1. $$\lim_{x\to0}(\cos x)^{-\frac{1}{x^2}} = \lim_{x\to0}\left(\left(1-1+\cos x\right)^{\frac{1}{-1+\cos x}\cdot(-1+\cos x)}\right)^{-1/x^2} = e^{\lim\limits_{x\to0}\frac{1-\cos x}{x^2}} =$$

$$(1+x)^{1/x}\xrightarrow[x\to0]{} e$$

$$= e^{\lim\limits_{x\to0}\frac{\frac{x^2}{2}}{x^2}} = e^{\frac12} = \sqrt{e}$$

$\lim e^{f(x)} = e^{\lim f(x)}$ Только потому, что e — непрерывна !

$(g(x))^{f(x)} = (\lim g)^{\lim f}$ g и f должны быть непрерывны

$\boxed{\text{2-й способ:}}$

$$a^b = e^{\ln(a^b)} = e^{b\ln a} =$$

$$\lim_{x\to0}(\cos x)^{-1/x^2} = \lim e^{-\frac{\ln\cos x}{x^2}} = \exp\lim_{x\to0}\left(-\frac{\ln\cos x}{x^2}\right) = \exp\lim_{x\to0}\left(-\frac{\ln(\overbrace{\cos x-1}^{\to 0}+1)}{x^2}\right) =$$

$$\exp\lim\left(-\frac{\cos x-1}{x^2}\right) = \exp\left(\frac12\right)$$

2) $$\lim_{x\to0}(\sin x+\cos x)^{-\frac{1}{\lg(1-2x\sin x)}} = \exp\lim_{x\to0} -\frac{\ln(\sin x+\cos x)}{\lg(1-2x\sin x)} = \exp\lim -\frac{\ln(1-1+\sin x+\cos x)}{\lg(1-2x\sin x)} =$$

$$= \exp\lim_{x\to0} -\frac{\sin x+\cos x-1}{\frac{-2x\sin x}{\ln 10}} = \exp\lim_{x\to0}\frac{\ln 10\cdot(\sin x+\cos x-1)}{2x\cdot x} = \exp\frac{\ln 10\left(\lim\frac{\sin x}{x^2} - \lim\frac{1-\cos x}{x^2}\right)}{2} =$$

$$= \exp\frac{\ln 10\left(\lim\limits_{x\to0}\frac1x - \frac12\right)}{2} = \exp(\infty) = e^{\infty}$$ ← неопределенность

част. пределы

$$\lim_{x\to0^-} = \exp(-\infty) = 0$$

$\neq$ $\Rightarrow$ предела нет

$$\lim_{x\to0^+} = \exp(+\infty) = +\infty$$

$$\ln(\sin x+\cos x)=\ln\big(x+o(x)+1+o(x)\big)=x+o(x)+o(x)+o(x)=o(x)=x+o(x)\sim x$$

$$\ln(1+x)\sim x \Rightarrow \ln(1+x)=x+o(x)$$

$$\sin x\sim x \Rightarrow \sin x = x+o(x)$$

$$\cos x\sim 1-\frac{x^2}{2} \Rightarrow \cos x = 1-\underbrace{\frac{x^2}{2}+o(x^2)}_{o(x)}=1+o(x)$$

3) $$\lim_{x\to-1}\frac{\sqrt{\pi}-\sqrt{\arccos x}}{\sqrt{x+1}}=\lim_{x\to-1}\frac{\pi-\arccos x}{\sqrt{x+1}\,(\underbrace{\sqrt{\pi}+\sqrt{\arccos x}}_{\to 2\sqrt{\pi}})}=\frac{1}{2\sqrt{\pi}}\lim_{x\to-1}\frac{\pi-\arccos x}{\sqrt{x+1}}\ (=)$$

замена $t=\pi-\arccos x$, $x=\cos(\pi-t)=-\cos t$

$t\to0^+$

можем т.к. $\sqrt{x}$ – непрерыв

$$(=)\ \frac{1}{2\sqrt{\pi}}\lim_{t\to0^+}\frac{t}{\sqrt{1-\cos t}}=\frac{1}{2\sqrt{\pi}}\sqrt{\lim\frac{t^2}{1-\cos t}}=\frac{\sqrt{2}}{2\sqrt{\pi}}$$

если $f\sim f_1$, то $\sqrt{f}\sim\sqrt{f_1}$

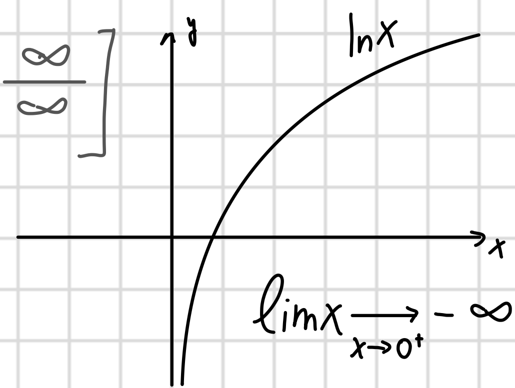
$\boxed{\text{2-й способ}}$

$$\lim_{x\to-1}\frac{\pi-\arccos x}{\sqrt{x+1}}=\lim_{x\to-1}\frac{\sin(\pi-\arccos x)}{\sqrt{x+1}}=\lim_{x\to-1}\frac{\overset{\to0}{\sin(\arccos x)}}{\sqrt{x+1}}\ (=)$$

$x\sim\sin x$

$$(=)\ \lim_{x\to-1}\frac{\sin(\arccos x)}{\sqrt{x-1}}\overset{\text{О.Т.Т}}{=}\lim_{x\to-1}\frac{\sqrt{1-x^2}}{\sqrt{x+1}}=\lim_{x\to-1}\sqrt{1-x}=\sqrt{2}$$

4) $$\lim_{x\to0^-}\frac{\ln(2x^2-x)}{\ln(x^4+x^2-x)}\ (=)\left[\frac{\infty}{\infty}\right]$$



$$\circleddash \lim_{x\to 0^-} \frac{\ln(-x(1-2x))}{\ln(-x(1-x^3-x))} = \lim_{x\to 0^-} \frac{\ln(-x)+\ln(1-2x)}{\ln(-x)+\ln(1-x^3-x)} = \lim_{x\to 0^-} \frac{1+\frac{\ln(1-2x)}{\ln(-x)}}{1+\frac{\ln(1-x^3-x)}{\ln(-x)}} =$$

$\frac{\ln(1-2x)}{\ln(-x)} \to 0$, $\frac{\ln(1-x^3-x)}{\ln(-x)} \to 0$

$$= \frac{1}{1} = 1$$

или $\ln(2x^2-x) \sim \ln(-x)$

$\ln(x^4+x^2-x) \sim \ln(-x)$

5) Найти эквивалент вида Ax^{α}

1. $f(x) = 3\sin^2(x^2) - 5x^7$, а) $x \to 0$

б) $x \to \infty$

а) $\sin^2 x \sim x^2,\ x\to 0 \Rightarrow \sin x^2 = x^2 + o(x^2)$

$$f(x) = 3(x^4+o(x^4)) - 5x^7 = 3x^4 - \underbrace{5x^7}_{5x^3\cdot x^4 = o(x^4)} + o(x^4) = 3x^4 + o(x^4) \Rightarrow \boxed{f \sim 3x^4}$$

$5x^3 \to 0$

б) $$\frac{f(x)}{x^7} = \frac{3\overbrace{\sin^2(x^2)}^{\text{огранич.}}}{x^7} - 5 \longrightarrow 0-5 = -5 \Rightarrow \boxed{f(x) \sim -5x^7}$$

2. $f(x) = \sqrt[5]{1+x^2} - \frac{e^x}{\sqrt[3]{\cos x}}$, $x \to 0$

$$f(x) = \underbrace{(1+x^2)^{1/5}}_{1+\frac{1}{5}x^2+o(x^2)} - \underbrace{e^x}_{1+o(1)} \cdot \underbrace{(\cos x)^{-1/3}}_{1-\frac{x^2}{2}+o(x^2)} = 1 + \frac{1}{5}x^2 + o(x^2) - (1+o(1))\cdot\left(1-\frac{x^2}{2}+o(x^2)\right)^{-1/3} \circleddash$$

$$\boxed{\begin{aligned}&(1+x)^{\alpha} - 1 \sim \alpha x\\ &(1+x)^{\alpha} = 1+\alpha x + o(x)\end{aligned}}$$

$$\circleddash\ 1 + \frac{1}{5}x^2 + o(x^2) - (1+o(1))\cdot\left[\left(1+\left(-\frac{1}{3}\right)\cdot\left(-\frac{x^2}{2}+o(x^2)\right)\right) + \overbrace{o\left(-\frac{x^2}{2}\right) + o(x^2)}^{o(x^2)}\right] =$$

$$= 1 + \frac{1}{5}x^2 + o(x^2) - (1+o(1))\cdot\left[1+\frac{x^2}{6}+o(x^2)\right] = \cancel{1} + \frac{1}{5}x^2 + o(x^2) - \cancel{1} - \frac{x^2}{6} - o(x^2) - o(1) -$$

$$- \underbrace{\frac{x^2}{6}\cdot o(1)}_{o(x^2)} - \underbrace{o(1)\ \ldots}_{o(x^2)} = \frac{1}{30} + o(x^2) + o(1) \circleddash$$

$e^x - 1 \sim x$

$e^x = 1 + x + o(x)$

$\Leftrightarrow \frac{1}{30}x^2 + o(x^2) - x + o(x) = -x + o(x) + \underbrace{\frac{1}{30}x^2 + o(x^2)}_{o(x)} = -x + o(x) \Rightarrow f \sim -x$