Software Requirements Specification for Inverted Pendulum Control Systems (IPCS)

Morteza Mirzaei

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Contents

Revision History

Date	Version	Notes
2024-02-06	1.0	Initial Release.
2024-04-13	2.0	Addressed reviewer comments.

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	**unit**	**SI**
m	length	metre
kg	mass	kilogram
s	time	second
N	force	N ($\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$)
rad	angle	radian

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

symbol	**unit**	**description**
m	kg	Mass of the pendulum
M	kg	Mass of the cart
l	m	Length to pendulum center of mass
g	$\mathrm{m\,s^{-2}}$	Acceleration due to gravity
$F(t)$	N	Horizontal force exerted on the cart over time
$x(t)$	m	The distance which cart transfer over time
$\dot{x}(t)$	$\mathrm{m\,s^{-1}}$	Velocity of the cart over time
$\ddot{x}(t)$	$\mathrm{m\,s^{-2}}$	Acceleration of the cart over time
$\theta(t)$	rad	Pendulum angle from vertical over time
$\dot{\theta}(t)$	$\mathrm{rad\,s^{-1}}$	Angular velocity of the pendulum over time
$\ddot{\theta}(t)$	$\mathrm{rad\,s^{-2}}$	Angular acceleration of the pendulum over time

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
IPCS	Inverted Pendulum Control Systems
TM	Theoretical Model

1.4 Mathematical Notation

Vectors are represented with an arrow on top. For example, $\vec{x}$ denotes a vector, while m represents a scalar. By default, a vector $\vec{x}$ belongs to $\mathbb{R}^2$, and if it belongs to higher dimensions, it is explicitly specified in the text. The symbol $\cdot$ signifies the dot product between two vectors.

2 Introduction

The inverted pendulum problem is a classic challenge in control theory, involving the stabilization of an upright pendulum atop a moving pivot. Its inherent instability makes it a compelling theoretical exercise, demanding precise control to prevent the pendulum from falling. This problem acts as a benchmark for testing and refining control algorithms, influencing advancements in areas like autonomous vehicles, robotics, and stability control for devices such as self-balancing scooters. The insights gained from solving this problem have practical applications, shaping developments in real-world systems that demand stability and dynamic control. The Inverted Pendulum Control Systems (IPCS) software aims to simulate the physical system of an inverted pendulum and design a control system to keep the pendulum upright. This document outlines the requirements for the Inverted Pendulum Control Systems software, providing a clear and unambiguous description of the software's functionalities, features, and constraints.

2.1 Purpose of Document

The primary purpose of this document is to provide a clear and unambiguous description of the software's functionalities, features, and constraints, serving as a bridge between the client's expectations and the development team's understanding. By meticulously documenting the project's requirements, the SRS enables effective communication between stakeholders, including clients, developers, testers, and project managers. It acts as a blueprint that helps in minimizing misunderstandings, mitigating risks, and ensuring that the final software product aligns with the client's needs and specifications. Additionally, it serves as a valuable reference point for developers, guiding them in the creation of a robust and reliable software solution while facilitating effective project management and quality assurance processes.

2.2 Scope of Requirements

In this project, the scope is delimited to two dimensions. The pendulum is affixed to the cart in a manner enabling free rotation about the pivot, while the cart possesses horizontal mobility. The primary focus of this project is on simulating system behavior and designing a control system for said simulation, with a specific emphasis on efficient execution. For further elucidation, readers are directed to the assumptions section.

2.3 Characteristics of Intended Reader

To effectively comprehend this material, readers or reviewers should be familiar with the principles of physics, specifically in the context of applying Newton's laws of motion. Additionally, a strong understanding of calculus, particularly in differentiation and integration, is essential. Proficiency in these subjects is typically attained through undergraduate engineering courses or at a high school level.

2.4 Organization of Document

The rest of this document is structured as follows. Section 3 discusses the general context and description of the system. Section 4 details the specific system description, goals, and definitions. Section 5 covers system requirements. Section 6 outlines likely changes for the system. Section 7 discusses unlikely changes. Section 8 covers the traceability of the requirements.

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

3.1 System Context

The system consists of three primary components: the *System Dynamics* module, responsible for simulating the physical state of the pendulum; the *Control System*, which receives the pendulum's state information and applies external forces to the cart to maintain the pendulum's upright position; and the *Visualizer* or GUI (Graphical User Interface), which presents the system's state information to the user. The interaction between these components and the user is outlined in Figure 1. Users provide initial conditions for the system, such as the mass of the cart and pendulum, as well as the angular position of the pendulum. In the output, users can view the state of the system in different time steps in real-time.

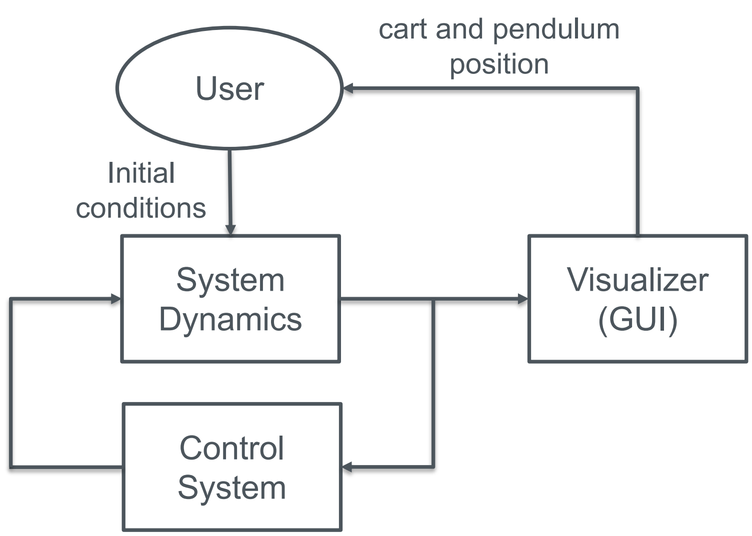


Figure 1: System Context

- User Responsibilities:
  - Provide required inputs, including cart and pole specifications, as well as initial conditions of the system.
- Inverted Pendulum Control Systems Responsibilities:
  - Calculate the physical state of the system for different time steps.
  - Display and visualize the calculated physical state to the user.
  - Implement control strategies to stabilize the pendulum and maintain its upright position.

3.2 User Characteristics

The intended users of the system do not require extensive technical expertise; rather, basic computer literacy is sufficient for running the program and observing the simulation. However, for users wishing to delve deeper into result analysis, a solid foundation in undergraduate-level mathematics and physics is necessary. Proficiency in mathematical concepts such as calculus as well as a fundamental understanding of physics principles, will enable users to effectively interpret and analyze the simulation outcomes.

3.3 System Constraints

There are no system constraints for this project.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

Inverted Pendulum Control Systems is intended to solve the classic challenge in control theory known as the inverted pendulum problem. This problem involves the stabilization of an upright pendulum atop a moving pivot, characterized by its inherent instability that demands precise control to prevent the pendulum from falling. Basically, we are going to simulate the physical system of an inverted pendulum and build a controller to keep the pendulum upright. It serves as a benchmark for testing and refining control algorithms, influencing advancements in areas like autonomous vehicles, robotics, and stability control for devices such as self-balancing scooters. The insights gained from solving this problem have practical applications, shaping developments in real-world systems that demand stability and dynamic control.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Inverted Pendulum: An inverted pendulum refers to a pendulum where its center of mass is positioned above its pivot point. Due to its unstable nature, it tends to topple over without external assistance.
- Friction: Friction denotes the resistance force encountered when one solid object slides or rolls over another.
- Control System: A control system is a system that manages, commands, directs, or regulates the behavior of other devices or systems.
- State: The state of a system refers to a set of quantities that determine the behavior of the system over time.
- Stabilization: Stabilization refers to the process of maintaining the pendulum within a small deviation from the upright position.
- Force Equilibrium: A body achieves force equilibrium when the total sum of all forces acting upon it equals zero.

4.1.2 Physical System Description

The physical system of Inverted Pendulum Control Systems, as shown in Figure 2, includes the following elements:

PS1: Pendulum

PS1a: rod (with lengh of l)

PS1b: ball (with mass of m)

PS2: Cart (with mass of M)

4.1.3 Goal Statements

Given the cart and the pendulum specifications (mass of cart, mass of ball, length of rod) and the initial conditions, the goal statements are:

GS1: Simulate the state of the system, position of the cart and the angle of the pendulum arm, over time.

GS2: Implement Control Algorithm that effectively stabilizes the inverted pendulum system and maintain the pendulum within the upright position.

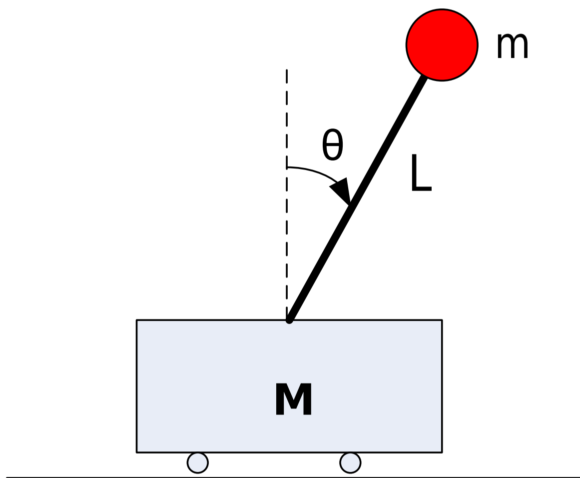
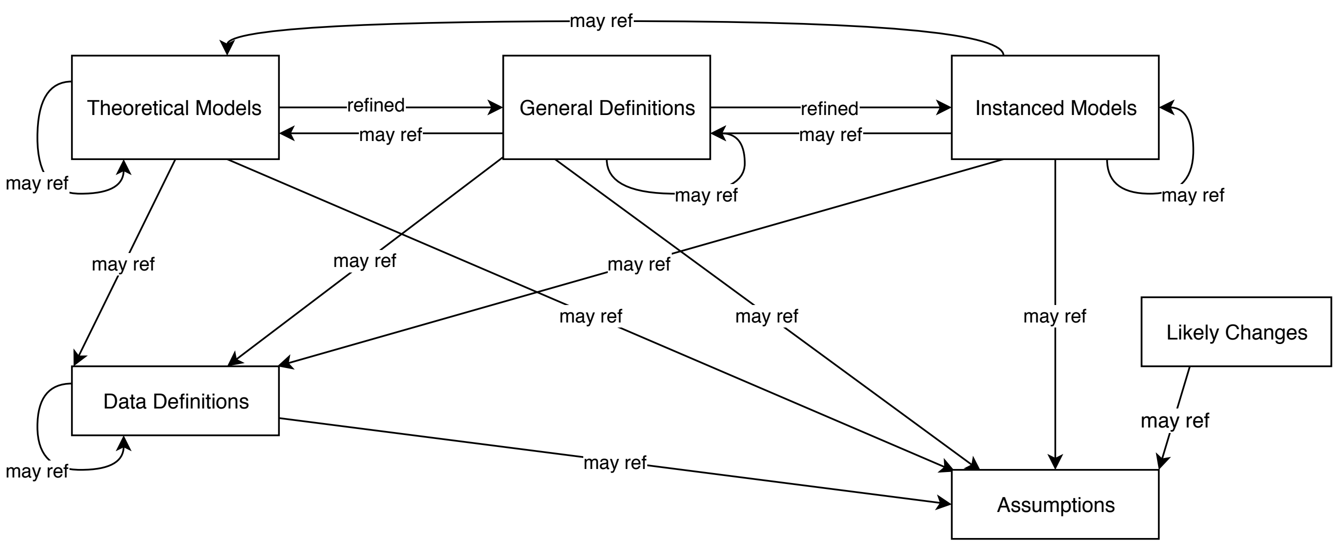


Figure 2: Inverted Pendulum Diagram

4.2 Solution Characteristics Specification



The instance models that govern Inverted Pendulum Control Systems are presented in Subsection 4.4.6. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Types

N/A.

4.3 Scope Decisions

N/A.

4.4 Modelling Decisions

4.4.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [TM], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: One degree of freedom for Pole (Rotation).

A2: One degree of freedom for Cart (Forward and Backward).

A3: The rod is considered massless.

A4: Classical mechanics holds (Newton Laws).

A5: No friction for the cart.

A6: No air resistance.

A7: Every body is considered as a rigid body.

4.4.2 Theoretical Models

This section focuses on the general equations and laws that Inverted Pendulum Control Systems is based on.

RefName: TM1

Label: Newton's Second Law

Equation: $F = ma$

Description: Newton's second law of motion pertains to the behavior of objects for which all existing forces are not balanced. The second law states that the acceleration of an object is dependent upon two variables, the net force acting upon the object which is a, and the mass of the object which is m. The acceleration of an object depends directly upon the net force acting upon the object, and inversely upon the mass of the object. As the force acting upon an object is increased, the acceleration of the object is increased. As the mass of an object is increased, the acceleration of the object is decreased.
F is the net force acting upon the object in N,
a is the acceleration of the object in $\mathrm{m\,s^{-2}}$,
m is the mass of that object in kg.

Notes: None.

Source: https://www.physicsclassroom.com/class/newtlaws/Lesson-3/Newton-s-Second-Law

Ref. By: IM1

Preconditions for TM:NSL: A4

Derivation for TM:NSL: Not Applicable

RefName: TM2

Label: Acceleration of a function (the rate of change of the rate of change)

Equation: $\frac{d^2}{dx^2} f(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

Description: Given any continous function $f(x)$, the acceleration of the function is defined as the rate of change of the rate of change of the function. This is equivalent to taking the second derivative of the function with respect to x.

Notes: None.

Source: https://en.wikipedia.org/wiki/Second_derivative

Ref. By: DD1

Preconditions for TM:AF: None.

Derivation for TM:AF: Not Applicable

4.4.3 General Definitions

This section collects the laws and equations that will be used in building the instance models. The global coordinate system referenced in the subsequent equations is positioned within the plane. Consequently, the cart's position, which consistently remains grounded relative to the general coordinate system, is expressed as $\vec{r}_c = x_c(t)\hat{i}$.

Number	GD1
Label	**Pendulum position in 2D space**
SI Units	$(\mathrm{m}, \mathrm{m})^T$
Equation	$\vec{r}_P = (x_c(t) - l\sin\theta(t))\hat{i} + (l\cos\theta(t))\hat{j}$
Description	The position of the pendulum in 2D space is given by the above equation, where $\vec{r}_P$ is the position vector of the pendulum, $x_c(t)$ is the position of the cart (m) at time t, l is the length of the rod (m), and $\theta(t)$ is the angle of the pendulum arm with respect to the vertical axis (rad) at time t.
Source	N/A
Ref. By	GD2

Derivation of the cart motion equation

In reference to the relative coordinate system depicted in Figure 3, the position of the pendulum can be defined as $\vec{r}_p = (-l\sin\theta(t))\hat{i} + (l\cos\theta(t))\hat{j}$. The values $l\sin\theta(t)$ and $l\cos\theta(t)$ are straightforwardly derived from the definitions of trigonometric functions within a right triangle. The negative sign preceding $l\sin\theta(t)$ accounts for the pendulum's angle increasing counter-clockwise from the vertical axis, with its zero position at the top.

The global coordinate system is situated somewhere within the plane. Consequently, the cart's position, which consistently remains grounded relative to the general coordinate system, is expressed as $\vec{r}_c = x_c(t)\hat{i}$. By combining the cart's position with that of the pendulum, we can ascertain the pendulum's position relative to the global coordinate system. Thus, $\vec{r}_p = \vec{r}_c + \vec{r}_p = (x_c(t) - l\sin\theta(t))\hat{i} + (l\cos\theta(t))\hat{j}$.

Number	GD2
Label	**Pendulum acceleration in 2D space**
SI Units	$(\mathrm{m\,s^{-2}}, \mathrm{m\,s^{-2}})^T$
Equation	$\vec{a}_P = (\ddot{x}_c(t) + l\dot{\theta}^2(t)\sin\theta(t) - l\ddot{\theta}(t)\cos\theta(t))\hat{\imath} + (-l\dot{\theta}^2(t)\cos\theta(t) - l\ddot{\theta}(t)\sin\theta(t))\hat{\jmath}$
Description	The acceleration of the pendulum in 2D space is given by the above equation, where $\vec{a}_P$ is the acceleration vector of the pendulum, $\ddot{x}$ is the acceleration of the cart ($\mathrm{m\,s^{-2}}$), $\dot{\theta}(t)$ is the angular velocity of the pendulum arm ($\mathrm{rad\,s^{-1}}$), $\ddot{\theta}(t)$ is the angular acceleration of the pendulum arm ($\mathrm{rad\,s^{-2}}$), and l is the length of the rod (m).
Source	GD1, DD1
Ref. By	IM1

Derivation of the cart motion equation

The acceleration of the pendulum can be obtained by computing the second derivative of its position vector. Denoted as $\vec{r}_P = (x_c(t) - l\sin\theta(t))\hat{\imath} + (l\cos\theta(t))\hat{\jmath}$, this represents the pendulum's position vector. Its first derivative is given by $\frac{d}{dt}\vec{r}_P = (\dot{x}_c(t) - l\dot{\theta}(t)\cos\theta(t))\hat{\imath} + (-l\dot{\theta}(t)\sin\theta(t))\hat{\jmath}$. Further differentiation leads to the second derivative of the position vector, expressed as $\frac{d^2}{dt^2}\vec{r}_P = (\ddot{x}_c(t) + l\dot{\theta}^2(t)\sin\theta(t) - l\ddot{\theta}(t)\cos\theta(t))\hat{\imath} + (-l\dot{\theta}^2(t)\cos\theta(t) - l\ddot{\theta}(t)\sin\theta(t))\hat{\jmath}$.

4.4.4 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

Number	DD1
Label	**Acceleration**
Symbol	a
SI Units	$\mathrm{m\,s^{-2}}$
Equation	$a = \ddot{x}$
Description	Acceleration is the second derivative of position to time. a is the acceleration of the object in $\mathrm{m\,s^{-2}}$, x is the position of the object in m.
Sources	TM2
Ref. By	GD2

4.4.5 Data Types

This section is not applicable.

4.4.6 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.4.4 to replace the abstract symbols in the models identified in Sections 4.4.2 and 4.4.3.

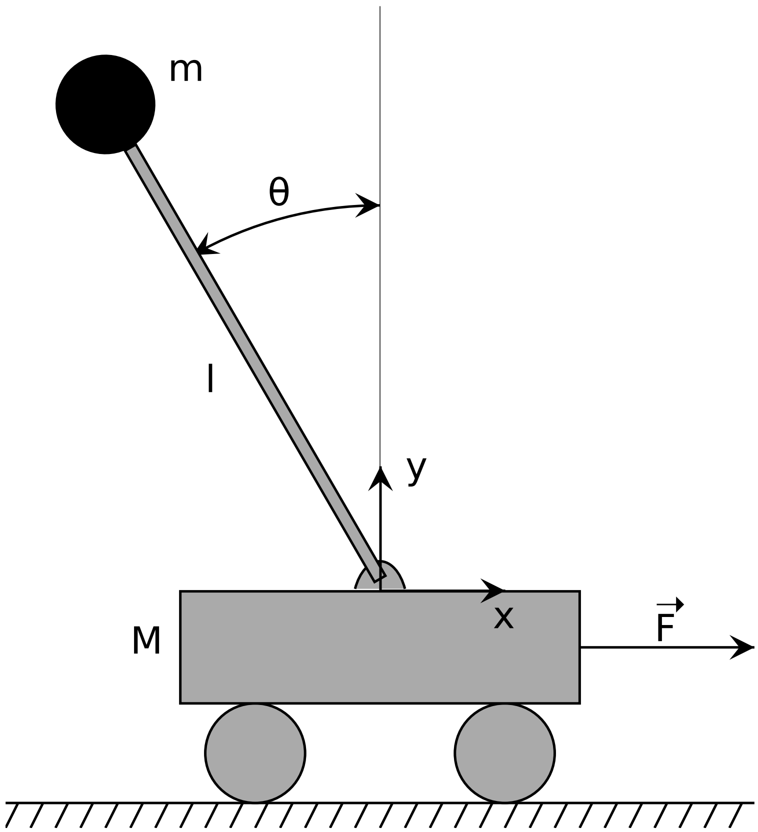


Figure 3: A schematic drawing of the inverted pendulum on a cart Inverted pendulum (2024).

The goals GS1 are solved instance model IM1.

Number	IM1
Label	**the cart and pendulum motion equation**
Input	$F(t)$, M, m, l
Output	$x_c(t)$ and $\theta(t)$, such that: $F(t) = (M+m)\ddot{x}_c(t) - ml\ddot{\theta}(t)\cos\theta(t) + ml(\dot{\theta}^2(t))\sin\theta(t)$ $l\ddot{\theta}(t) + g\sin\theta(t) = \ddot{x}_c(t)\cos\theta(t)$
Description	m is the mass of the pendulum (kg). M is the mass of the cart (kg). l is the length of pendulum (m). $F(t)$ is the force applied to the cart (N) at time t. $x_c(t)$ is the position of the cart (m) at time t. $\theta(t)$ is the angle of the pendulum arm with respect to the vertical axis (rad) at time t.
Sources	Inverted pendulum (2024)
Ref. By	-

Derivation of the cart motion equation

If we write the second law of Newton (TM1) for the cart in the x-axis, we obtain $F - R_x = (M+m)\ddot{x}$, where R_x represents the reaction forces at the joint. It is known that $R_x = m(\ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta)$ by TM1 and GD2. By substituting R_x into the first equation, we derive the equation of motion for the cart (first one).

Based on the second law of Newton (TM1), we can write the equation of motion for the pendulum as $\Sigma\vec{F} = m\vec{a_P}$. We define a unit vector $\vec{x_b} = \cos\theta\hat{i} + \sin\theta\hat{j}$, which is perpendicular to the pendulum at all times. Therefore, $\vec{x_b}^T \cdot \Sigma\vec{F} = -mg\sin\theta$, indicating that $-mg\sin\theta$ is the only force acting on the pendulum in the perpendicular direction. Additionally, we can calculate the $\vec{a_P}$ (GD2) of the pendulum in the perpendicular direction as $\vec{x_b}^T \cdot m\vec{a_P} = m(\ddot{x}\cos\theta - l\ddot{\theta})$. By equating these two equations, we obtain the equation of motion for the pendulum (second one).

4.4.7 Input Data Constraints

Table 1 shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 1 are listed in Table 2.

Table 1: Input Variables

Var	**Physical Constraints**	**Software Constraints**	**Typical Value**	**Uncertainty**
m	$0 < m$	$m_{\text{min}} \leq m \leq m_{\text{max}}$	0.2 kg	–
M	$0 < M$	$m_{\text{min}} \leq M \leq m_{\text{max}}$	0.5 kg	–
l	$0 < l$	$l_{\text{min}} \leq l \leq l_{\text{max}}$	0.3 m	–
x_i	-	-	0 m	–
θi	-	-	0 rad	–

Table 2: Specification Parameter Values

Var	**Value**
m_{min}	0.01 kg
m_{max}	100.0 kg
l_{min}	0.01 m
l_{max}	10.0 m

4.4.8 Properties of a Correct Solution

A correct solution should visually represent the motion of the pendulum in a manner consistent with Newton's laws.

Table 3: Output Variables

Var	Physical Constraints
$\theta(t)$	$0 \leq \theta(t) < 2\pi$ rad
$x(t)$	-

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1: Check validity of the input data, like the masses of the cart and pendulum, and the initial positions of both the cart and pendulum, based on the specifications provided in Table 1.

R2: The system shall calculate the position of the cart over the simulation time. (IM1)

R3: The system shall calculate the angular position of the pendulum over the simulation time. (IM1)

R4: The system shall implement control strategies to stabilize the pendulum and ensure it remains upright after a reasonable number of time-steps.

R5: The system shall provide real-time visualization of the pendulum's state to the user.

5.2 Nonfunctional Requirements

NFR1: **Accuracy** The relative error in the location of the cart and pendulum for Inverted Pendulum Control Systems will be compared with that of the benchmark program. Refer to the Verification and Validation Plan for details.

NFR2: **Usability** There results of the simulation shall be clearly visualized to the user.

NFR3: **Maintainability** Main modules shall be independent and easily replaceable with clear API.

NFR4: **Portability** Inverted Pendulum Control Systems should be able to run on both Linux and MacOS operating systems.

5.3 Rationale

The assumptions made for the inverted pendulum problem are geared towards simplifying the system's dynamics to facilitate analysis and solution development. Among these, pivotal assumptions entail treating the pole as possessing a single degree of rotational freedom (A1) and the cart as having one degree of freedom for its forward and backward motion (A2). These simplifications significantly facilitate the process of formulating equations governing the motion of both the cart and the pendulum (IM1).

6 Likely Changes

LC1: Inverted Pendulum Control Systems make use of a frictionless cart and the ground (A5).

7 Unlikely Changes

None.

8 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 4 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 5 shows the dependencies of instance models, requirements, and data constraints on each other. Table 6 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	TM1	TM2	GD1	GD2	DD1	IM1
TM1						
TM2						
GD1						
GD2			X		X	
DD1		X				
IM1	X			X		

Table 4: Traceability Matrix Showing the Connections Between Items of Different Sections

	TM1	TM2	GD1	GD2	DD1	IM1
R1						
R2					X	X
R3					X	X
R4					X	X
R5						

Table 5: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	TM1	TM2	GD1	GD2	DD1	IM1
A1			X	X		X
A2						X
A3				X		X
A4	X					X
A5			X	X		X
A6			X	X		X
A7			X	X		X

Table 6: Traceability Matrix Showing the Connections Between Assumptions and Other Items

9 Development Plan

N/A.

10 Values of Auxiliary Constants

N/A.

References

Inverted pendulum. Inverted pendulum — Wikipedia, the free encyclopedia, 2024. URL https://en.wikipedia.org/wiki/Inverted_pendulum. [Online; accessed 5-February-2024].