EFFICIENT CONNECTIONS

Ramon Ferrer i Cancho and Ricard Solé have proposed a method to produce graphs with a fixed number of nodes N and having, at the same time, few links and a small typical distance between nodes. The idea is to construct a graph G by changing the number and the arrangement of the links in order to minimize the following cost function:

$$c(G,\alpha) = \alpha \frac{L}{(N+1)/3} + (1-\alpha)\frac{\sum_{i<j} a_{ij}}{\binom{N}{2}}$$

where $A = a_{ij}$ is the adjacency matrix of the graph and $\alpha \in [0,1]$ is a parameter to tune the weight of the two different contributions. The first term in $c(G,\alpha)$ is the average distance L between two nodes in the graph, normalized by the maximum value L can take in a connected graph with N nodes, namely $L^{\text{chain}} = \frac{N+1}{3}$. The second term is the graph density, i.e. the ratio between the number of links in the graph and the maximum possible value of this number.

Your task is to demonstrate how graphs can be constructed that minimize this cost function for different values of α and numerically analyze the degree distributions that they entail.