

## 7.1 From Fully Connected Layers to Convolutions

### MEMO:

#### Invariance

- Recognize objects should not be overly concerned with the precise location of the object in the image
  - Translation invariance: Our network should respond similarly to the same patch, regardless of where it appears in the image

#### Locality:

- $$[\mathbf{H}]_{i,j} = u + \sum_{a=-\Delta}^{\Delta} \sum_{b=-\Delta}^{\Delta} [\mathbf{V}]_{a,b} [\mathbf{X}]_{i+a,j+b}.$$
- As such, we reduced the number of parameters by another four orders of magnitude.
- The price paid for this drastic reduction in parameters is that our features are now translation invariant and that our layer can only incorporate local information, when determining the value of each hidden activation

#### Channels:

- we blissfully ignored that images consist of three channels: red, green, and blue. In sum, images are not two-dimensional objects but rather third-order tensors, characterized by a height, width, and channel, e.g., with shape 1024 x 1024 x 3 pixels.
- While the first two of these axes concern spatial relationships, the third can be regarded as assigning a multidimensional representation to each pixel location.
- To support multiple channels in both inputs (X) and hidden representations (H), we can add a fourth coordinate to V.

- $$[\mathbf{H}]_{i,j,d} = \sum_{a=-\Delta}^{\Delta} \sum_{b=-\Delta}^{\Delta} \sum_c [\mathbf{V}]_{a,b,c,d} [\mathbf{X}]_{i+a,j+b,c},$$

## 7.2 Convolutions for Images

```
pip install d2l==1.0.3
```



Show hidden output

```
import torch
from torch import nn
from d2l import torch as d2l

def corr2d(X, K):
    h, w = K.shape
    Y = torch.zeros((X.shape[0] - h + 1, X.shape[1] - w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            Y[i, j] = (X[i:i + h, j:j + w] * K).sum()
    return Y

X = torch.tensor([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
K = torch.tensor([[0.0, 1.0], [2.0, 3.0]])
corr2d(X, K)

tensor([[19., 25.],
        [37., 43.]])

class Conv2D(nn.Module):
    def __init__(self, kernel_size):
```

```

    super().__init__()
    self.weight = nn.Parameter(torch.rand(kernel_size))
    self.bias = nn.Parameter(torch.zeros(1))

    def forward(self, x):
        return corr2d(x, self.weight) + self.bias

X = torch.ones((6, 8))
X[:, 2:6] = 0
X
=> tensor([[1., 1., 0., 0., 0., 0., 1., 1.],
          [1., 1., 0., 0., 0., 0., 1., 1.],
          [1., 1., 0., 0., 0., 0., 1., 1.],
          [1., 1., 0., 0., 0., 0., 1., 1.],
          [1., 1., 0., 0., 0., 0., 1., 1.],
          [1., 1., 0., 0., 0., 0., 1., 1.]])

K = torch.tensor([[1.0, -1.0]])

Y = corr2d(X, K)
Y
=> tensor([[ 0.,  1.,  0.,  0.,  0., -1.,  0.],
          [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
          [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
          [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
          [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
          [ 0.,  1.,  0.,  0.,  0., -1.,  0.]])

corr2d(X.t(), K)
=> tensor([[0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.],
          [0., 0., 0., 0., 0.]])

# Construct a two-dimensional convolutional layer with 1 output channel and a
# kernel of shape (1, 2). For the sake of simplicity, we ignore the bias here
conv2d = nn.LazyConv2d(1, kernel_size=(1, 2), bias=False)

# The two-dimensional convolutional layer uses four-dimensional input and
# output in the format of (example, channel, height, width), where the batch
# size (number of examples in the batch) and the number of channels are both 1
X = X.reshape((1, 1, 6, 8))
Y = Y.reshape((1, 1, 6, 7))
lr = 3e-2 # Learning rate

for i in range(10):
    Y_hat = conv2d(X)
    l = (Y_hat - Y) ** 2
    conv2d.zero_grad()
    l.sum().backward()
    # Update the kernel
    conv2d.weight.data[:] -= lr * conv2d.weight.grad
    if (i + 1) % 2 == 0:
        print(f'epoch {i + 1}, loss {l.sum():.3f}')

=> epoch 2, loss 10.034
    epoch 4, loss 1.696
    epoch 6, loss 0.289
    epoch 8, loss 0.051
    epoch 10, loss 0.009

conv2d.weight.data.reshape((1, 2))

```

```
→ tensor([[ 0.9797, -0.9875]])
```

## ✓ Self-exercise:

```
X = torch.tensor([[1., 0., 0., 0., 0.],
                  [0., 1., 0., 0., 0.],
                  [0., 0., 1., 0., 0.],
                  [0., 0., 0., 1., 0.],
                  [0., 0., 0., 0., 1.]])
```

```
Y = corr2d(X, K)
Y
```

```
→ tensor([[ 1.,  0.,  0.,  0.],
          [-1.,  1.,  0.,  0.],
          [ 0., -1.,  1.,  0.],
          [ 0.,  0., -1.,  1.],
          [ 0.,  0.,  0., -1.]])
```

```
corr2d(X.t(), K)
```

```
→ tensor([[ 1.,  0.,  0.,  0.],
          [-1.,  1.,  0.,  0.],
          [ 0., -1.,  1.,  0.],
          [ 0.,  0., -1.,  1.],
          [ 0.,  0.,  0., -1.]])
```

```
corr2d(X, K.t())
```

```
→ tensor([[ 1., -1.,  0.,  0.,  0.],
          [ 0.,  1., -1.,  0.,  0.],
          [ 0.,  0.,  1., -1.,  0.],
          [ 0.,  0.,  0.,  1., -1.]])
```

Some discussion:

1. Applying kernel K to the diagonal edges will highlight the changes in the diagonal edges
2. Transposing X does not make any changes since the kernel detects the same diagonal edges
3. Transposing K will change to detect vertical edges and it will change the direction of detection diagonal.

## ✓ 7.3 Padding and Stride

```
def comp_conv2d(conv2d, X):
    X = X.reshape((1,1)+X.shape)
    Y = conv2d(X)
    return Y.reshape(Y.shape[2:])
conv2d = nn.LazyConv2d(1, kernel_size=3, padding=1)
X = torch.rand(size=(8,8))
comp_conv2d(conv2d, X).shape
```

```
→ torch.Size([8, 8])
```

```
conv2d = nn.LazyConv2d(1, kernel_size=(5,3), padding=(2,1))
comp_conv2d(conv2d, X).shape
```

```
→ torch.Size([8, 8])
```

```
conv2d = nn.LazyConv2d(1, kernel_size=3, padding=1, stride=2)
comp_conv2d(conv2d, X).shape
```

```
→ torch.Size([4, 4])
```

```
conv2d = nn.LazyConv2d(1, kernel_size=(3, 5), padding=(0,1), stride=(3,4))
comp_conv2d(conv2d, X).shape
```

```
→ torch.Size([2, 2])
```

## ✓ 7.4 Multiple Input and Multiple Output Channels

```
def corr2d_multi_in(X, K):
    return sum(d2l.corr2d(x, k) for x, k in zip(X, K))
```

```
X = torch.tensor([[[[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]],
                    [[1.0, 2.0, 3.0], [4.0, 5.0, 6.0], [7.0, 8.0, 9.0]]]])
K = torch.tensor([[[[0.0, 1.0], [2.0, 3.0]], [[1.0, 2.0], [3.0, 4.0]]]])
```

```
corr2d_multi_in(X, K)
```

```
→ tensor([[ 56.,  72.],
          [104., 120.]])
```

```
def corr2d_multi_in_out(X, K):
    return torch.stack([corr2d_multi_in(X, k) for k in K], 0)
```

```
K = torch.stack((K, K + 1, K + 2), 0)
K.shape
```

```
→ torch.Size([3, 2, 2, 2])
```

```
corr2d_multi_in_out(X, K)
```

```
→ tensor([[[[ 56.,  72.],
              [104., 120.]],

             [[ 76., 100.],
              [148., 172.]],

             [[ 96., 128.],
              [192., 224.]]]])
```

```
def corr2d_multi_in_out_1x1(X, K):
    c_i, h, w = X.shape
    c_o = K.shape[0]
    X = X.reshape((c_i, h * w))
    K = K.reshape((c_o, c_i))
    Y = torch.matmul(K, X)
    return Y.reshape((c_o, h, w))
```

```
X = torch.normal(0, 1, (3, 3, 3))
K = torch.normal(0, 1, (2, 3, 1, 1))
Y1 = corr2d_multi_in_out_1x1(X, K)
Y2 = corr2d_multi_in_out(X, K)
assert float(torch.abs(Y1 - Y2).sum()) < 1e-6
```

## ✓ 7.5 Pooling

```
def pool2d(X, pool_size, mode='max'):
    p_h, p_w = pool_size
    Y = torch.zeros((X.shape[0] - p_h + 1, X.shape[1] - p_w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            if mode == 'max':
                Y[i, j] = X[i: i + p_h, j: j + p_w].max()
            elif mode == 'avg':
```

```

        Y[i, j] = X[i: i + p_h, j: j + p_w].mean()
    return Y

X = torch.tensor([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
pool2d(X, (2, 2))

→ tensor([[4., 5.],
          [7., 8.]])

pool2d(X, (2, 2), 'avg')

→ tensor([[2., 3.],
          [5., 6.]])

X = torch.arange(16, dtype=torch.float32).reshape((1, 1, 4, 4))
X

→ tensor([[[[ 0., 1., 2., 3.],
              [ 4., 5., 6., 7.],
              [ 8., 9., 10., 11.],
              [12., 13., 14., 15.]])]])

pool2d = nn.MaxPool2d(3)
pool2d(X)

→ tensor([[[[10.]])]])

pool2d = nn.MaxPool2d(3, padding=1, stride=2)
pool2d(X)

→ tensor([[[[ 5., 7.],
              [13., 15.]])]])

pool2d = nn.MaxPool2d((2, 3), stride=(2, 3), padding=(0, 1))
pool2d(X)

→ tensor([[[[ 5., 7.],
              [13., 15.]])]])

X = torch.cat((X, X + 1), 1)
X

→ tensor([[[[ 0., 1., 2., 3.],
              [ 4., 5., 6., 7.],
              [ 8., 9., 10., 11.],
              [12., 13., 14., 15.]],

              [[ 1., 2., 3., 4.],
              [ 5., 6., 7., 8.],
              [ 9., 10., 11., 12.],
              [13., 14., 15., 16.]])]])

pool2d = nn.MaxPool2d(3, padding=1, stride=2)
pool2d(X)

→ tensor([[[[ 5., 7.],
              [13., 15.],

              [ 6., 8.],
              [14., 16.]])]])

```

## ✓ 7.6 LeNet

```

def init_cnn(module):
    """Initialize weights for CNNs."""
    if type(module) == nn.Linear or type(module) == nn.Conv2d:

```

```

nn.init.xavier_uniform_(module.weight)

class LeNet(d2l.Classifier):
    """The LeNet-5 model."""
    def __init__(self, lr=0.1, num_classes=10):
        super().__init__()
        self.save_hyperparameters()
        self.net = nn.Sequential(
            nn.LazyConv2d(6, kernel_size=5, padding=2), nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.LazyConv2d(16, kernel_size=5), nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.Flatten(),
            nn.LazyLinear(120), nn.Sigmoid(),
            nn.LazyLinear(84), nn.Sigmoid(),
            nn.LazyLinear(num_classes))

@d2l.add_to_class(d2l.Classifier)
def layer_summary(self, X_shape):
    X = torch.randn(*X_shape)
    for layer in self.net:
        X = layer(X)
        print(layer.__class__.__name__, 'output shape:\t', X.shape)

model = LeNet()
model.layer_summary((1, 1, 28, 28))

```

```

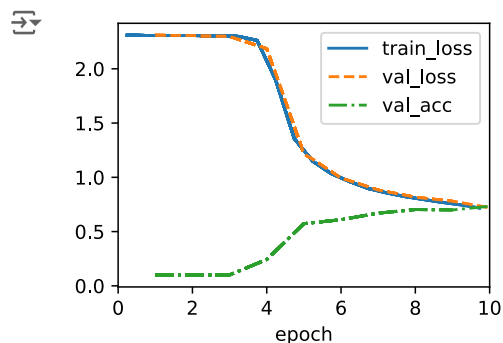
→ Conv2d output shape:      torch.Size([1, 6, 28, 28])
  Sigmoid output shape:     torch.Size([1, 6, 28, 28])
  AvgPool2d output shape:   torch.Size([1, 6, 14, 14])
  Conv2d output shape:      torch.Size([1, 16, 10, 10])
  Sigmoid output shape:     torch.Size([1, 16, 10, 10])
  AvgPool2d output shape:   torch.Size([1, 16, 5, 5])
  Flatten output shape:     torch.Size([1, 400])
  Linear output shape:      torch.Size([1, 120])
  Sigmoid output shape:     torch.Size([1, 120])
  Linear output shape:      torch.Size([1, 84])
  Sigmoid output shape:     torch.Size([1, 84])
  Linear output shape:      torch.Size([1, 10])

```

```

trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128)
model = LeNet(lr=0.1)
model.apply_init([next(iter(data.get_dataloader(True)))[0]], init_cnn)
trainer.fit(model, data)

```



## ✓ Self Exercise:

Let's modernize LeNet. Implement and test the following changes:

Replace average pooling with max-pooling. Replace the softmax layer with ReLU.

```

def init_cnn(module):
    """Initialize weights for CNNs."""
    if type(module) == nn.Linear or type(module) == nn.Conv2d:

```

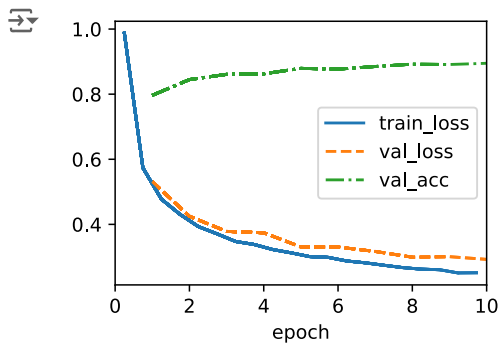
```

nn.init.xavier_uniform_(module.weight)

class LeNet2(d2l.Classifier):
    """The LeNet-5 model."""
    def __init__(self, lr=0.1, num_classes=10):
        super().__init__()
        self.save_hyperparameters()
        self.net = nn.Sequential(
            nn.LazyConv2d(6, kernel_size=5, padding=2), nn.ReLU(),
            nn.MaxPool2d(kernel_size=2, stride=2),
            nn.LazyConv2d(16, kernel_size=5), nn.ReLU(),
            nn.MaxPool2d(kernel_size=2, stride=2),
            nn.Flatten(),
            nn.LazyLinear(120), nn.ReLU(),
            nn.LazyLinear(84), nn.ReLU(),
            nn.LazyLinear(num_classes))

trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128)
model = LeNet2(lr=0.1)
model.apply_init([next(iter(data.get_dataloader(True)))[0]], init_cnn)
trainer.fit(model, data)

```



Takeaway: We could see that using max-pooling and ReLU gives a much lower train loss at much faster rate.

## ✓ 8.2 Networks Using Blocks

```

def vgg_block(num_convs, out_channels):
    layers = []
    for _ in range(num_convs):
        layers.append(nn.LazyConv2d(out_channels, kernel_size=3, padding=1))
        layers.append(nn.ReLU())
    layers.append(nn.MaxPool2d(kernel_size=2, stride=2))
    return nn.Sequential(*layers)

class VGG(d2l.Classifier):
    def __init__(self, arch, lr=0.1, num_classes=10):
        super().__init__()
        self.save_hyperparameters()
        conv_blks = []
        for (num_convs, out_channels) in arch:
            conv_blks.append(vgg_block(num_convs, out_channels))
        self.net = nn.Sequential(
            *conv_blks, nn.Flatten(),
            nn.LazyLinear(4096), nn.ReLU(), nn.Dropout(0.5),
            nn.LazyLinear(4096), nn.ReLU(), nn.Dropout(0.5),
            nn.LazyLinear(num_classes))
        self.net.apply(d2l.init_cnn)

VGG(arch=((1, 64), (1, 128), (2, 256), (2, 512), (2, 512))).layer_summary(
    (1, 1, 224, 224))

```

```

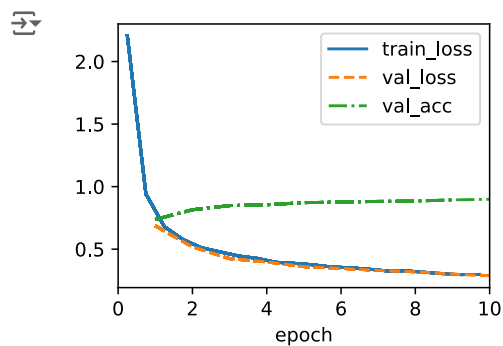
Sequential output shape:      torch.Size([1, 64, 112, 112])
Sequential output shape:      torch.Size([1, 128, 56, 56])
Sequential output shape:      torch.Size([1, 256, 28, 28])
Sequential output shape:      torch.Size([1, 512, 14, 14])
Sequential output shape:      torch.Size([1, 512, 7, 7])
Flatten output shape:         torch.Size([1, 25088])
Linear output shape:           torch.Size([1, 4096])
ReLU output shape:             torch.Size([1, 4096])
Dropout output shape:          torch.Size([1, 4096])
Linear output shape:           torch.Size([1, 4096])
ReLU output shape:             torch.Size([1, 4096])
Dropout output shape:          torch.Size([1, 4096])
Linear output shape:           torch.Size([1, 10])

```

```

model = VGG(arch=((1, 16), (1, 32), (2, 64), (2, 128), (2, 128)), lr=0.01)
trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128, resize=(224, 224))
model.apply_init([next(iter(data.get_dataloader(True)))[0]], d2l.init_cnn)
trainer.fit(model, data)

```



## 8.6 Residual Network (ResNet) and ResNeXt

```

import torch
from torch import nn
from torch.nn import functional as F
from d2l import torch as d2l

class Residual(nn.Module):
    """The Residual block of ResNet models."""
    def __init__(self, num_channels, use_1x1conv=False, strides=1):
        super().__init__()
        self.conv1 = nn.LazyConv2d(num_channels, kernel_size=3, padding=1,
                                     stride=strides)
        self.conv2 = nn.LazyConv2d(num_channels, kernel_size=3, padding=1)
        if use_1x1conv:
            self.conv3 = nn.LazyConv2d(num_channels, kernel_size=1,
                                         stride=strides)
        else:
            self.conv3 = None
        self.bn1 = nn.LazyBatchNorm2d()
        self.bn2 = nn.LazyBatchNorm2d()

    def forward(self, X):
        Y = F.relu(self.bn1(self.conv1(X)))
        Y = self.bn2(self.conv2(Y))
        if self.conv3:
            X = self.conv3(X)
        Y += X
        return F.relu(Y)

blk = Residual(3)
X = torch.randn(4, 3, 6, 6)
blk(X).shape

```



```
→ torch.Size([4, 3, 6, 6])
```

```
blk = Residual(6, use_1x1conv=True, strides=2)
blk(X).shape
```

```
→ torch.Size([4, 6, 3, 3])
```

```
class ResNet(d2l.Classifier):
    def b1(self):
        return nn.Sequential(
            nn.LazyConv2d(64, kernel_size=7, stride=2, padding=3),
            nn.LazyBatchNorm2d(), nn.ReLU(),
            nn.MaxPool2d(kernel_size=3, stride=2, padding=1))

@d2l.add_to_class(ResNet)
def block(self, num_residuals, num_channels, first_block=False):
    blk = []
    for i in range(num_residuals):
        if i == 0 and not first_block:
            blk.append(Residual(num_channels, use_1x1conv=True, strides=2))
        else:
            blk.append(Residual(num_channels))
    return nn.Sequential(*blk)

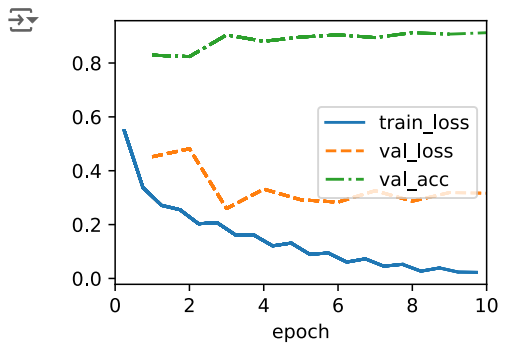
@d2l.add_to_class(ResNet)
def __init__(self, arch, lr=0.1, num_classes=10):
    super(ResNet, self).__init__()
    self.save_hyperparameters()
    self.net = nn.Sequential(self.b1())
    for i, b in enumerate(arch):
        self.net.add_module(f'b{i+2}', self.block(*b, first_block=(i==0)))
    self.net.add_module('last', nn.Sequential(
        nn.AdaptiveAvgPool2d((1, 1)), nn.Flatten(),
        nn.LazyLinear(num_classes)))
    self.net.apply(d2l.init_cnn)
```

```
class ResNet18(ResNet):
    def __init__(self, lr=0.1, num_classes=10):
        super().__init__(((2, 64), (2, 128), (2, 256), (2, 512)),
                          lr, num_classes)
```

```
ResNet18().layer_summary((1, 1, 96, 96))
```

```
→ Sequential output shape:      torch.Size([1, 64, 24, 24])
Sequential output shape:      torch.Size([1, 64, 24, 24])
Sequential output shape:      torch.Size([1, 128, 12, 12])
Sequential output shape:      torch.Size([1, 256, 6, 6])
Sequential output shape:      torch.Size([1, 512, 3, 3])
Sequential output shape:      torch.Size([1, 10])
```

```
model = ResNet18(lr=0.01)
trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128, resize=(96, 96))
model.apply_init([next(iter(data.get_dataloader(True)))[0]], d2l.init_cnn)
trainer.fit(model, data)
```



```
class ResNeXtBlock(nn.Module):
    """The ResNeXt block."""
    def __init__(self, num_channels, groups, bot_mul, use_1x1conv=False,
                 strides=1):
        super().__init__()
        bot_channels = int(round(num_channels * bot_mul))
        self.conv1 = nn.LazyConv2d(bot_channels, kernel_size=1, stride=1)
        self.conv2 = nn.LazyConv2d(bot_channels, kernel_size=3,
                                   stride=strides, padding=1,
                                   groups=bot_channels//groups)
        self.conv3 = nn.LazyConv2d(num_channels, kernel_size=1, stride=1)
        self.bn1 = nn.LazyBatchNorm2d()
        self.bn2 = nn.LazyBatchNorm2d()
        self.bn3 = nn.LazyBatchNorm2d()
        if use_1x1conv:
            self.conv4 = nn.LazyConv2d(num_channels, kernel_size=1,
                                       stride=strides)
            self.bn4 = nn.LazyBatchNorm2d()
        else:
            self.conv4 = None

    def forward(self, X):
        Y = F.relu(self.bn1(self.conv1(X)))
        Y = F.relu(self.bn2(self.conv2(Y)))
        Y = self.bn3(self.conv3(Y))
        if self.conv4:
            X = self.bn4(self.conv4(X))
        return F.relu(Y + X)
```

```
blk = ResNeXtBlock(32, 16, 1)
X = torch.randn(4, 32, 96, 96)
blk(X).shape
```

```
→ torch.Size([4, 32, 96, 96])
```