Beamer Example

eqs

Fujitsu Research

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Uniform convergence

Definition

A sequence of functions $f_n \colon \mathbb{R} \to \mathbb{R}$ *converges uniformly* to a function $f \colon \mathbb{R} \to \mathbb{R}$ if for all $\epsilon > 0$ there exists an $N \in \mathbb{N}$ such that $n \geq N$ implies

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Theorem

Let $f_n\colon \mathbb{R} \to \mathbb{R}$ be continuous and converge uniformly to $f\colon \mathbb{R} \to \mathbb{R}$. Then f is continuous.