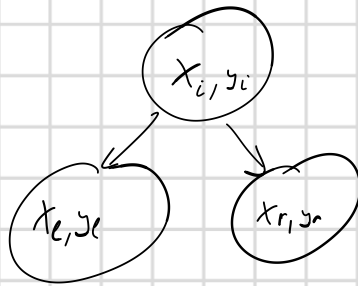


# Декартово древо (Cartesian tree)

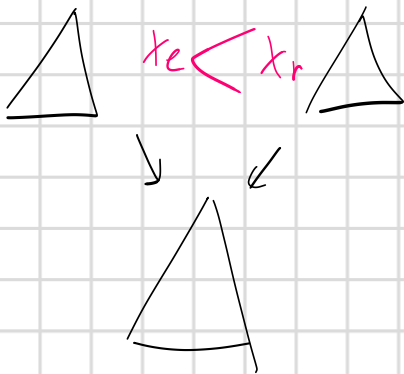


$x_i$  - ключ  
 $y_i$  - приоритет

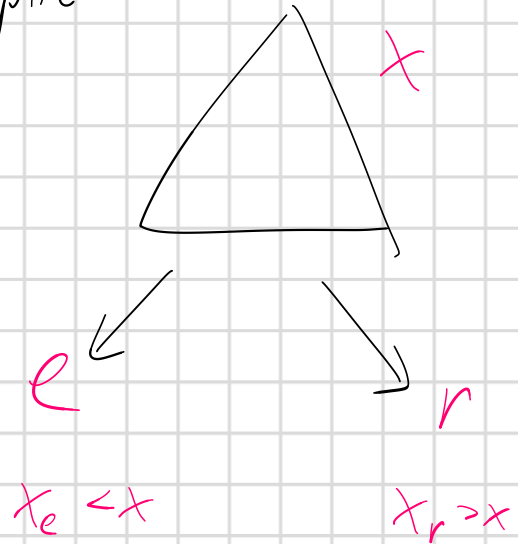
$$\begin{cases} x_l < x \\ x_r > x \end{cases} \quad \begin{cases} y > y_l \\ y > y_r \end{cases}$$

$\exists y_i$  строго больше

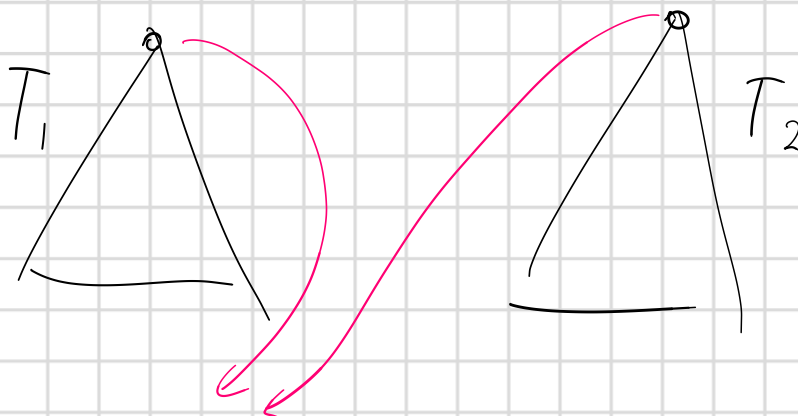
• merge



• split



merge



$\exists$  корень  $y_{max}$

```

node {
  x
  y
  node * l
  node * r
}

```

Node merge (l, r)

if (l = Null): return r

if (r = Null): return l

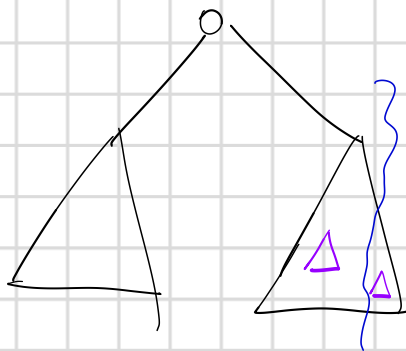
if (l.y > r.y):

l.r = merge(l.r, r)

return l;

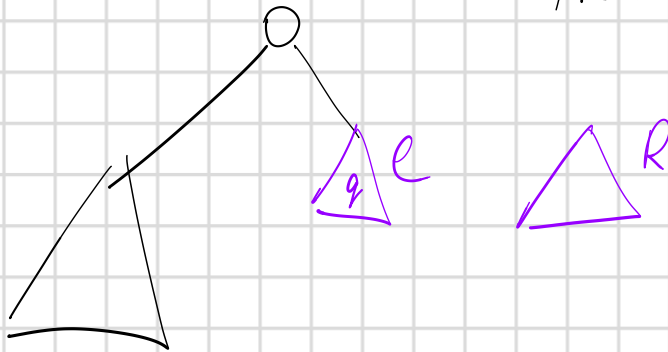
См. merge. long run. аналитический

split



$\langle L, R \rangle \text{ split}(T)$

Значит.



$\langle L, R \rangle \text{ split}(T, x)$

if (!T)

$L = R = \text{Null}$

if (T.x < x)

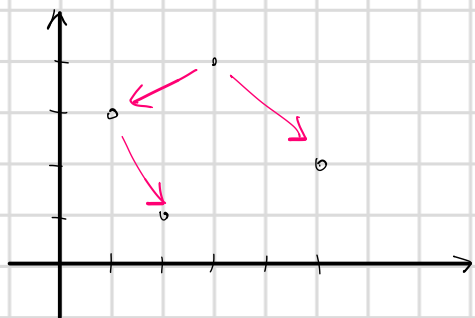
$q, R$

$\text{split}(T.R, x)$

$L = T$

$L.R = q$

(1, 3) (2, 1) (5, 2) (3, 4)



insert(x)

$L, R = \text{split}(T, x)$

$t = \text{merge}(\text{merge}(L, x), x)$

remove(x)

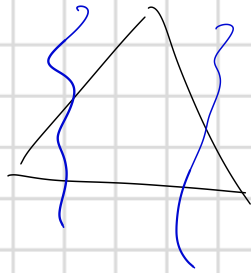
$L \leq x$

$q, R = \text{split}(T, x)$

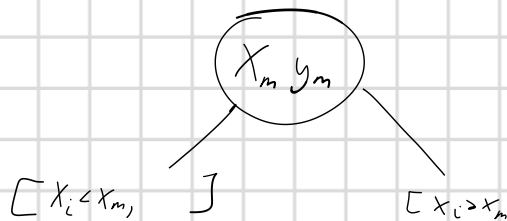
$L_{\text{mid}} = \text{split}(q, x-1)$

$T = \text{merge}(L, R)$

$sum(l, r)$



$\exists (x_1, y_1) (x_2, y_2) \dots (x_n, y_n)$



носитель  $O(n \log n)$

$y$  - случайная перестановка

$$\forall v: E(d(v)) = O(\log n)$$

$$d_{ij} = \begin{cases} 1, & i \text{ precedes } j \\ 0 & \end{cases}$$

$$d(v) = \sum_i d_{i,v}$$

$$E(d(v)) = E\left(\sum_i d_{i,v}\right) = \sum_i E(d_{i,v}) = \sum_i P(i \text{ precedes } v) \quad \text{⊖}$$

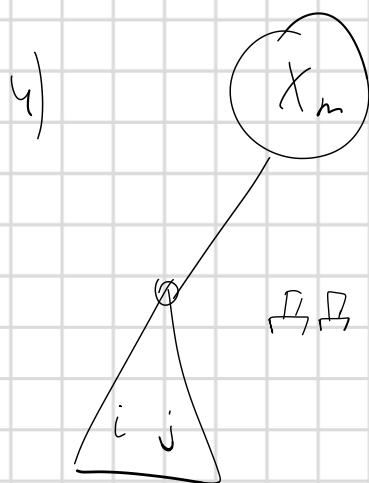
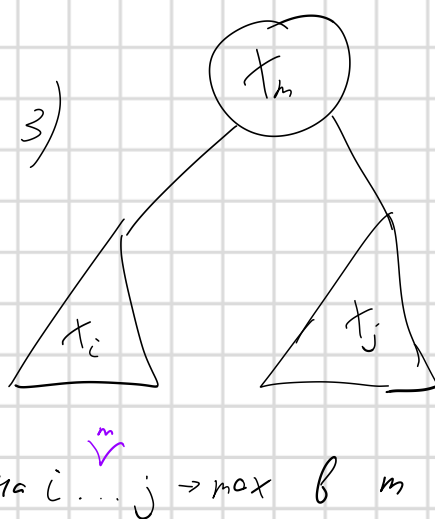
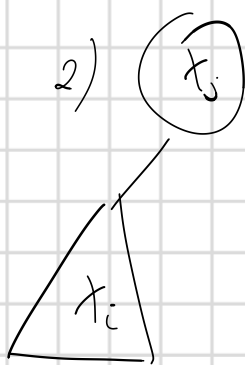
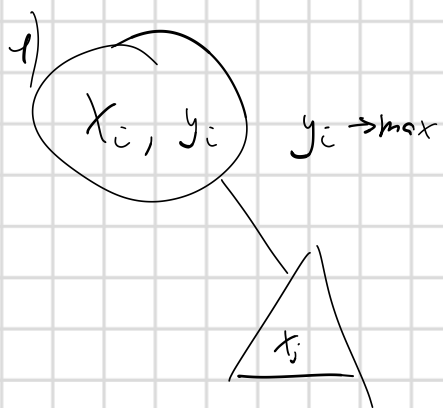
$$\exists x_1 < x_2 < \dots < x_n$$

$$y_1 \ y_2 \dots y_n$$

Лемма()

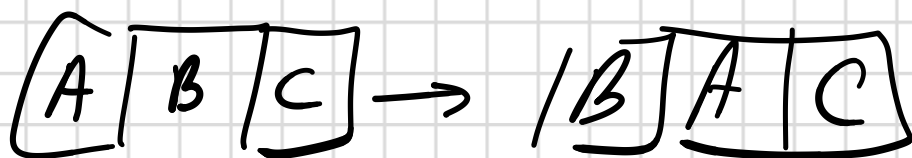
$$i < j$$

$$i \text{ precedes } j \Leftrightarrow y_i = \max(y_i, \dots, y_j)$$



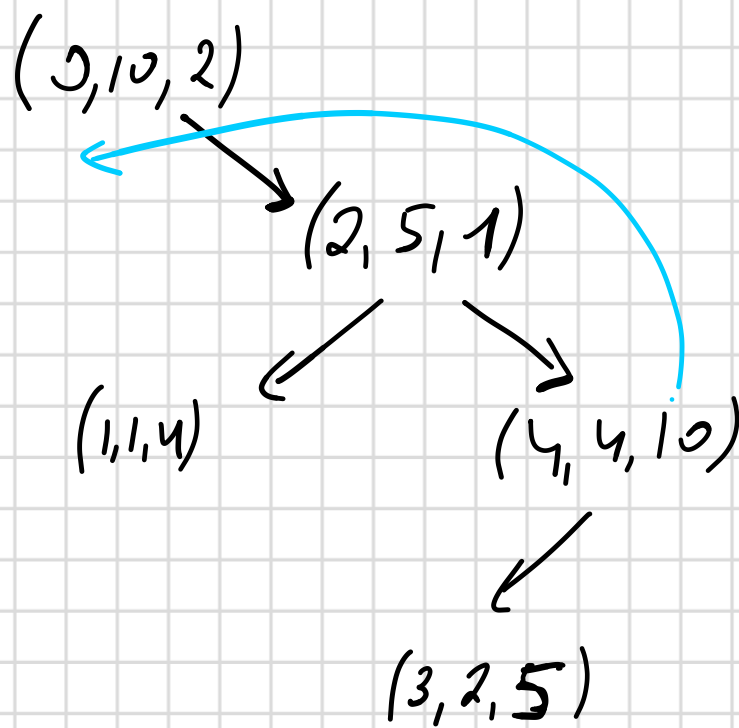
$$\sum_i p(y_i = \max(y_i \dots y_v)) = \sum \frac{1}{|i-v|+1} \leq \sum \frac{1}{i} = O(\log n)$$

Аннотация к работе

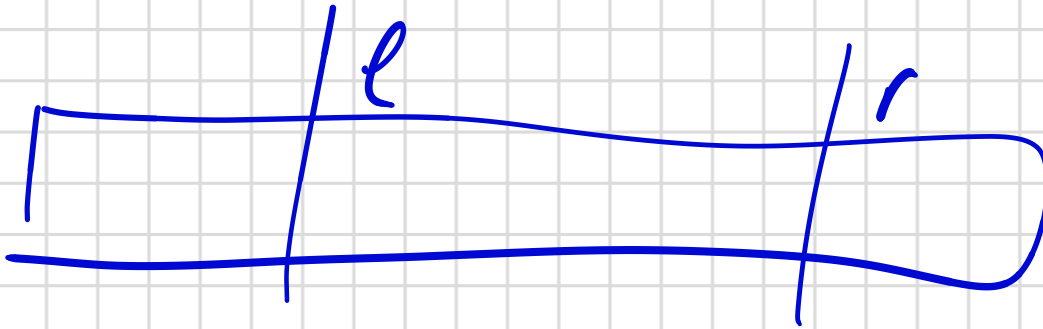


y	10	1	5	2	4
val	2	4	1	5	10
t	0	1	2	3	4

Node  
 { val  
 y  
 size  
 }



K. 61  $\rightarrow n-1$



split( $T, vol$ )

if ( $T \rightarrow l \rightarrow size > vol$ ):