Problem Statement and Goals
Inverted Pendulum Control Systems

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Table 1: Revision History

Date	Developer(s)	Change
2024-01-19	Morteza Mirzaei	Write the first version of the initial document for reverse pendulum.

1 Problem Statement

The inverted pendulum problem is a classic challenge in control theory, involving the stabilization of an upright pendulum atop a moving pivot. Its inherent instability makes it a compelling theoretical exercise, demanding precise control to prevent the pendulum from falling. This problem acts as a benchmark for testing and refining control algorithms, influencing advancements in areas like autonomous vehicles, robotics, and stability control for devices such as self-balancing scooters. The insights gained from solving this problem have practical applications, shaping developments in real-world systems that demand stability and dynamic control.

1.1 Problem

Mathematically, the dynamics of an inverted pendulum can be described by a set of differential equations derived from Newton's laws of motion. Let x be the horizontal position of the cart, θ the angle of the pendulum with respect to the vertical axis. The equations of motion for the inverted pendulum are given by:

$$(M+m)\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta = F$$

Where: M is the mass of the cart, m is the mass of the pendulum point mass, g is the acceleration due to gravity, F is the horizontal force applied to the cart, and L is the length of the pendulum.

The challenge in the inverted pendulum problem is to design a control law F such that the system remains stable, and the pendulum stays balanced in the inverted position despite its inherent instability.

1.2 Inputs and Outputs

The inverted pendulum problem involves the following key inputs:

1. Angle of the pendulum with the axis perpendicular to the cart (θ)
2. Angular velocity of the pendulum ($\dot{\theta}$)
3. Position of the cart along the axis (x)
4. Speed of the cart along the axis ($\dot{x}$)

The primary output of the inverted pendulum problem is the force (F), representing the force applied to the cart along the x-axis.

2 Goals

1. **Stability Achievement:** Develop a control algorithm that effectively stabilizes the inverted pendulum system, maintaining the pendulum within a deviation of less than 20 degrees from the upright position after 20 seconds of operation.
2. **Real-time Responsiveness:** Design the controller to exhibit real-time responsiveness, ensuring swift and accurate adjustments to the applied forces based on feedback from sensors, with the requirement that the angular speed remains below 10 degrees per second for smooth and controlled movements.
3. **Recovery from Instability:** Implement a feature that enables the system to autonomously recover from instability caused by external forces, bringing the pendulum back to a stable position within 20 seconds of deviation.

3 Stretch Goals

1. **Generalizability:** Extend the control solution's versatility beyond the traditional simple round-shaped pendulum to encompass more advanced shapes, specifically focusing on configurations where the center of mass of the complex pendulum dynamically changes through rotation. This goal aims to enhance the control algorithm's adaptability, allowing it to effectively handle a broader range of inverted pendulum systems with varying complexities and geometries.