ASE 389 : Homework 2 : Balancing the Atlas Humanoid
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Question 1:

$$\sum_{i=1}^{n} w_i \| J_i \ddot{q} + \dot{J}_i \dot{q}_m - \ddot{x}_i^d \|^2 + \lambda_q \| \ddot{q} \|^2 + \lambda_f \| fr \|^2$$

$$\rightarrow \frac{1}{2} \begin{bmatrix} \ddot{q} & fr \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} \ddot{q} \\ fr \end{bmatrix} + \begin{bmatrix} a \\ 0 \end{bmatrix}^T \begin{bmatrix} \ddot{q} \\ fr \end{bmatrix}$$

$$\rightarrow w_1 (J_1^2 \ddot{q}^2 + J_1 \ddot{q} \dot{J} \dot{q}_m - J_1 \ddot{q} \ddot{x}_1^d + \dot{J}^2 \dot{q}_m^2 - \dot{J}_1 \dot{q}_m \ddot{x}_1^d + \ddot{x}_1^{d2}) + \dots$$

$$\rightarrow w_1 \ddot{q}^T J_1^T J_1 \ddot{q} + w_2 \ddot{q}^T J_2^T J_2 \ddot{q} + \dots$$

$$\rightarrow \ddot{q}^T (w_1 J_1^T J_1 + w_2 J_2^T J_2 + \dots) \ddot{q} = \ddot{q}^T (\tfrac{1}{2} A) \ddot{q}$$

$$\boxed{A = w_1 J_1^T J_1 + w_2 J_2^T J_2 + \dots w_n J_n^T J_n + \lambda_q}$$

$$2 w_1 [(\dot{q}_1^T \dot{J}_1^T J_1 - \ddot{x}_1^{dT} J_1) + (\dot{q}_2^T \dot{J}_2^T J_2 - x_2^{dT} J_2) + \dots] \ddot{q}$$

$$\boxed{\begin{aligned} a = 2 w_1 [& (\dot{q}_1^T J_1^T J_1 - x_1^d J_1) + (\dot{q}_2^T \dot{J}_2^T J_2 - x_2^{dT} J_2) + \dots \\ & \dots + (\dot{q}_n^T J_n^T J - x_n^d J_n)] \end{aligned}}$$

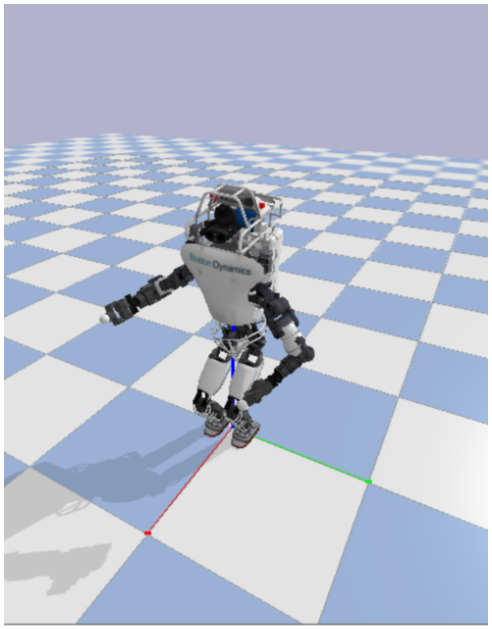
Question 2:

$$A \ddot{q} + b + g = \begin{bmatrix} 0_{6x1} \\ T_{cmd} \end{bmatrix} + J_c^T fr \quad \longleftrightarrow \quad [C \;\; D] \begin{bmatrix} \ddot{q} \\ Fr \end{bmatrix} = c$$

$$A \ddot{q} - J_c^T fr = \begin{bmatrix} 0_{6x1} \\ T_{cmd} \end{bmatrix} - b - g \quad \longleftrightarrow \quad C \ddot{q} + D fr = c$$

$$\boxed{\begin{aligned} C &= A \\ D &= -J_c^T \\ c &= \begin{bmatrix} 0_{6x1} \\ T_{cmd} \end{bmatrix} - b - g \end{aligned}}$$

Running Atlas simulation in pybullet :



← Atlas remains standing and balanced