Theory of Neuronal Dynamics and Application to Machine Learning Based on Reservoir Computing

Summer 2024

Forecastability in chaotic neural dynamics using reservoir computing

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GitHub-directory: Click Here to get the code on GitHub

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Result Visualizations: Problem A

Problem-A-1a:

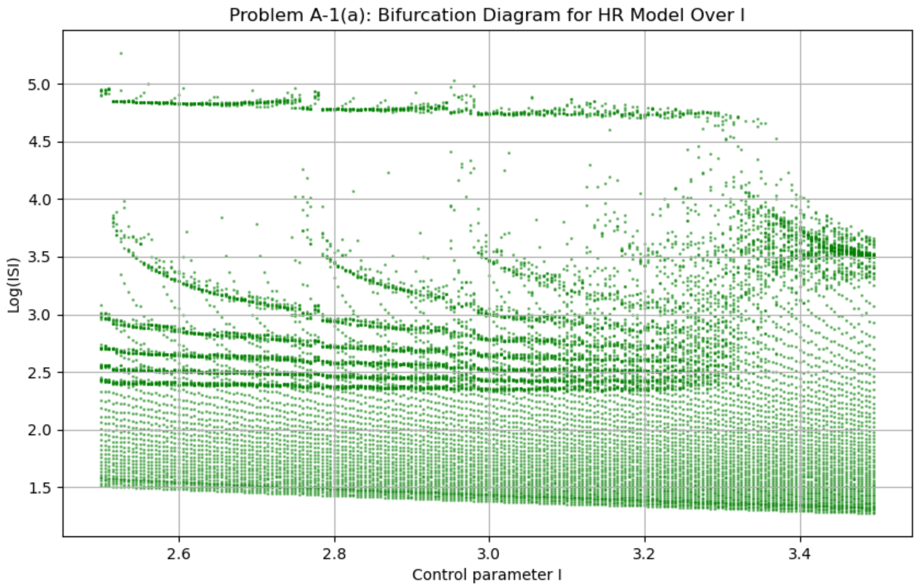


Figure -1: Problem-A-1a (A bifurcation diagram of the HR neuron model for the control parameter I)

Problem-A-1b:

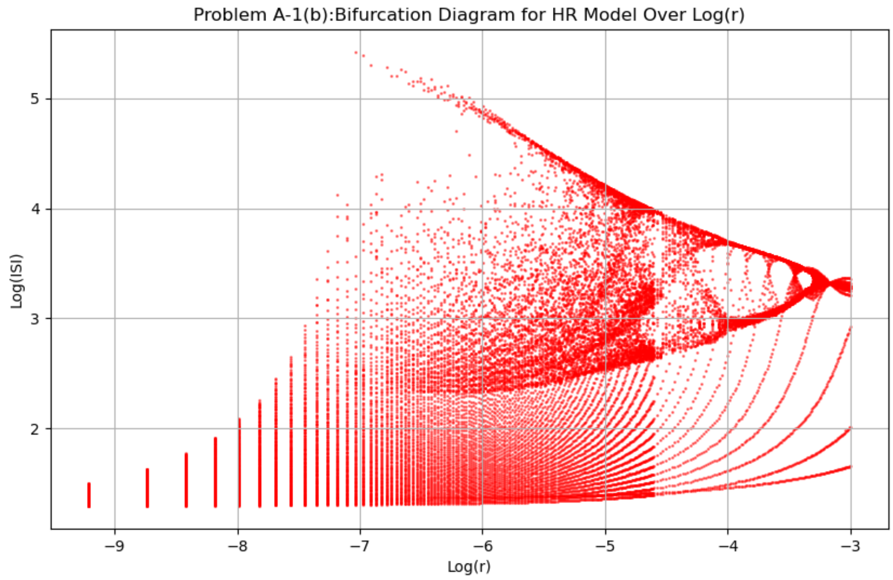


Figure -2: Problem -A-1b (Bifurcation diagram of the HR neuron model for the control parameter r)

Problem-A-2a:

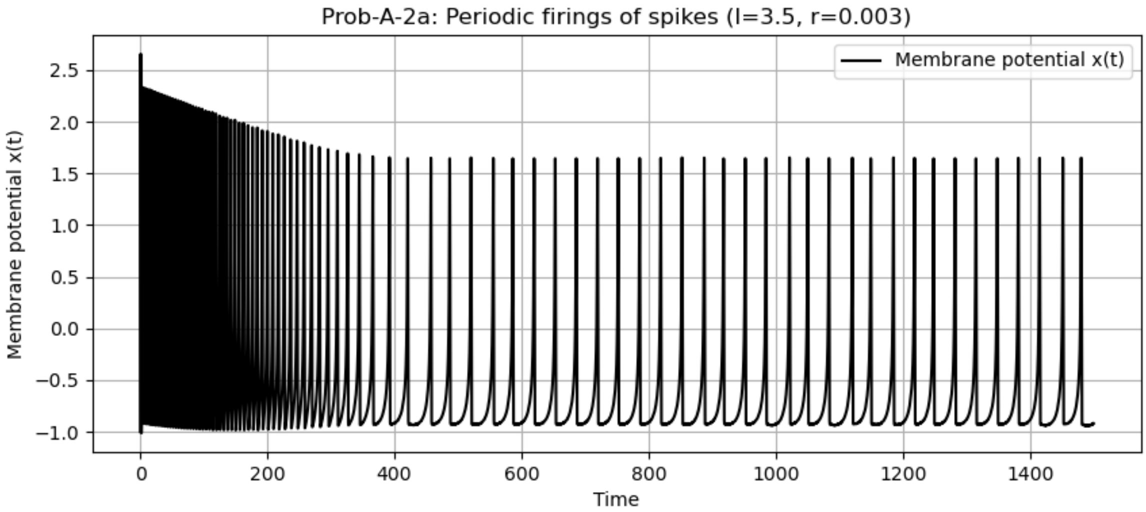


Figure-3: Problem-A-2a (Time series of the membrane potential variable)

Problem-A-2b:

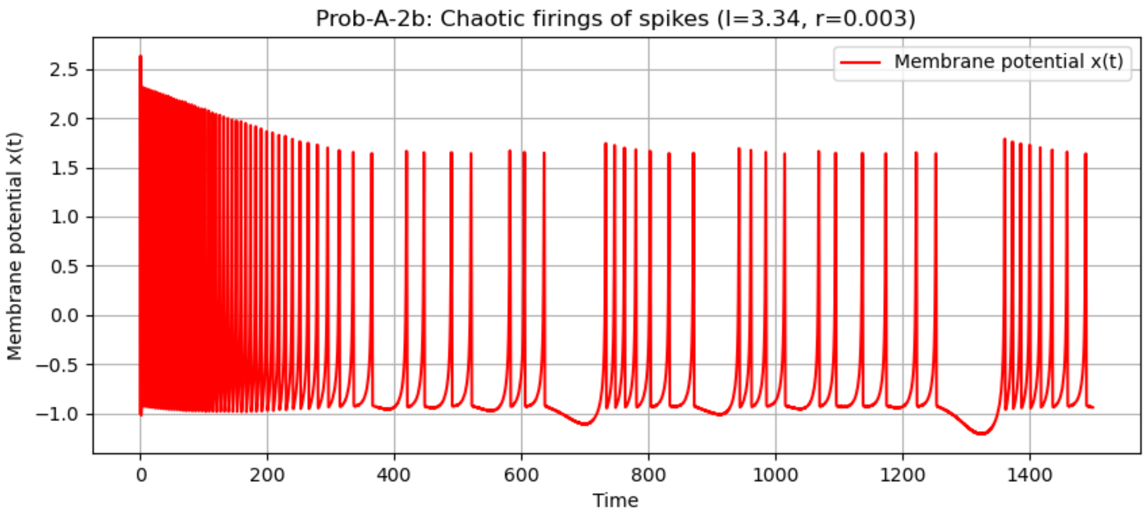


Figure-4: Problem-A-2b (Time series of the membrane potential variable)

Problem-A-2c:

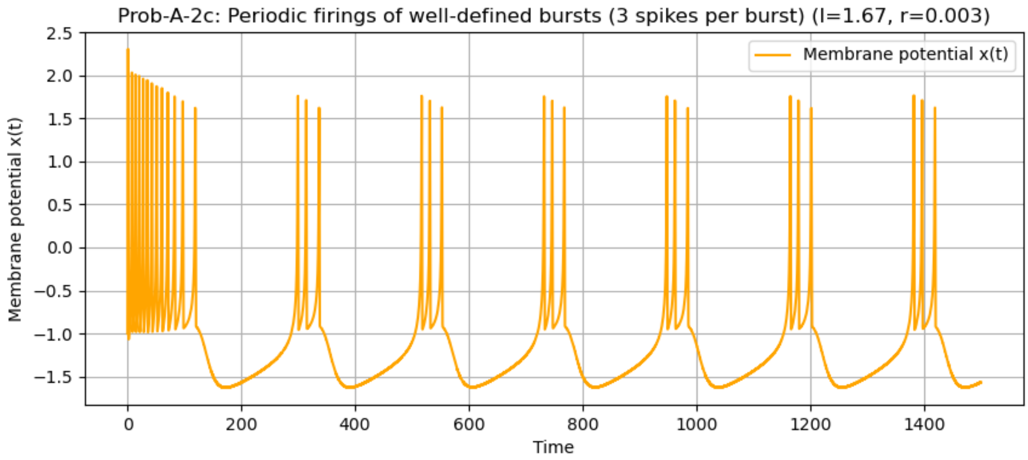


Figure-5: Problem-A-2c (Time series of the membrane potential variable)

Problem-A-2d:

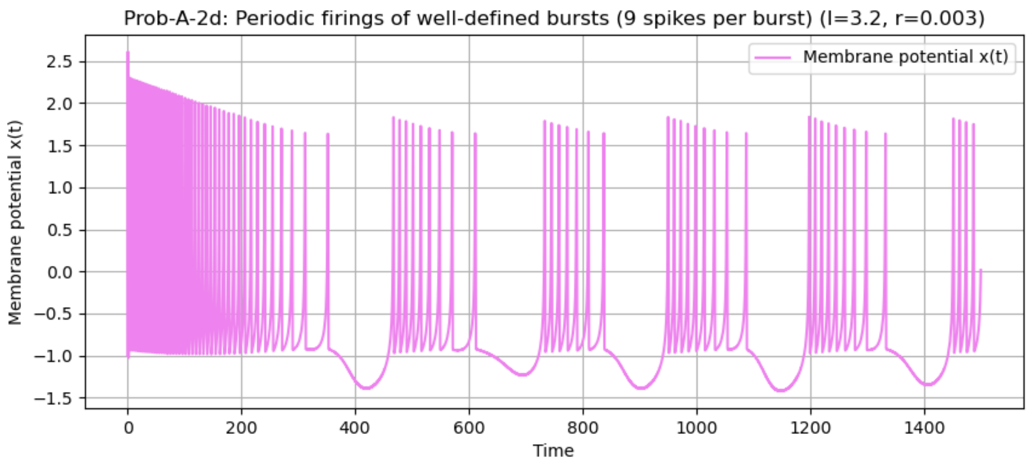


Figure-6: Problem-A-2d (Time series of the membrane potential variable)

Problem-A-2e:

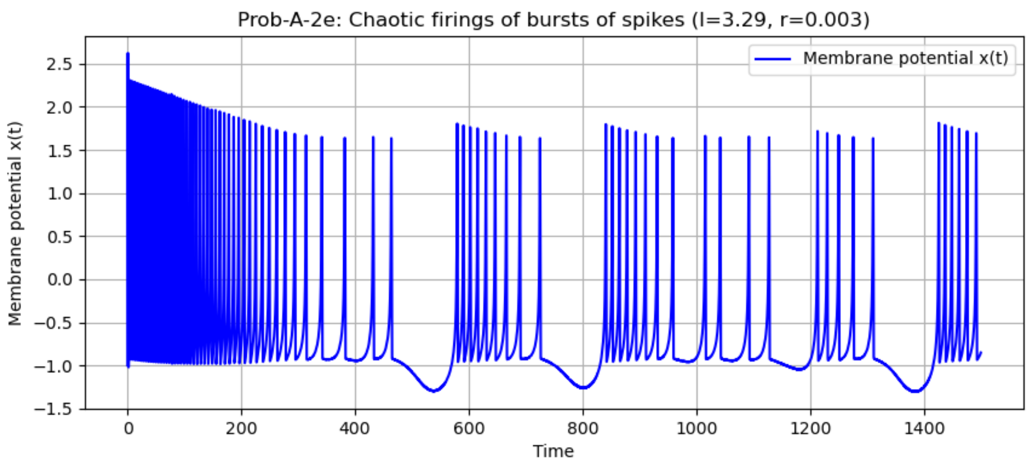
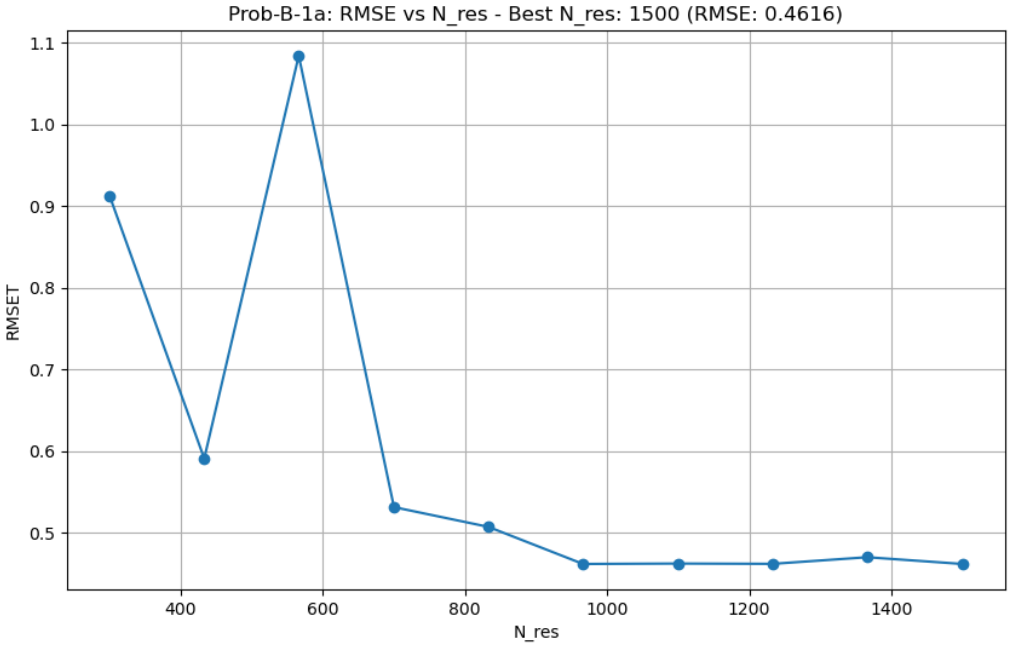
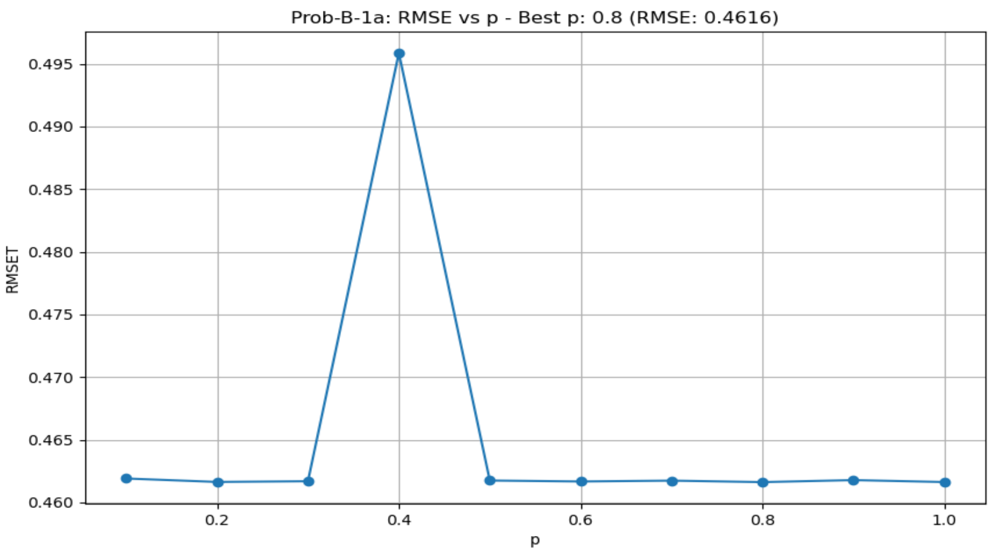


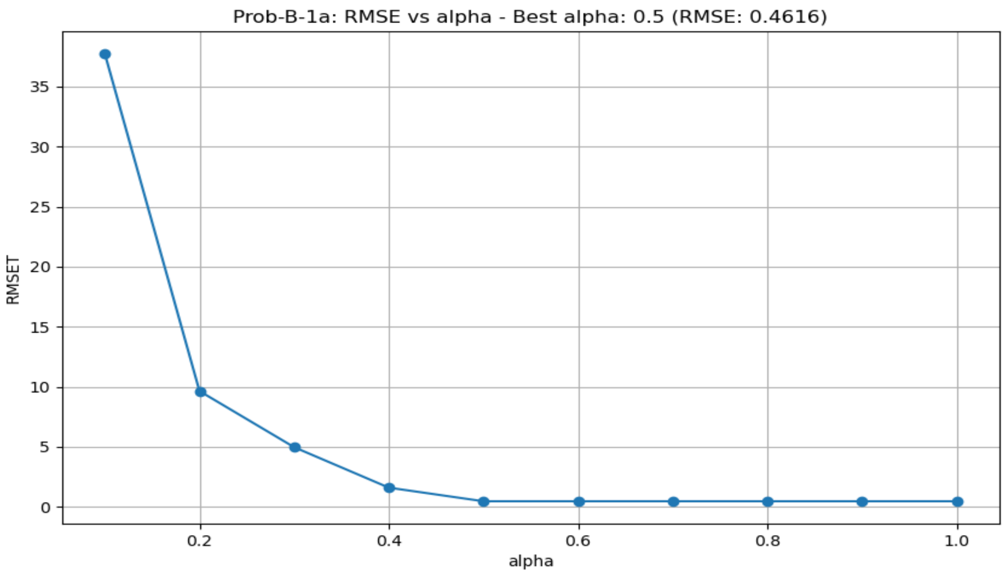
Figure-7: Problem-A-2e (Time series of the membrane potential variable)

Result Visualizations: Problem B

Problem-B-1a







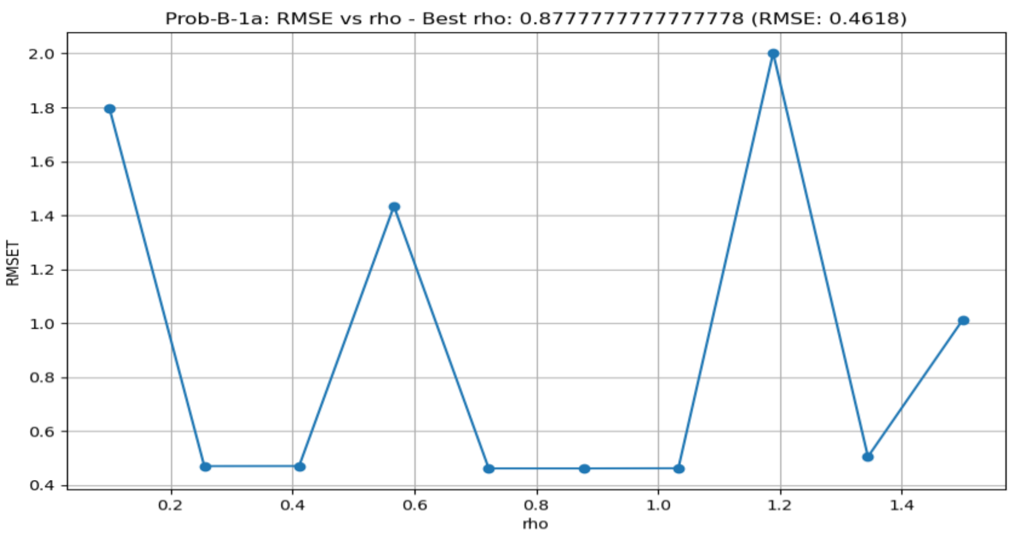


Figure-8: Problem-B-1a (Root Mean Square Error against each hyperparameter N_{res}, p, alpha, and rho)

Results:

Minimum RMSEs for 2a:

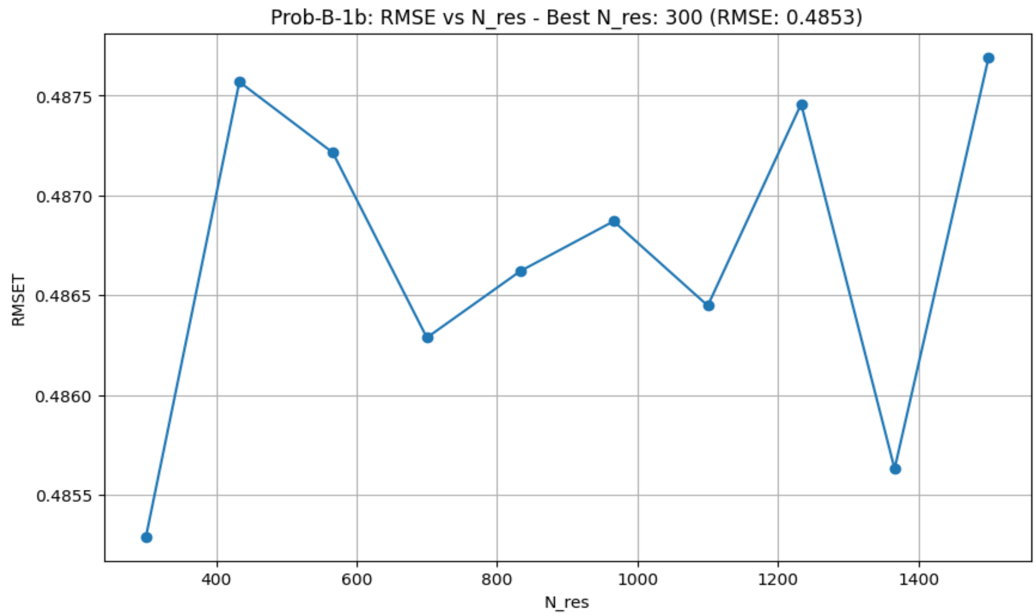
N_res=1500, RMSE=0.4616

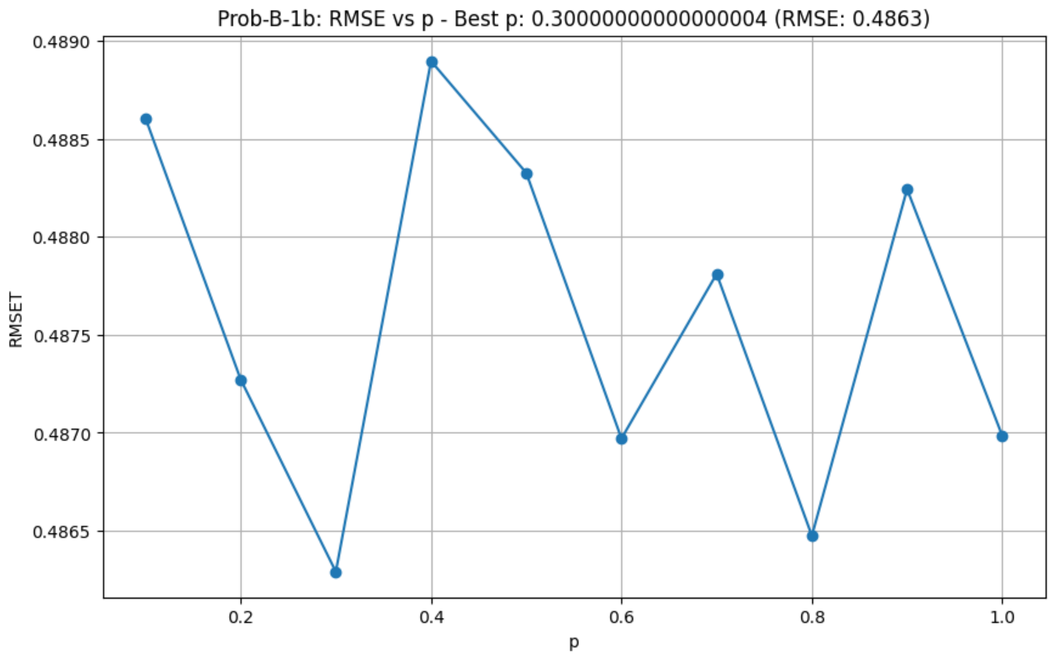
p=0.8, RMSE=0.4616

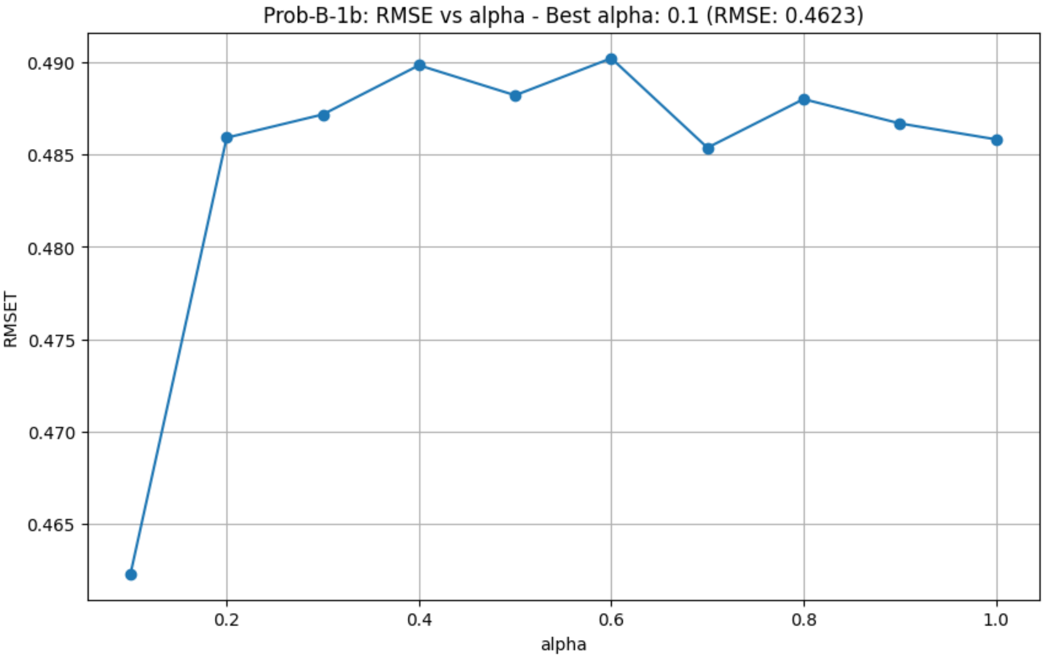
alpha=0.5, RMSE=0.4616

rho=0.8777777777777778, RMSE=0.4618

Problem-B-1b


Prob-B-1b: RMSE vs N_res - Best N_res: 300 (RMSE: 0.4853)
RMSET
0.4875
0.4870
0.4865
0.4860
0.4855
400
600
800
1000
1200
1400
N_res


Prob-B-1b: RMSE vs p - Best p: 0.30000000000000004 (RMSE: 0.4863)
RMSET
0.4890
0.4885
0.4880
0.4875
0.4870
0.4865
0.2
0.4
0.6
0.8
1.0
p



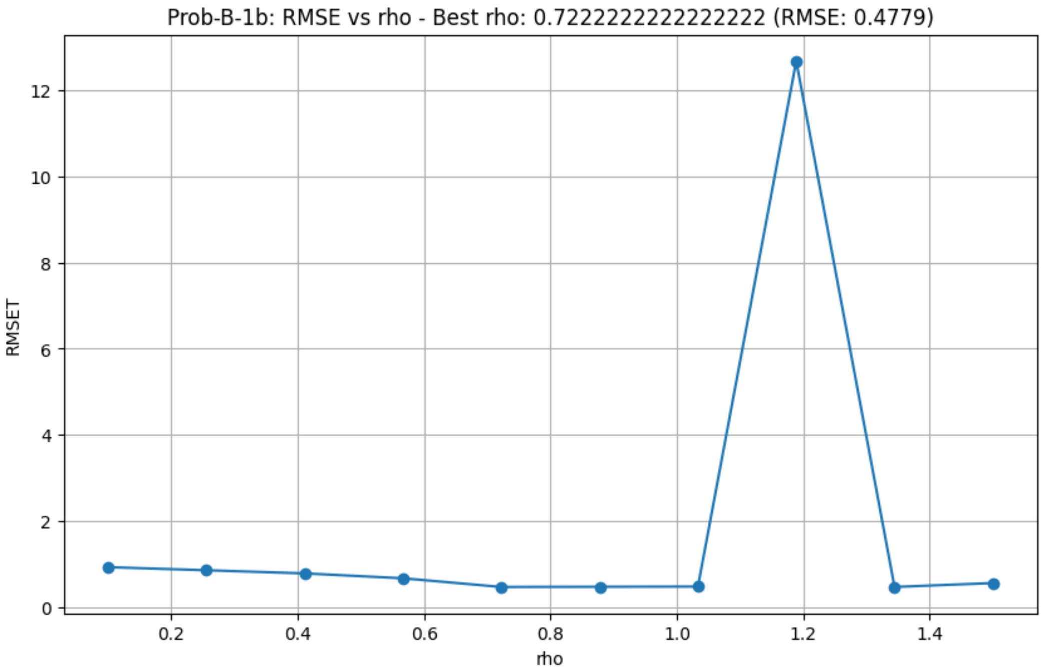


Figure-9: Problem-B-1b (Root Mean Square Error against each hyperparameter N_{res}, p, alpha, and rho)

Results:

Minimum RMSEs for 2b:

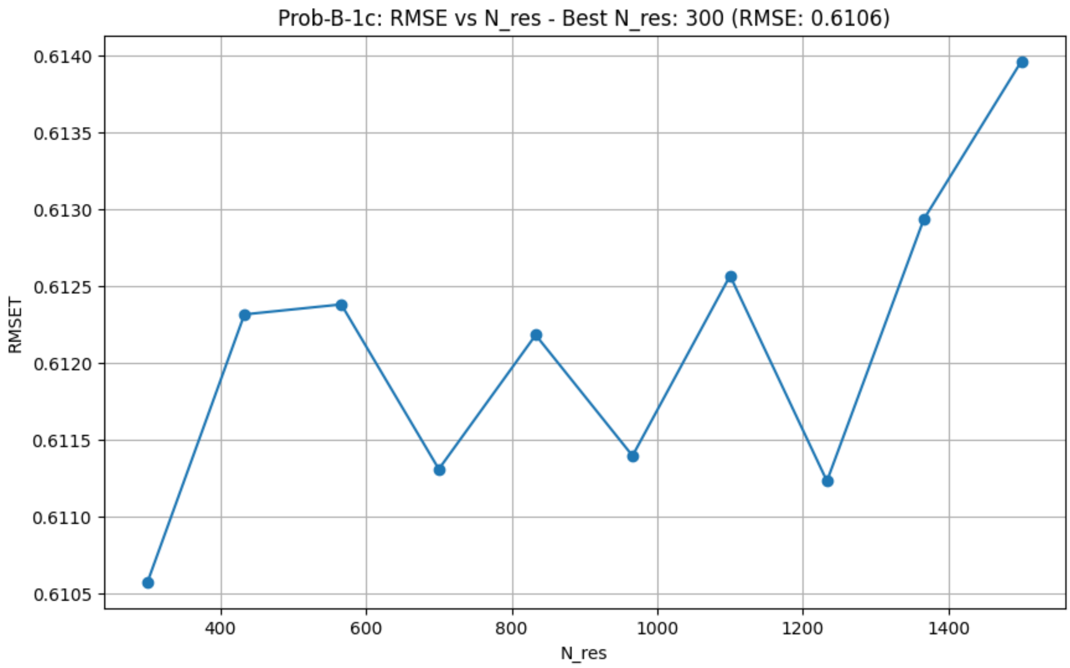
N_res=300, RMSE=0.4853

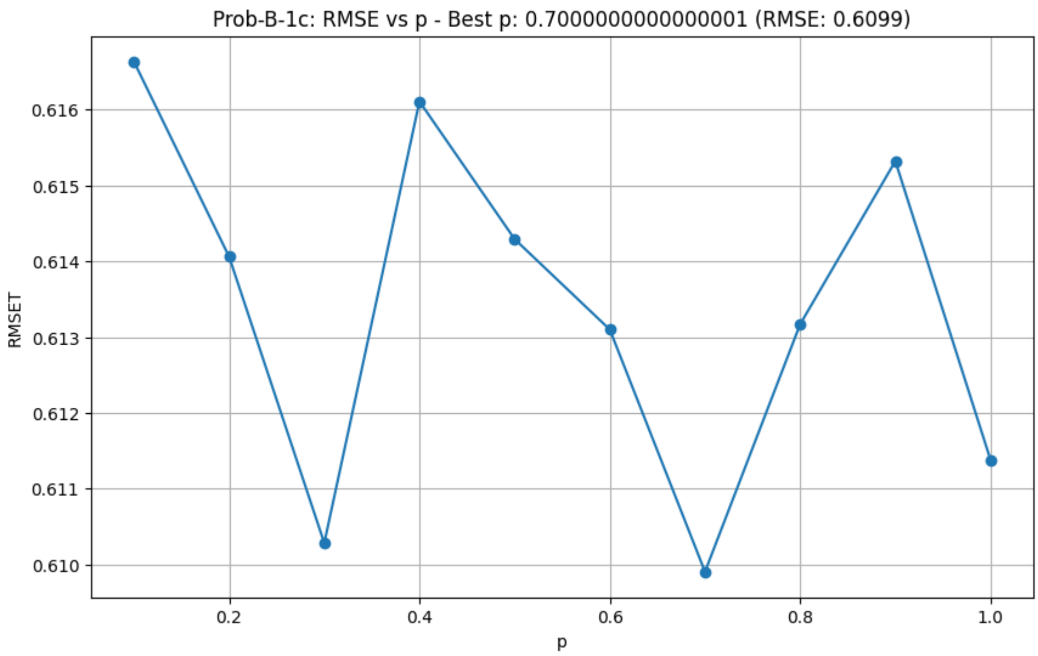
p=0.30000000000000004, RMSE=0.4863

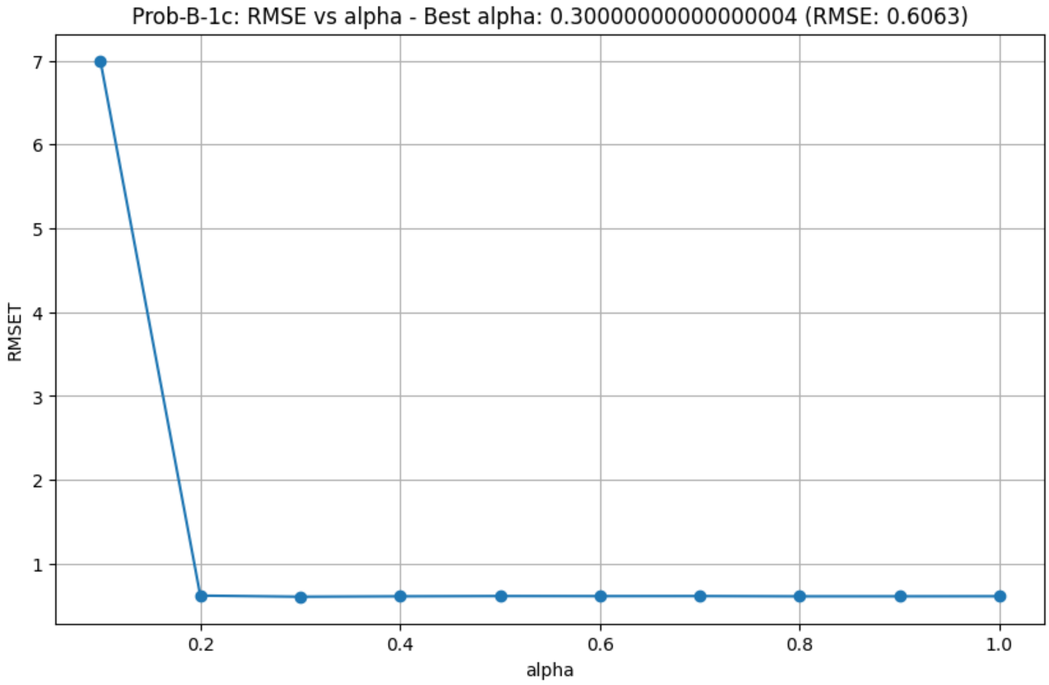
alpha=0.1, RMSE=0.4623

rho=0.7222222222222222, RMSE=0.4779

Problem-B-1c


Prob-B-1c: RMSE vs N_res - Best N_res: 300 (RMSE: 0.6106)
RMSET
N_res
0.6140
0.6135
0.6130
0.6125
0.6120
0.6115
0.6110
0.6105
400
600
800
1000
1200
1400


Prob-B-1c: RMSE vs p - Best p: 0.7000000000000001 (RMSE: 0.6099)
RMSET
p
0.616
0.615
0.614
0.613
0.612
0.611
0.610
0.2
0.4
0.6
0.8
1.0



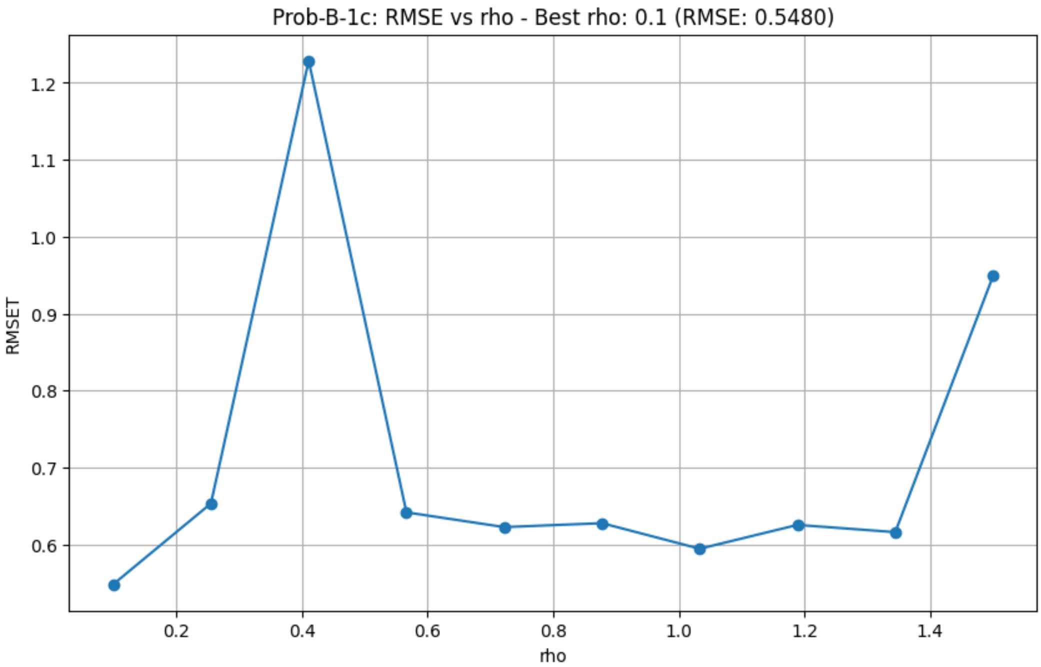


Figure-9: Problem-B-1c (Root Mean Square Error against each hyperparameter N_{res}, p, alpha, and rho)

Results:

Minimum RMSEs for 2c:

N_res=300, RMSE=0.6106

p=0.7000000000000001, RMSE=0.6099

alpha=0.30000000000000004, RMSE=0.6063

rho=0.1, RMSE=0.5480

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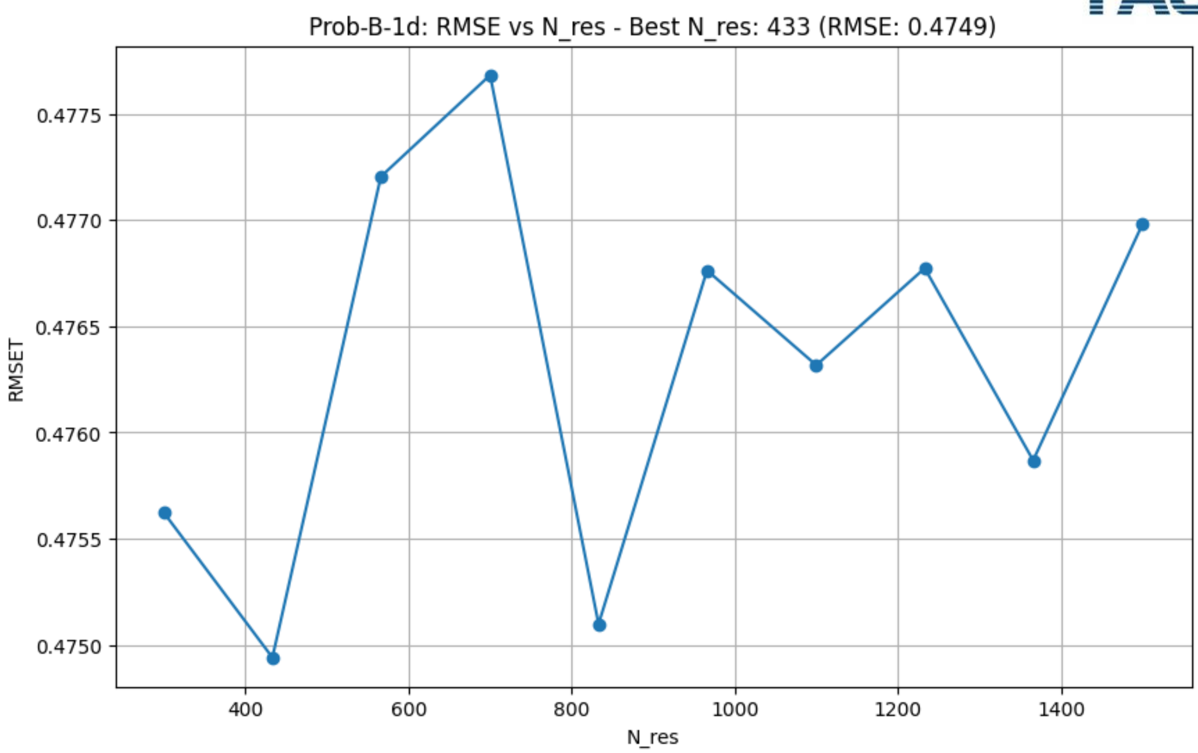
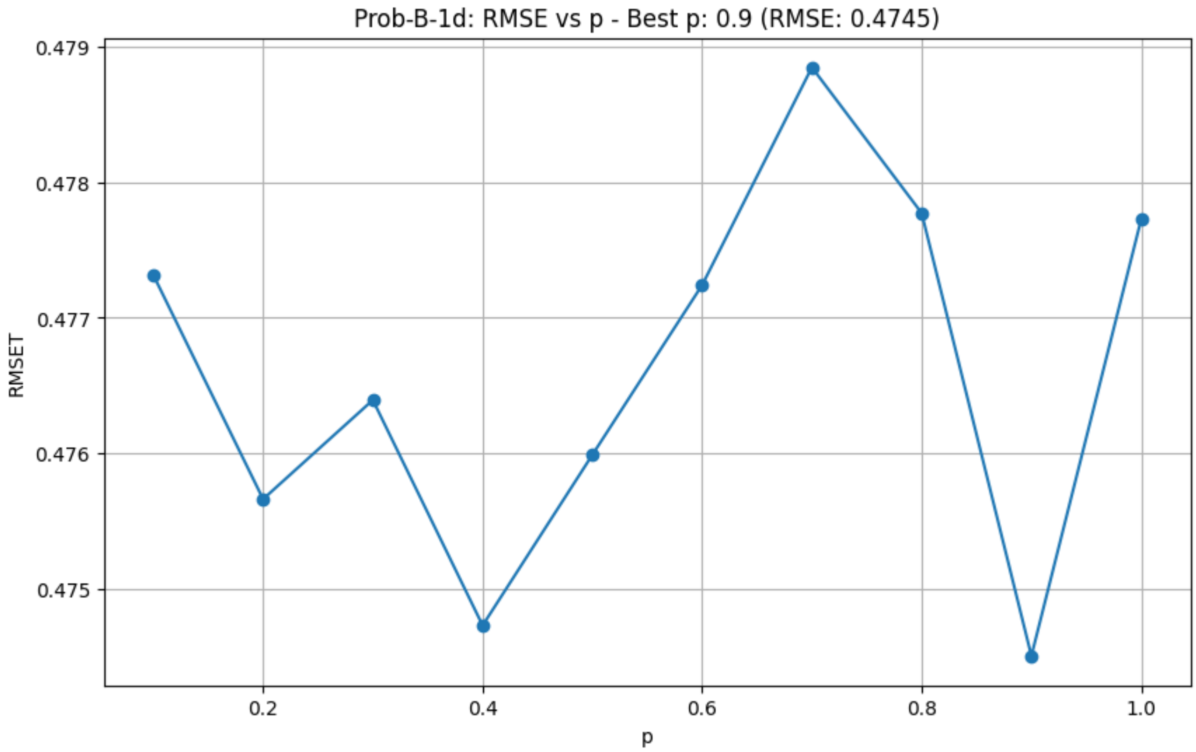
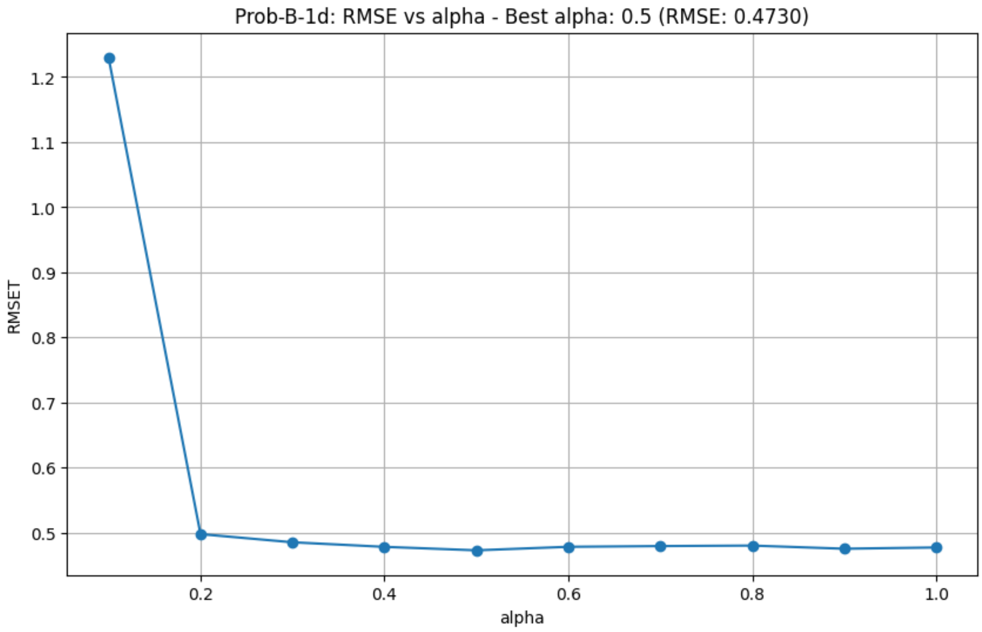
Problem-B-1d


Prob-B-1d: RMSE vs N_res - Best N_res: 433 (RMSE: 0.4749)
RMSET
N_res
0.4750
0.4755
0.4760
0.4765
0.4770
0.4775
400
600
800
1000
1200
1400


Prob-B-1d: RMSE vs p - Best p: 0.9 (RMSE: 0.4745)
RMSET
p
0.475
0.476
0.477
0.478
0.479
0.2
0.4
0.6
0.8
1.0



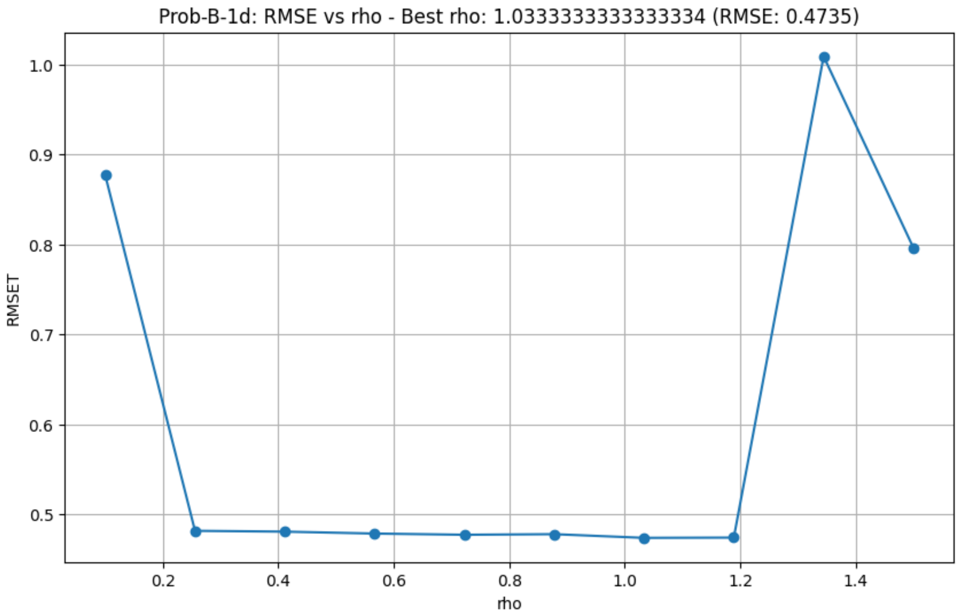


Figure-10: Problem-B-1d (Root Mean Square Error against each hyperparameter N_{res}, p, alpha, and rho)

Results:

Minimum RMSEs for 2e:

N_res=433, RMSE=0.4749

p=0.9, RMSE=0.4745

alpha=0.5, RMSE=0.4730

rho=1.0333333333333334, RMSE=0.4735

Problem-B-2a

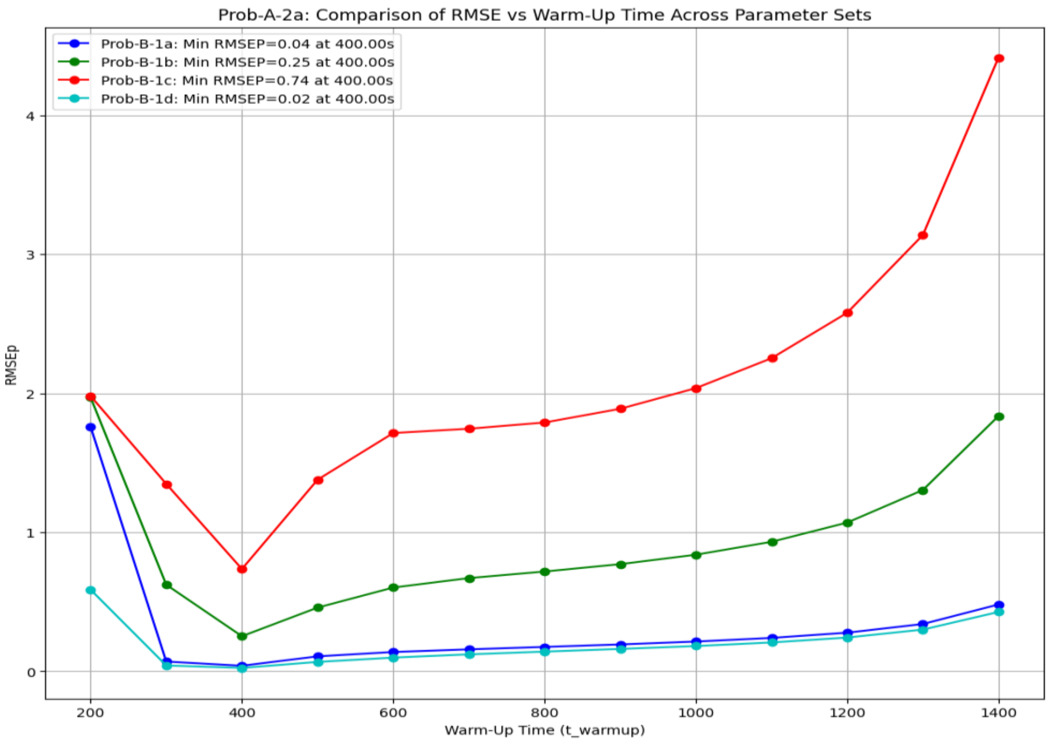


Figure-11: Problem-B-2a (Root Mean Square Error in the prediction phase against the Warm-up time)

Problem-B-2b

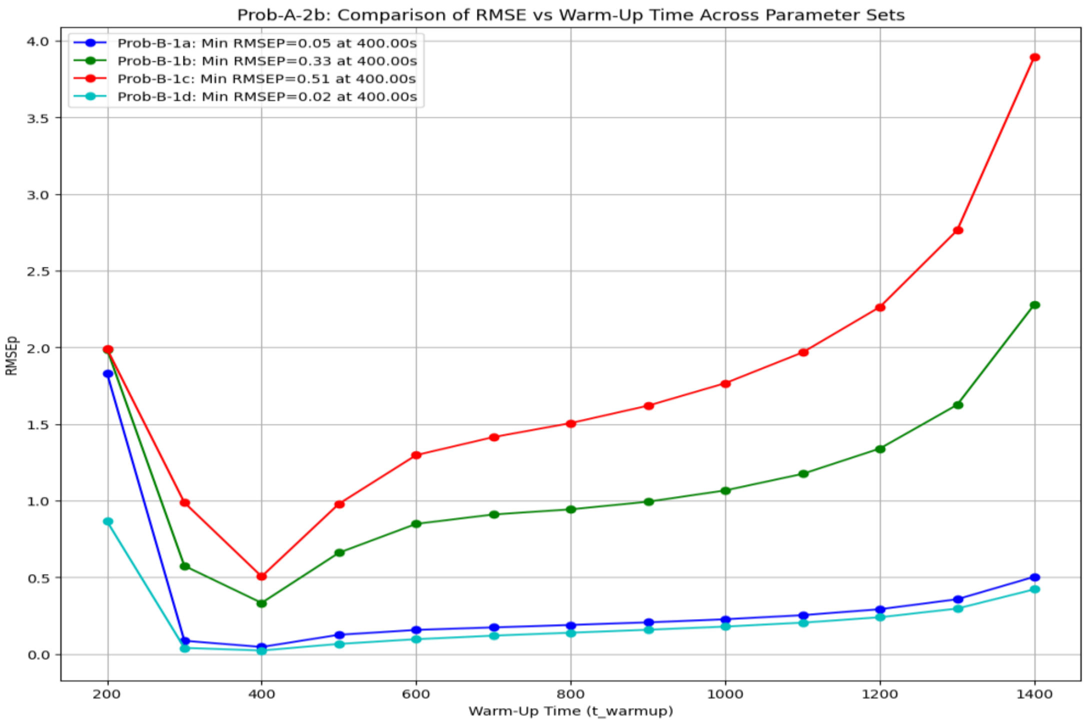


Figure-12: Problem-B-2b (Root Mean Square Error in the prediction phase against the Warm-up time)

Problem-B-2c

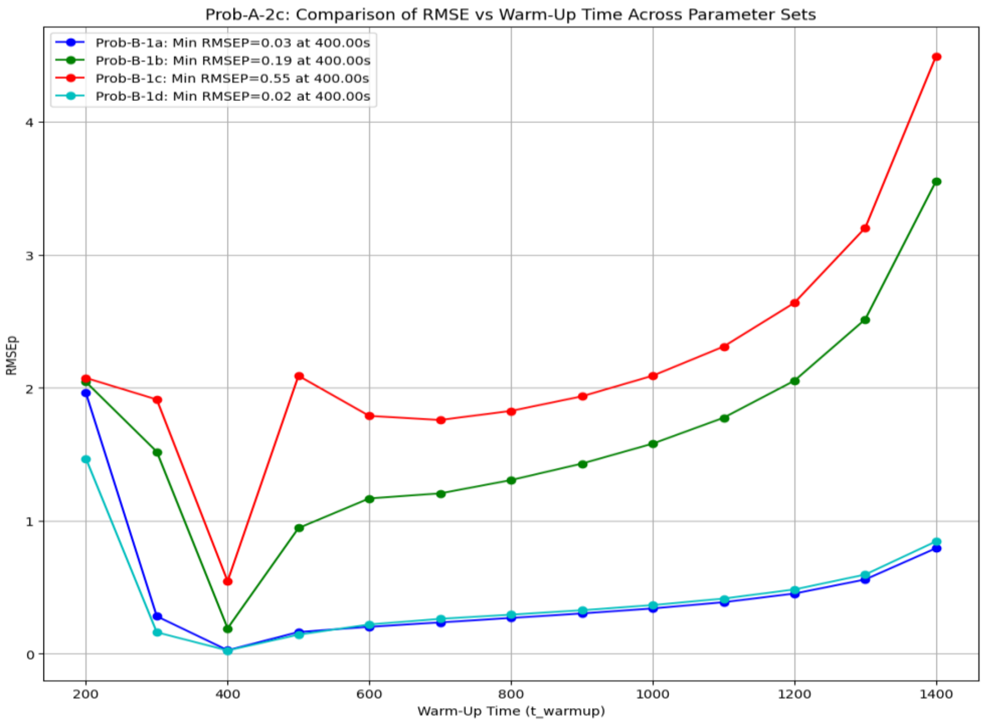


Figure-13: Problem-B-2c (Root Mean Square Error in the prediction phase against the Warm-up time)

Problem-B-2d

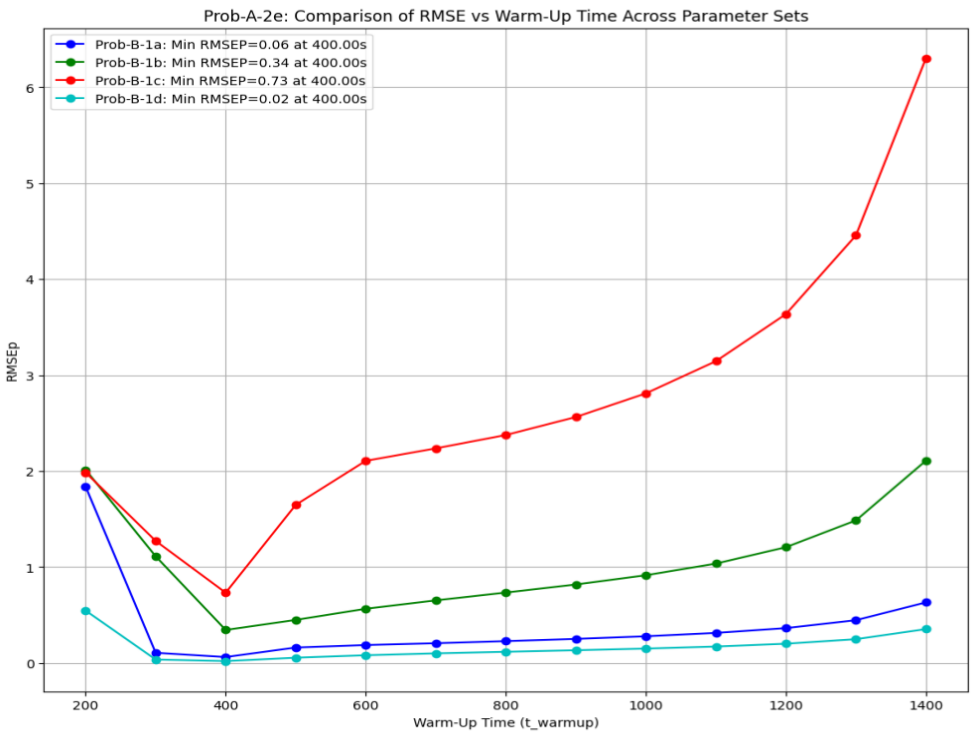


Figure-14: Problem-B-2d (Root Mean Square Error in the prediction phase against the Warm-up time)

For Prob-B-2(a,b,c,d): Each line in the graph represents a unique set of optimized hyperparameters found in Problem B 1(a), 1(b), 1(c), and 1(d), allowing comparison across different configurations and their impact on the RMSE during the prediction phase. We take the minimal RMSE value to find out optimal hyperparameters for each problem.

Problem-B-3a

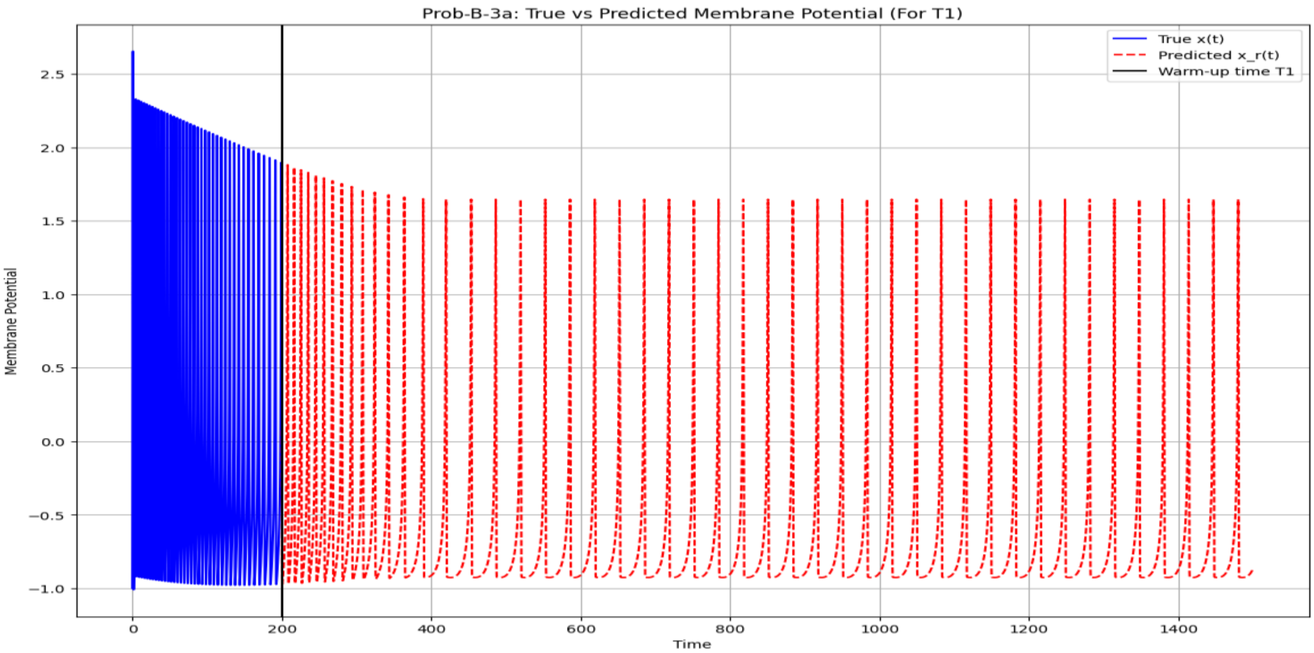


Figure-15: Problem-B-3a (Membrane Potential against Warm-up time T_1)

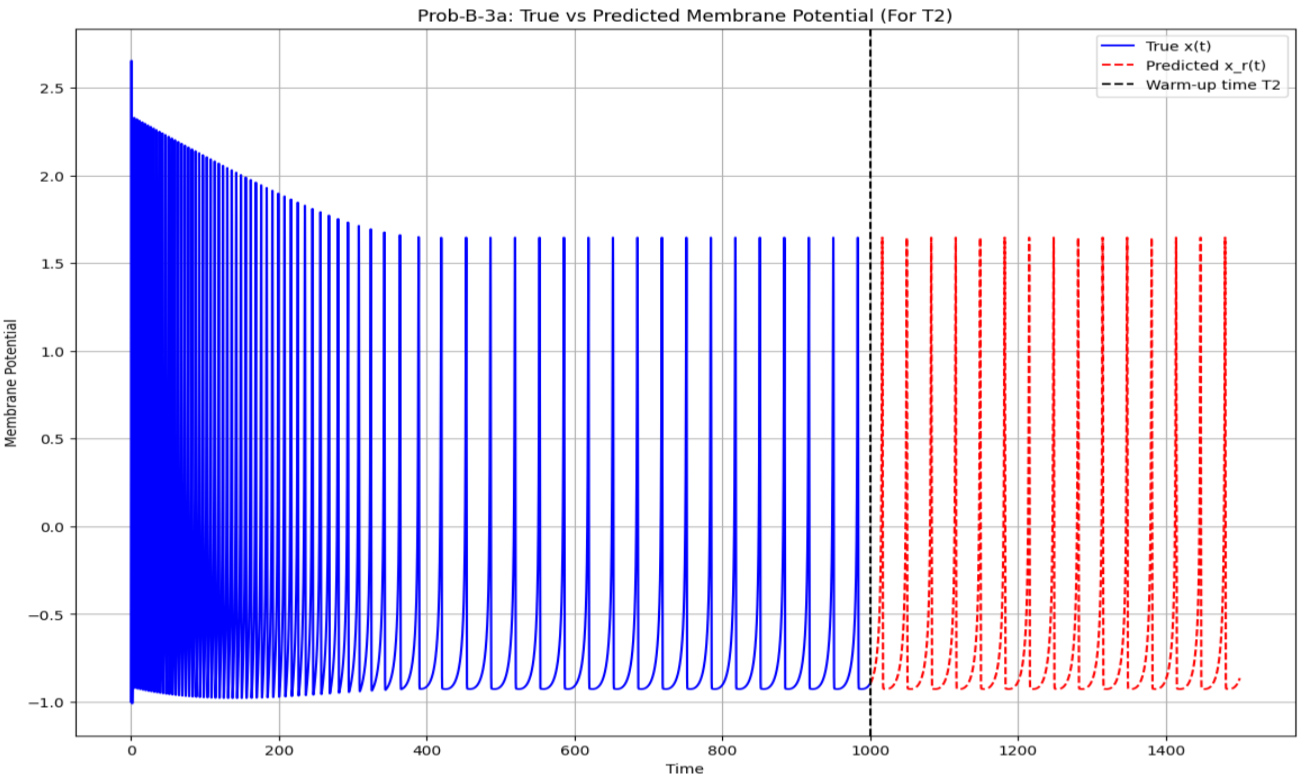


Figure-16: Problem-B-3a (Membrane Potential against Warm-up time T_2)

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Problem-B-3b

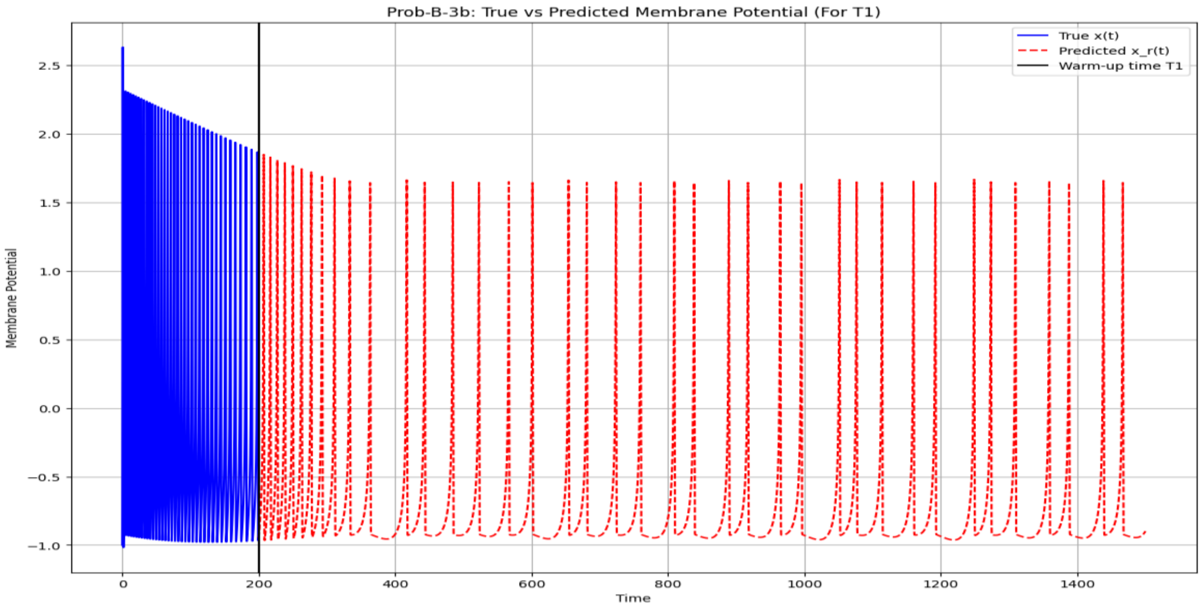

Figure-17: Problem-B-3b (Membrane Potential against Warm-up time T_1)

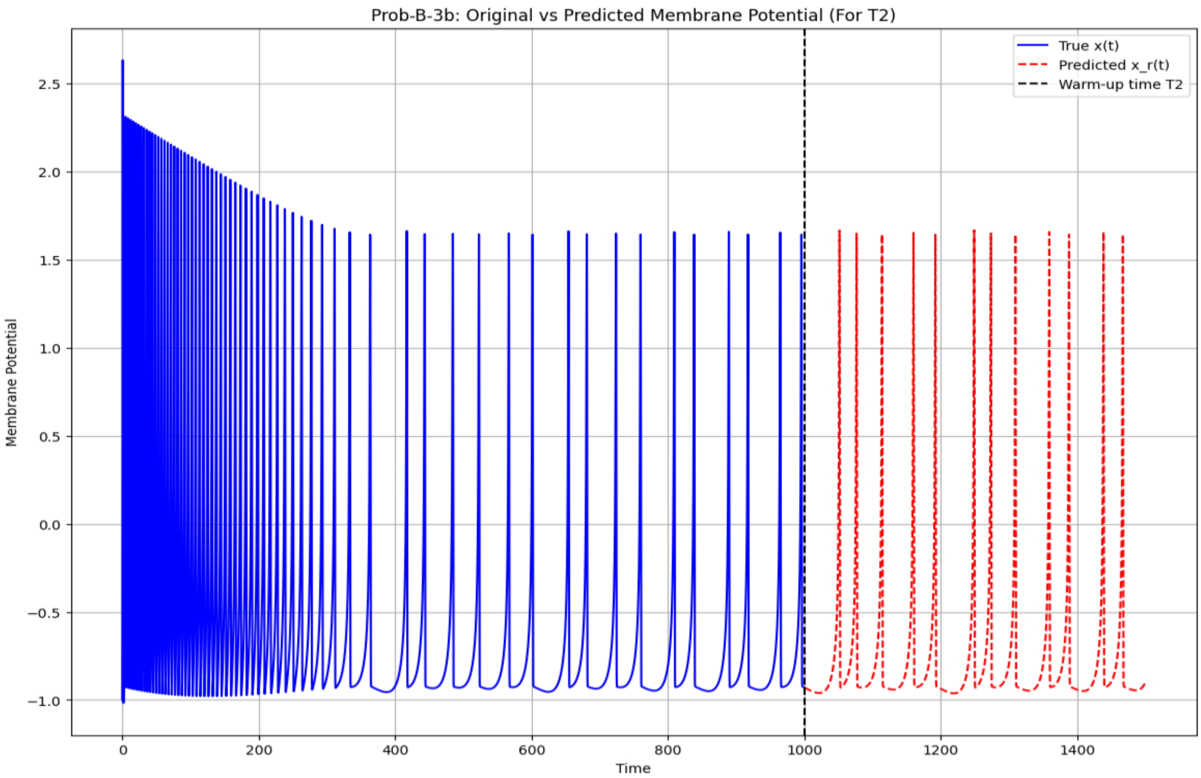

Figure-18: Problem-B-3b (Membrane Potential against Warm-up time T_2)

Problem-B-3c

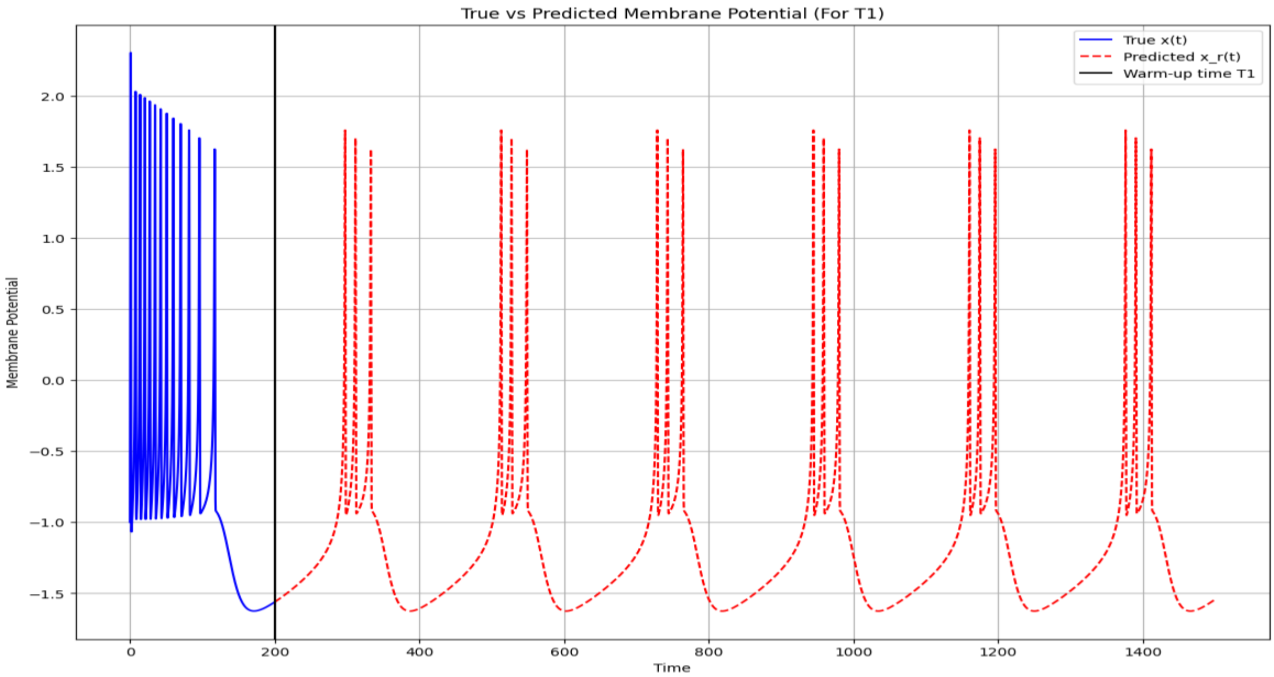

Figure-19: Problem-B-3c (Membrane Potential against Warm-up time T_1)

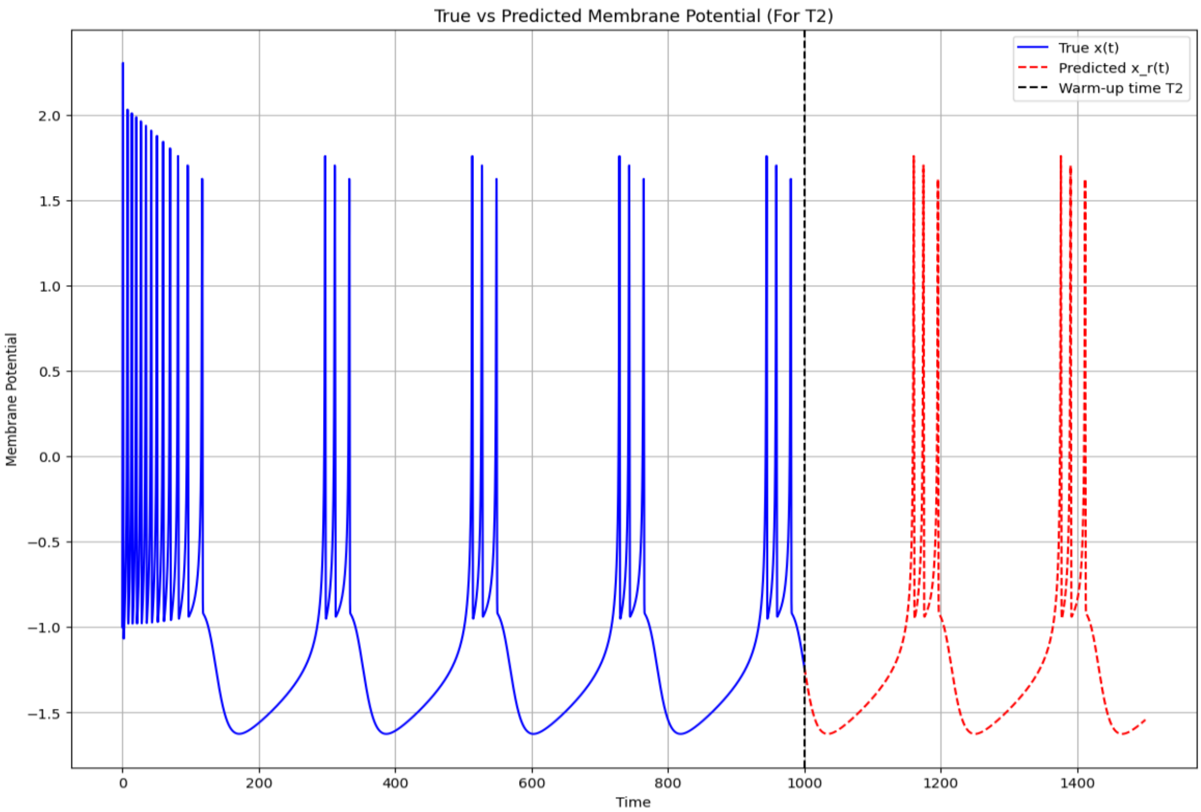

Figure-20: Problem-B-3c (Membrane Potential against Warm-up time T_2)

Problem-B-3d

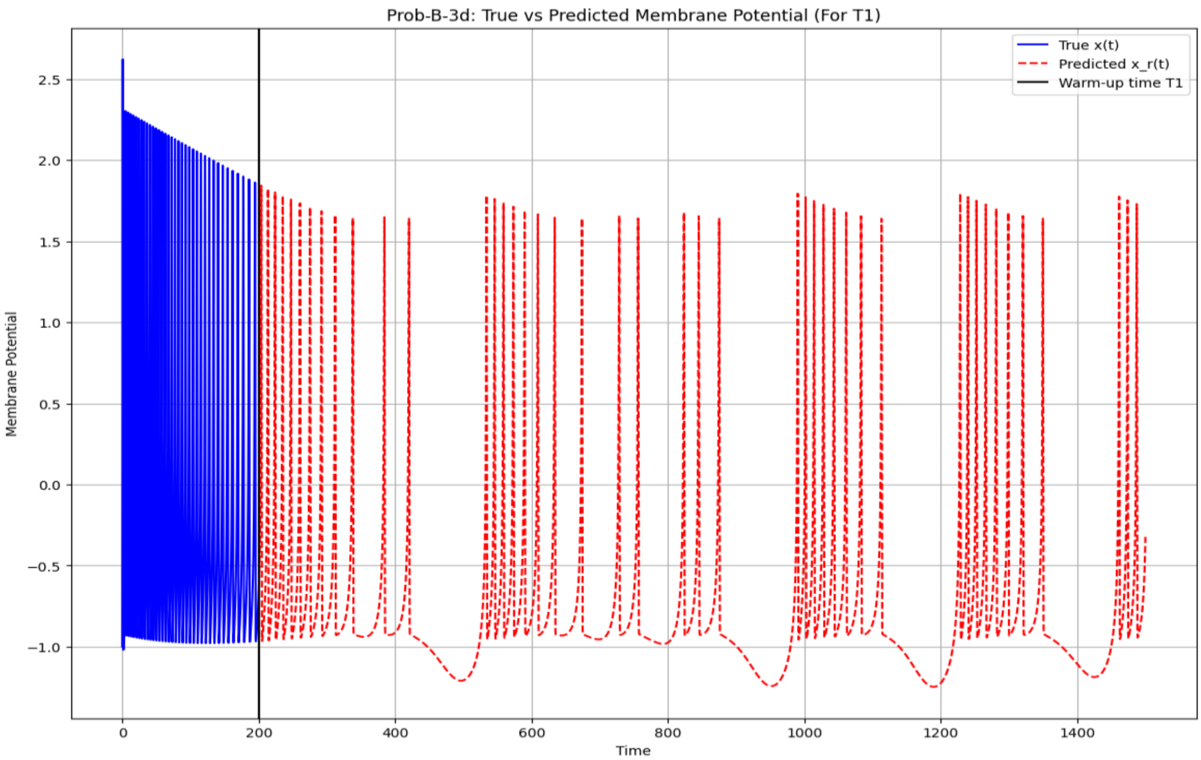

Figure-21: Problem-B-3d (Membrane Potential against Warm-up time T_1)

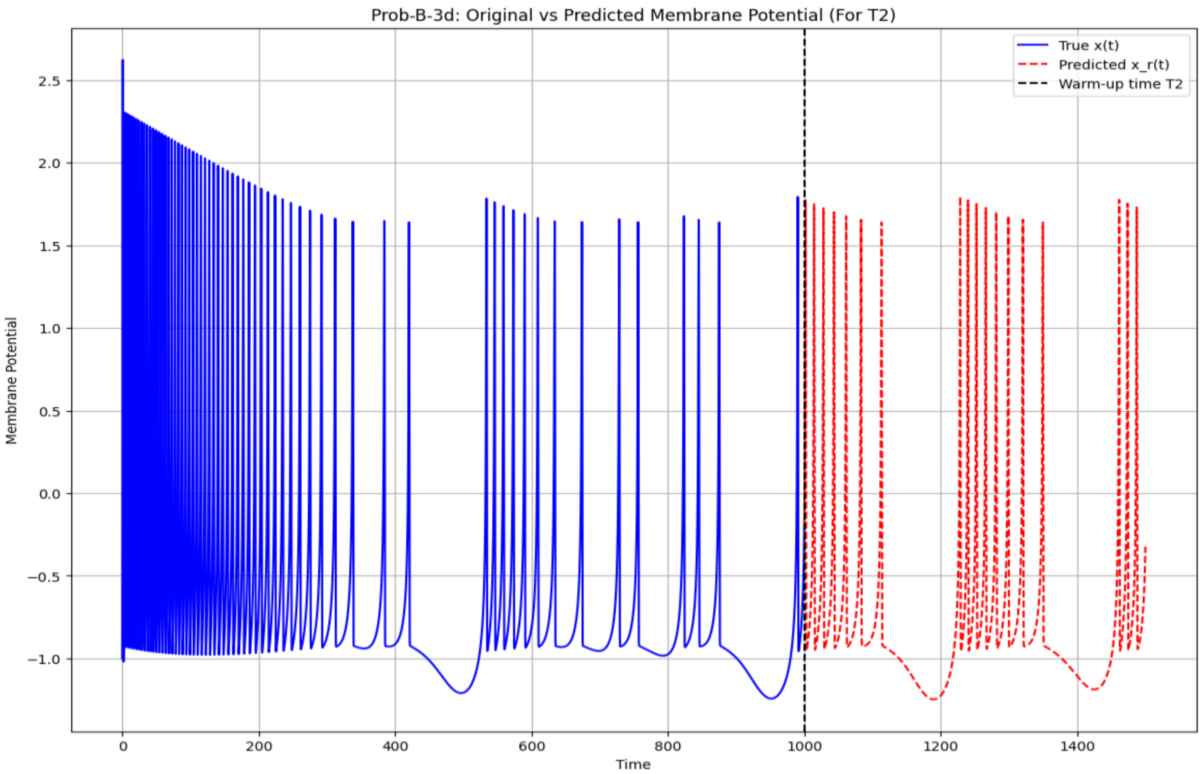

Figure-22: Problem-B-3d (Membrane Potential against Warm-up time T_2)

Problem-B-4

From Previous analysis we chose optimized hyperparameters with respect to lowest RMSE as follows:

optimized_params = {'N_res': 904, 'p': 0.8, 'alpha': 0.5, 'rho': 0.875}

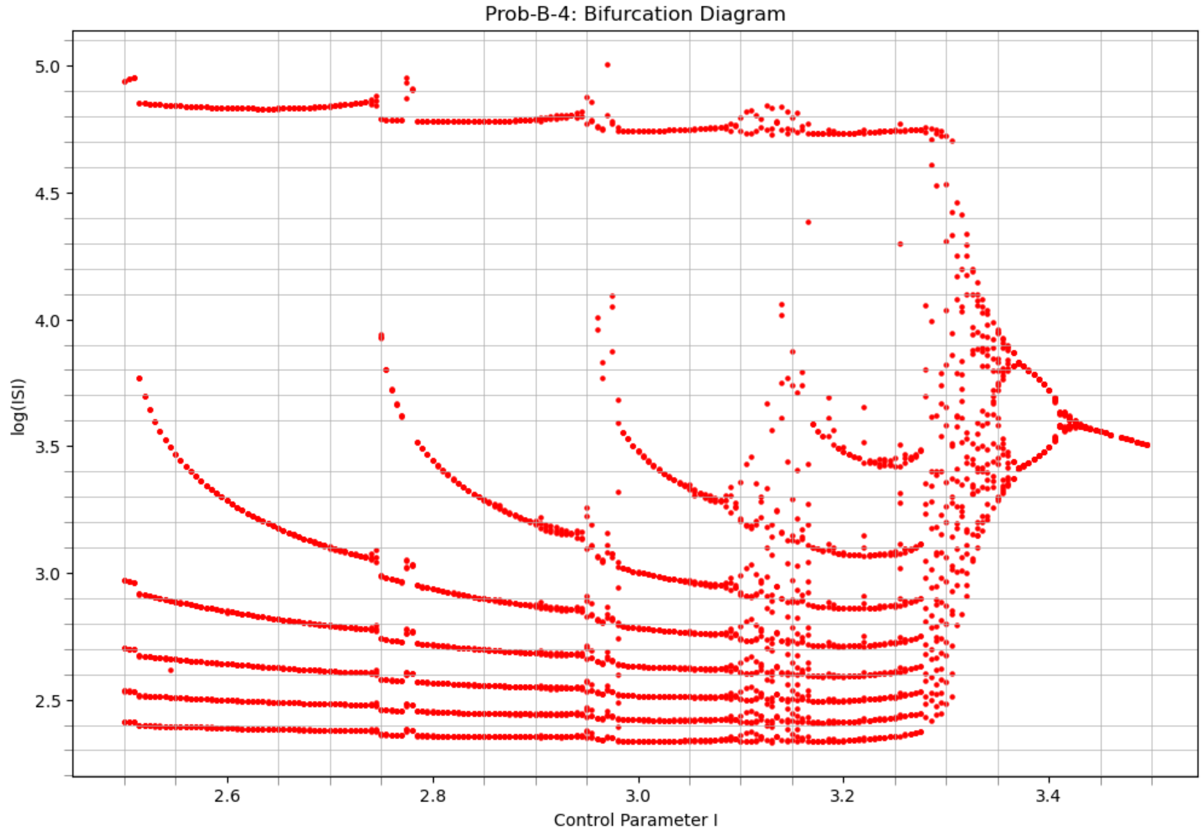

Figure-23: Problem-B-4 (Membrane Potential against Control parameter I)

Comparison between the bifurcation diagram obtained in Problem-A-1(a) and in Problem-B-4:

Overview

This analysis revisits the bifurcation diagrams derived from the direct simulations of the Hindmarsh-Rose neuron model (Problem A 1(a)) and contrasts them with those generated through predictions by an Echo State Network (ESN) (Problem B 4). The focus is on understanding the variation in dynamics captured by both approaches as influenced by the control parameter I.

Detailed Observations

1. Bifurcation Diagram in Figure 1 from Direct Simulation (Problem A 1(a)):

   i) Characteristics: The bifurcation diagram generated from the direct simulation exhibits a dense and intricate pattern with numerous branching points. These branches highlight a diverse range of dynamical states, transitioning between periodic and chaotic behaviors as I varies.

   ii) Implication: The complexity observed in this diagram reflects the model's sensitivity and responsiveness to changes in I, capturing a wide spectrum of neuronal behaviors critical for understanding neurophysiological processes.

2. Bifurcation Diagram in Figure 23 from ESN Predictions (Problem B 4):

    i) Characteristics: In contrast, the diagram obtained from ESN predictions presents a more streamlined and less varied pattern. The branches are notably fewer and the transitions between different dynamical states appear more muted.

    ii) Implication: This simplified pattern suggests that while the ESN grasps the overarching trends of the Hindmarsh-Rose dynamics, it may smooth over some of the finer, more complex behaviors inherent to the original model.

The bifurcation diagrams from Problem A 1(a) and Problem B 4 illustrate a marked difference in the level of detail each approach captures. The direct simulation provides a richer, more detailed view of the neuron model's dynamics, while the ESN, though effective for broader trend prediction, may not fully replicate the intricate dynamics, particularly in regions characterized by rapid, chaotic changes. This comparative analysis underscores the need for enhancements in neural network training strategies or model architectures to bridge the gap between broad-scale predictive accuracy and the capture of detailed dynamical nuances.