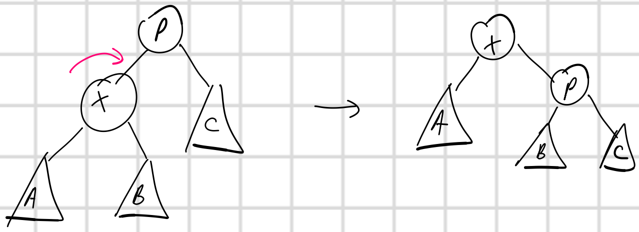
Splay-tree

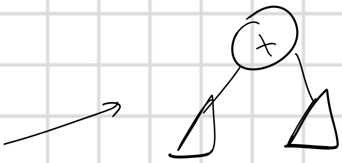
$\tilde{T} = O(\log n)$



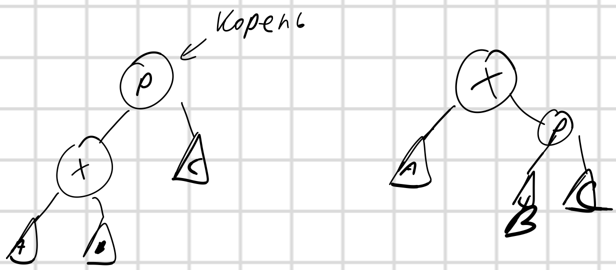
Инвариант дерева поиска

find:

x splay

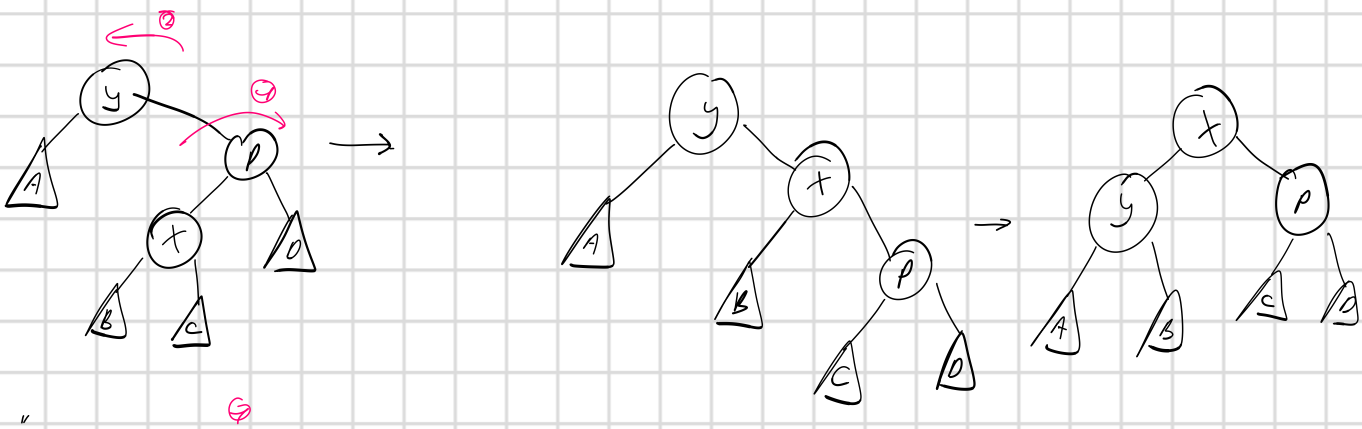


„Зиг"

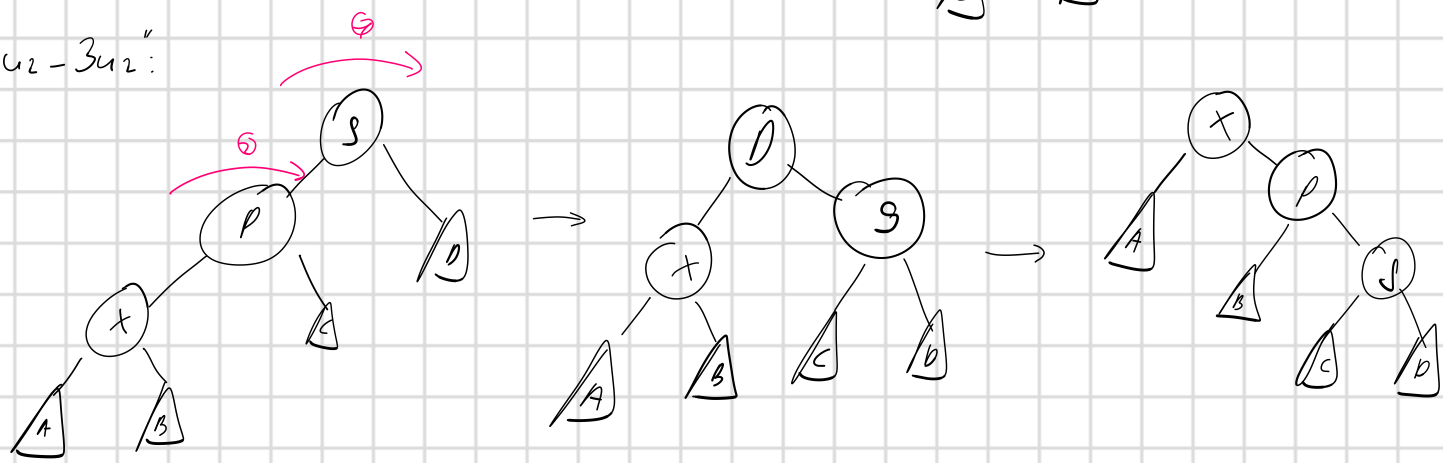


A↑
x↑
p↓
C↓

„Зиг-Заг"



„Зиг-Зиг":



▷ $S(v)$ – кол-во вершин в поддереве

$r(v) = \log_2(S(v))$

$\tilde{T} = T + \Delta\varphi$

$\Delta\varphi_0 = 0 \quad \varphi_i \geq 0$

$\sum \tilde{T} = \sum T + \varphi_n$

$\exists\, \varphi = \sum_v r(v)$

$$\tilde{T} = T + \underbrace{\overset{\text{стало}}{\sum r'(v)} - \overset{\text{было}}{\sum r(v)}}_{\Delta\varphi}$$

$r'(x)$

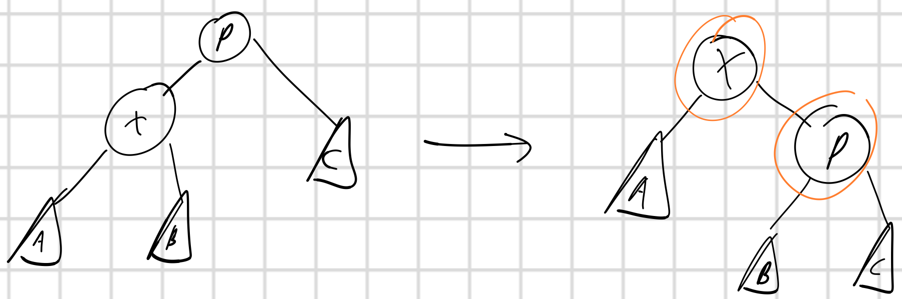
$1 + 3(r'(t) - r(x)) = \tilde{T}$

$\tilde{T}'(\text{зиг}) = 1 + 3(r'(x) - r(x))$

$\tilde{T}(\text{зиг-заг}) = 3(r'(x) - r(x))$

$\tilde{T}(\text{зиг-зиг}) = 3(r'(x) - r(x))$

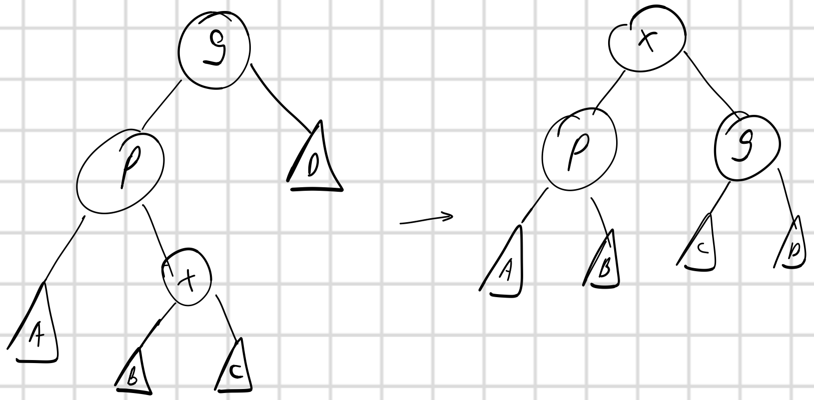
$$3(r'(x) - r(x)) + 3(r''(x) - r'(x)) + \dots = 1 + 3(r(t) - r(x))$$



— изменили размер

$$1 + \cancel{r'(x)} - r(x) + r'(p) - \cancel{r(p)} = 1 + r'(p) - r(x) \leq 1 + r'(x) - r(x) = 1 + 3(r'(x) - r(x))$$

т.к. они одинаковы стали



$$\widetilde{T} = 2 + (r'(x) - r(x)) + (r'(g) - r(g)) + (r'(p) - \underbrace{r(p)}_{< r(x)})$$

$$2 + r'(g) + \cancel{r'(p)} - 2\,r(x) \le 2(r'(x) - \cancel{r(x)}) \le 3(r'(x) - r(x))$$

$$2 + r'(g) + r'(p) - 2r'(x) \le 0$$

$$r'(g) + r'(p) - 2r'(x) \le -2$$

$$\log_2 \underbrace{\frac{s'(g)}{s'(x)}}_{a} \cdot \underbrace{\frac{s'(p)}{s'(x)}}_{b} \le -2$$

$$a \cdot b \le 1/4 \quad ?$$

$$a + b \le 1$$

$$\frac{a+b}{2} \ge \sqrt{ab}$$

$$\left(\frac{a+b}{2}\right)^2 \ge ab.$$