ASE 389P.4 Methods of Orbit Determination
Term Project

Junette Hsin
Masters Student, Aerospace Engineering and Engineering Mechanics, University of Texas, Austin, TX 78712

The theory and algorithms are derived and computer program to establish the trajectory of an Earth-orbiting satellite is developed. The assumptions for the study are:

- **Three tracking stations taking apparent range and range-rate data are available for tracking the satellite. Apparent quantities imply that the one-way light time between signal transmission and reception were modeled into the measurement (i.e. the effect is dealt with).**
- **The force model used to generate the truth is the EGM96 gravity field of degree and order 20, attitude-dependent solar radiation pressure, and atmospheric drag.**
- **The satellite is a box-wing shaped with one Sun-pointed solar panel with known component sizes, material properties, and orientation. The spacecraft -Z axis (in the spacecraft body reference frame) is always Nadir-pointed and has the antenna.**

Problem 1

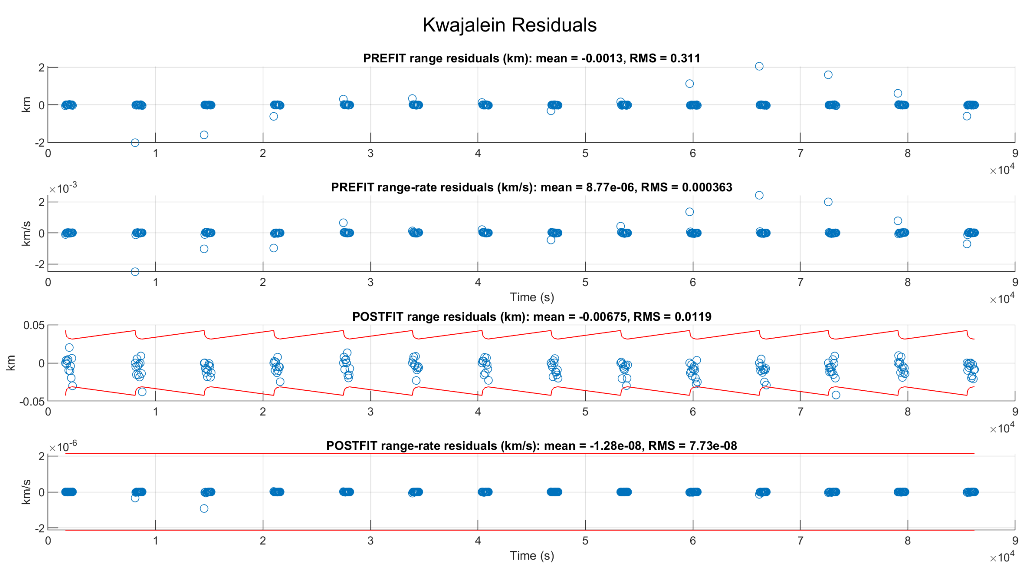
Prefit residuals for all data and all sensors.

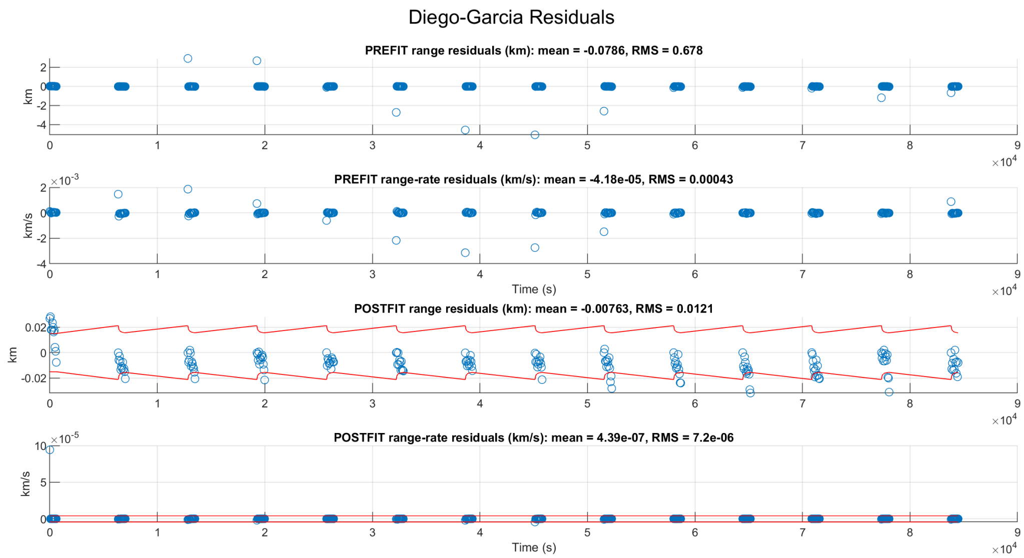
Problem 2

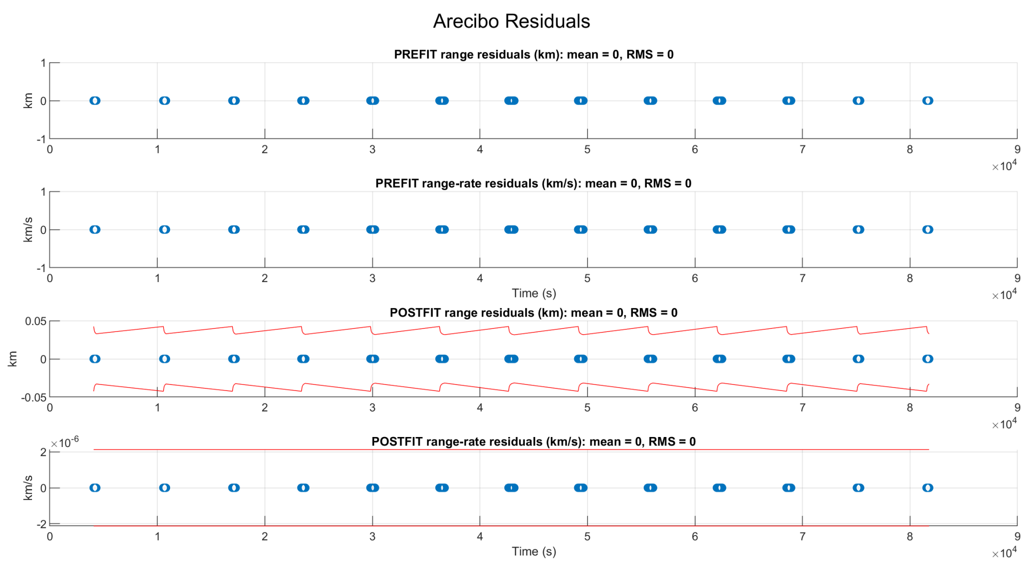
Postfit residuals (bounded by 3-σ bounds of the innovations covariance) for all data and all sensors

Solution for Problems 1 and 2

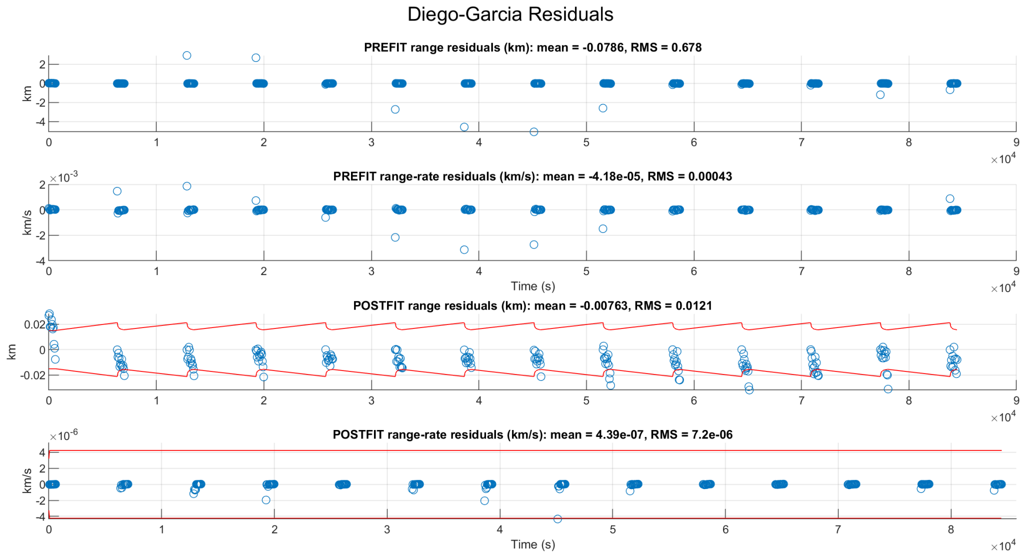
The prefit and postfit residuals for all stations are provided below (all data from all stations were used to filter the best estimate). The postfit residuals mostly lie within the 3-sigma covariance bounds, which are plotted as red lines.





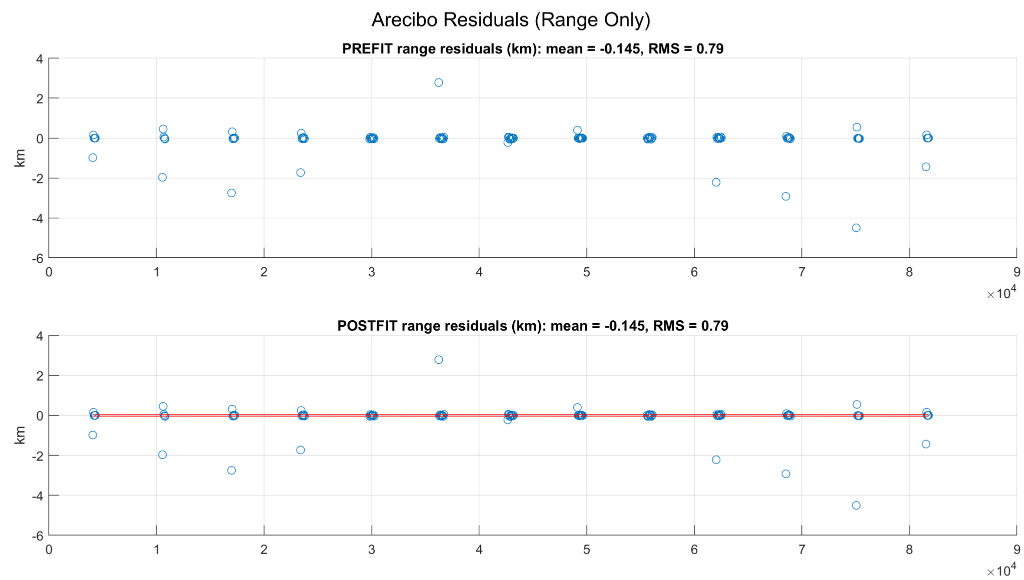


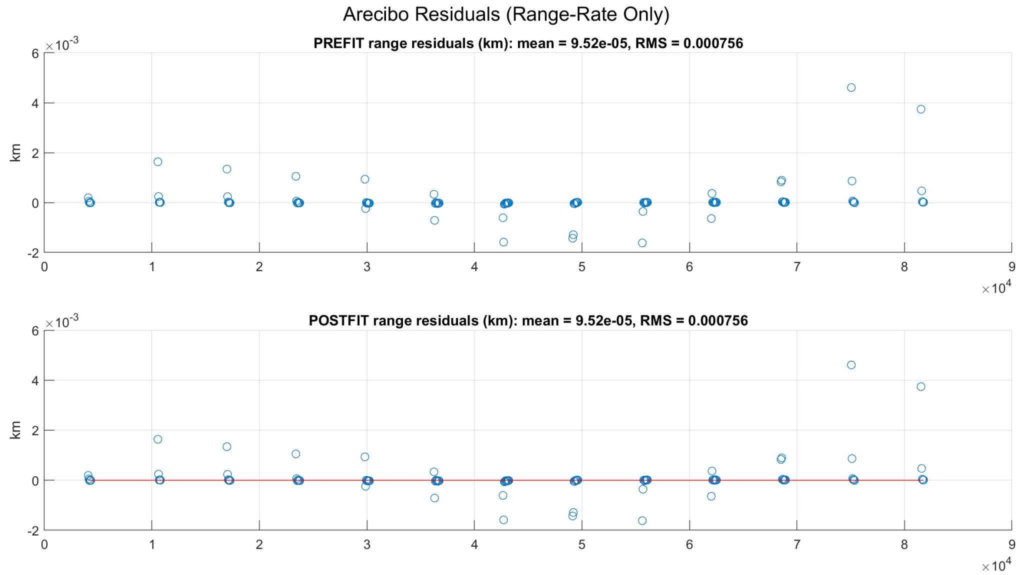
The Diego-Garcia plots have one residual for range-rate which lies far outside the 3-sigma covariance bounds. Zooming in reveals that most of the residuals lie within the 3-sigma covariance bounds, although some residuals



There seems to be a station bias for Kwajalein and Diego-Garcia, particularly in ranging. For the next phase of the project, I am planning on estimating the bias for each station to remove its effect on the estimation process and the residuals.

Arecibo residuals have 0 mean and 0 RMS. This is strange to me and I believe there is something going wrong as I expected there to be more noise due to both the estimation process and measurements themselves. When plotting the Arecibo residuals for the range-only (Case A) and range-rate (Case B) simulations, the residuals show some noisy behavior characteristic of sensors.





This warrants further investigation, which I will complete during the next phase of the project.

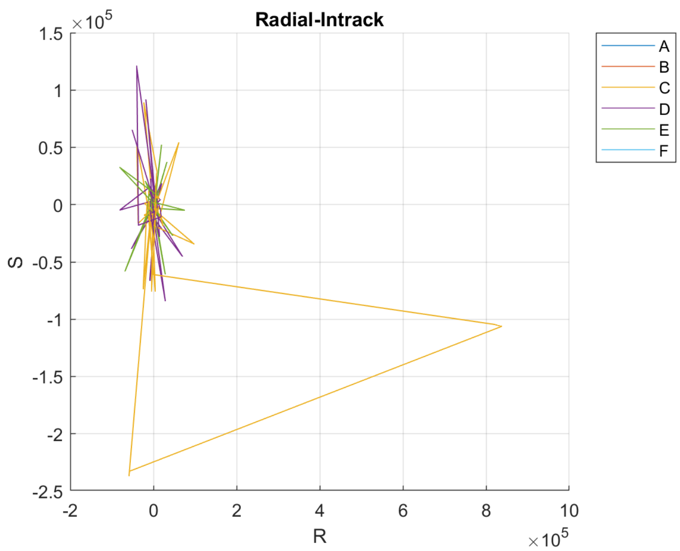
Problem 3

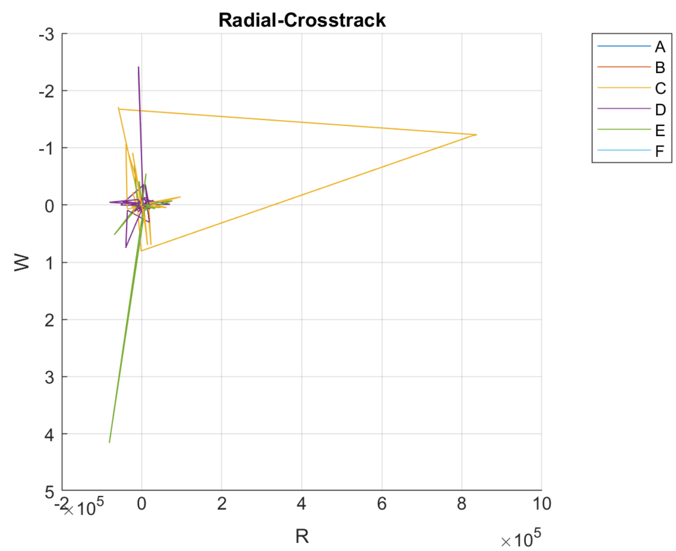
A plot in each of the following frames: Radial-Crosstrack (spacecraft motion is going into the page). Radial-Intrack (looking face-on to the orbital plane), and Crosstrack-Intrack (looking edge-on to the orbital plane) with all of the following ellipses simultaneously. Label each ellipse with the case number from below so we know what is being plotted. Be mindful of your units. Show things in an adequate scale.

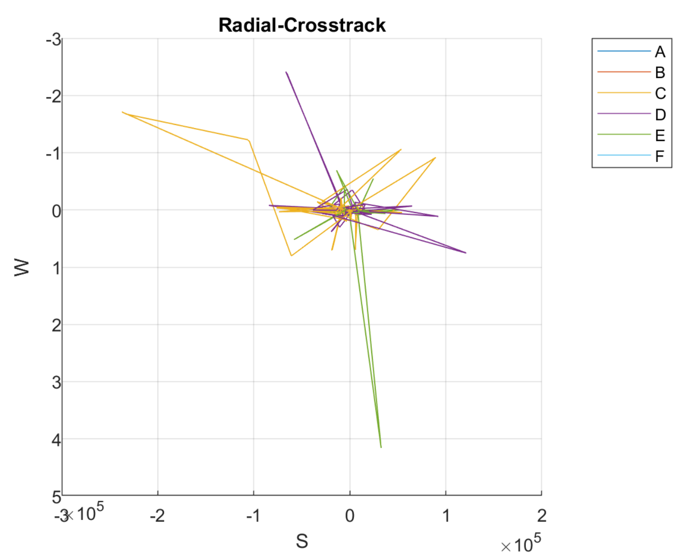
(a) fit range only for all sensors
(b) fit range-rate only for all sensors
(c) fit Kwajalein only for all data types
(d) fit Diego Garcia only for all data types
(e) fit Arecibo only for all data types
(f) fit the long-arc (all data and all sensors)
(g) fit the short arc (only the last day of data for all sensors)

Solution

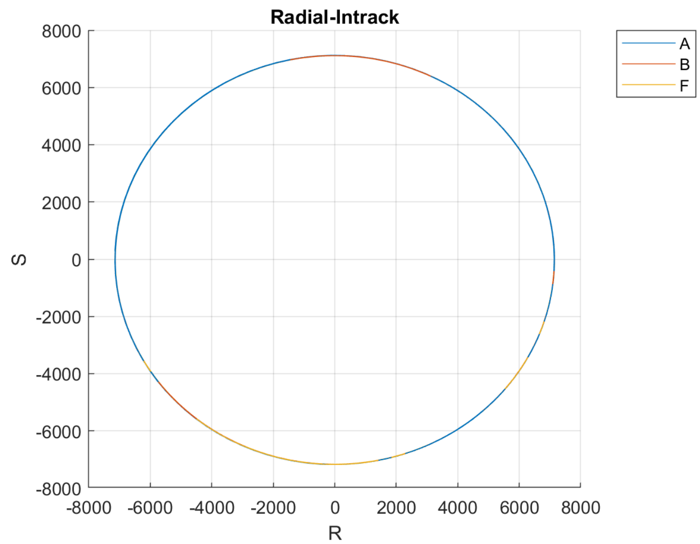
The radial-intrack, radial-crosstrack, and intrack-crosstrack plots for Cases A - F are provided below. For Case F, the "long arc" is really just 24 hours for the first phase of this project. **The units for all axes are in kilometers.**

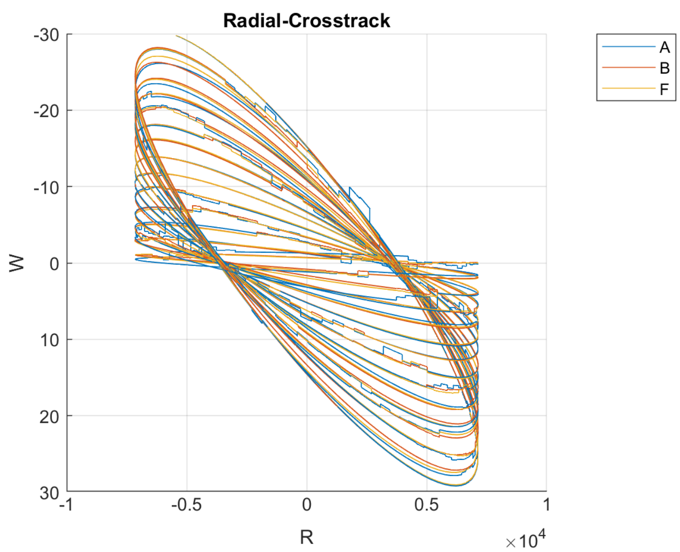

Radial-Intrack
A
B
C
D
E
F
S
R

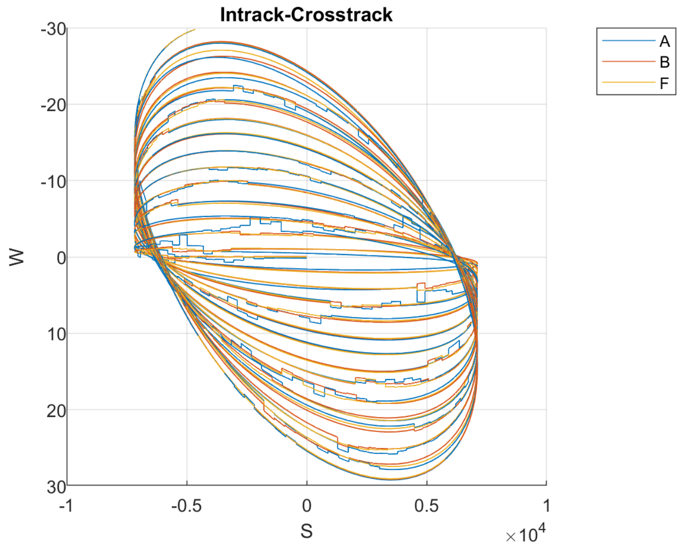

Radial-Crosstrack
A
B
C
D
E
F
W
R


Radial-Crosstrack
A
B
C
D
E
F
W
S

Cases C, D, and E were poorly fit with only one station data. When Cases C, D, and E are removed, the ellipses of each orbit are possible to recognize:







I suspect that something is going wrong with the station-only estimation and orbit propagation.

	Case A	Case B	Case C	Case D	Case E	Case F
x (km)	-6330.33	-6330.25	-10041.40	1567.22	-1924.64	-6330.25
y (km)	3306.96	3306.89	8472.56	4791.99	-3742.66	3306.89
z (km)	127.98	127.77	986.39	15103.25	-1536.00	127.82

Table 1 Predicted Position of JahSat after 24 hours in ECI frame

When the final best estimate of Case F is converted into orbital elements, the eccentricity is 0.00403122939916287. That is quite circular, which possibly indicates to me that the satellite is either nearing the end of orbit raising and nearing its final orbital position, or is about to be raised into a graveyard orbit as a delta-V maneuver is planned at 24 hours.

For my orbit determination process I am using a [6x1] state vector composed of satellite position and velocity in ECI frame. I am working with units in kilometers and kilometers/second. My dynamics include 20x20 spherical harmonics gravity field, lunisolar perturbations, SRP modeled as pressure on the solar panel, and cannonball drag.

I then used a batch filter to refine the initial condition. The batch filter was initialized with the following station noise errors (converted into km and km/s in the code):

Number	Description	Range σ (m)	Range Rate σ (mm)
1	Kwajalein	10	0.5
2	Diego Garcia	5	1
3	Arecibo	10	0.5

The state covariance was initialized for position error for 10 km and velocity error of 10 m/s:

$$P_0 = \begin{bmatrix} 10^2 I_{3x3} & 0_{3x3} \\ 0_{3x3} & (10^{-3})^2 I_{3x3} \end{bmatrix}$$

The algorithm for the Batch filter was taken from Born, Section 4.6: Computational Algorithm for the Batch Processor [1]. The first 28 measurements were iterated in the batch filter until the updated state changed by less than 0.1 m.

Then, I used an Extended Kalman Filter to filter and update the best estimate of the trajectory, which also taken from Born, Section 4.7.3: The Extended Sequential Computational Algorithm. The last updated covariances from the batch filter were used to initialize the EKF. The prefit residuals were computed using the dynamically propagated state, and the postfit residuals were computed using the measurement-updated state.

For the range and range-only simulations, I updated the EKF to use a reduced dimension G matrix (mapping of state space to measurement space). The observation-state H matrix was also updated to reflect range or range-only measurements.

The process noise Q was initially set at 1e-20 along the diagonal, but the estimate improved vastly when I increased the power. I saw no better performance past 1e-12 for the diagonal elements and I didn't want to jack up the process noise too much to cover my modeling sins. For the next phase of the project, I am planning on including light-time correction.

Appendix

HW5 MATLAB code

Note: The equations of motion and functions to transform from ECI to ECEF and back were submitted as part of Homework 4, and so are left out of this appendix.

finalproj_EOM_batch.m:

```
% HW 5
% Junette Hsin

clear; clc
addpath(genpath('mice'));
addpath(genpath('spice_data'));

% Load SPICE kernel file
cspice_furnsh( 'spice_data/naif0011.tls' )
cspice_furnsh( 'spice_data/de421.bsp' )
cspice_furnsh( 'spice_data/pck00010.tpc ')

format long g

%% Parameters

global muE RE muS AU muM eE wE J2 J3 J4 Cd Cs eop_data
global A m p p0 r0_drag H CD

% initial state initial guess (M --> KM)
CD  = 1.88;
X0 = [ 6984.45711518852
1612.2547582643
13.0925904314402
-1.67667852227336
7.26143715396544
0.259889857225218 ];

% ballpark right answer
% X0 = [   6978.25607108059
%          1616.30079732436
%          19.7187784486054
%         -1.66208620583392
%          7.26104892573667
%          0.270612926043287 ];

nX  = length(X0);

% initialize STM
STM0 = eye(length(X0));
STM0 = reshape(STM0, [length(X0)^2 1]);
XSTM0 = [X0; STM0];

% Constants
muE = 398600.4415;          % Earth Gravitational Parameter (km^3/s^2)
RE  = 6378.1363;            % Earth Radius (km)
muS = 132712440018;         % Sun's Gravitational Parameter (km^3/s^2)
AU  = 149597870.7;          % 1 Astronomical Unit (km)
muM = 4902.800066;          % Moon's Gravitational Parameter (km^3/s^2)
eE  = 0.081819221456;       % Earth's eccentricity
wE  = 7.292115146706979e-5; % Earth's rotational velocity (rad/s)
m   = 2000;                 % satellite mass (kg)
Cd  = 0.04;                 % diffuse reflection
Cs  = 0.04;                 % specular reflection

J2 = 1.08262617385222e-3;
J3 = -2.53241051856772e-6;
J4 = -1.61989759991697e-6;

global r_KJL_ECEF r_DGO_ECEF r_ACB_ECEF
```

```
% Station coords. Convert M --> KM
r_KJL_ECEF = [-6143584   1364250   1033743]' / 1000;  % Kwajalein
r_DGO_ECEF = [ 1907295   6030810  -817119 ]' / 1000;  % Diego
r_ACB_ECEF = [ 2390310  -5564341   1994578]' / 1000;  % Arecibo

global LEO_DATA_Apparent
% Load observation data
load('LEO_DATA_Apparent.mat')

% Station data
ID_STA    = 1;
i_STA     = find(LEO_DATA_Apparent(:, 1) == ID_STA);
Yobs_KJL  = LEO_DATA_Apparent(i_STA, :);
t_KWJ     = Yobs_KJL(:,2);

ID_STA    = 2;
i_STA     = find(LEO_DATA_Apparent(:, 1) == ID_STA);
Yobs_DGO  = LEO_DATA_Apparent(i_STA, :);
t_DGO     = Yobs_DGO(:,2);

ID_STA    = 3;
i_STA     = find(LEO_DATA_Apparent(:, 1) == ID_STA);
Yobs_ACB  = LEO_DATA_Apparent(i_STA, :);
t_ACB     = Yobs_DGO(:,2);

eop_data = load('finals_iau1980.txt');

% Atmospheric drag
r    = norm(X0(1:3));            % km
H    = 88667.0 / 1000;           % m --> km
r0_drag   = (700 + RE);          % m --> km
p0   = 3.614e-13 * 1e9;          % kg/m3 --> kg/km^3
p    = p0*exp( -(r-r0_drag)/H );
A    = 15 / 1e6;                 % km^2

global Cnm Snm

% Gravity
Cnm = zeros(181,181);
Snm = zeros(181,181);
fid = fopen('GGM03S.txt','r');
for n=0:180
for m=0:n
temp = fscanf(fid,'%d %d %f %f %f %f',[6 1]);
Cnm(n+1,m+1) = temp(3);
Snm(n+1,m+1) = temp(4);
end
end

%% Convert t0 to ET, i.e. seconds past J2000, the base time variable for SPICE. function calls.

%  Epoch for initial conditions
% epoch = 23 March 2018, 08:55:03 UTC ;
t0       = 'March 23, 2018, 08:55:03 UTC';
abcorr   = 'NONE';

%  Convert the epoch to ephemeris time.
et_t0    = cspice_str2et( t0 );

% extract observation epochs
epochs = LEO_DATA_Apparent(:,2);
epochs = et_t0 + epochs;


%% Derive A and H matrices

X  = sym('X', [length(X0) 1]);
```

```
dX = fn.EOM(et_t0, X);

% compute partials
Amat    = jacobian( dX, X );
Amat_fn = matlabFunction(Amat);

% DO EVERYTHING IN ECI FRAME

X  = sym('X', [nX; 1]);
XS = sym('XS', [nX; 1]);

r_site = [X(1)-XS(1); X(2)-XS(2); X(3)-XS(3)];
v_site = [X(4)-XS(4); X(5)-XS(5); X(6)-XS(6)];
d      = norm(r_site);
v      = dot(v_site, r_site/norm(r_site));

Htmat      = sym(zeros(2,nX));
Htmat(1,:) = simplify(gradient(d, X));
Htmat(2,:) = simplify(gradient(v, X));
Ht_fn      = matlabFunction(Htmat);

Ht_r_fn  = matlabFunction(Htmat(1,:));
Ht_rr_fn = matlabFunction(Htmat(2,:));

%% integrate EOM

% set ode45 params
rel_tol = 1e-10;        % 1e-14 accurate; 1e-6 coarse
abs_tol = 1e-10;
options = odeset('reltol', rel_tol, 'abstol', abs_tol );

% Set run state
run_state = 2;
disp('Running sim ...')

if run_state == 1
[t, X] = ode45(@fn.EOM, [epochs(1) : 60 : epochs(end)], X0, options);
elseif run_state == 2
[t, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), [epochs(1) : 60 : epochs(end)],
    XSTM0, options);
X = XSTM(:, 1:6);
end
disp('Pos and Vel end: ')
disp(X(end, 1:6)');

XSTM_ref0   = XSTM;
t_XSTM_ref0 = t;


%% Setting up filters

% weighting matrices m --> km, mm --> km
global R_KJL R_DGO R_ACB
R_KJL = [(10e-3)^2 0; 0 (0.5e-6)^2];
R_DGO = [(5e-3)^2  0; 0 (1e-6)^2];
R_ACB = [(10e-3)^2 0; 0 (0.5e-6)^2];

% initial covariance error
% 10 km std - position
% 10 m/s    - velocity
P0 = [ 10^2*eye(3), zeros(3);
zeros(3),  (10e-3)^2*eye(3) ];
Lambda0 = inv(P0);
global Lambda_KJL0 Lambda_DGO0 Lambda_ACB0
Lambda_KJL0 = Lambda0;
Lambda_DGO0 = Lambda0;
Lambda_ACB0 = Lambda0;

```

```
global N_KJL0 N_DGO0 N_ACB0
N0     = Lambda0*X0;
N_KJL0 = N0;
N_DGO0 = N0;
N_ACB0 = N0;


%% Batch all stations to refine IC

Xstar  = 100*ones(1,3);
XSTM   = XSTM_ref0;
t_XSTM = t_XSTM_ref0;
XSTM0  = XSTM_ref0(1,:)';
iter   = 0;
N_prev = N0;
Lambda_prev = Lambda0;

% Batch first 28 measurements

while norm(Xstar(1:3)) > 0.1

% keep track of iterations
iter = iter + 1;
sprintf('iter = %d', iter)

% Test - All stations
[Ycalc_all, Lambda, N] = ...
fn.batch_LSQ(LEO_DATA_Apparent, t_XSTM, XSTM, et_t0, Ht_fn, Lambda_prev, N_prev);

%     % Test - Kwajalein
%     [Ycalc_all, Lambda, N] = ...
%         fn.batch_LSQ(Yobs_KJL, t_XSTM, XSTM, et_t0, Ht_fn, Lambda0, N0);

% Solve normal equation
Xstar = inv(Lambda) * N;

% update covariance
Lambda_prev = Lambda0;
N_prev      = N0;

% update initial conditions
XSTM0(1:nX) = XSTM0(1:nX) + Xstar;
[t, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), [epochs(1) : 60 : epochs(end)],
    XSTM0, options);

disp('xhat pos'); Xstar(1:3)
disp('xhat pos norm'); norm(Xstar(1:3))
disp('x IC pos'); XSTM0(1:3)

end

Lambda_batch = Lambda;
XSTM_batch   = XSTM;
XSTM0_batch  = XSTM0;
```

finalproj_EKF.m:

```
% HW 5
% Junette Hsin

% finalproj_EOM_batch;

%% EKF - all observations

STATIONS = 0;
% for STATIONS = 1:3
for DATA = 1:2

%          DATA = 0;
```

```
if STATIONS == 0     % use all station data
Yobs_STA = LEO_DATA_Apparent;
elseif STATIONS == 1
Yobs_STA = Yobs_KJL;
elseif STATIONS == 2
Yobs_STA = Yobs_DGO;
else % STATIONS == 3
Yobs_STA = Yobs_ACB;
end

et_obs      = Yobs_STA(:,2) + et_t0;
XSTM_prev   = XSTM0_batch;

iter        = 0;
P_prev      = inv(Lambda_batch);

X_EKF       = [];
t_X_EKF     = [];
Y_prefit    = [];
Y_postfit   = [];

Lambda_mat  = [];
sigma3      = [];

for i = 1:length(et_obs)

% keep track of iterations
iter = iter + 1;
sprintf('iter = %d', iter)

% Propagate state
if      i == 1 && et_obs(1) == et_t0; t_prop = et_obs(i);
elseif  i == 1;                       t_prop = [et_t0 : 60 : et_obs(1) ];
else;                                 t_prop = [et_obs(i-1) : 60 : et_obs(i)];
end

% EKF. All data, range, or range-only
if DATA == 0
[t_XSTM, XSTM, Xstar, Y_pre, Y_post, P, Lambda] = fn.EKF(Yobs_STA, XSTM_prev, nX, ...
epochs(1), t_prop, options, Amat_fn, Ht_fn, P_prev);
elseif DATA == 1
% EKF for range only
[t_XSTM, XSTM, Xstar, Y_pre, Y_post, P, Lambda] = fn.EKF_r(Yobs_STA, XSTM_prev, nX, ...
epochs(1), t_prop, options, Amat_fn, Ht_r_fn, P_prev);
else % DATA == 2
% EKF for range-rate only
[t_XSTM, XSTM, Xstar, Y_pre, Y_post, P, Lambda] = fn.EKF_rr(Yobs_STA, XSTM_prev, nX, ...
epochs(1), t_prop, options, Amat_fn, Ht_rr_fn, P_prev);
end

% save states from current iteration
if i == 1 && et_obs(1) == et_t0; X_EKF = XSTM(1:nX)';
else;                            X_EKF = [X_EKF; XSTM(:, 1:nX)];
end

t_X_EKF    = [t_X_EKF; t_XSTM];
Y_prefit   = [Y_prefit; Y_pre];
Y_postfit  = [Y_postfit; Y_post];

% update measurement for next iteration
XSTM_prev = [Xstar; STM0];
P_prev    = P;

% innovations covariance
Lambda_mat = [Lambda_mat; Lambda];

% 3-sigma STD
if DATA == 0; sigma3 = [sigma3; sqrt(Lambda(1,1))*3, sqrt(Lambda(2,2))*3];
```

```
else;           sigma3 = [sigma3; sqrt(Lambda(1,1))*3];
end

end

% Propagate to last period of time
t_prop = [ et_obs(end) : 60 : et_t0 + 60*60*24 ];
[t_XSTM, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), t_prop, XSTM_prev, options);
Xi    = XSTM(end,1:nX)';
STMi = XSTM(end,nX+1:end);
STMi = reshape(STMi, [nX nX]);

% Time update + process noise
dt     = t_prop(end) - t_prop(1);
Q      = diag( (10000e-10)^2 * [1 1 1] );
Gamma = [diag( dt^2/2 * [1 1 1] ); diag([dt dt dt])];
P_noise = Gamma * Q * Gamma';
Pi_bar = STMi * P_prev * STMi' + P_noise;

X_EKF_prop    = [X_EKF; XSTM(:,1:nX)];
t_X_EKF_prop = [t_X_EKF; t_XSTM];


%% Plot satellite position

ftitle = 'JahSat Orbit';
figure('name', ftitle);
plot3(XSTM_ref0(:,1), XSTM_ref0(:,2), XSTM_ref0(:,3)); hold on; grid on;
plot3(XSTM_batch(:,1), XSTM_batch(:,2), XSTM_batch(:,3));
plot3(X_EKF(:,1), X_EKF(:,2), X_EKF(:,3)); hold on; grid on;
plot3(X_EKF(1,1), X_EKF(1,2), X_EKF(1,3), 'o')
xlabel('x (km)'); ylabel('y (km)'); zlabel('z (km)');
legend('initial', 'batch', 'EKF', 'prop')
title(ftitle)


%% Calculate residuals

Ypre_KJL = [];        Ypre_DGO = [];        Ypre_ACB = [];

Ypost_KJL = [];       Ypost_DGO = [];       Ypost_ACB = [];

sigma3_KJL = [];      sigma3_DGO = [];      sigma3_ACB = [];

% Extract states that correspond with station measurements
for i = 1:length(Y_postfit)

ti   = Y_postfit(i, 2);
ti   = ti + et_t0;
i_X = find(t_X_EKF == ti);

i_STA = Y_postfit(i, 1);

if i_STA == 1
Ypre_KJL    = [Ypre_KJL; Y_prefit(i,:)];
Ypost_KJL   = [Ypost_KJL; Y_postfit(i,:)];
sigma3_KJL = [sigma3_KJL; sigma3(i,:)];
elseif i_STA == 2
Ypre_DGO    = [Ypre_DGO; Y_prefit(i,:)];
Ypost_DGO   = [Ypost_DGO; Y_postfit(i,:)];
sigma3_DGO = [sigma3_DGO; sigma3(i,:)];
else
Ypre_ACB    = [Ypre_ACB; Y_prefit(i,:)];
Ypost_ACB   = [Ypost_ACB; Y_postfit(i,:)];
sigma3_ACB = [sigma3_ACB; sigma3(i,:)];
end

end
```

```matlab

t_KJL = Yobs_KJL(:,2);
t_DGO = Yobs_DGO(:,2);
t_ACB = Yobs_ACB(:,2);

if STATIONS == 0
if DATA == 0
res_all_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, 'Kwajalein Residuals');
res_all_plot_STA(t_DGO, Yobs_DGO, Ypre_DGO, Ypost_DGO, sigma3_DGO, 'Diego-Garcia Residuals');
res_all_plot_STA(t_ACB, Yobs_ACB, Yobs_ACB, Yobs_ACB, sigma3_ACB, 'Arecibo Residuals');
elseif DATA == 1
res_r_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, 'Kwajalein Residuals (Range Only)')
res_r_plot_STA(t_DGO, Yobs_DGO, Ypre_DGO, Ypost_DGO, sigma3_DGO, 'Diego-Garcia Residuals (Range Only)')
res_r_plot_STA(t_ACB, Yobs_ACB, Ypre_ACB, Ypost_ACB, sigma3_ACB, 'Arecibo Residuals (Range Only)')
else
res_rr_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, 'Kwajalein Residuals (Range-Rate Only)')
res_rr_plot_STA(t_DGO, Yobs_DGO, Ypre_DGO, Ypost_DGO, sigma3_DGO, 'Diego-Garcia Residuals (Range-Rate Only)')
res_rr_plot_STA(t_ACB, Yobs_ACB, Ypre_ACB, Ypost_ACB, sigma3_ACB, 'Arecibo Residuals (Range-Rate Only)')
end
elseif STATIONS == 1
res_all_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, 'Kwajalein Residuals');
elseif STATIONS == 2
res_all_plot_STA(t_DGO, Yobs_DGO, Ypre_DGO, Ypost_DGO, sigma3_DGO, 'Diego-Garcia Residuals');
else % STATIONS == 3
res_all_plot_STA(t_ACB, Yobs_ACB, Yobs_ACB, Yobs_ACB, sigma3_ACB, 'Arecibo Residuals');
end


%% radial-intrack-cross-track frame transformation for best estimate

% radial-intrack-cross-track frame transformation for best estimate
if STATIONS == 0 && DATA == 0
T_best = fn.ECItoRSW_T(Xi);
end

% transform all measurements
X_RSW = [];
for i = 1:length(X_EKF_prop)
X_RSW = [X_RSW; [T_best * X_EKF_prop(i, 1:3)']' ];
end

if STATIONS == 0
if DATA == 0
X_ALL_RSW = X_RSW;
save('X_ALL_RSW.mat')
elseif DATA == 1
X_R_RSW = X_RSW;
save('X_R_RSW.mat')
else
X_RR_RSW = X_RSW;
save('X_RR_RSW.mat')
end
elseif STATIONS == 1
X_KJL_RSW = X_RSW;
save('X_KJL_RSW.mat')
elseif STATIONS == 2
X_DGO_RSW = X_RSW;
save('X_DGO_RSW.mat')
else
X_ACB_RSW = X_RSW;
save('X_ACB_RSW.mat')
end
```

```
%       end
end

%% Plot radial-intrack-crosstrack

% Start figure
ftitle = 'Radial-Intrack-Crosstrack';
figh = figure('name', ftitle);
plot3(X_R_RSW(:,1), X_R_RSW(:,2), X_R_RSW(:,3)); hold on; grid on;
plot3(X_RR_RSW(:,1), X_RR_RSW(:,2), X_RR_RSW(:,3));
%       plot3(X_KJL_RSW(:,1), X_KJL_RSW(:,2), X_KJL_RSW(:,3));
%       plot3(X_DGO_RSW(:,1), X_DGO_RSW(:,2), X_DGO_RSW(:,3));
%       plot3(X_ACB_RSW(:,1), X_ACB_RSW(:,2), X_ACB_RSW(:,3));
plot3(X_ALL_RSW(:,1), X_ALL_RSW(:,2), X_ALL_RSW(:,3));
%       legend('A', 'B', 'C', 'D', 'E', 'F')
legend('A', 'B', 'F')
xlabel('R')
ylabel('S')
zlabel('W')


%% Subfunctions

function res_all_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, ftitle)

[dpre_err_KJL, dpre_rms_KJL, vpre_err_KJL, vpre_rms_KJL] = calc_res_all(Yobs_KJL, Ypre_KJL);
[dpost_err_KJL, dpost_rms_KJL, vpost_err_KJL, vpost_rms_KJL] = calc_res_all(Yobs_KJL, Ypost_KJL);

plot_res_all(ftitle, t_KJL, sigma3_KJL, dpre_err_KJL, dpre_rms_KJL, vpre_err_KJL, vpre_rms_KJL,
    ...
dpost_err_KJL, dpost_rms_KJL, vpost_err_KJL, vpost_rms_KJL)

end

function res_r_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, ftitle)

[dpre_err_KJL, dpre_rms_KJL]   = calc_res_r(Yobs_KJL, Ypre_KJL);
[dpost_err_KJL, dpost_rms_KJL] = calc_res_r(Yobs_KJL, Ypost_KJL);

plot_res_r(ftitle, t_KJL, sigma3_KJL, dpre_err_KJL, dpre_rms_KJL, dpost_err_KJL, dpost_rms_KJL)

end

function res_rr_plot_STA(t_KJL, Yobs_KJL, Ypre_KJL, Ypost_KJL, sigma3_KJL, ftitle)

[vpre_err_KJL, vpre_rms_KJL]   = calc_res_rr(Yobs_KJL, Ypre_KJL);
[vpost_err_KJL, vpost_rms_KJL] = calc_res_rr(Yobs_KJL, Ypost_KJL);

plot_res_r(ftitle, t_KJL, sigma3_KJL, vpre_err_KJL, vpre_rms_KJL, vpost_err_KJL, vpost_rms_KJL)

end

function [d_err_STA, d_rms_STA, v_err_STA, v_rms_STA] = calc_res_all(Yobs_STA, Ycalc_STA)

% Calculate residuals
d_err_STA = Yobs_STA(:,3) - Ycalc_STA(:,3);
d_rms_STA = rms(d_err_STA);
v_err_STA = Yobs_STA(:,4) - Ycalc_STA(:,4);
v_rms_STA = rms(v_err_STA);

end

function [d_err_STA, d_rms_STA] = calc_res_r(Yobs_STA, Ycalc_STA)

% Calculate residuals
d_err_STA = Yobs_STA(:,3) - Ycalc_STA(:,3);
d_rms_STA = rms(d_err_STA);
```

```
end

function [v_err_STA, v_rms_STA] = calc_res_rr(Yobs_STA, Ycalc_STA)

% Calculate residuals
v_err_STA = Yobs_STA(:,4) - Ycalc_STA(:,3);
v_rms_STA = rms(v_err_STA);

end

function plot_res_all(ftitle, t_STA, sigma3_STA, dpre_err_STA, dpre_rms_STA, vpre_err_STA, vpre_rms_STA, ...
dpost_err_STA, dpost_rms_STA, vpost_err_STA, vpost_rms_STA)

figure('name', ftitle);
subplot(4,1,1)
scatter(t_STA, dpre_err_STA); hold on; grid on;
title({sprintf('PREFIT range residuals (km): mean = %.3g, RMS = %.3g', mean(dpre_err_STA), dpre_rms_STA)} );
    ylabel('km')
    subplot(4,1,2)
    scatter(t_STA, vpre_err_STA); hold on; grid on;
    title({sprintf('PREFIT range-rate residuals (km/s): mean = %.3g, RMS = %.3g', mean(vpre_err_STA), vpre_rms_STA)} );
        xlabel('Time (s)')
        ylabel('km/s')
        subplot(4,1,3)
        scatter(t_STA, dpost_err_STA); hold on; grid on;
        plot(t_STA, sigma3_STA(:,1), 'r');
        plot(t_STA, -sigma3_STA(:,1), 'r');
        title({sprintf('POSTFIT range residuals (km): mean = %.3g, RMS = %.3g', mean(dpost_err_STA), dpost_rms_STA)} );
            ylabel('km')
            subplot(4,1,4)
            scatter(t_STA, vpost_err_STA); hold on; grid on;
            plot(t_STA, sigma3_STA(:,2), 'r');
            plot(t_STA, -sigma3_STA(:,2), 'r');
            title({sprintf('POSTFIT range-rate residuals (km/s): mean = %.3g, RMS = %.3g', mean(vpost_err_STA), vpost_rms_STA)} );
                xlabel('Time (s)')
                ylabel('km/s')
                sgtitle(ftitle);

                end

                function plot_res_r(ftitle, t_STA, sigma3_STA, dpre_err_STA, dpre_rms_STA, dpost_err_STA, dpost_rms_STA)

                figure('name', ftitle);
                subplot(2,1,1)
                scatter(t_STA, dpre_err_STA); hold on; grid on;
                title({sprintf('PREFIT range residuals (km): mean = %.3g, RMS = %.3g', mean(dpre_err_STA), dpre_rms_STA)} );
                    ylabel('km')
                    subplot(2,1,2)
                    scatter(t_STA, dpost_err_STA); hold on; grid on;
                    plot(t_STA, sigma3_STA(:,1), 'r');
                    plot(t_STA, -sigma3_STA(:,1), 'r');
                    title({sprintf('POSTFIT range residuals (km): mean = %.3g, RMS = %.3g', mean(dpost_err_STA), dpost_rms_STA)} );
                        ylabel('km')
                        sgtitle(ftitle);

                        end

                        function plot_res_rr(ftitle, t_STA, sigma3_STA,
```

```
vpre_err_STA, vpre_rms_STA, vpost_err_STA,
vpost_rms_STA)

figure('name', ftitle);
subplot(2,1,1)
scatter(t_STA, vpre_err_STA); hold on; grid on;
title({sprintf('PREFIT range-rate residuals (km/s
): mean = %.3g, RMS = %.3g', mean(
vpre_err_STA), vpre_rms_STA)} );
    ylabel('km')
    subplot(2,1,2)
    scatter(t_STA, vpost_err_STA); hold on;
        grid on;
    plot(t_STA, sigma3_STA(:,1), 'r');
    plot(t_STA, -sigma3_STA(:,1), 'r');
    title({sprintf('POSTFIT range-rate
        residuals (km/s): mean = %.3g, RMS =
        %.3g', mean(vpost_err_STA),
        vpost_rms_STA)} );
        ylabel('km')
        sgtitle(ftitle);

        end
```

batch_LSQ.m:

```
function [Ycalc_STA, Lambda, N] = ...
batch_LSQ(Yobs_STA, t_XSTM, XSTM, et_t0, Ht_fn, Lambda0, N0)

global wE R_KJL R_DGO R_ACB
global r_KJL_ECEF r_DGO_ECEF r_ACB_ECEF

% Initialize calculated Y
Ycalc_STA = zeros(size(Yobs_STA));
Ycalc_STA(:, 1:2) = Yobs_STA(:, 1:2);

% Set up covariance
nX     = length(N0);
Lambda = Lambda0;
N      = N0;

% for i = 1:length(Yobs_STA)
for i = 1:28

% find index for same time state and observation
Yi   = Yobs_STA(i, :);
ti   = Yi(2) + et_t0;
i_X  = find(t_XSTM == ti);

% get JD time
JD_UTC = cspice_et2utc(ti, 'J', 10);
JD_UTC = str2num(extractAfter(JD_UTC, 'JD '));

% observation covariance
if Yi(1) == 1
R = R_KJL;
r_STA_ECEF = r_KJL_ECEF;
elseif Yi(1) == 2
R = R_DGO;
r_STA_ECEF = r_DGO_ECEF;
else
R = R_ACB;
r_STA_ECEF = r_ACB_ECEF;
end

% Convert station to ECI frame
r_STA_ECI  = fn.ECEFtoECI(JD_UTC, r_STA_ECEF);
v_KJL_ECEF = [0; 0; 0];
a_ECEF     = v_KJL_ECEF + cross([ 0 0 wE ]', r_STA_ECEF);
```

```
v_STA_ECI   = fn.ECEFtoECI(JD_UTC, a_ECEF); % Technically wrong. Look in Vallado
XSi         = [r_STA_ECI; v_STA_ECI];

% Extract states (all in ECI)
Xi   = XSTM( i_X, 1:nX)';
STMi = XSTM( i_X, nX+1 : nX+nX^2 );
STMi = reshape(STMi, [nX nX]);

% compute H [2x7]
Hi = Ht_fn(Xi(1), Xi(2), Xi(3), Xi(4), Xi(5), Xi(6), XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi
    (6)) * STMi;

% Accumulate observation
Ycalc_STA(i,3:4) = Hi * Xi;

% Obtain y difference
yi = Yobs_STA(i,3:4)' - fn.G_fn(Xi, XSi);

% Accumulate covariance
Lambda = Lambda + Hi' * inv(R) * Hi;
N      = N + Hi' * inv(R) * yi;

end

end
```

EKF.m:

```
function [t_XSTM, XSTM, Xstar, Y_prefit, Y_postfit, Pi, Lambda] = ...
EKF(Yobs_STA, XSTM_prev, nX, et_t0, t_prop, options, Amat_fn, Ht_fn, P_prev)

global wE R_KJL R_DGO R_ACB
global r_KJL_ECEF r_DGO_ECEF r_ACB_ECEF

% Integrate ref trajectory and STM from t = i-1 (prev) to t = i (curr)
if length(t_prop) > 1
[t_XSTM, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), t_prop, XSTM_prev, options);
Xi   = XSTM(end,1:nX)';
STMi = XSTM(end,nX+1:end);
else
t_XSTM = t_prop;
XSTM = XSTM_prev;
Xi   = XSTM(1:nX);
STMi = XSTM(nX+1:end);
end
STMi = reshape(STMi, [nX nX]);

% % find index for same time state and observation
t_Y   = Yobs_STA(:,2) + et_t0; % time after initial epoch
ti_X  = t_XSTM(end);
i_Y   = find(t_Y == ti_X);
Yi    = Yobs_STA(i_Y, :);

% get JD time
JD_UTC = cspice_et2utc(t_XSTM(end), 'J', 10);
JD_UTC = str2num(extractAfter(JD_UTC, 'JD '));

% observation covariance
if Yi(1) == 1
R = R_KJL;
r_STA_ECEF = r_KJL_ECEF;
elseif Yi(1) == 2
R = R_DGO;
r_STA_ECEF = r_DGO_ECEF;
else
R = R_ACB;
r_STA_ECEF = r_ACB_ECEF;
end

```

```
% Convert station to ECI frame
r_STA_ECI   = fn.ECEFtoECI(JD_UTC, r_STA_ECEF);
v_KJL_ECEF = [0; 0; 0];
a_ECEF      = v_KJL_ECEF + cross([ 0 0 wE ]', r_STA_ECEF);
v_STA_ECI   = fn.ECEFtoECI(JD_UTC, a_ECEF); % Technically wrong. Look in Vallado
XSi         = [r_STA_ECI; v_STA_ECI];

% Time update + process noise
dt      = t_prop(end) - t_prop(1);
% Q      = diag( (1e-10)^2 * [1 1 1] );
Q       = diag( (10000e-10)^2 * [1 1 1] );
Gamma = [diag( dt^2/2 * [1 1 1] ); diag([dt dt dt])];
P_noise = Gamma * Q * Gamma';
Pi_bar = STMi * P_prev * STMi' + P_noise;

% Y prefit
Y_prefit(1:2) = Yi(1:2);
Y_prefit(3:4) = fn.G_fn(Xi, XSi)';

% Observation-state matrix
Hti = Ht_fn(Xi(1), Xi(2), Xi(3), Xi(4), Xi(5), Xi(6), XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi
    (6));

% Gain matrix
Ki = Pi_bar * Hti' * inv( Hti * Pi_bar * Hti' + R );

% Obtain y difference
yi = Yi(3:4)' - fn.G_fn(Xi, XSi);

% Measurement and reference orbit update
xhat  = Ki * yi;
Xstar = Xi + xhat;
Pi     = ( eye(nX) - Ki * Hti ) * Pi_bar;
% Pi     = ( eye(nX) - Ki * Hti ) * Pi_bar * ( eye(nX) - Ki * Hti )' + Ki * R * Ki';

% Innovation (information) covariance
Lambda = (Hti * Pi * Hti' + R);

% Y postfit
Y_postfit(1:2) = Yi(1:2);
Y_postfit(3:4) = fn.G_fn(Xstar, XSi)';

end
```

ECItoRSW_t.m:

```
function T = ECItoRSW_T(X_ECI)

r_ECI = X_ECI(1:3);
v_ECI = X_ECI(4:6);

R = r_ECI / norm(r_ECI);
W = cross(r_ECI, v_ECI) / norm(cross(r_ECI, v_ECI));
S = cross(W, R);

T = [R'; S'; W'];

end
```

References

[1] Bob Schutz, G. H. B., Byron Tapley, *Statistical Orbit Determination*, Academic Press, 2004.

[2] Vallado, D. A., and McClain, W. D., *Fundamentals of Astrodynamics and Applications*, 4th ed., Microcosm Press, 2013.

[3] Vallado, D. A., "Fundamentals of Astrodynamics and Applications 4th Ed Consolidated Errata," `https://celestrak.com/software/vallado/ErrataVer4.pdf`, 2019.

[4] Montenbruck, O., and Gill, E., *Satellite Orbits: Models, Methods and Applications*, Springer, Berlin, Heidelberg, 2000.

[5] "Standard Rapid EOP Data since 02. January 1973 (IAU1980)," `https://datacenter.iers.org/data/7/finals.all`, 2021.