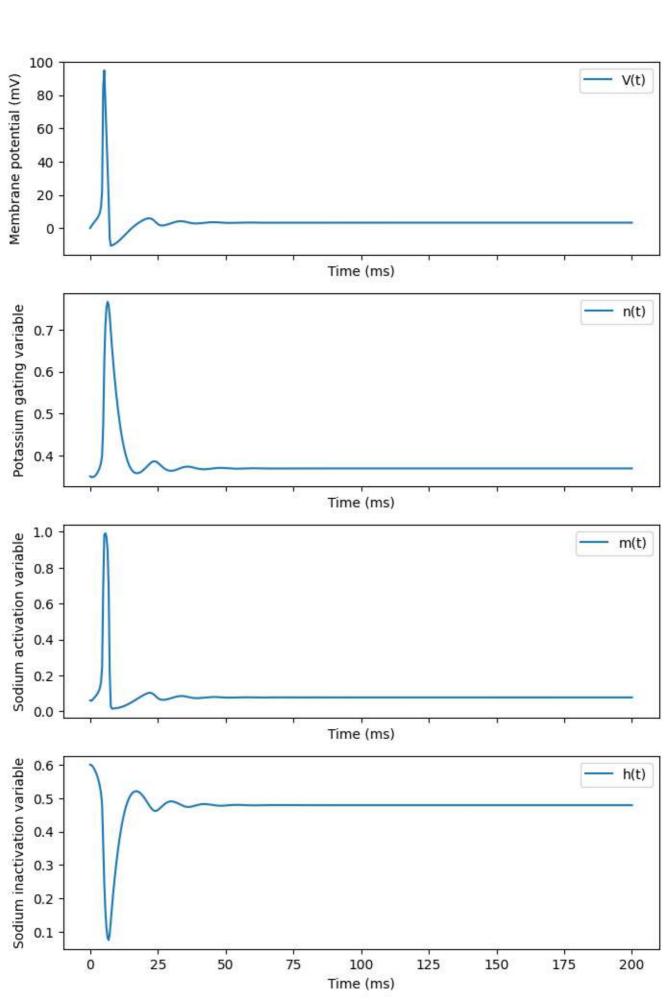
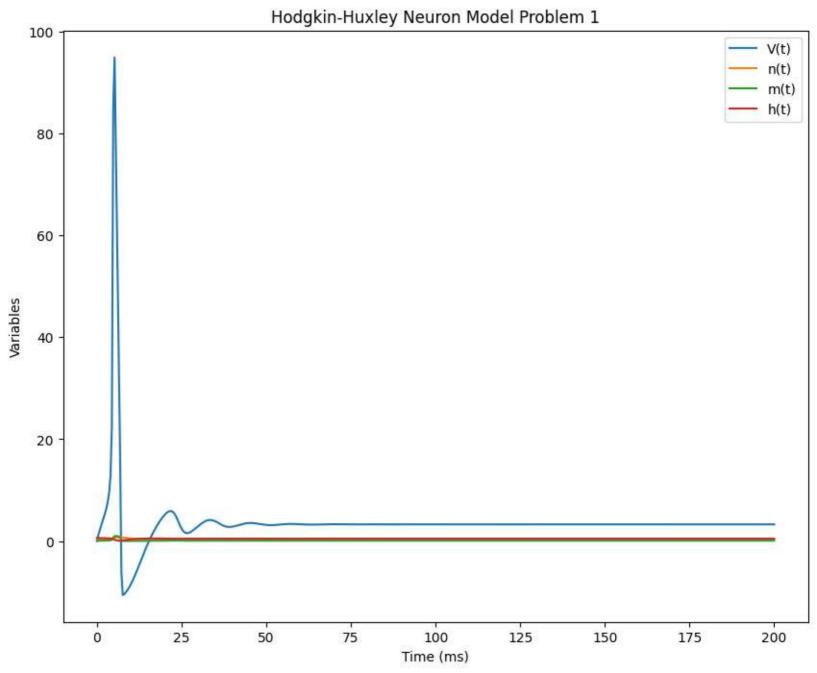
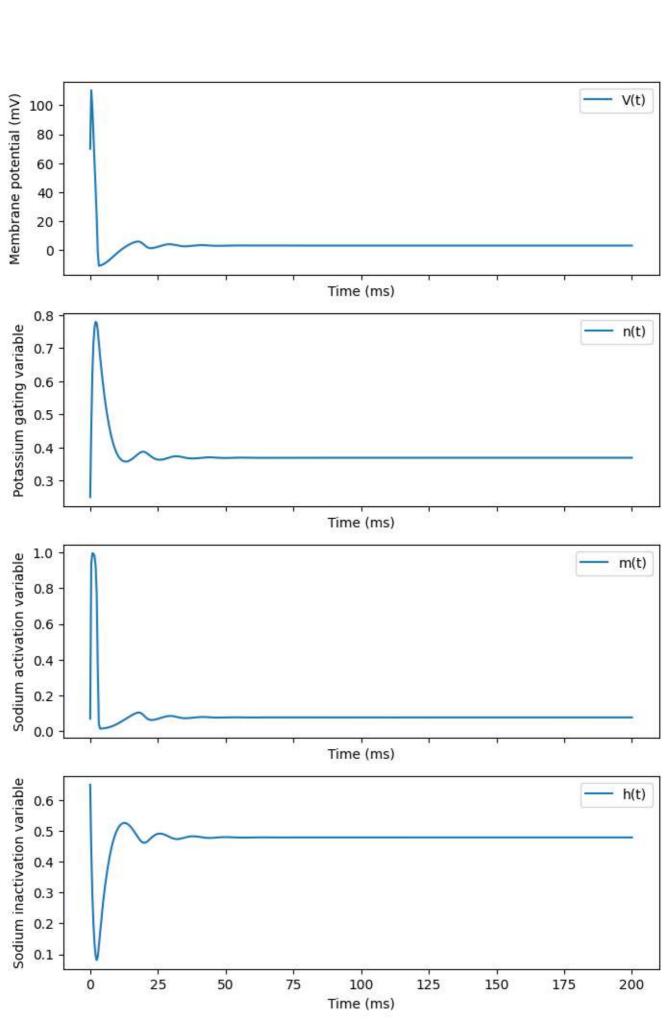
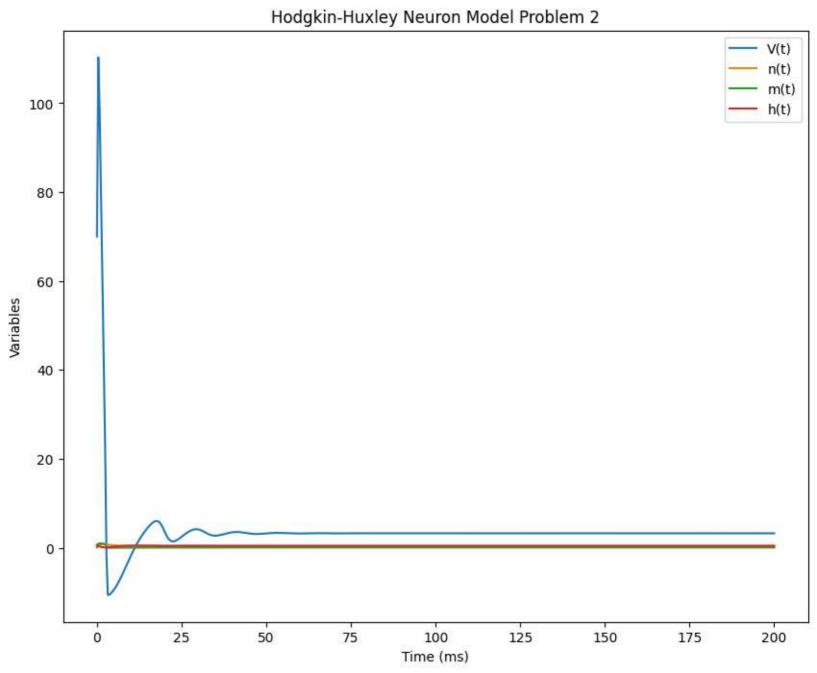
Hodgkin-Huxley Neuron Dynamics for Problem 1

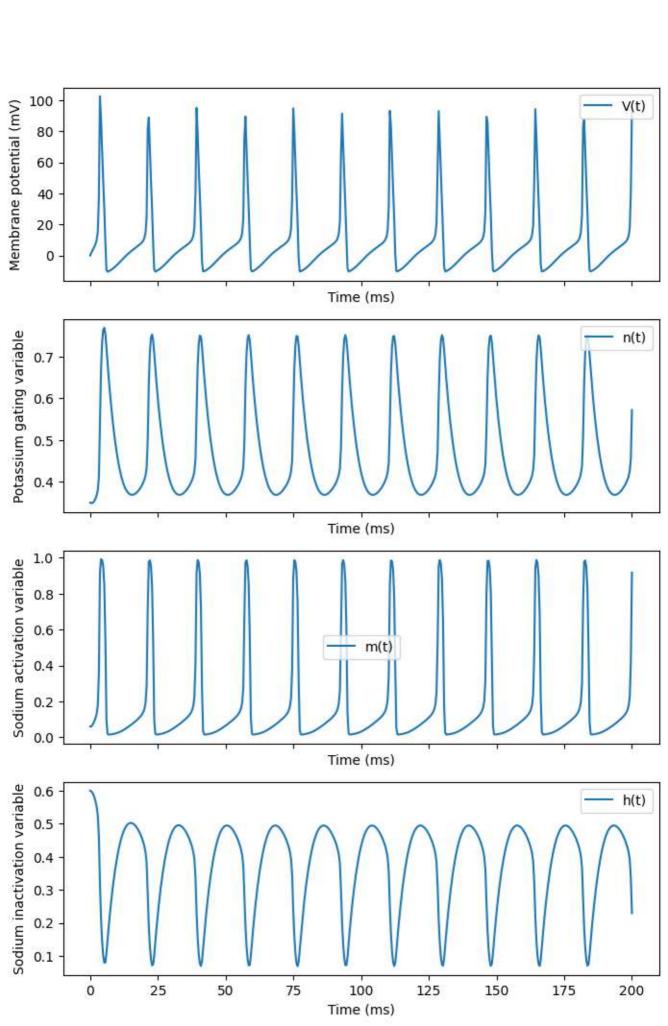
Mirza Shaheen Iqubal (22998316)

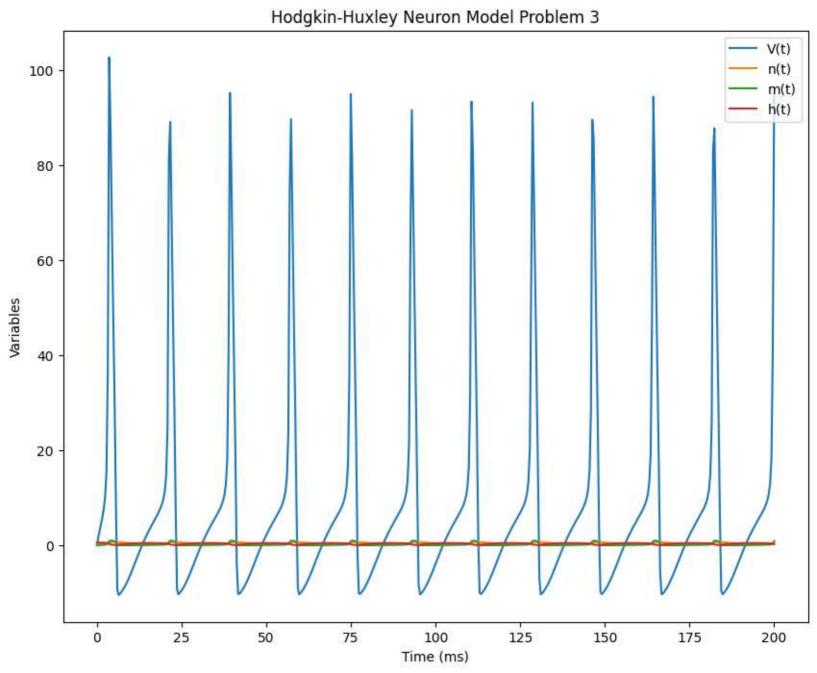


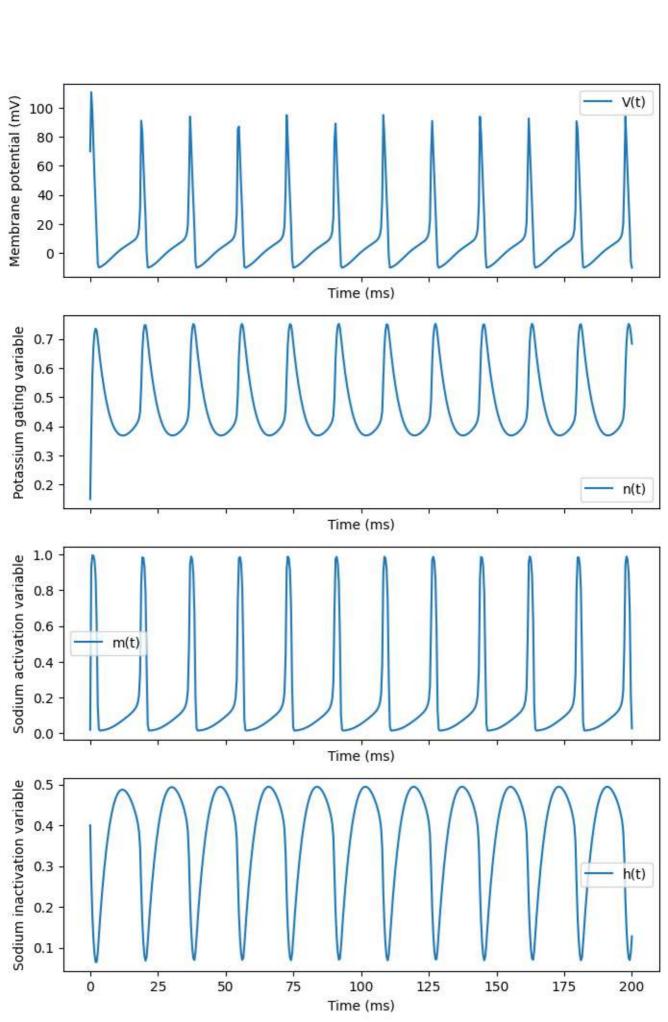


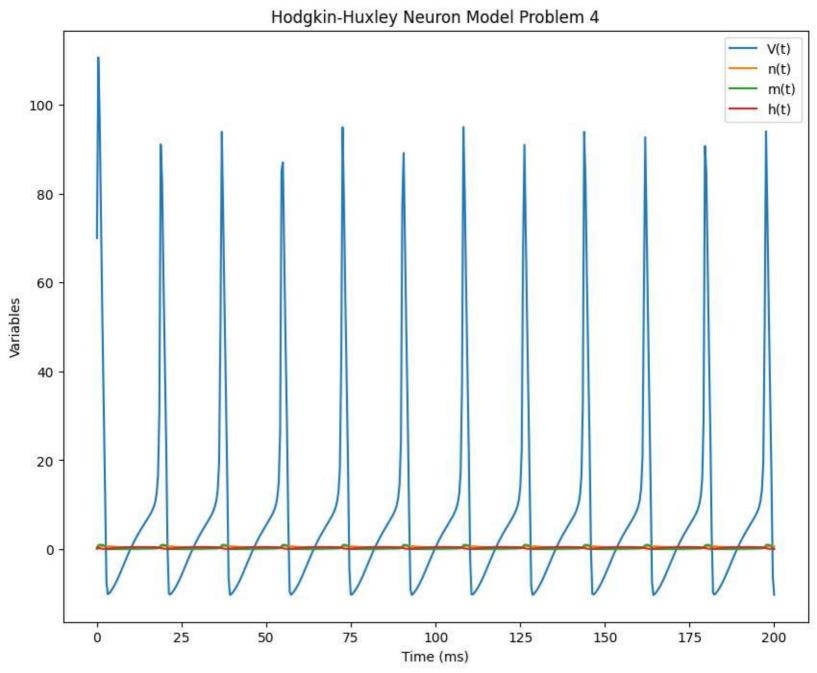












Part 2: Timescale separation and dimension reduction

Problem 1

Consider the following set of coupled ODEs, each characterized by the timescales ε_1 and ε_2 ,

$$\begin{cases} \frac{dx}{dt} = \varepsilon_1^{-1} \left[-x(t) + y(t) + I(t) \right] \\ \frac{dy}{dt} = \left[-y(t) + x(t)^2 + A \right] / \varepsilon_2 \end{cases}$$
(3)

(a) If $\varepsilon_1 \leq \varepsilon_2$, then the system can be reduced to:

$$\varepsilon_1 \frac{dx}{dt} = -x(t) + x(t)^2 + I(t) + A \tag{4}$$

(b) If $\varepsilon_2 \leq \varepsilon_1$, then the system can be reduced to:

$$\frac{dy}{dt} = \frac{1}{\varepsilon_2} \left[-y(t) + \left[y(t) + I(t) \right]^2 + A \right] \tag{5}$$

(c) If $\varepsilon_1 = \varepsilon_2$, then none of the ODEs in (a) and (b) above can be correct.

Problem 2

A channel with gating variable R(t), given by $\frac{dR}{dt} = \varepsilon_1^{-1} \left[-R(t) + R_0(V) \right]$ influences the voltage V(t) given by $\varepsilon_2 \frac{dV}{dt} = -\left[V(t) - V_0 \right] + R(t)^2 A$.

A reduction of dimension

- (a) is possible, and the result is $\varepsilon_1 \frac{dR}{dt} = -R(t) + R_0 [V_0 + R(t)^2 A]$ if $\varepsilon_2 \le \varepsilon_1$.
- (b) is impossible if $\varepsilon_1 = \varepsilon_2$.
- (c) is possible, and the result is $\varepsilon_2 \frac{dV}{dt} = -V(t) + V_0 + R_0(V)^2 A$ if $\varepsilon_2 \ge \varepsilon_1$.