Total Type Error Localization and Recovery with Holes

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Reality check

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Most of the code we write is
ill-typed

Localization and recovery

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- **recover**: even when an error is encountered, continue to provision downstream services and find other errors

i.e., code completion, type hints, and other downstream services shouldn't suddenly all fail because of one error!

Observations in practice

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OCaml: the first branch is right

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1 | let x = if true then 5 else false in f x
                                ^^^^^
Error: This expression has type bool but an expression
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Haskell: 5 is the problem

$$\text{let } f = \lambda b : \text{bool}. \cdots \text{ in let } x = \text{if } \text{true} \text{ then } \boxed{5} \text{ else } \text{false} \text{ in } f\ x$$

```
Main.hs:4:26: error: [GHC-39999]
    • No instance for ‘Num Bool’ arising from the literal ‘5’
  |
4 | _ = let x = if True then 5 else False in f x
  |                          ^
```

Rust: maybe 5 is also the problem?

$$\text{let } f = \lambda b : \text{bool}. \ \cdots \ \text{in let } x = \text{if true then } 5 \text{ else false in } f\ x$$

```
error[E0308]: ‘if‘ and ‘else‘ have incompatible types
 -> src/main.rs:2:30
  |
6 | let x = if true  5  else  false ; f(x);
  |                  -        ^^^^^ expected integer, found ‘bool‘
  |                  |
  |                  expected because of this
```

Hazel: the whole thing's all messed up!

$$\text{let } f = \lambda b : \text{bool}.\ \cdots \text{ in let } x = \text{if true then } 5 \text{ else false in } f\ x$$

```
let f = fun b : Bool -> ⬡ in
let x = if true then 5 else false in f(x)
```

Γ EXP ? If expression Branches have inconsistent types: Int , Bool

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- localization can be varied, often guessing about *user intent*
- recovery necessitates reasoning without complete knowledge about types
- decisions can influence other downstream decisions

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$$\Gamma \vdash e : \tau$$

$$\Downarrow$$

If a type error appears **anywhere**,
the program is meaningless **everywhere**.

The goal

We'd like a way to formally specify type checkers that are capable of localizing and recovering from errors

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> These semantics should give meaning to *all well-typed and ill-typed programs.*

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Uniting *local* and *global* type inference for principled total type error localization and recovery:

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Uniting *local* and *global* type inference for principled total type error localization and recovery:

- the **marked lambda calculus**: a judgmental framework for bidirectional type error localization and recovery

- **type hole inference**: a global, constraint-based system that is neutral in error localization and recovery

Marked lambda calculus: a tutorial

Start: a small gradually typed lambda calculus*

$$\begin{array}{rcl} \tau & ::= & ? \mid \mathsf{num} \mid \mathsf{bool} \mid \tau \rightarrow \tau \\ e & ::= & x \mid \lambda x : \tau .\ e \mid e\ e \mid n \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \end{array}$$

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with a standard bidirectional type system

$\Gamma \vdash e \Rightarrow \tau$ (e synthesizes type τ)

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*We need only the *static semantics* for ill-typed programs!

Typing variable occurrences

Synthesizing the type of a variable is standard:

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$$\textsc{SVar}\ \frac{\boxed{x : \tau \in \Gamma}}{\Gamma \vdash x \Rightarrow \tau}$$

But what if $x \notin \mathrm{dom}(\Gamma)$?

Typing variable occurrences

How do we handle this failure case?

```
let rec syn ctx e =
  match e with
    Var x ->
      match Ctx.lookup ctx x with
        Some ty -> ty
        None    -> ???
    ...
```

Typing variable occurrences

How do we handle this failure case?

```
let rec syn ctx e =
  match e with
    Var x ->
      match Ctx.lookup ctx x with
        Some ty -> ty
        None    -> failwith (x ++ " is unbound")
    ...
```

Typing variable occurrences

How do we handle this failure case?

```
let rec syn ctx e =
  match e with
    Var x ->
      match Ctx.lookup ctx x with
        Some ty -> Ok(ty)
        None    -> Error(UnboundError( … ))
    ...
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We've *localized* the error: "this occurrence of x is unbound!"

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Solution. $?$

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How can we *recover*?

Solution. $?$ $\leftarrow$ unknown type

From type checking to marking

Idea. Augment the type checking process with *marking*!

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- use the unknown type to encapsulate missing type information

$$\frac{}{? \sim \tau} \qquad \frac{}{\tau \sim ?} \qquad \frac{}{\tau \sim \tau} \qquad \frac{\tau_1 \sim \tau_1' \qquad \tau_2 \sim \tau_2'}{\tau_1 \to \tau_2 \sim \tau_1' \to \tau_2'}$$

A two-layer calculus

Supplement the *unmarked* syntax

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into a *marked* one that contains *error marks*:

$$\check{e} \ ::= \ x \mid \lambda x : \tau .\, \check{e} \mid \check{e}\ \check{e} \mid n \mid \text{true} \mid \text{false} \mid \text{if } \check{e} \text{ then } \check{e} \text{ else } \check{e} \mid (\!|x|\!)_{\square}$$

$(\!|x|\!)_{\square}$ is a *marked term* denoting a free occurrence of x

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Extend the original typing judgments

$$\Gamma \vdash e \Rightarrow \tau \quad (e \text{ synthesizes type } \tau)$$
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into the bidirectional **marking judgments**:

$$\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau \quad (e \text{ is marked into } \check{e} \text{ and synthesizes type } \tau)$$
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Type-based semantic services use marked terms!

$$\Gamma \vdash_{M} \check{e} \Rightarrow \tau \quad (\check{e} \text{ synthesizes type } \tau)$$
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Marking free variables

One typing rule for variables

$$\dfrac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau}\ \textsc{SVar}$$

becomes two synthetic marking rules:

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Marking local inconsistencies

The standard subsumption principle:

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… and the rest

$$
\begin{array}{rcll}
e & ::= & \cdots & \\
& | & (\!|\check{e}_1|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e}_2 & \check{e}_1 \text{ synthesizes non-matched arrow type} \\
& | & (\!|\mathbf{if}\ \check{e}_1\ \mathbf{then}\ \check{e}_2\ \mathbf{else}\ \check{e}_3|\!)_{\not\sqcup} & \text{branches synthesize inconsistent types} \\
& | & (\!|\lambda x : \tau.\ \check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}} & \text{analysis against non-matched arrow type} \\
& | & (\!|\lambda x : \tau.\ \check{e}|\!)_{:} & \text{ascription inconsistent with domain}
\end{array}
$$

A total marking

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$$\Downarrow$$

Theorem 2.1. Totality

For all Γ and e, $\exists \check{e}, \tau$ s.t. $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$.
For all Γ, e, and τ, $\exists \check{e}$ s.t. $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau$.
(Every unmarked term can be marked under any context!)

A sensible marking

Theorem 2.2. Well-Formedness

If $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$, then $\Gamma \vdash_{M} \check{e} \Rightarrow \tau$ and $\check{e}^{\square} = e$.
If $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau$, then $\Gamma \vdash_{M} \check{e} \Leftarrow \tau$ and $\check{e}^{\square} = e$.
(Marking only adds marks, *i.e.*, marking then erasing is identity!)

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Theorem 2.3(1). Well-Typed Terms

If $\Gamma \vdash_{U} e \Rightarrow \tau$, then $\exists \check{e}$ s.t. $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$ and $\check{e}$ markless.
If $\Gamma \vdash_{U} e \Leftarrow \tau$, then $\exists \check{e}$ s.t. $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau$ and $\check{e}$ markless.
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A sensible marking

e

(*marking*)

$$\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$$
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(*erasure*) $\_^{\square}$

$$\Gamma \vdash_{M} \check{e} \Rightarrow \tau$$
$$\Gamma \vdash_{M} \check{e} \Leftarrow \tau \dashrightarrow \text{(downstream services)}$$

(*marked*)

A sensible marking

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(marked)

Metatheory completely mechanized in Agda [hazelgrove/error-localization-agda]

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- Begin with a bidirectional gradually* typed language
  - *Only need the static parts for marking ill-typed programs!
- Derive marking rules from each typing rule
  - Consider the “success” case
  - Consider the “failure” cases, introducing error marks
- *Not* prescriptive *w.r.t.* localization strategy
  - We formalize three possible localization strategies for if-then-else with inconsistent branches

Marking global inconsistencies?

Consider this program:

$$\lambda f : ?\,.\ f\ (f + 1)$$

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Marking global inconsistencies?

Consider this program:

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$f : ?$, so the bidirectional system operates gradually, but f is

1. applied as a function (a function?)
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Global, constraint-based type checking would have caught this!

Layers upon layers

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Get the best of both worlds by layering **constraint-based inference** atop the marked lambda calculus

- The marked lambda calculus localizes and recovers predictably from *local inconsistencies*

- **Type hole inference** solves and marks *global inconsistencies*
  - Downstream service to supplement the marked lambda calculus

Type hole inference in Hazel

```
fun f : ! -> f (f + 1)
```

Γ TYP ? Empty type hole conflicting constraints Int Int ->

Type hole inference in Hazel

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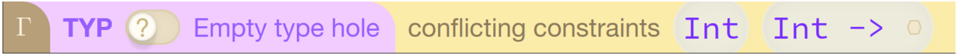

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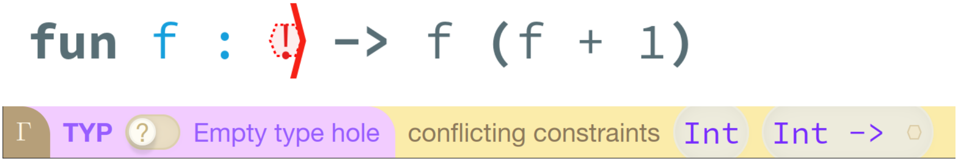
Type hole inference in Hazel



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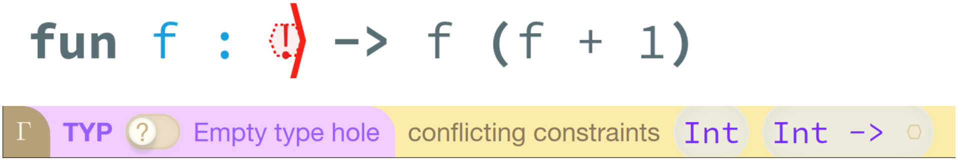

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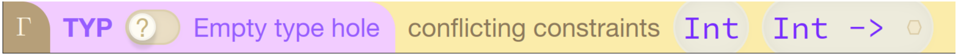
3. When unsolvable, maintain partial solutions: possible type fillings, which are offered to the user for selection

Type hole inference in Hazel

```
fun f : ! -> f (f + 1)
```



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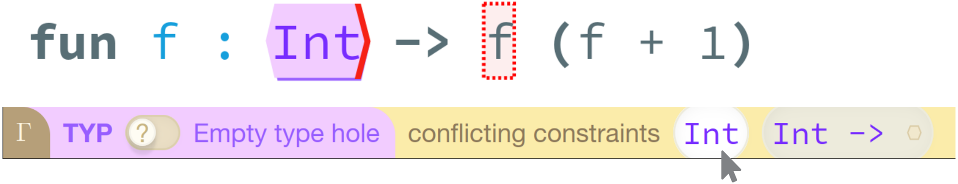

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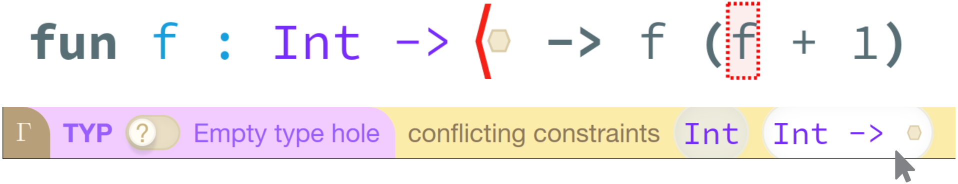

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Consider using these techniques for your next language!

Marking many ways

MKSIF

$$\frac{}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \hookrightarrow \qquad\qquad \Rightarrow}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

Marking many ways

$$\dfrac{\Gamma \vdash e_1 \hookrightarrow \check{e}_1 \Leftarrow \mathsf{bool}}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \hookrightarrow \qquad \Rightarrow}\ \textsc{MKSIf}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

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MKSInconsistentBranches

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. . . .

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MKSIf'

(prioritize first branch)

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. . . .

MKSIf'

(prioritize first branch) (blame second branch)

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Add type provenances p to the *marked language*:

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\check{e} & ::= & \cdots \mid (\!| x |\!)^u_\square \mid (\!| \check{e} |\!)^{\Rightarrow,\ u}_{\blacktriangleright_{\not\to}} \mid (\!| \check{e} |\!)^u_{\not\sim}
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To unify $\tau_1 \approx \tau_2$, recursively merge PotentialTypeSet(τ_1) and PotentialTypeSet(τ_2) without substituting

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and solving gives

$$\text{PotentialTypeSet}(?^{1}) = \{\}$$

Unification with inconsistencies

$$\lambda f : ?^{1} \,.\, f\ (f + 1)$$

yields the (inconsistent) constraints

$$\{?^{1} \approx \underset{\wedge}{?^{\to_L(1)}} \to ?^{\to_R(1)}, \mathbf{num} \approx ?^{1}, \mathbf{num} \approx ?^{\to_L(1)}\}$$

and solving gives

$$\text{PotentialTypeSet}(?^{1}) = \{\underline{\{?^{\to_L(1)}\} \to \{?^{\to_R(1)}\}}\}$$

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