrt[n]{2}\cdot\frac{1}{n}\cdot\frac{x}{2}} = 0$$

10. $$\lim_{x\to 0} \frac{1}{x} \ln\left(\frac{2x+3}{3-2x}\right) = \lim_{x\to 0} \frac{1}{x} \ln\left(1-1+\frac{2x+3}{3-2x}\right) =$$

$$= \lim_{x\to 0} \frac{1}{x}\left(\frac{2x+3}{3-2x}-1\right) = \lim_{x\to 0} \frac{1}{x}\left(\frac{4x}{3-2x}\right) =$$

$$= \lim_{x\to 0} \frac{4x}{3x-2x^2} = \lim_{x\to 0} \frac{4}{3-2x} = \frac{4}{3}$$

11. $$\lim_{x\to 0} \frac{\log_2(2+x)-\log_2(2+2x)}{x\,\underbrace{\ln(2+x)}_{\to \ln 2}} = \lim_{x\to 0} \frac{\log_2\left(\frac{2+x}{2+2x}+1-1\right)}{x\ln 2} =$$

$$= \lim_{x\to 0} \frac{\frac{2+x}{2+2x}-1}{x\cdot\ln^2 2} = \lim_{x\to 0} \frac{-x}{(2+2x)\,x\ln^2 2} = -\frac{1}{2\ln^2 2}$$

12. $$\lim_{x\to 0} \frac{\sin 5x-\sin 7x}{\operatorname{tg} 3x-\operatorname{tg} 4x} = \lim_{x\to 0} \frac{\frac{\sin 5x}{x}-\frac{\sin 7x}{x}}{\frac{\operatorname{tg} 3x}{x}-\frac{\operatorname{tg} 4x}{x}} =$$

$$= \frac{\lim\limits_{x\to 0}\left(\frac{\sin 5x}{x}-\frac{\sin 7x}{x}\right)}{\lim\limits_{x\to 0}\left(\frac{\operatorname{tg} 3x}{x}-\frac{\operatorname{tg} 4x}{x}\right)} = \frac{5-7}{3-4} = 2$$

обязательно так.

здесь нельзя так!

13. $$\lim_{x\to 0} \frac{\operatorname{tg} x-\sin x}{x^3} = \lim_{x\to 0} \frac{\operatorname{tg} x\,(1-\cos x)}{x\cdot x^2} = \frac{1}{2}$$

$\to 1$ $\qquad$ $\to \frac{1}{2}$

14. $$\lim_{x\to 0^+} \frac{\cos 2x - \cos 3x}{\sqrt[5]{1+2x} - \sqrt[6]{1-3x}} = \lim \frac{1-\cos(3\sqrt{x}) - (1-\cos(2\sqrt{x}))}{\sqrt[5]{1+2x} - 1 - (\sqrt[6]{1-3x} - 1)} =$$

$$= \frac{\lim \frac{1-\cos(3\sqrt{x})}{x} - \lim \frac{1-\cos(2\sqrt{x})}{x}}{\lim \frac{\sqrt[5]{1+2x}-1}{x} - \lim \frac{\sqrt[6]{1-3x}-1}{x}} = \frac{\frac{9}{2} - \frac{4}{2}}{\frac{1}{5}\cdot 2 - \frac{1}{6}(-3)}$$

15. $$\lim_{x\to\infty} x^2\left(\cos\frac{1}{x} - \cos\frac{3}{x}\right) = \lim_{x\to\infty} x^2\left(-2\sin\overset{\to 0}{\frac{2}{x}}\sin\left(\overset{\to 0}{-\frac{1}{x}}\right)\right) =$$

$$= \lim_{x\to\infty} x^2\,(-2)\,\frac{2}{x}\left(-\frac{1}{x}\right) = 4$$

16. $$\lim_{x\to 0} \frac{\ln(1+2x+x^2) + \ln(1-2x+x^2)}{x^2} \neq \lim_{x\to 0} \frac{2x+x^2+(-2x+x^2)}{x^2} = \lim \frac{2x^2}{x^2} = 2$$

//

$$\lim_{x\to 0} \frac{1}{x^2}\cdot \ln(1+\cancel{2x}-\cancel{2x}+x^4-2x^2) = \lim_{x\to 0} \frac{x^4-2x^2}{x^2} = -2$$