

Неопределенный интеграл

$$1. \int \frac{dx}{x+5} = \ln|x+5| + C$$

$$2. \int \frac{dx}{3x-5} = \frac{1}{3} \int \frac{dx}{x-\frac{5}{3}} = \frac{1}{3} \ln|x-\frac{5}{3}| + C$$

$$3. \int \frac{dx}{3x^2+5} = \frac{1}{3} \cdot \int \frac{dx}{x^2+\frac{5}{3}} \ominus$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctg \frac{x}{a} + C$$

$$\ominus \frac{1}{3} \cdot \frac{1}{\sqrt{\frac{5}{3}}} \cdot \arctg \frac{x}{\sqrt{\frac{5}{3}}} + C$$

$$4. \int \frac{dx}{3x^2-5} = \frac{1}{3} \int \frac{dx}{x^2-\frac{5}{3}} \ominus$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\ominus \frac{1}{3} \cdot \frac{1}{2 \cdot \sqrt{\frac{5}{3}}} \cdot \ln \left| \frac{x-\sqrt{\frac{5}{3}}}{x+\sqrt{\frac{5}{3}}} \right| + C$$

$$5. \int \frac{x dx}{3x+5} = \int \frac{3x+5-5}{3(3x+5)} dx = \int \frac{3x+5}{3(3x+5)} - \frac{5}{3(3x+5)} dx = \int \frac{1}{3} - \frac{5}{3(3x+5)} dx = \frac{x}{3} - \frac{5}{3} \int \frac{1}{3x+5} dx \ominus$$

$$\ominus \frac{x}{3} - \frac{5}{9} \ln|x+\frac{5}{3}| + C$$

$$6. \int \frac{x^2 dx}{3x+5} = \int \frac{9x^2-25+25}{9(3x+5)} dx = \frac{1}{9} \int \frac{(3x-5)(3x+5)+25}{(3x+5)} dx = \frac{1}{9} \int (3x-5) + \frac{25}{3x+5} dx \ominus$$

$$\ominus \frac{1}{9} \left(\int 3x-5 dx + 25 \int \frac{1}{3x+5} dx \right) = \frac{1}{9} \left(\frac{3}{2} x^2 - 5x + \frac{25}{3} \ln|x+\frac{5}{3}| \right) + C$$

$$7. \int \sqrt{3x+5} dx = \frac{1}{3} \int \sqrt{3x+5} d(3x+5) = \frac{1}{3} \frac{(3x+5)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$8. \int \frac{dx}{\sqrt{3x^2+5}} \ominus$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x+\sqrt{x^2+a^2}| + C$$

$$\ominus \int \frac{dx}{\sqrt{3(x^2+\frac{5}{3})}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{x^2+\frac{5}{3}}} = \frac{1}{\sqrt{3}} \ln|x+\sqrt{x^2+\frac{5}{3}}| + C$$

$$9. \int \frac{x dx}{\sqrt{3x^2+5}} = \int \frac{d(3x^2+5)}{6\sqrt{3x^2+5}} = \frac{1}{6} \int \frac{d(3x^2+5)}{\sqrt{3x^2+5}} \ominus$$

$$d(3x^2+5) = 6dx$$

$$\ominus \int \frac{1}{\sqrt{t}} = \frac{1}{6} \int \frac{dt}{\sqrt{t}} = \frac{1}{6} \frac{\sqrt{t}}{\frac{1}{2}} = \frac{1}{3} \cdot \sqrt{3x^2+5} + C$$

$$10. \int \cos(t-6x) dx = \int \cos(t-6x) d(t-6x) = \frac{1}{6} \cdot \sin(t-6x) + C$$

$$11. \int \tan x dx = \int \frac{\sin x}{\cos x} dx \ominus d \cos x = -\sin x dx \ominus \int \frac{-1}{\cos x} d \cos x = -\ln |\cos x| + C$$

$$12. \int \tan^2 x dx = \int \frac{(1-\cos^2 x)}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int 1 dx \ominus$$

$$\int \frac{dx}{\cos^2 x} = \tan x + C$$

$$\ominus \tan x - x + C$$

$$13. \int e^{\cos x} \cdot \sin x dx = - \int e^{\cos x} d \cos x = -e^{\cos x} + C$$

$$d \cos x = -\sin x dx$$

$$13. \int \frac{\arctan x}{1+x^2} dx = \int \arctan x d \arctan x = \frac{\arctan^2 x}{2} + C$$

$$d(\arctan x) = \frac{dx}{1+x^2}$$

$$14. \int \frac{\sqrt{\arcsin x}}{1-x^2} dx = \int \sqrt{\arcsin x} d(\arcsin x) = \frac{\arcsin^{\frac{3}{2}} x}{\frac{3}{2}} + C$$

$$15. \int \frac{dx}{x^2+x+1} = \int \frac{dx}{x^2+x+\frac{1}{4}-\frac{3}{4}} = \int \frac{dx}{(x+\frac{1}{2})^2-\frac{3}{4}} = \int \frac{d(x+\frac{1}{2})}{(x+\frac{1}{2})^2-\frac{3}{4}} \ominus$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\ominus \frac{1}{2 \cdot \frac{\sqrt{3}}{2}} \ln \left| \frac{x+\frac{1}{2}-\frac{\sqrt{3}}{2}}{x+\frac{1}{2}+\frac{\sqrt{3}}{2}} \right|$$

$$16. \int \underbrace{x}_{u} \underbrace{\cos 2x}_{dv} dx$$

$$u = x \quad du = dx$$

$$dv = \cos 2x dx$$

$$v = \int \cos 2x dx = \frac{\sin(2x)}{2}$$

$$\int x \cos(2x) dx = x \cdot \frac{\sin(2x)}{2} - \int \frac{\sin(2x)}{2} dx = x \cdot \frac{\sin(2x)}{2} + \frac{1}{2} \frac{\cos(2x)}{2} + C$$

$$17. \int (3x-2) \cdot e^{-x} dx \ominus$$

$$u = 3x-2 \quad du = 3dx$$

$$dv = e^{-x} dx$$

$$v = \int e^{-x} = -e^{-x}$$

$$\ominus (3x-2)(-e^{-x}) - \int -3e^{-x} dx = (3x-2)(-e^{-x}) - 3e^{-x} + C$$

$$18. \int \sin^2 x dx \ominus$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\ominus \int \frac{1 - \cos 2x}{2} dx = \frac{1}{4} \int 1 - \cos 2x d(2x) = \frac{1}{4}(2x) - \frac{1}{4} \int \cos 2x d(2x) = \frac{1}{2}x - \frac{1}{4} \sin 2x + C$$

$$2. \int \sin^3 x dx = \int \sin^2 x \cdot \sin x dx = -\int \sin^2 x d(\cos x) = -\int 1 - \cos^2 x d(\cos x) = -(\cos x - \int \cos^2 x d(\cos x)) = -\cos x + \frac{\cos^3 x}{3} + C$$

$$3. \int \sin^4 x dx = \int \left(\frac{1 - \cos 2x}{2} \right)^2 dx = \frac{1}{4} \int 1 - 2\cos 2x + \cos^2 2x dx = \frac{1}{4}x - \frac{1}{2} \int \cos 2x d(2x) + \frac{1}{8} \int 1 + \cos 4x dx = \frac{1}{4}x - \frac{\sin 2x}{4} + \frac{1}{8}x + \frac{1}{32} \sin 4x + C$$

$$4. \int \sin^5 x dx = \int \sin^4 x \cdot \sin x dx = \int (\sin^2 x)^2 \cdot \sin x dx = -\int (1 - t^2) dt$$

$$5. \int \sin^2 x \cdot \cos^3 x dx = \int \sin^2 x \cdot \cos^2 x \cdot \cos x dx$$

$$6. \int \frac{x dx}{(x+1)(x+2)(x-3)} =$$

$$\frac{x}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3} \quad \Bigg| \quad (x+1)(x+2)(x-3)$$

$$x = A(x+2)(x-3) + B(x+1)(x-3) + C(x+1)(x+2)$$

$$x = A(x^2 - x - 6) + B(x^2 - 2x - 3) + C(x^2 + 3x + 2)$$

$$\begin{aligned} \text{hpu } x^2: & \begin{cases} A+B+C=0 \\ -A-2B+3C=1 \\ -6A-3B+2C=0 \end{cases} \\ \text{hpu } x^1: & \\ \text{hpu } x^0: & \end{aligned}$$

2 Cnosed:

$$x=-1: -1 = -4A \quad A = \frac{1}{4}$$

$$x=-2: -2 = 5B$$

$$x=3: 3 = 20C$$

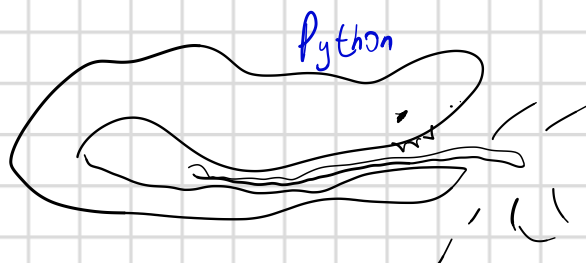
$$\int \frac{x dx}{(x+1)(x+2)(x-3)} = \frac{1}{4} \ln|x+1| - \frac{2}{5} \ln|x+2| + \frac{3}{20} \ln|x-3| + C$$

$$7. \int \frac{3x^4 + 3x^3 + 2}{x^3 + x^2 - x - 1} dx$$

$3x^4 + 3x^3 + 2$	$x^3 + x^2 - x - 1$
$3x^4 + 3x^3 - 3x^2 - 3x$	
$3x^2 + 3x + 2$	$3x$

$$\frac{3x^2 + 3x + 2}{x^3 + x^2 - x - 1} = \frac{3x^2 + 3x + 2}{(x^2 - 1)(x + 1)} = \frac{3x^2 + 3x + 2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$3x^2 + 3x + 2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$



$$x=-1: 2 = -2C$$

$$x=1: 2 = 4A$$

$$x=0: 2 = A - B - C \Rightarrow B=1$$



$$\textcircled{=} \int \left(3x + \frac{2}{x-1} + \frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx = \frac{3}{2} x^2 + 2 \ln|x-1| + \ln|x+1| - \frac{1}{x+1} + C$$

$$8. \int \frac{dx}{x^3+1} = \int \frac{dx}{(x+1)(x^2-x+1)}$$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$x = -1: 1 = 3A, A = \frac{1}{3}$$

$$x = 0: 1 = A + C \Rightarrow C = \frac{2}{3}$$

$$\text{hpu } x^2: 0 = A + B \Rightarrow B = -\frac{1}{3}$$

$$\int \frac{dx}{(x+1)(x^2-x+1)} = \int \left(\frac{1/3}{x+1} + \frac{1/3}{x^2-x+1} \right) dx = \frac{1}{3} \ln|x+1| + \frac{1}{3} \int \frac{-x+2}{x^2-x+1} dx \quad \textcircled{=}$$

$$\textcircled{=} \frac{1}{3} \ln|x+1| - \frac{1}{3} \int \frac{x-2}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx = \frac{1}{3} \ln|x+1| - \frac{1}{3} \int \frac{x-\frac{1}{2}-\frac{3}{2}}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx \quad \textcircled{=}$$

$$\textcircled{=} \frac{1}{3} \ln|x+1| - \frac{1}{6} \int \frac{2x-1}{x^2-x+1} dx + \frac{1}{2} \int \frac{dx}{(x-\frac{1}{2})^2 + \frac{3}{4}} = \frac{1}{3} \ln|x+1| - \frac{1}{6} \ln(x^2-x+1) + \frac{1}{2} \cdot \frac{2}{\sqrt{3}} \arctan \frac{2(x-\frac{1}{2})}{\sqrt{3}}$$

$$9. \int \frac{dx}{\sqrt[3]{x+1}} = \int \frac{6t^5 dt}{t^2+t^3} = 6 \int \frac{t^3 dt}{1+t} = 6 \int \frac{t^3-1+1}{1+t} dt = 6 \int (t^2-t+1) \left(\frac{1}{1+t} \right) dt \quad \textcircled{=}$$

$$\exists x=t^6$$

$$\textcircled{=} 2t^3 - 3t^2 + 6t - 6 \ln|1+t| + C =$$

$$\int \frac{x dx}{(x^2+3x+1)^2} \rightarrow \int \frac{t+3}{(t^2-a^2)^2} dt$$

$$\int \frac{dt}{(t^2-a^2)^2} = \frac{1}{a^2} \int \frac{a^2-t^2+t^2}{(t^2-a^2)^2} dt =$$

$$10. \int \frac{2x^3+x^2+5x+1}{(x^2+3)(x^2-x+1)} dx$$

$$\frac{2x^3 + x^2 + 5x + 1}{(x^2+3)(x^2-x+1)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2-x+1} \quad / \cdot (x^2+3)(x^2-x+1)$$

$$2x^3 + x^2 + 5x + 1 = (Ax+B)(x^2-x+1) + (Cx+D)(x^2+3)$$

$$2x^3 + x^2 + 5x + 1 = Ax^3 - Ax^2 + Ax + Bx^2 - Bx + B + Cx^3 + 3Cx + Cx^2 + 3D$$

$$\begin{cases} A+C=2 \\ B+D-A=1 \\ A-B+3C=5 \\ 3D+B=1 \end{cases} \quad \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ -1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 3 & 0 & 5 \\ 0 & 1 & 0 & 3 & 1 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 3 & 1 & 6 \\ 0 & 0 & -1 & 2 & -2 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 3 & 1 & 6 \\ 0 & 0 & 0 & 7 & 0 \end{array} \right) \quad \begin{aligned} D &= 0 \\ C &= 2 \\ B &= 1 \\ A &= 0 \end{aligned}$$

$$\int \frac{2x^3 + x^2 + 5x + 1}{(x^2+3)(x^2-x+1)} dx = \int \frac{dx}{x^2+3} + 2 \int \frac{x}{x^2-x+1} dx = \frac{1}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} + * \quad \textcircled{=}$$

$$* = 2 \int \frac{x}{x^2-x+1} dx = \int \frac{x}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx = \int \frac{x+\frac{1}{2}-\frac{1}{2}}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx = \int \frac{x-\frac{1}{2}}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx + \int \frac{\frac{1}{2}}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx ;$$

$$\int \frac{t dt}{t^2 + \frac{3}{4}} = \frac{1}{2} \int \frac{du}{u}, \quad u = t^2 + \frac{3}{4}$$

$$\textcircled{=} \frac{1}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} + \ln|x^2-x+1| + \frac{2}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} + C$$

$$2. \int \frac{dx}{5+\cos^2 x} \quad \textcircled{=}$$

$$\int t = \tan x \quad dt = \frac{1}{\cos^2 x} \quad dx = \cos^2 x dt$$

$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$

$$\cos^2 x = \frac{1}{t^{2x+1}}$$

$$\textcircled{=} \int \frac{\cos^2 x}{5 + \cos^2 x} dt = \int \frac{dt}{(t^2+4)(5+\frac{1}{t^2+1})} = \int \frac{dt}{5t^2+6} = \frac{1}{5} \int \frac{dt}{t^2+\frac{6}{5}} = \frac{1}{5} \cdot \frac{\sqrt{5}}{\sqrt{6}} \cdot \arctan \frac{t\sqrt{5}}{\sqrt{6}}$$

$$3. \int \frac{\cos x dx}{\sin^3 x + \cos^3 x} \stackrel{|\cos^2 x}{=} \int \frac{d \tan x}{t^3 x + 1} \leftarrow \text{огна нодол + поделителе}$$

$$4. \int \frac{dx}{3 \sin x + 4 \cos x - 5} \textcircled{=}$$

$$\textcircled{=} t = \tan \frac{x}{2} \quad \text{гм-ал. тп-ал. но-ка}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$dx = \frac{2dt}{1+t^2}$$

$$\textcircled{=} \int \frac{\frac{2dt}{1+t^2}}{3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) - 5} = 2 \int \frac{dt}{6t + 4 - 4t^2 + 5 + 5t^2} = 2 \int \frac{dt}{t^2 + 6t + 9} = 2 \int \frac{dt}{(t+3)^2}$$

$$= 2 \int \frac{d(t+3)}{(t+3)^2} = -\frac{2}{t+3} + C$$

$$5. \int \frac{\sqrt{x^2+1}}{x^2+2} dx \textcircled{=} \text{IX} = \tan t; \quad dt = \frac{1}{\cos^2 x} dx$$

$$\sqrt{a^2 - x^2} \quad \text{IX} = a \cdot \sin t$$

$$\sqrt{a^2 + x^2} \quad \text{IX} = a \cdot \tan t$$

$$\sqrt{x^2 - a^2} \quad \text{IX} = \frac{a}{\cos t} \quad \text{или} \quad x = a \cdot \sinh t$$

$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$

$$\textcircled{=} \int \frac{\sqrt{t^2+1}}{(t^2+2)\cos^2 t} dt = \int \frac{dt}{\cos t \cdot (\sin^2 t + 2\cos^2 t)} = \int \frac{\cos t dt}{\cos^3 t (\sin^2 t + 2\cos^2 t)} \textcircled{=} \int \frac{du}{(1-u^2)(u^2+2-2u^2)} \quad \text{IX} = \sin t$$

$$\textcircled{=} \int \frac{du}{(1-u^2)(u^2+2-2u^2)} = \int \frac{du}{(1-u^2)(2-u^2)} = \int \frac{(2-u^2)-(1-u^2)}{(1-u^2)(2-u^2)} du = \int \dots$$

$$6. \int \frac{dx}{(x^2+1)^{\frac{3}{2}}} = \int \frac{\overset{dx}{dt} dt}{\underbrace{\cos^2 t}_{x=\tan t} \underbrace{(\frac{1}{\cos^2 t})^{\frac{3}{2}}}_{y^2+1}} = \int \cos t dt = \sin(\arctan x) + C$$

2-ой способ:

Погружение Адамс:

$$t = (\sqrt{x^2+1})' = \frac{x}{\sqrt{x^2+1}}$$

$$t^2 = \frac{x^2}{x^2+1} \Rightarrow x^2 = \frac{t^2}{1-t^2} \quad x = \frac{t}{\sqrt{1-t^2}}$$

$$dx = \frac{\sqrt{1-t^2} + \frac{t^2}{\sqrt{1-t^2}}}{1-t^2} dt = \frac{dt}{(1-t^2)^{\frac{3}{2}}}$$

$$\int \frac{dx}{(x^2+1)^{\frac{3}{2}}} = \int \frac{dt}{(1-t^2)^{\frac{3}{2}}} (1-t^2)^{\frac{3}{2}} = t + C = \frac{x}{\sqrt{x^2+1}} + C$$

$$7. \int \frac{\sqrt{x^2+1}}{x^2} dx = \int \sqrt{x^2+1} d(-\frac{1}{x}) = -\frac{1}{x} \sqrt{x^2+1} + \int \frac{x dx}{x \sqrt{x^2+1}} = -\frac{1}{x} \sqrt{x^2+1} + \ln|x + \sqrt{x^2+1}| + C$$

$$8. \int \varphi(x) dx, \text{ где } \varphi(x) = \int_0^x e^{-t^2} dt \Leftrightarrow \varphi'(x) = e^{-x^2}$$

$$\int \varphi(x) dx = x \cdot \varphi(x) - \int x \cdot d(\varphi(x)) = x \cdot \varphi(x) - \int \underbrace{x}_{dx=x^2} \cdot \underbrace{e^{-x^2}}_{d(x^2) = -2x dx} dx = x \cdot \varphi(x) + \frac{1}{2} \cdot e^{-x^2} + C$$