

Lighting/Illumination/Reflection Model



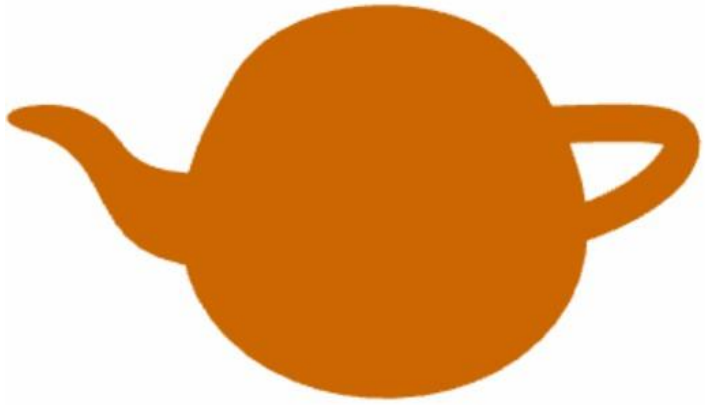
Some of the contents used in this lecture belong to...

- Dr Jon Shiach, Manchester Metropolitan University, “Direct Lighting Model”, <https://www.youtube.com/watch?v=7CdS8oOJtVA>
- Mr. Jacobson, University of Toronto, “WebGL Phong Shading”, <http://www.cs.toronto.edu/~jacobson/phong-demo/>
- RapidCompact, “Simplifying a 3D Mesh”, <https://rapidcompact.com/doc/cli/latest/Simplify/index.html>

Basic Terms

- **Illumination:** the transport of energy from light sources to surfaces & points
 - Local illumination
 - Global illumination
- **Lighting model or Illumination model:** Express the factors determining a surface's color or luminous intensity (outgoing or reflected light) at a particular 3D point

Effects of Lighting

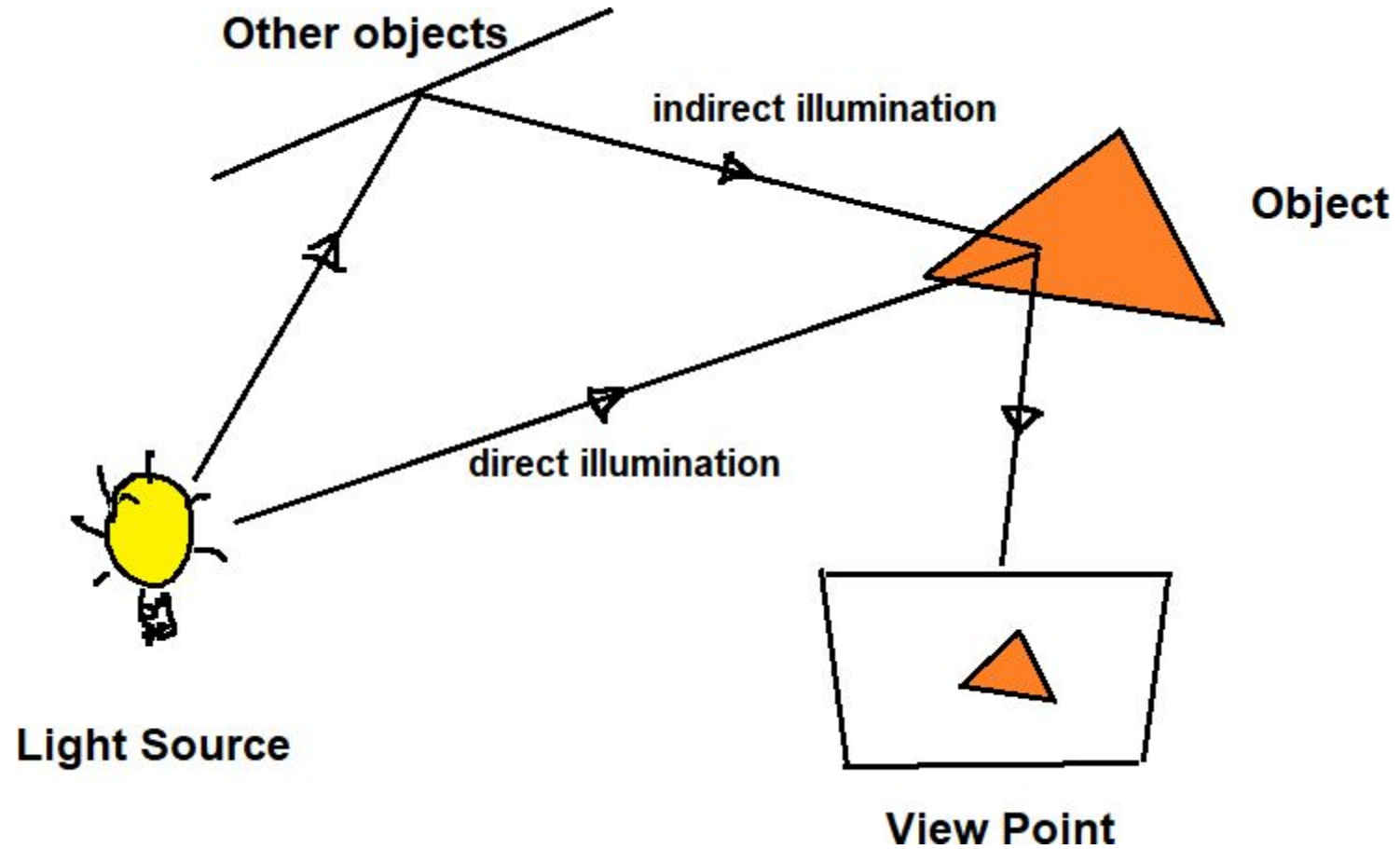


Pixel colours only



Pixel colours and lighting

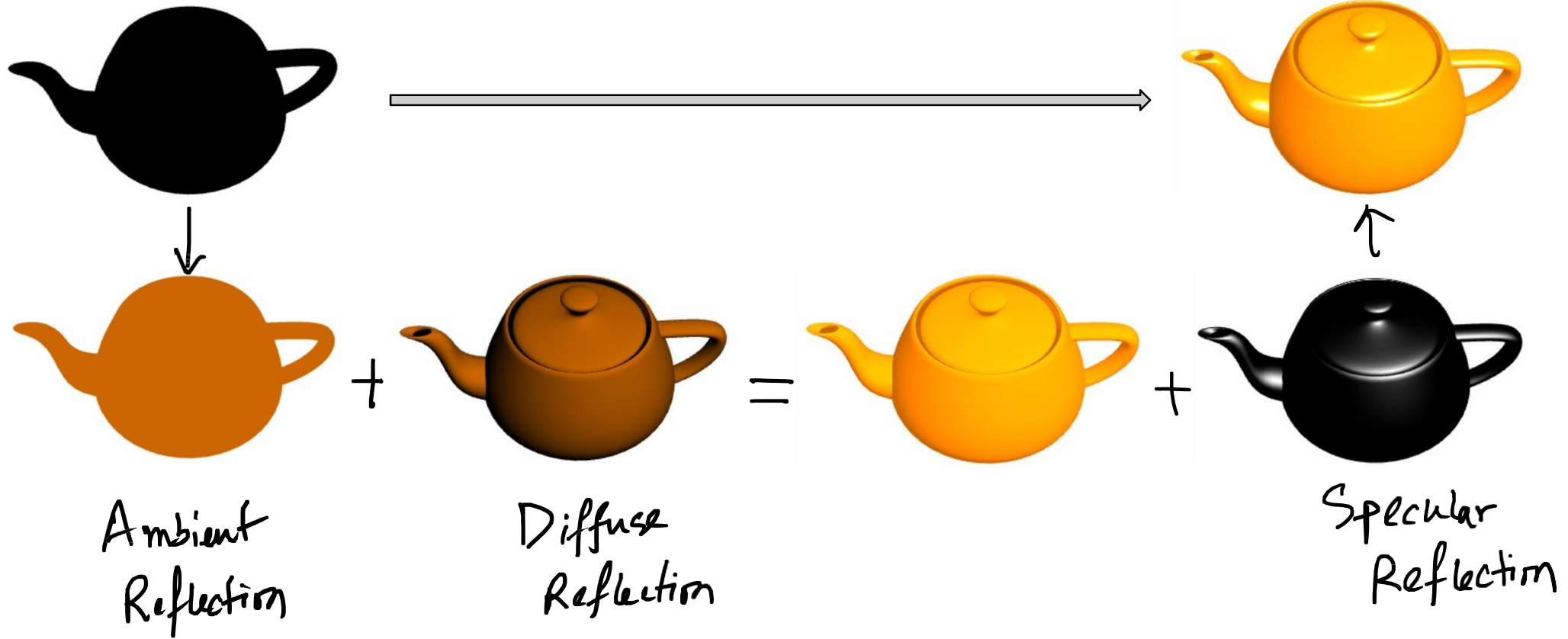
Illumination



Components of Illumination

- Two components of illumination:
 1. Light sources
 2. Surface properties
- Light source described by a luminance/intensity 'I'
 - Each color is described separately
 - $I = [I_r \ I_g \ I_b]$
- Types of Light Sources:
 1. Ambient Light
 2. Diffuse Light
 3. Spot Light

Phong's Reflection Model



Ambient Light

- No identifiable source or direction
- Product of multiple reflections of light from the many surfaces present in the environment
- Computationally inexpensive



Ambient Light

Categories:

1. Global ambient light
 - Independent of light source
 - Lights entire scene
 - Example: reflection of sunlight from several surfaces
2. Local ambient light
 - Contributed by additional light sources
 - Can be different for each light and primary color
 - Example: Reflection of fluorescent lamps from several surfaces

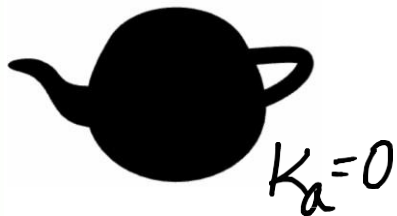
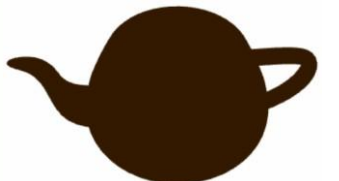
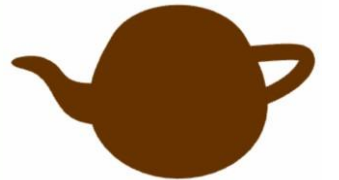
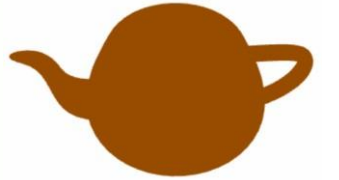
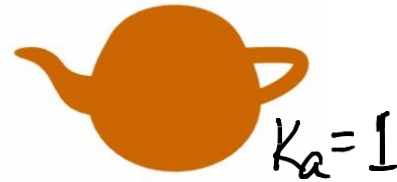
Ambient Reflection

- **Ambient reflection** is the reflection of light that does not come directly from a light source
- Even in a darkened room, we can make out the edges of objects – this is because of light bouncing off of objects
- Since Phong is a **direct** lighting model, we assume that ambient light falls equally on all objects, i.e.,

$$A = \underline{I_a} k_a //$$

where I_a is the intensity of the ambient light and $k_a \in [0, 1]$ is the **ambient coefficient**

- k_a is set to provide the right amount of ambient light for a scene, e.g., $k \rightarrow 1$ for bright scenes and $k \rightarrow 0$ for dark or nighttime scenes.



Ambient Reflection Coefficient

- Effect of adding ambient light to the diffuse light reflected by a sphere
- Diffuse source intensity is 1.0
Diffuse reflection coefficient is 0.4
Ambient source intensity is 1.0
- Moving from left to right the ambient reflection coefficient takes on values: 0.0, 0.1, 0.3, 0.5, and 0.7
 - Too little ambient light makes shadows too deep and harsh
 - Too much makes the picture look washed out and bland

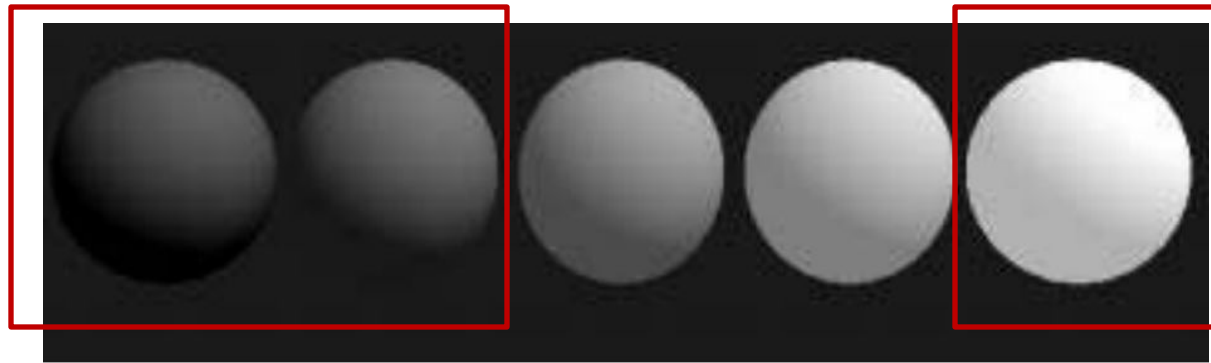
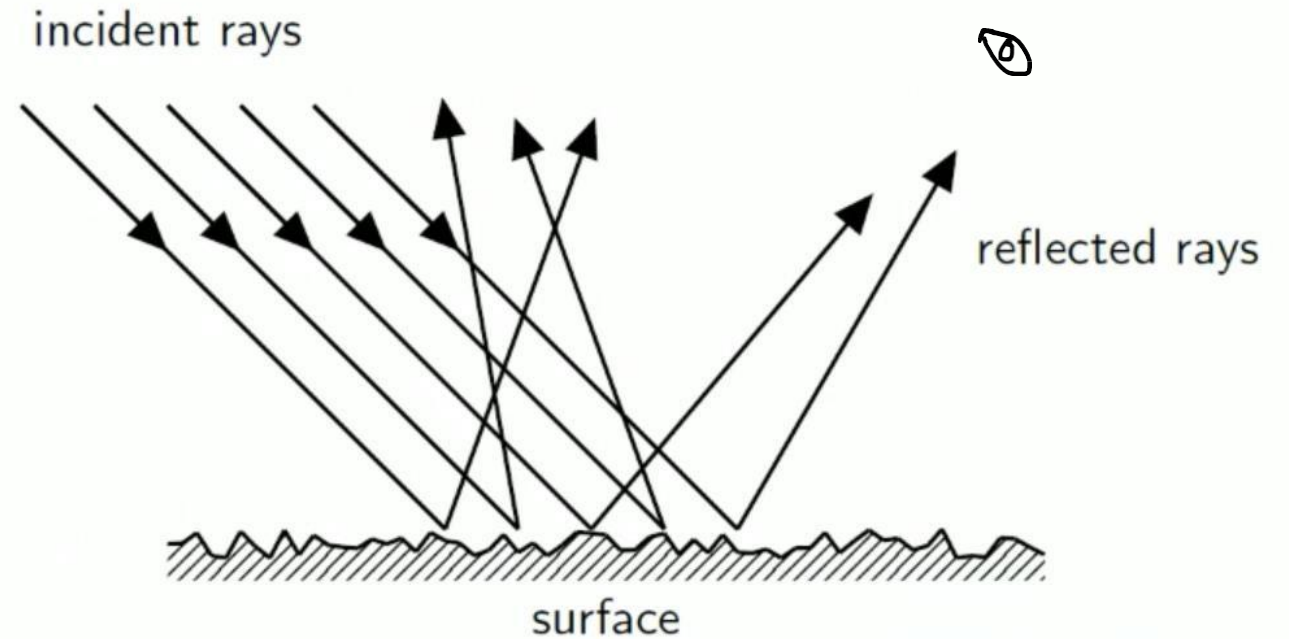


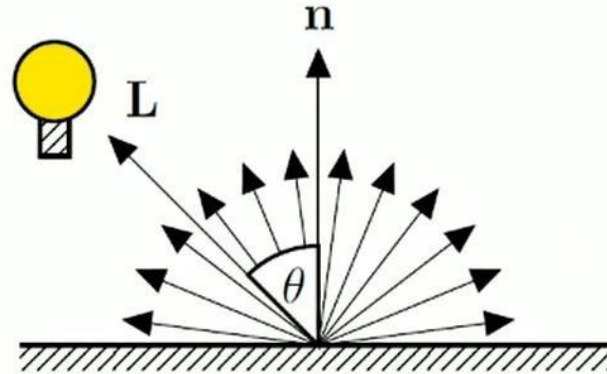
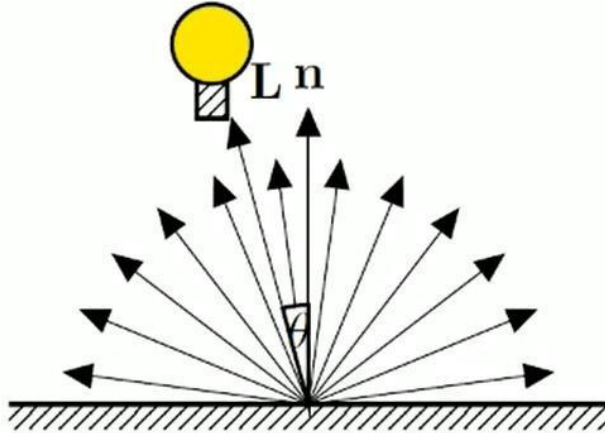
Figure 8.16. On the effect of ambient light.

Diffuse Reflection

- ▣ Parallel rays from light source
- ▣ Reflected rays are scattered
- ▣ Direction of reflection
- ▣ Some are visible, some are not



Diffuse Reflection - Phong's model

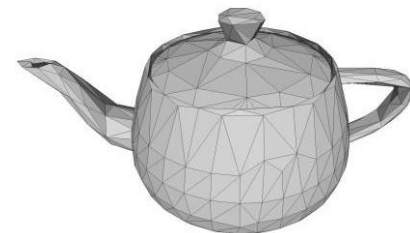
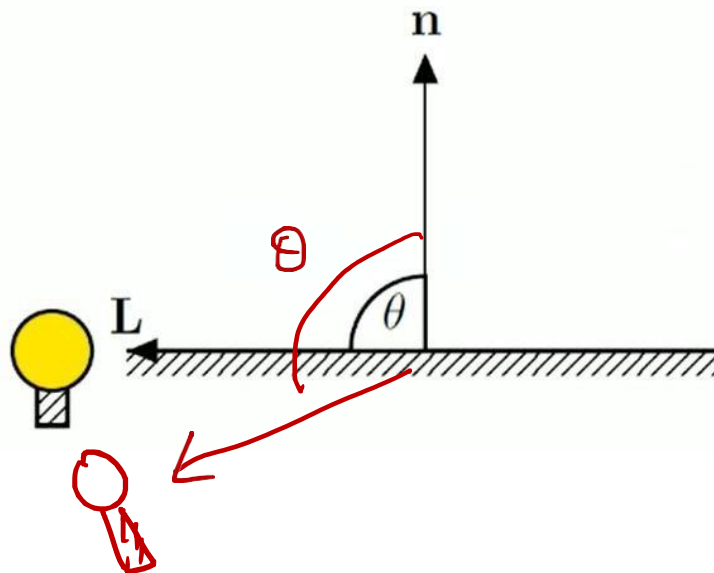
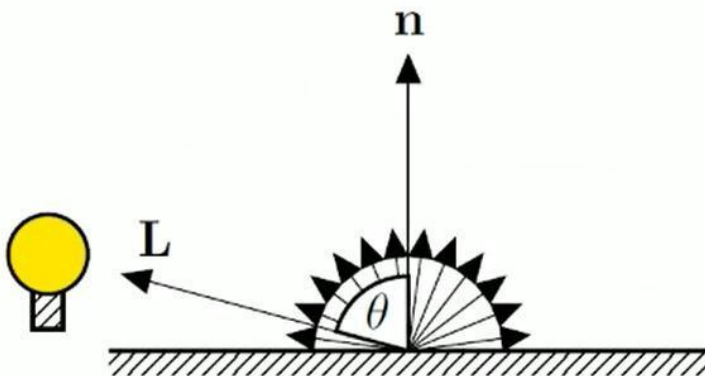


☐ Point light source

☐ Light reflected equally in all direction

☐ "Magnitude" of reflection depends on θ

☐ Direction of L reversed?
 $\rightarrow (-L)$



Diffuse Reflection - Phong's model



- Phong's diffuse reflection model depends upon the position of the light source relative to the surface

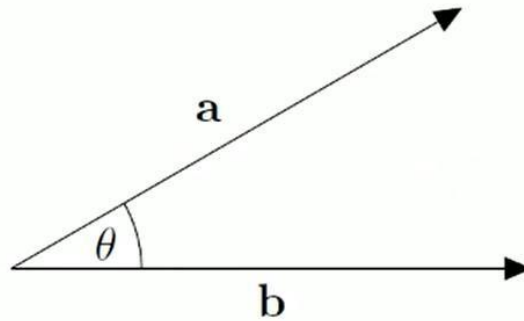
$$D = I_p k_d \max[\cos(\theta), 0]$$

where

- I_p is the intensity of the point light source
 - $k_d \in [0, 1]$ is the **diffuse coefficient**
 - θ is the angle between the lighting vector and the surface normal
- The $\max[\cos(\theta), 0]$ is used so that no light is reflected if the light source is behind the surface

Diffuse Reflection - Phong's model

$$\underline{\text{Dot product} \leftrightarrow \cos(\theta)}$$



- The definition of the dot product is

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos(\theta)$$

$$a \cdot b = \cos(\theta)$$

- If \mathbf{L} and \mathbf{n} are unit vectors then

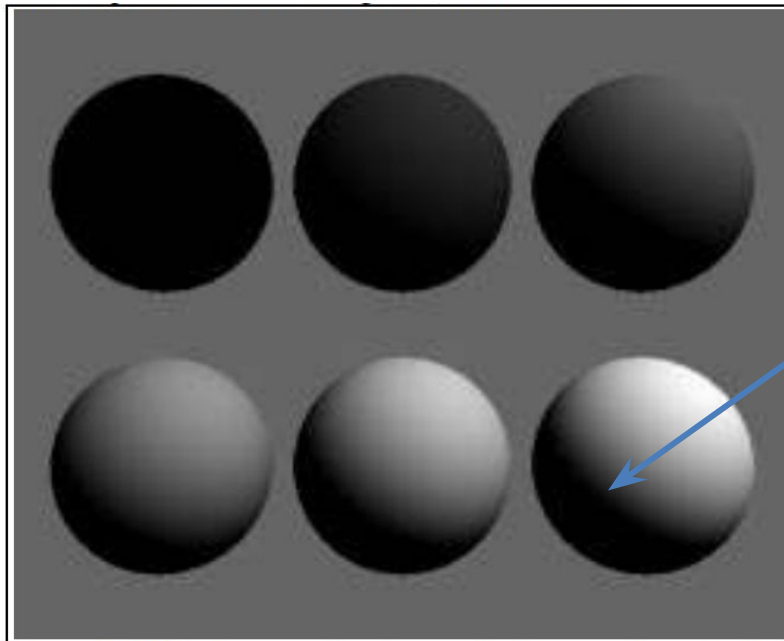
$$\mathbf{L} \cdot \mathbf{n} = \cos(\theta)$$

so we can replace the expensive cosine calculation by an easy dot product, i.e.,

$$D = I_p k_d \max(\mathbf{L} \cdot \mathbf{n}, 0)$$

Diffuse Reflection Coefficient

- $I_d = \max \{I_s k_d \cos\theta, 0\}$
- Source intensity is 1.0
- Background intensity is 0.4
- Sphere reflecting diffuse light, for six reflection coefficients: 0, 0.2, 0.4, 0.6, 0.8, and 1.



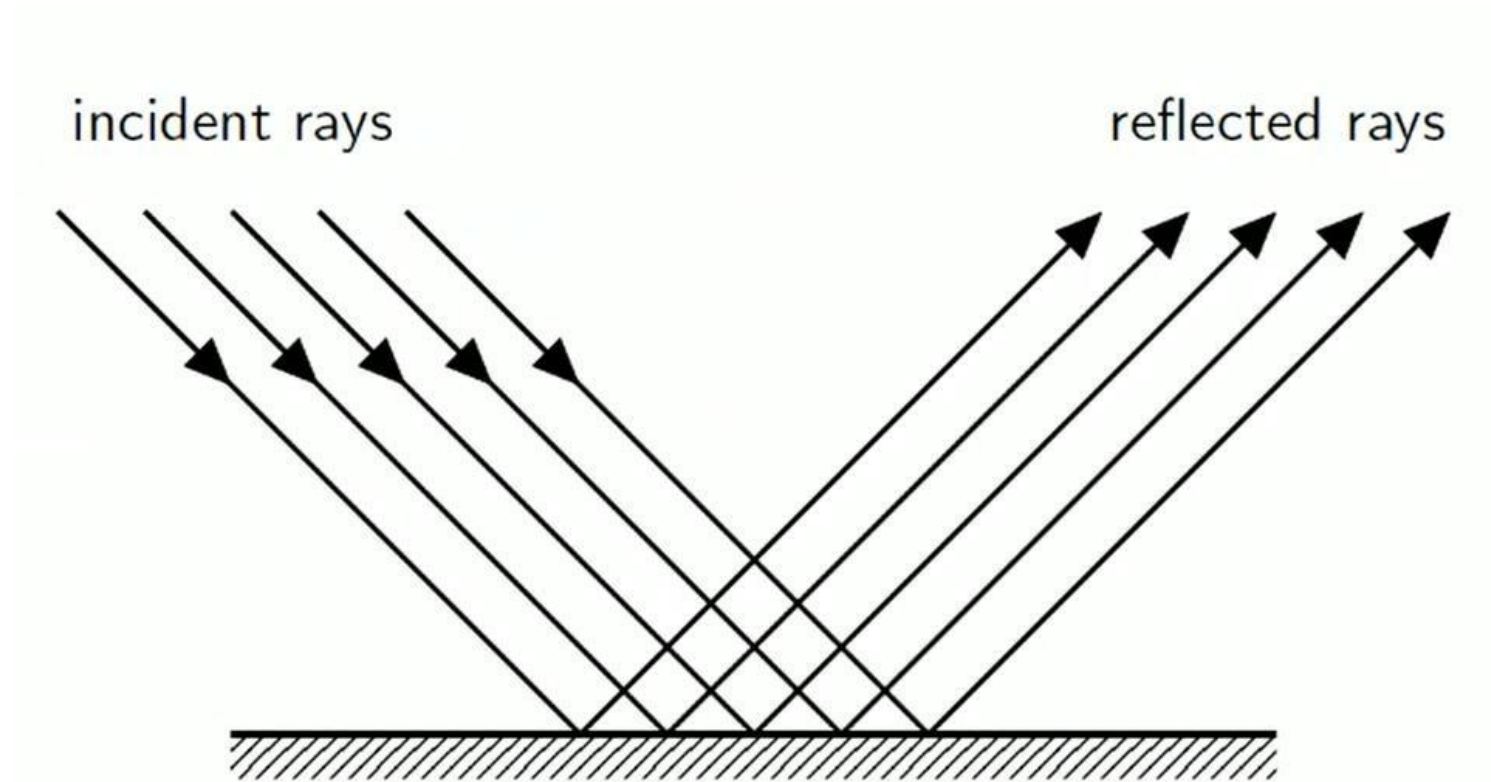
Angle θ between
surface normal and
incident light is $> 90^\circ$

□ What is the ambient
component here?

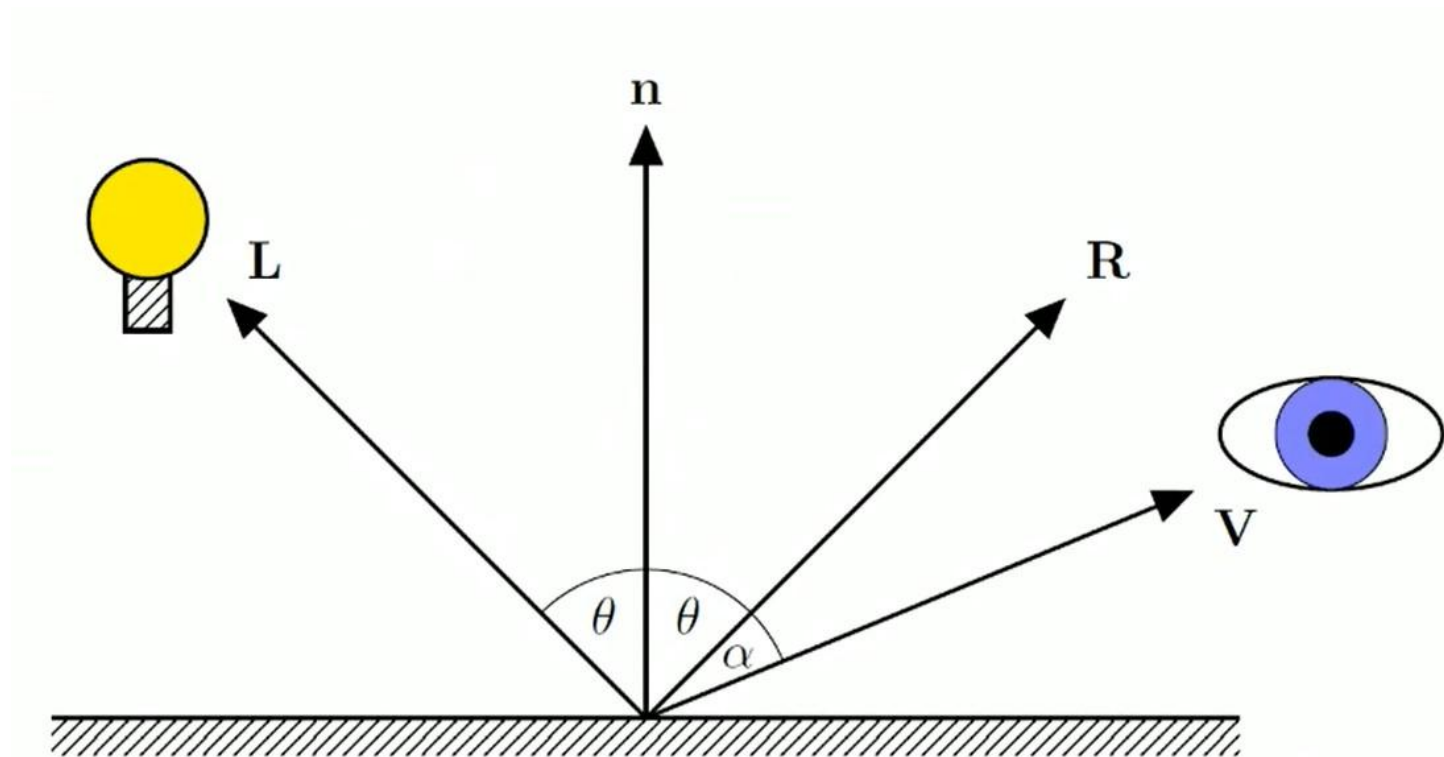
□ What is the specular
component?

Figure 8.11. Spheres with various reflection coefficients shaded with diffuse light.
(file: fig8.11.bmp)

Specular Reflection



Specular Reflection



Specular Reflection

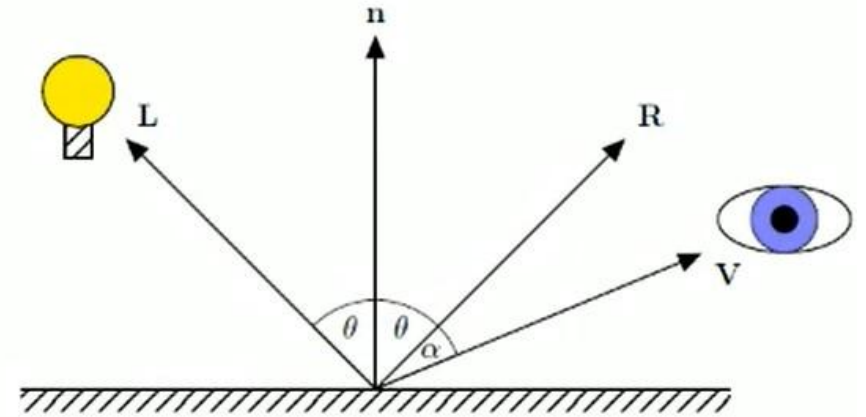


- Phong's specular model is

$$S = I_p k_s \cos^n(\alpha)$$

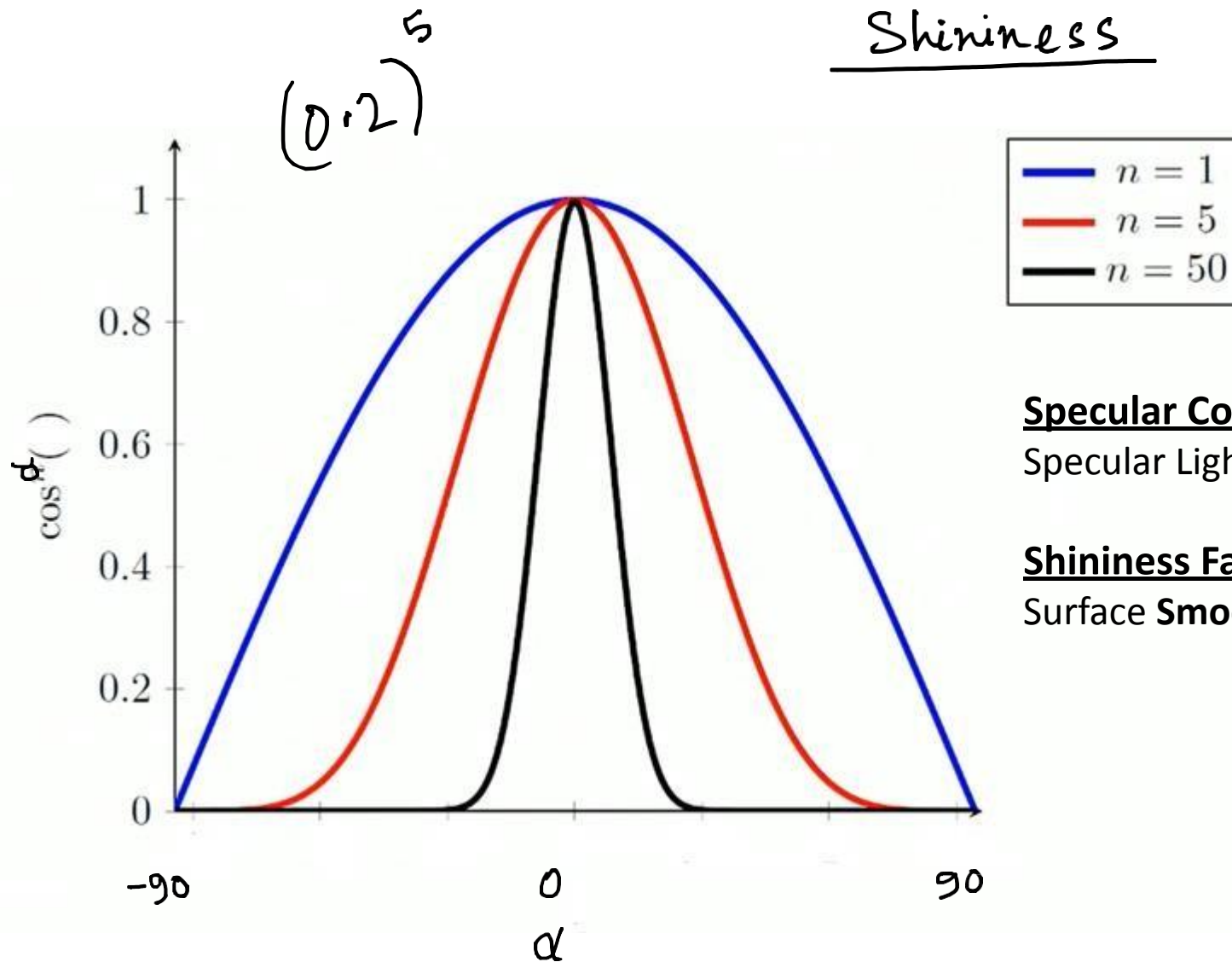
where

- $k_s \in [0, 1]$ is the **specular coefficient**
 - n is the **specular exponent** (shininess)
 - α is the angle between **R** and **V**
- The $\cos^n(\alpha)$ term determines the amount of light that is reflected



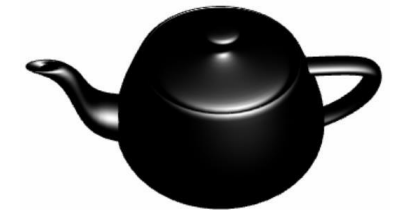
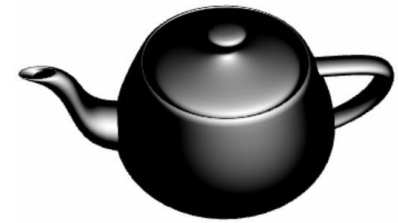
Specular Reflection

Shininess



Specular Coefficient - Determines Specular Light **Intensity**

Shininess Factor - Determines Surface **Smoothness**



Specular Reflection



$$S = I_p k_s \cos^n(\alpha)$$

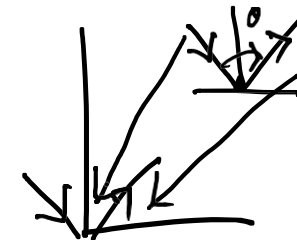
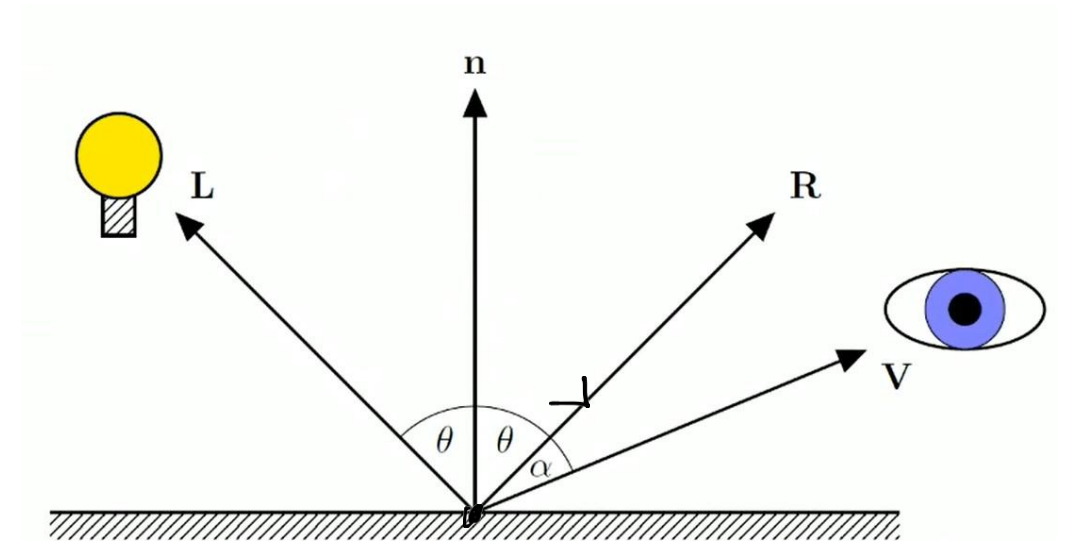
Calculate R , $\cos(\alpha)$

▣ $\cos(\alpha) = \underline{V \cdot R}$ [unit vectors]

▣ R

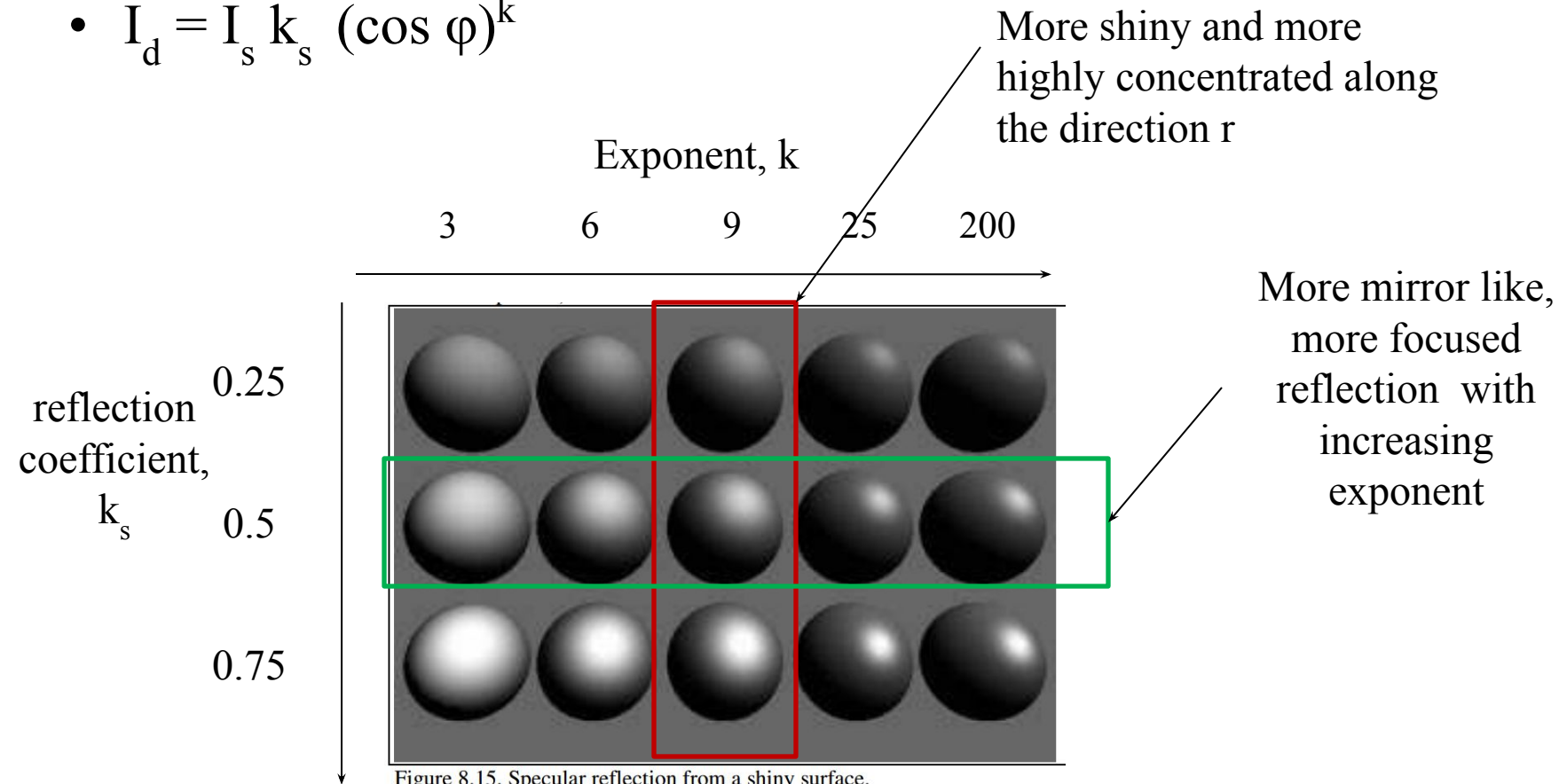
↙ Using Rotation

↙ Projection of vector

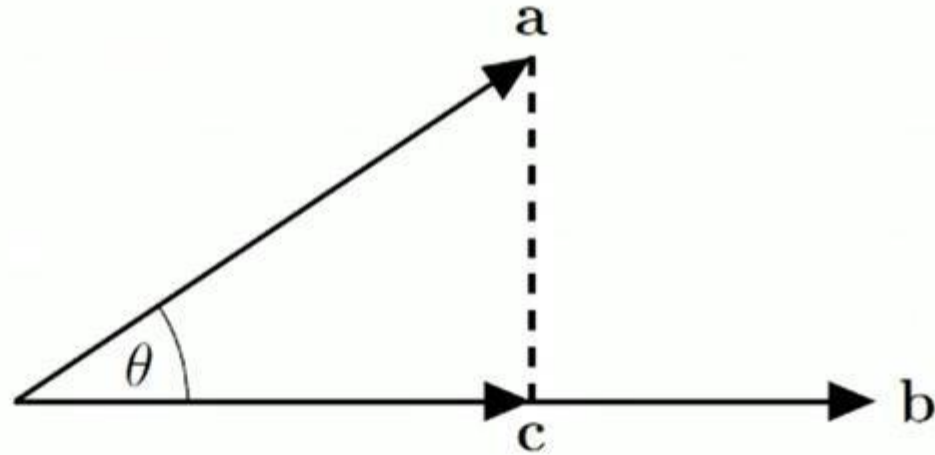


Specular Reflection Coefficient

- The ambient and diffuse reflection coefficients are 0.1 and 0.4 for all spheres.
- $I_d = I_s k_s (\cos \phi)^k$



Vector Projection



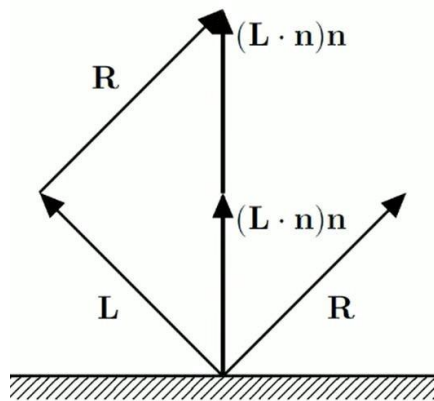
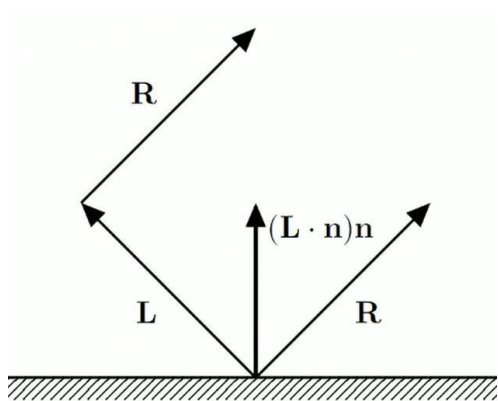
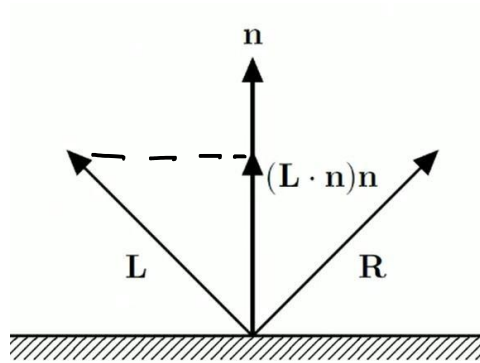
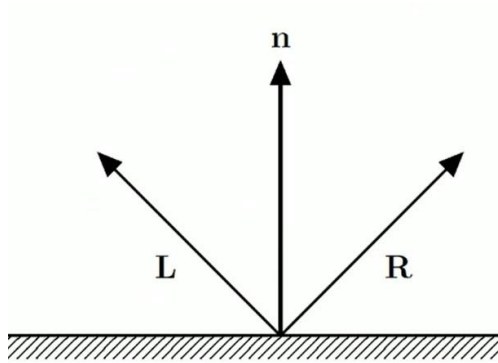
- Using the definition of a dot product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta) = |\mathbf{a}||\mathbf{b}| \frac{|\mathbf{c}|}{|\mathbf{a}|} = |\mathbf{b}||\mathbf{c}|$$

$$\therefore |\mathbf{c}| = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

- If \mathbf{b} is a unit vector then $|\mathbf{c}| = \mathbf{a} \cdot \mathbf{b}$ and

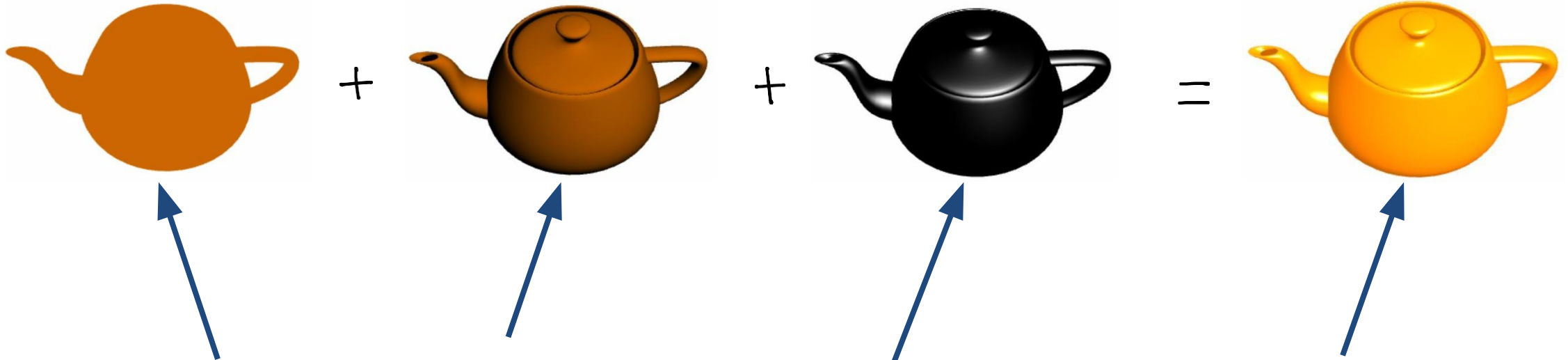
$$\mathbf{c} = \underline{(\mathbf{a} \cdot \mathbf{b})} \underline{\underline{\mathbf{b}}}$$



$$\mathbf{L} + \mathbf{R} = 2(\mathbf{L} \cdot \mathbf{n})\mathbf{n}$$

$$\therefore \mathbf{R} = 2(\mathbf{L} \cdot \mathbf{n})\mathbf{n} - \mathbf{L}$$

Phong's Reflection Model



The diagram illustrates the Phong's Reflection Model by showing the combination of three different surface properties to create a final glossy orange teapot. The components are:

- A flat orange teapot representing ambient reflection.
- A shaded orange teapot representing diffuse reflection.
- A glossy black teapot representing specular reflection.

These are summed together to produce the final glossy orange teapot, which represents the total color I .

$$I_a k_a + I_p [k_d \max \{(\mathbf{L} \cdot \mathbf{N}), 0\} + k_s \max \{(\mathbf{R} \cdot \mathbf{V})^k, 0\}] = I$$

Mathematical Calculation of Phong Model

We need to incorporate the effect light present in environment. Thus total reflected light also includes ambient component.

$$\text{Ambient Component} = I_a k_a$$

Total reflected light intensity from Q,

$$\begin{aligned} I &= \text{Ambient Component} + \text{Diffuse Component} + \text{Specular Component} \\ &= I_a k_a + I_p k_d (\mathbf{L} \cdot \mathbf{N}) + I_p k_s (\mathbf{R} \cdot \mathbf{V})^k \end{aligned}$$

More specifically,

$$I = I_a k_a + I_p [k_d \max \{(\mathbf{L} \cdot \mathbf{N}), 0\} + k_s \max \{(\mathbf{R} \cdot \mathbf{V})^k, 0\}]$$

Additional Issues

When there are n light sources in the scene, their effects are cumulative: Intensity at Q,

$$I = I_a k_a + \sum_{(i=1 \text{ to } n)} I_{pi} \{k_d (\mathbf{L} \cdot \mathbf{N}) + k_s (\mathbf{R} \cdot \mathbf{V})^k \}$$

The intensity of red, green and blue component of reflected light,

$$I_r = I_a k_{ar} + I_p k_{dr} (\mathbf{L} \cdot \mathbf{N}) + I_p k_s (\mathbf{R} \cdot \mathbf{V})^k$$

$$I_g = I_a k_{ag} + I_p k_{dg} (\mathbf{L} \cdot \mathbf{N}) + I_p k_s (\mathbf{R} \cdot \mathbf{V})^k$$

$$I_b = I_a k_{ab} + I_p k_{db} (\mathbf{L} \cdot \mathbf{N}) + I_p k_s (\mathbf{R} \cdot \mathbf{V})^k$$

k_s : coefficient for specular component which is same as the color of light source, not affected by surface color.

Attenuation

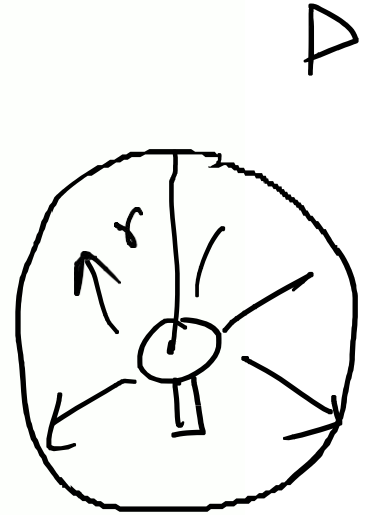
- **Attenuation** is the loss of light energy over space
- In Phong's model attenuation is account for by the variable f_{att} and applied to diffuse and specular components
- Theoretically is should follow the inverse square law, i.e.,

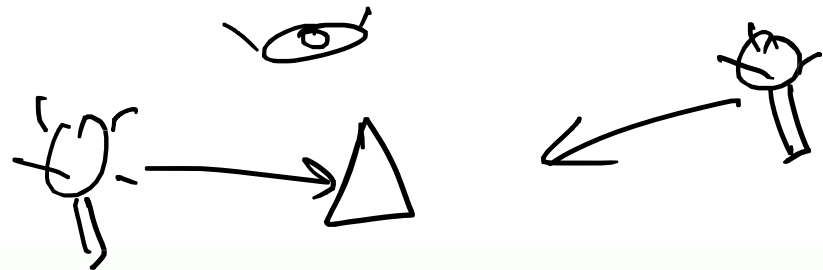
$$f_{att} = \frac{1}{d^2}$$

- In practice this removes too much light, Phong's model uses

$$f_{att} = \max \left[1 - \left(\frac{d}{r} \right)^2, 0 \right]$$

where r is the radius of the light source's sphere of influence





- Combining ambient, diffuse, specular and attenuation models results in Phong's model of reflection

$$\underline{I} = \underline{I_a K_a} + I_p f_{att} \left(K_d \max(L \cdot n, 0) + K_s (\max(V \cdot R, 0))^n \right)$$

- For multiple light sources, the diffuse and specular components are calculated for each light source and added together, i.e.,

$$\underline{I} = \underline{I_a K_a} + \sum_{i=1}^m I_{p_i} f_{att} \left(K_d \max(L_i \cdot n, 0) + K_s (\max(V \cdot R_i, 0))^n \right)$$

Shading

Whether an object, or some part of an object is obstructed by another object.

- Z buffer / Depth buffer

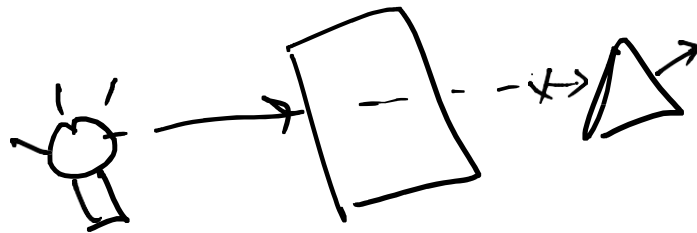
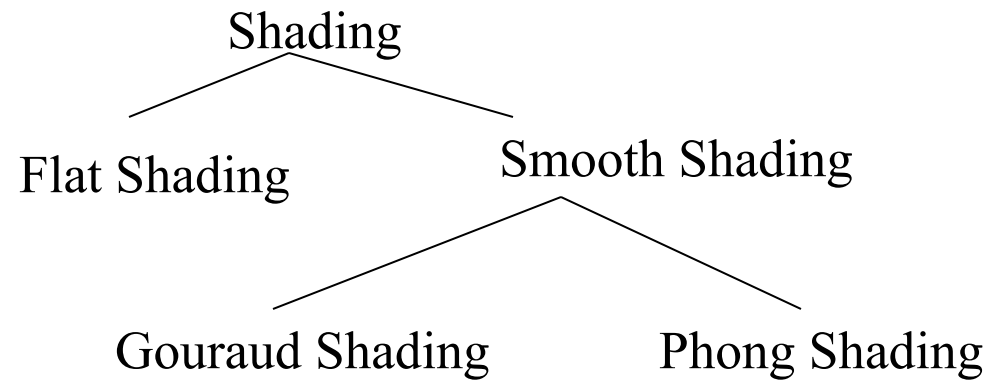


Photo: [Alex Canclini](#)



Shading

- The process of assigning colors to pixels.



Shading Model

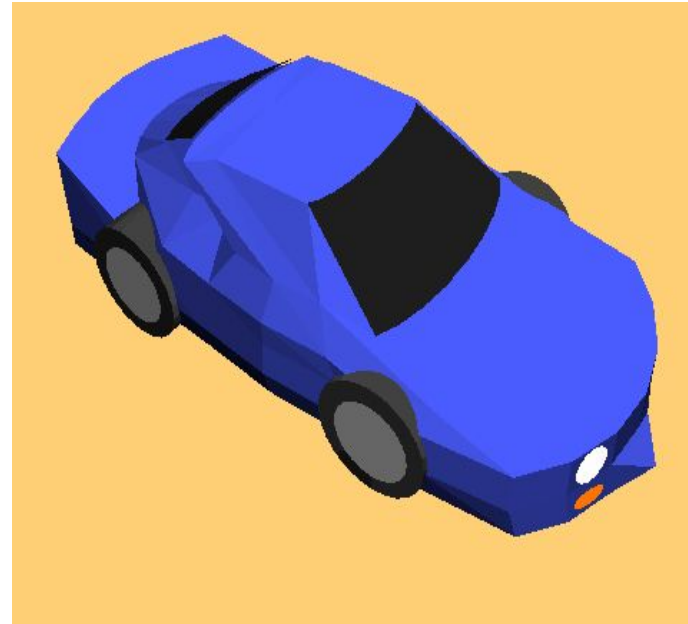
- Flat Shading
 - Compute Phong lighting once for entire polygon
- Gouraud Shading
 - Compute Phong lighting at the vertices and interpolate lighting values across polygon
- Phong Shading
 - Interpolate normals across polygon and perform Phong lighting across polygon

Flat Shading

- For each polygon
 - Determines a single intensity value at a chosen point on the polygon
 - Uses that value to shade the entire polygon
- Assumptions
 - Light source at infinity
 - Viewer at infinity
 - The polygon represents the actual surface being modeled

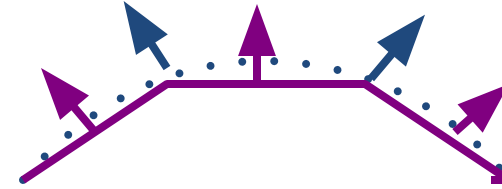
Problems of Flat Shading

- Specular highlights tends to get lost
- If chosen point on polygon is at location of the light source, then color of the polygon will be significantly distorted.



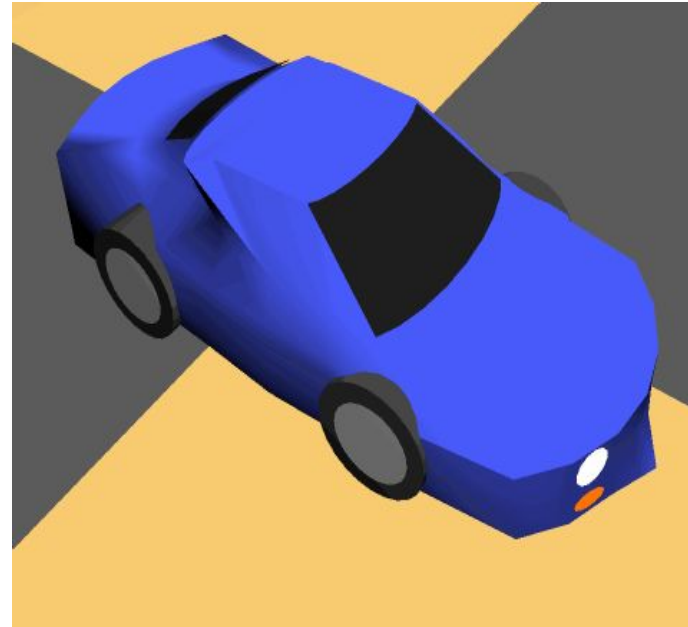
Smooth Shading

- Introduce vertex normals at each vertex
 - Used only for shading
 - Think of as a better approximation of the real surface that the polygons approximate
 - Finds color value for each point in the polygon individually
- Two types
 - Gouraud Shading
 - Phong Shading (do not confuse with Phong Lighting Model)

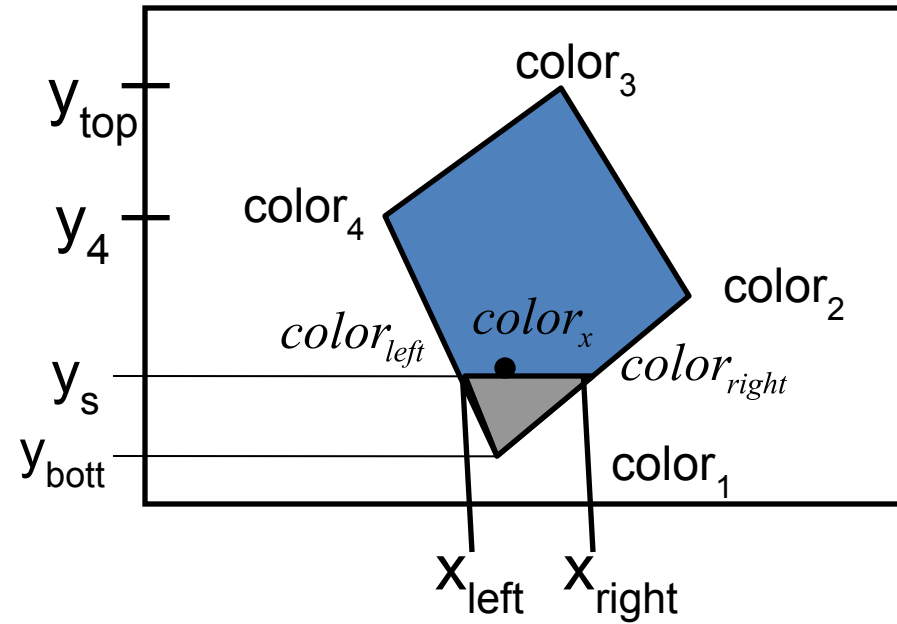


Gouraud Shading

- Most common approach
- Perform Phong lighting at the vertices
- Linearly interpolate the resulting colors over faces
 - Along edges
 - Along scanline



Gouraud Shading

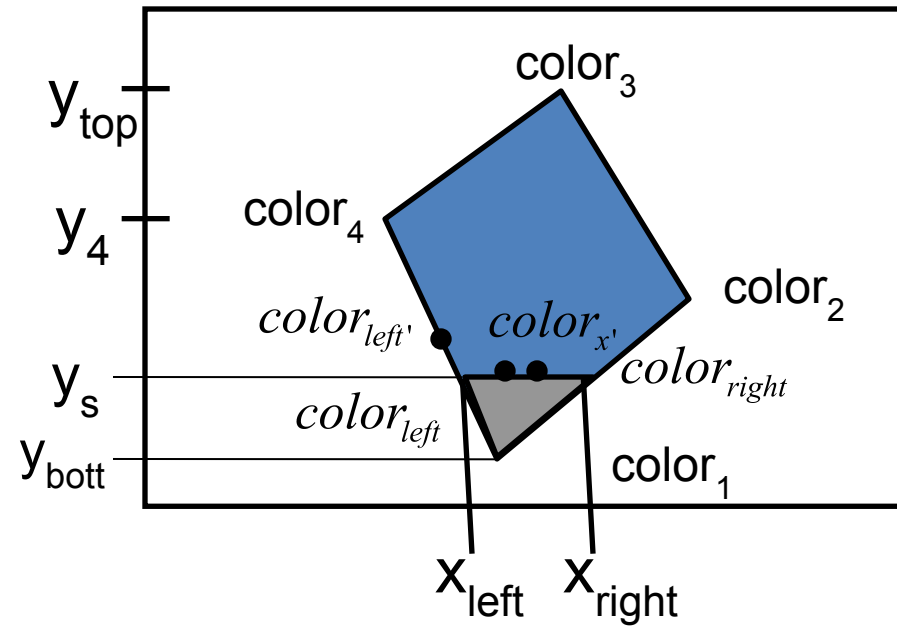


$$color_{left} = color_1 + (color_4 - color_1) \frac{y_s - y_{bott}}{y_4 - y_{bott}}$$

$$color_{right} = color_1 + (color_2 - color_1) \frac{y_s - y_{bott}}{y_2 - y_{bott}}$$

$$color_x = color_{left} + (color_{right} - color_{left}) \frac{x - x_{left}}{x_{right} - x_{left}}$$

Gouraud Shading



$$color_x = color_{\text{left}} + (color_{\text{right}} - color_{\text{left}}) \frac{x - x_{\text{left}}}{x_{\text{right}} - x_{\text{left}}}$$

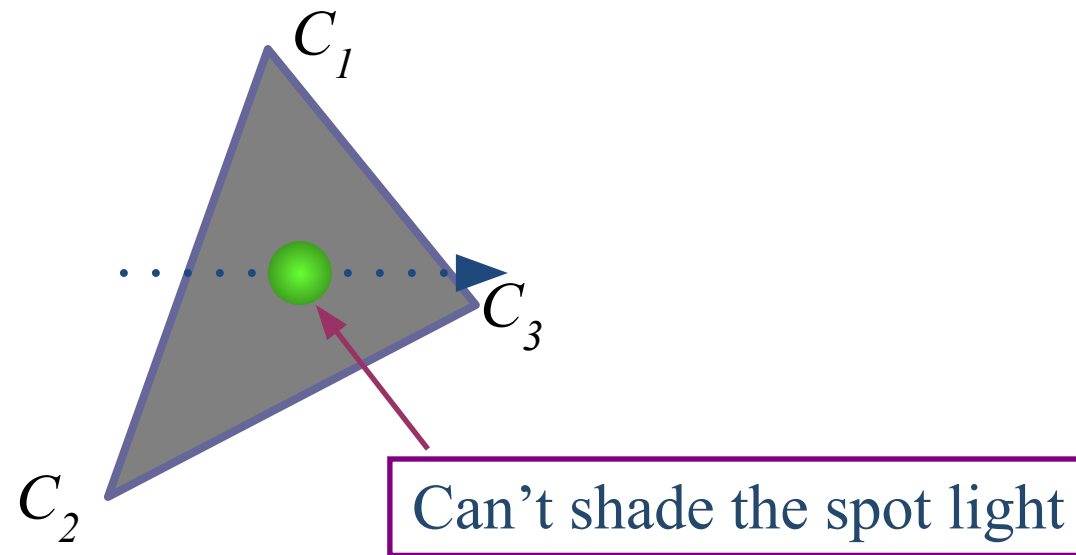
$$color_{x'} = color_x + K \Delta x$$

$$color_{\text{left}'} = color_{\text{left}} + K' \Delta y$$

Calculate the surface normals
along the scan line and the edge
using incremental approach

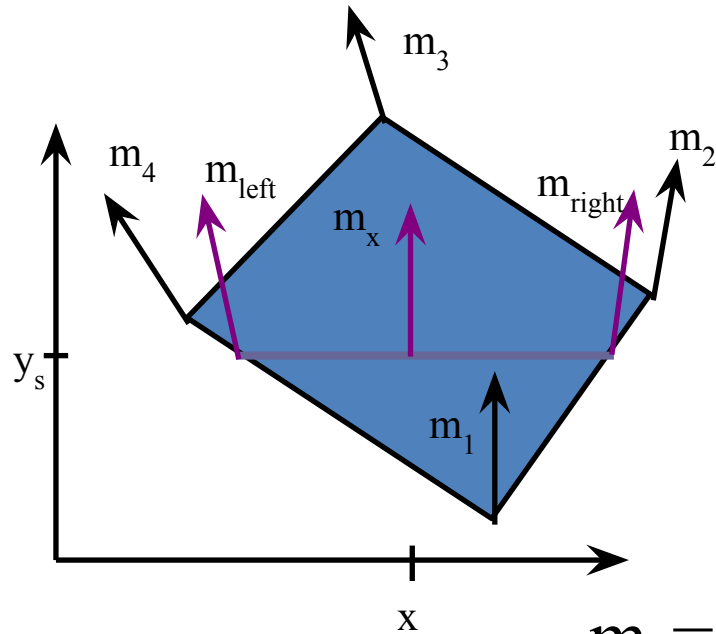
Problem of Gouraud Shading

- Often appears dull
- Lacks accurate specular component



Phong Shading

Interpolate normal vectors of face vertices at each pixel, then perform Phong lighting at each pixel.



$$m_{\text{left}} = m_1 + (m_4 - m_1) \frac{y_s - y_1}{Y_4 - y_1}$$

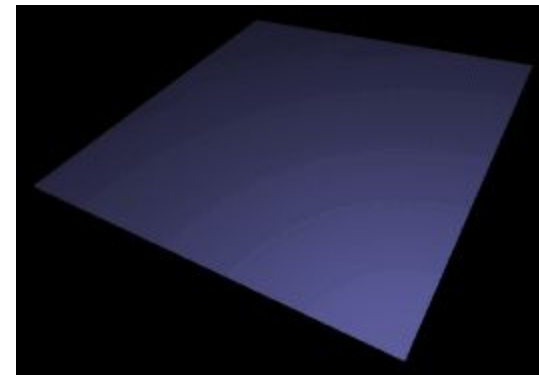
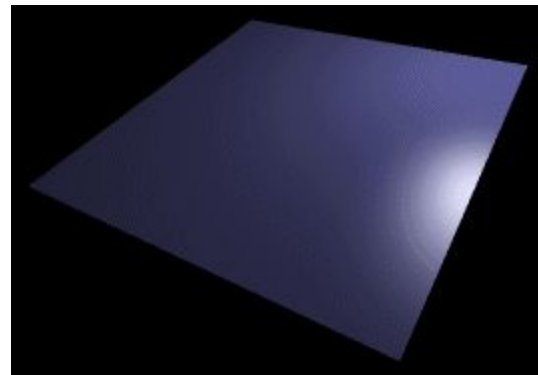
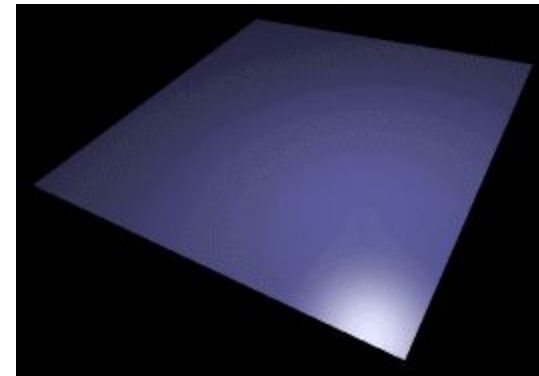
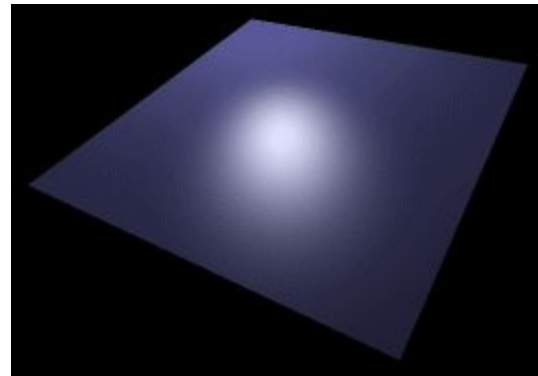
$$m_{\text{right}} = m_1 + (m_2 - m_1) \frac{y_s - y_1}{Y_2 - y_1}$$

$$m_x = m_{\text{left}} + (m_{\text{right}} - m_{\text{left}}) \frac{x - x_{\text{left}}}{x_{\text{right}} - x_{\text{left}}}$$

Calculate the surface normals along the scan line and the edge using incremental approach

Phong vs Gouraud Shading

- Phong shading is more smooth
- If a highlight does not fall on a vertex, Gouraud shading may miss it completely, but Phong shading does not.



Reference:

Computer Graphics: Principles and Practice: John F. Hughes, James D. Foley, Andries van Dam, Steven K. Feiner (2nd Edition)

Chapter: 16.1.1 - 16.1.4, 16.1.6, 16.2.1 - 16.2.5

