Было:

$F_\xi(a) = P(\xi \le a)$

$f_\xi(a) = P(\xi = a)$

$F_\xi(a) = \sum_{b \le a} f_\xi(a)$

$E\xi = \sum_{w\in\Omega} p(w)\,\xi(w) = \sum_a a \cdot P(\xi = a)$

$E(\xi + \eta) = E(\xi) + E(\eta)$

$E(\xi - E\xi) = E\xi - \overset{E\xi}{E E\xi} = 0$

$\xi \mapsto \eta = \xi - E\xi$

$D\xi = E(\xi - E\xi)^2 = E\xi^2 - (E\xi)^2$

$D(\xi+\eta) = E(\xi+\eta)^2 - (E(\xi+\eta))^2 = E\xi^2 + E\eta^2 + 2E\xi\cdot\eta - (E\xi)^2 - (E\eta)^2 - 2E\xi E\eta =$

$= D\xi + D\eta + 2(E\xi\eta - E\xi E\eta)$

$Cov(\xi, \eta) = E\xi\eta - E\xi E\eta$ ← Ковариация

$D\xi = Cov(\xi, \xi)$

$Corr(\xi, \eta) = \dfrac{Cov(\xi,\eta)}{\sqrt{D\xi\, D\eta}}$ ← Корреляция

Теорема:

$-1 \le Corr(\xi, \eta) \le 1$

$\triangleright$ Пусть $\alpha = \xi - \lambda\eta$

$D\alpha = E\xi^2 - 2\lambda E\xi\eta + \lambda^2 E\eta^2 - (E\xi)^2 + 2\lambda E\xi E\eta - \lambda^2 (E\eta)^2 \ge 0$

$$D_\lambda = E\xi + 2\lambda\, Cov(\xi,\eta) + \lambda^2 D\eta \geq 0$$

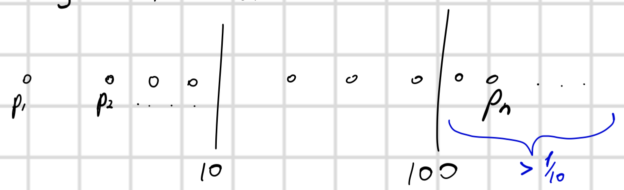
$$4\, Cov(\xi,\eta)^2 - 4 D\xi\, D\eta \leq 0 \qquad \xi = \lambda\eta + C \qquad \blacktriangleleft$$

Хвостовые неравенства

Пр.:

$\exists\ \xi,\ E\xi = 10,\ \xi \geq 0$

$P(\xi \geq 100) < \frac{1}{2}$?



Теорема (Нер-во Маркова)

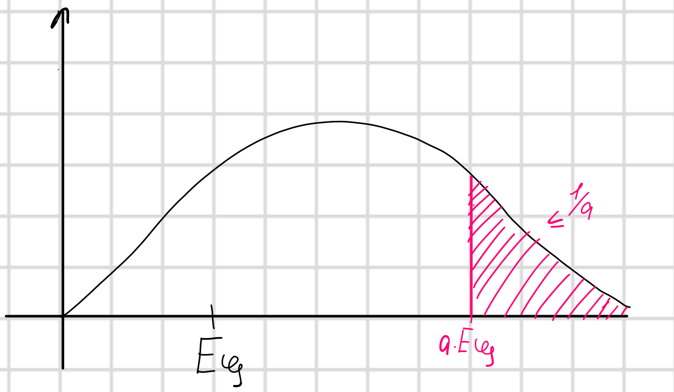
$\exists\ \xi \geq 0,\ \xi \not\equiv 0$

$$P(\xi \geq a \cdot E\xi) \leq \frac{1}{a}$$

$$\triangleright\ E\xi = \sum_v v \cdot P(\xi = v) = \underbrace{\sum_{v < a\cdot E\xi} v \cdot P(\xi = v)}_{\geq 0} + \sum_{v \geq a\cdot E\xi} v \cdot P(\xi = v) \ \textcircled{\geq}$$

$$\geq a \cdot E\xi \cdot \sum_{v \geq a\cdot E\xi} P(\xi = v)$$

$$\textcircled{\geq}\ a \cdot E\xi \cdot P(\xi \geq a \cdot E\xi) \qquad \blacktriangleleft$$



Если $a = \frac{c}{E\xi} \rightarrow P(\xi \geq c) \leq \frac{E\xi}{c}$

$$D\xi = E(\xi - E\xi)^2$$

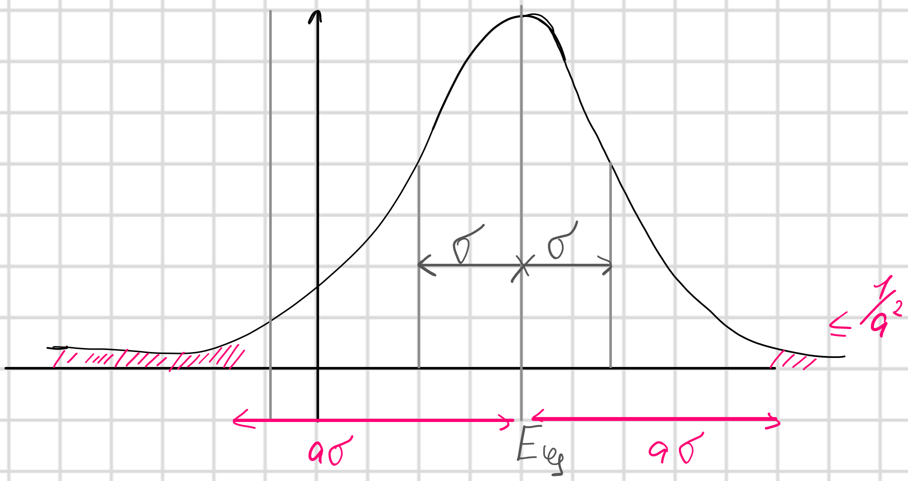
$$\eta = (\xi - E\xi)^2$$

$$P\left((\xi - E\xi)^2 \geq a \cdot D\xi\right) \leq 1/a^2$$

Теорема (Нер-во Чебышева)

$$P\left(|\xi - E\xi| \geq a\sigma\right) \leq 1/a^2$$ ← среднеквадратичное отклонение

$$P\left(|\xi - E\xi| \geq c\right) \leq \frac{D\xi}{c^2}$$



Пр.:

10 монет, # единиц ξ

$E\xi = 5$

$D\xi = 2{,}5$

$$P(\xi \leq 0) \leq P\left(|\xi - E\xi| \geq 5\right) \leq \frac{2{,}5}{25} \leq \frac{1}{10}$$

Пр.:

нечестная монета, $p \neq 1/2$

Вопрос: $p > 1/2$ или $p < 1/2$?

c единиц, и $n-c$ нулей

$\exists\, c < n/2$, $p > 1/2 \Rightarrow pn > n/2$

$$P(\xi = c) \leq P(\xi \leq c) \leq P\left(|\xi - pn| \geq pn - c\right) \leq P\left(|\xi - pn| \geq n/2 - c\right) \leq \frac{n}{4(n/2 - c)^2}$$

$\exists\ \xi_i,\ P(\xi_i = 1) = p,\ P(\xi_i = 0) = q$

$\forall\ \xi = \sum_{i=1}^{n} \xi_i,\ E\xi = n \cdot p = \mu$, тогда:

Теорема (Граница Чернова)

$$P(|\xi - \mu| \geq \delta\mu) \leq e^{-\mu\frac{\delta^2}{3}}$$