ASE 389P.4 Methods of Orbit Determination
Term Project

Junette Hsin
Masters Student, Aerospace Engineering and Engineering Mechanics, University of Texas, Austin, TX 78712

The theory and algorithms are derived and computer program to establish the trajectory of an Earth-orbiting satellite is developed. The assumptions for the study are:

- **Three tracking stations taking apparent range and range-rate data are available for tracking the satellite. Apparent quantities imply that the one-way light time between signal transmission and reception were modeled into the measurement (i.e. the effect is dealt with).**
- **The force model used to generate the truth is the EGM96 gravity field of degree and order 20, attitude-dependent solar radiation pressure, and atmospheric drag.**
- **The satellite is a box-wing shaped with one Sun-pointed solar panel with known component sizes, material properties, and orientation. The spacecraft -Z axis (in the spacecraft body reference frame) is always Nadir-pointed and has the antenna.**

Problem 1 and 2

PREFIT and POSTFIT residuals for all data and all sensors.

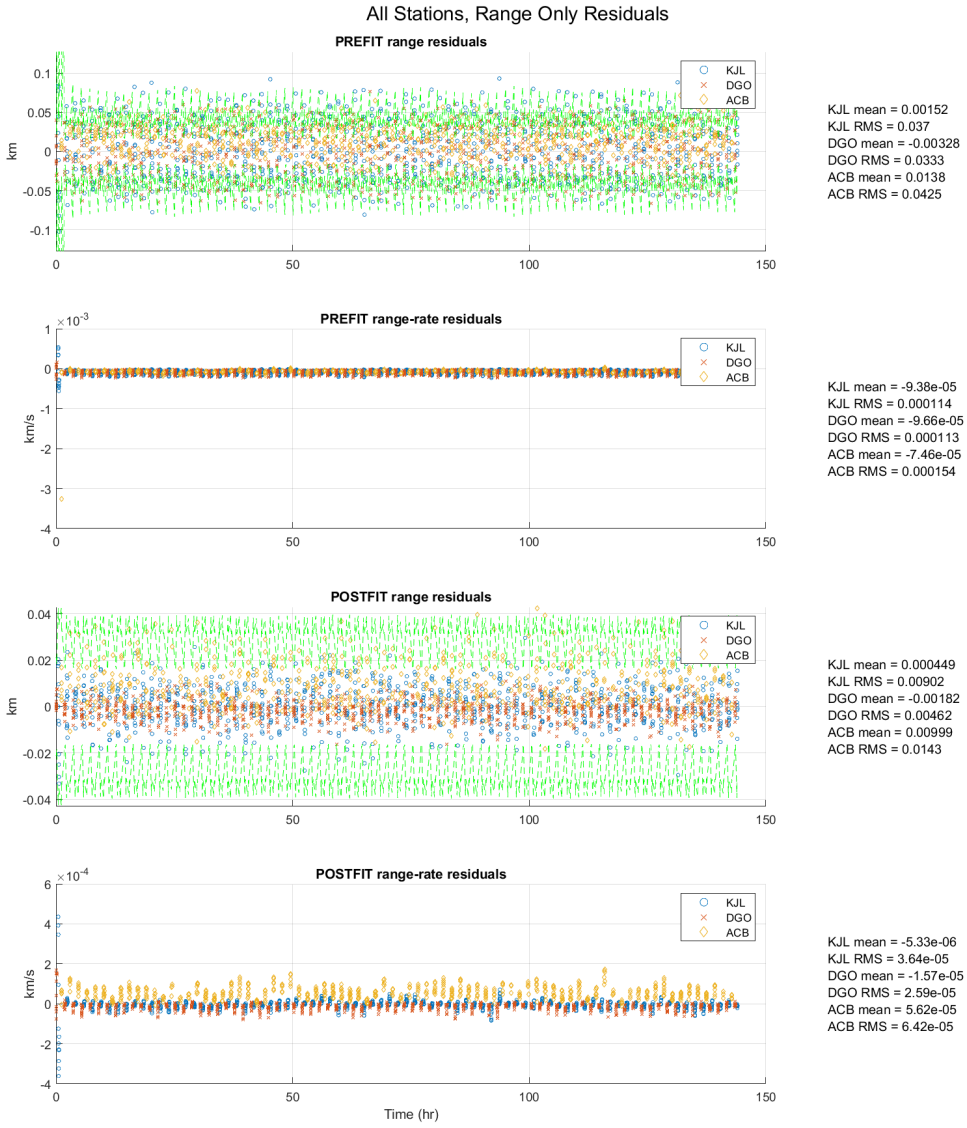


Fig. 1 Case A: Range Only Prefit and Postfit Residuals

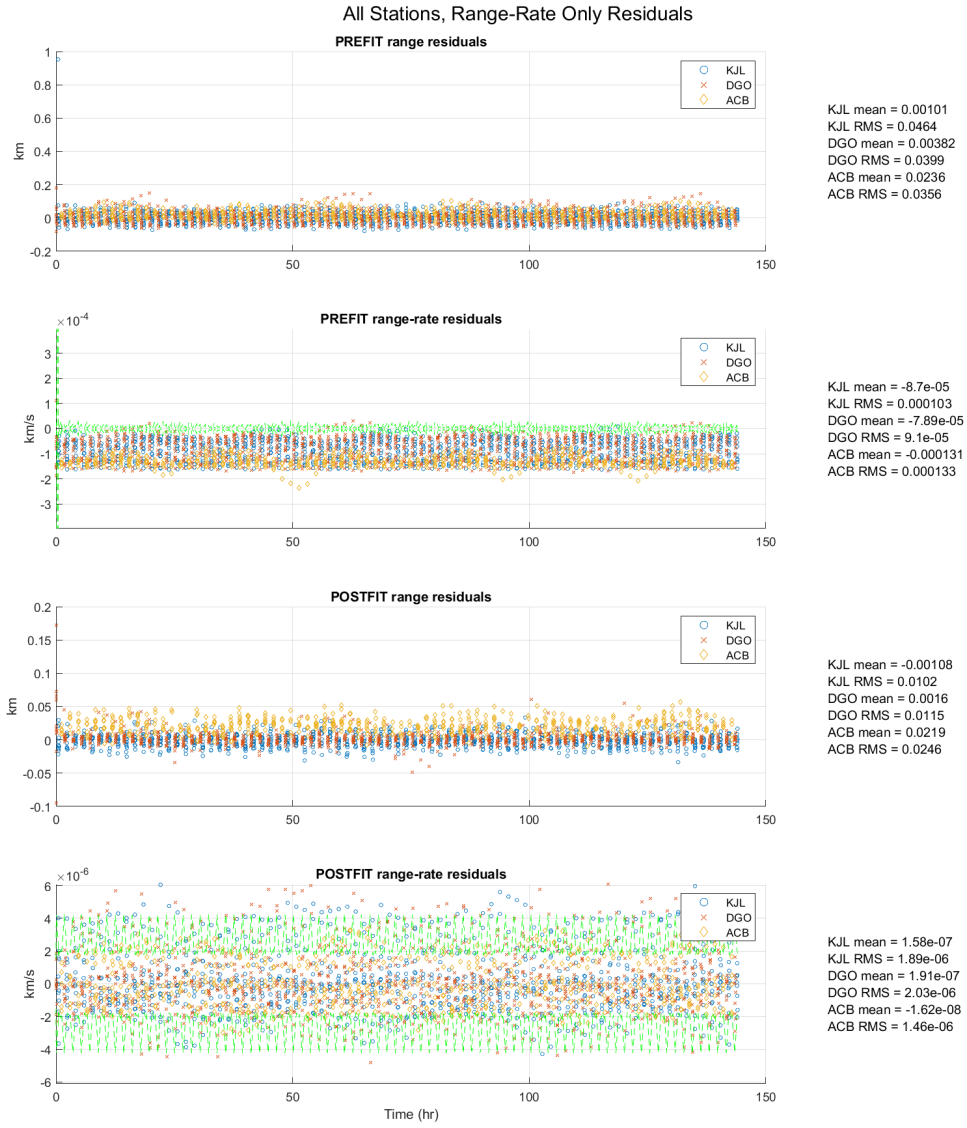


Fig. 2 Case B: Range-Rate Only Prefit and Postfit Residuals

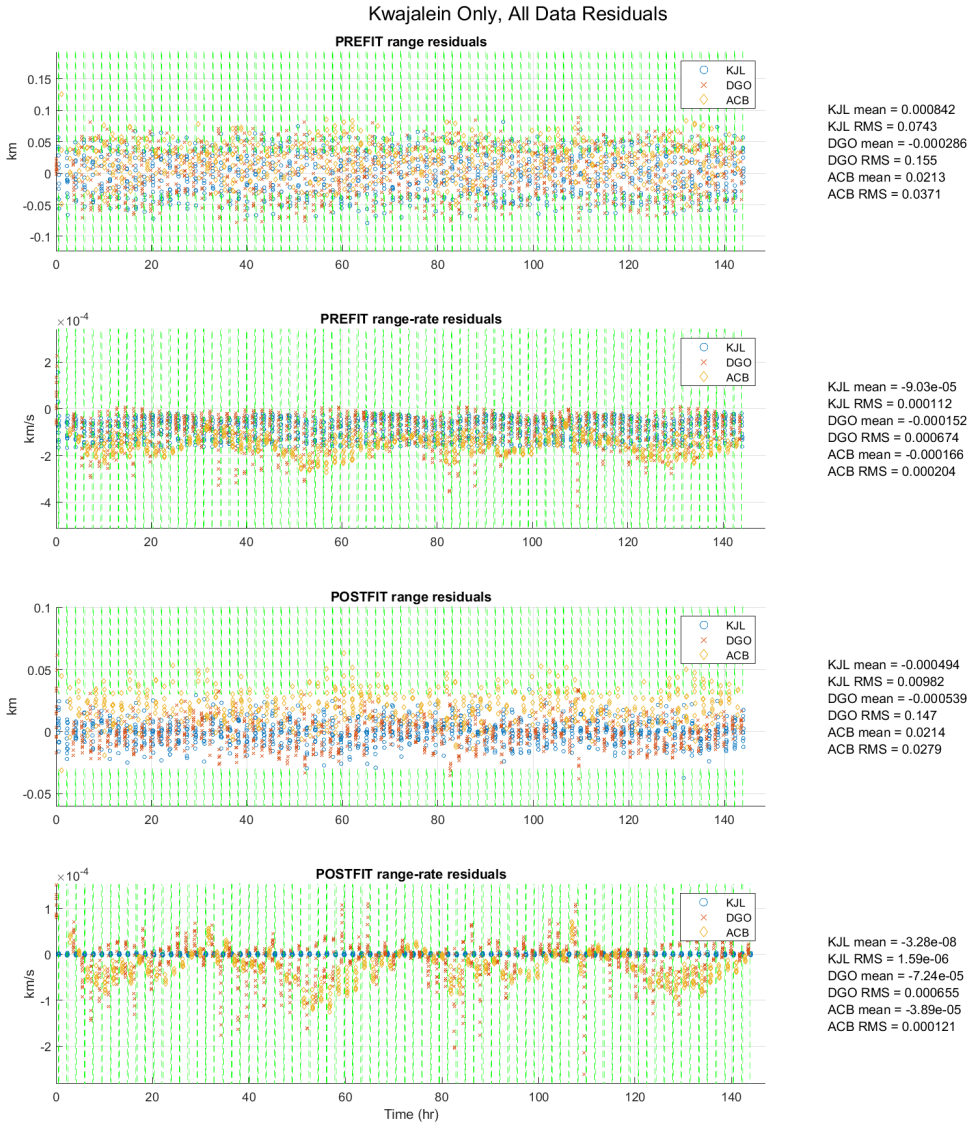


Fig. 3 Case C: Kwajalein Only Prefit and Postfit Residuals

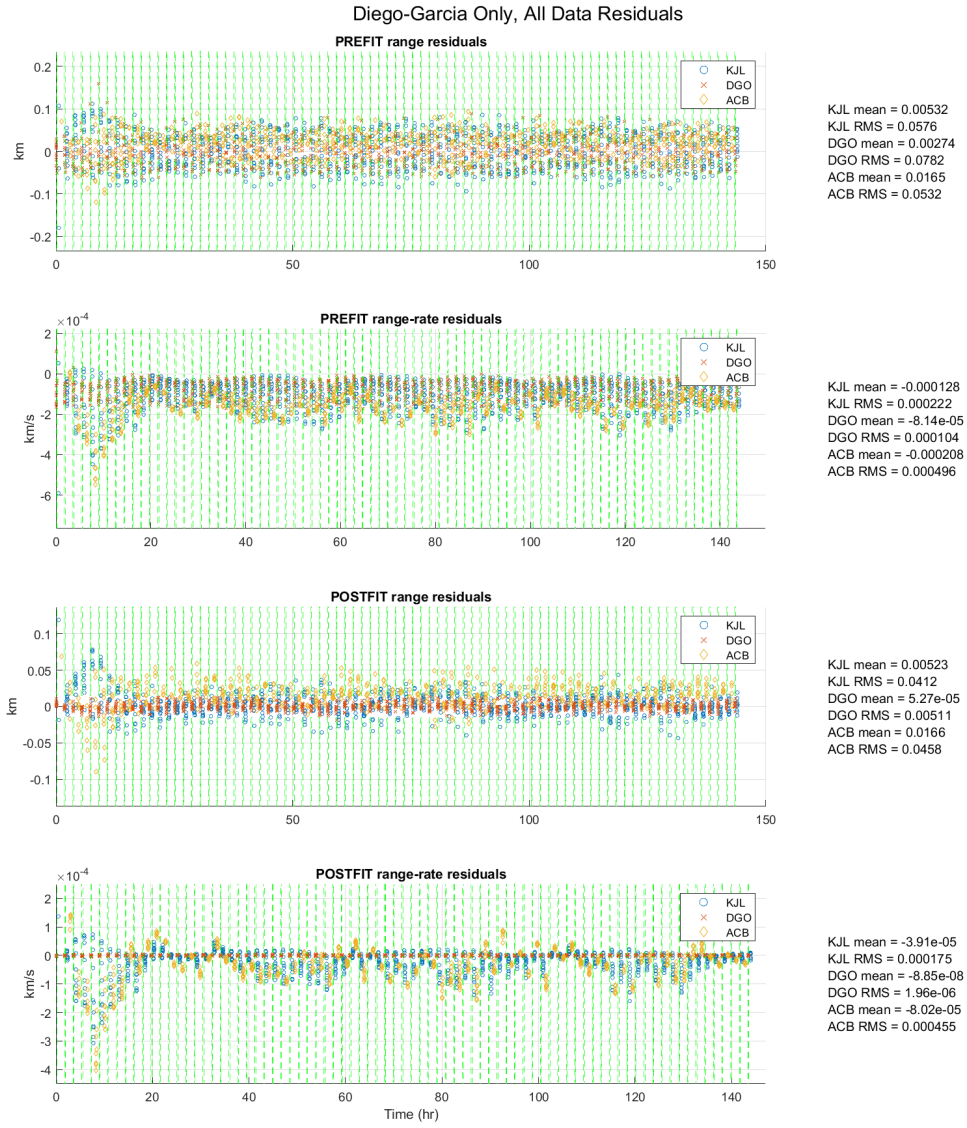


Fig. 4 Case D: Diego Garcia Only Prefit and Postfit Residuals

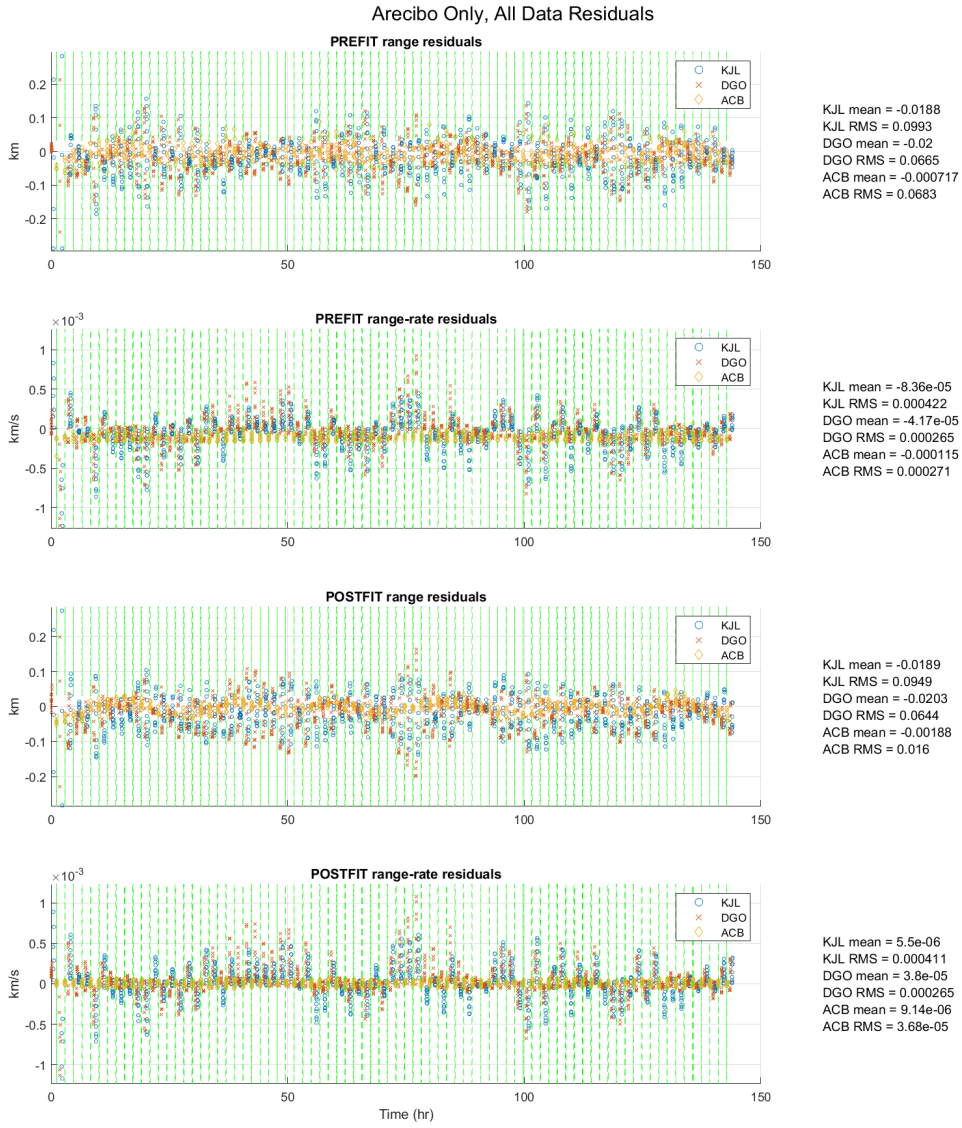


Fig. 5 Case E: Arecibo Only Prefit and Postfit Residuals

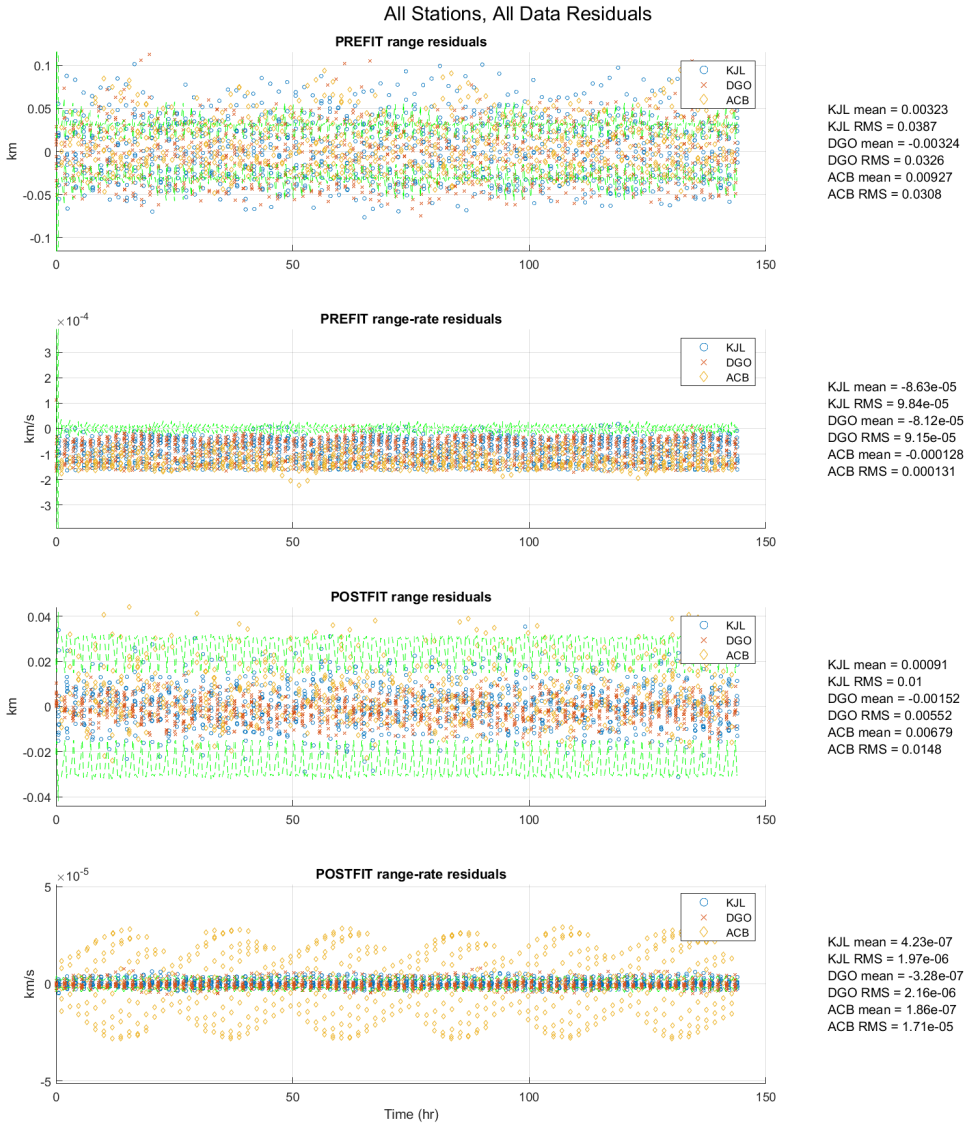


Fig. 6 Case F: Long-Arc Only Prefit and Postfit Residuals

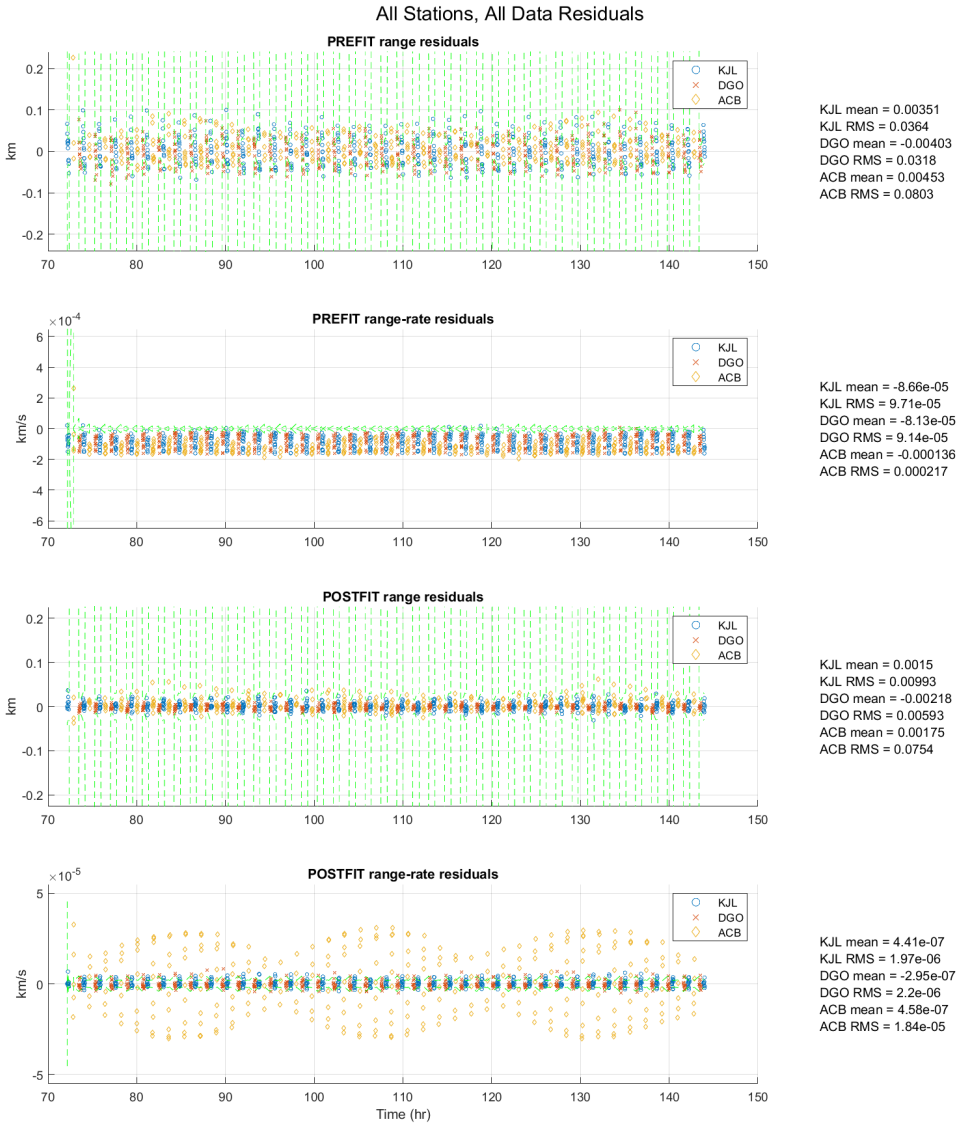


Fig. 7 Case G: Short-Arc Only Prefit and Postfit Residuals

Problem 3

A plot in each of the following frames: Radial-Crosstrack (spacecraft motion is going into the page). Radial-Intrack (looking face-on to the orbital plane), and Crosstrack-Intrack (looking edge-on to the orbital plane) with all of the following ellipses simultaneously. Label each ellipse with the case number from below so we know what is being plotted. Be mindful of your units. Show things in an adequate scale.

(a) fit range only for all sensors

(b) fit range-rate only for all sensors

(c) fit Kwajalein only for all data types

(d) fit Diego Garcia only for all data types

(e) fit Arecibo only for all data types

(f) fit the long-arc (all data and all sensors)

(g) fit the short arc (only the last day of data for all sensors)

Solution

The radial-intrack, radial-crosstrack, and intrack-crosstrack plots for Cases A - F are provided below. For Case F, the "long arc" is really just 24 hours for the first phase of this project. **The units for all axes are in kilometers.**

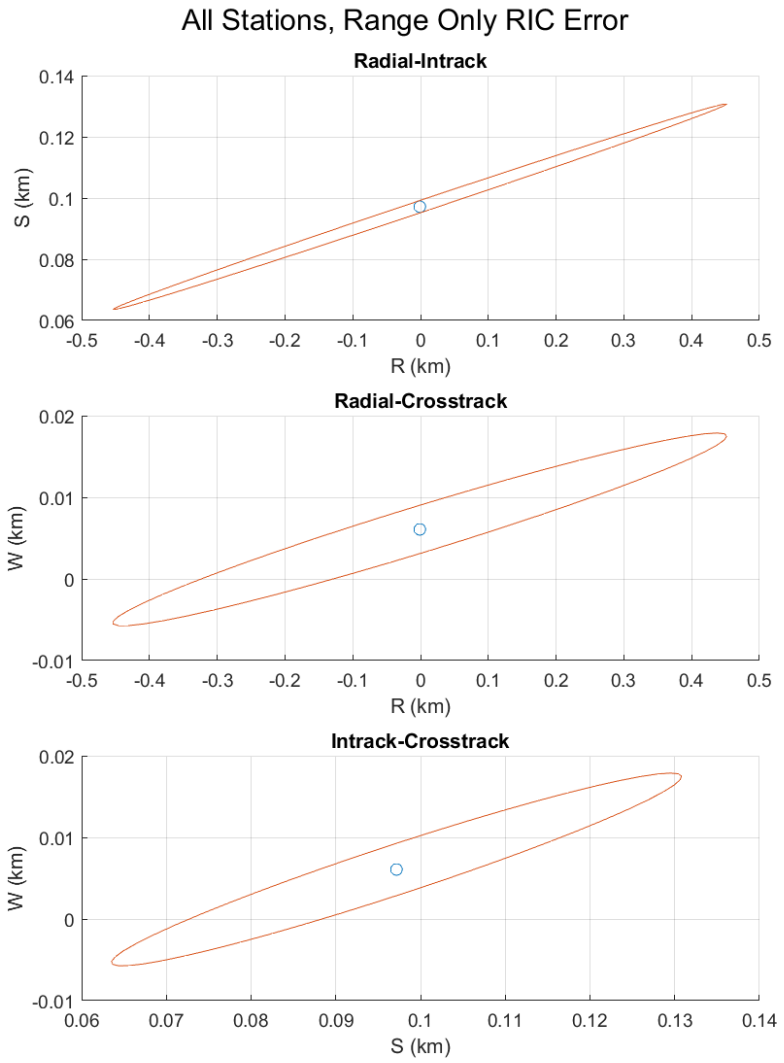


Fig. 8 Case A: Range Only RIC Error

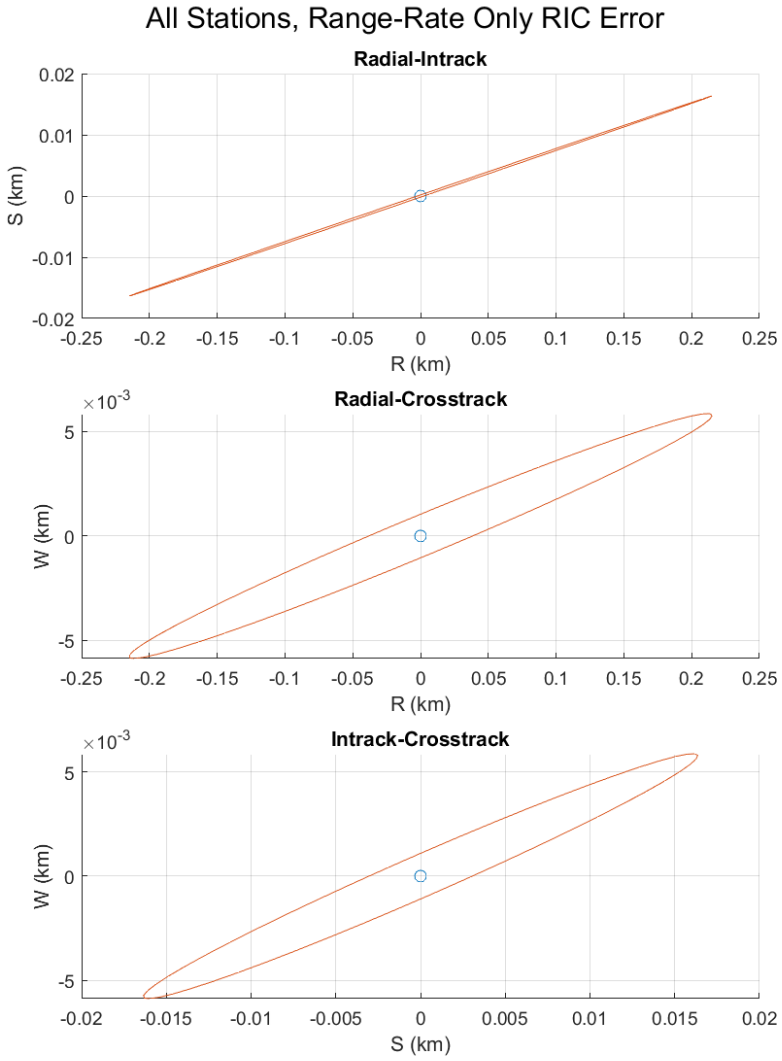


Fig. 9 Case B: Range-Rate Only RIC Error

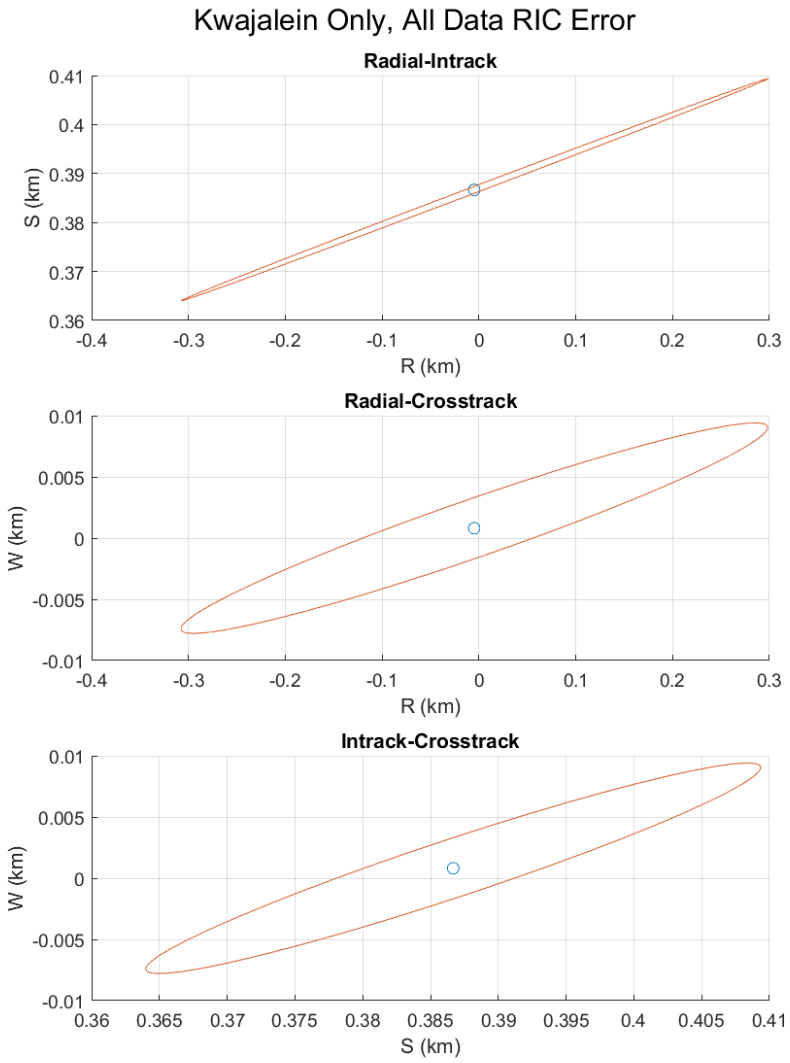


Fig. 10 Case C: Kwajalein Only RIC Error

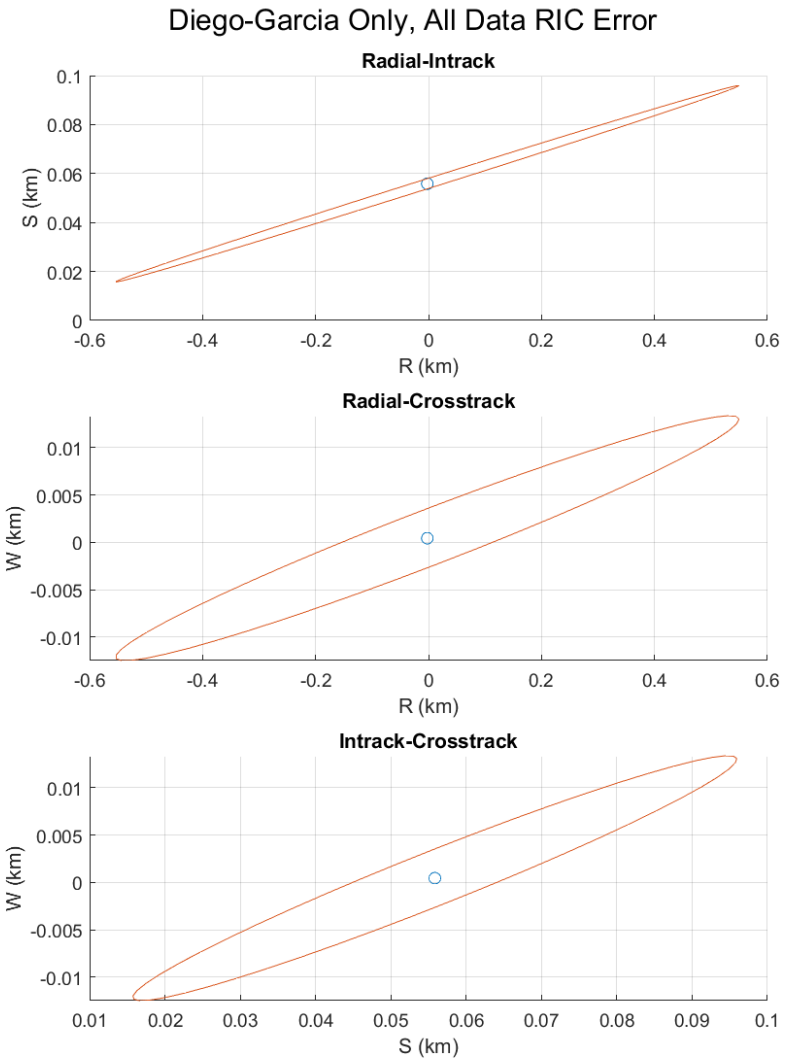


Fig. 11 Case D: Diego Garcia Only RIC Error

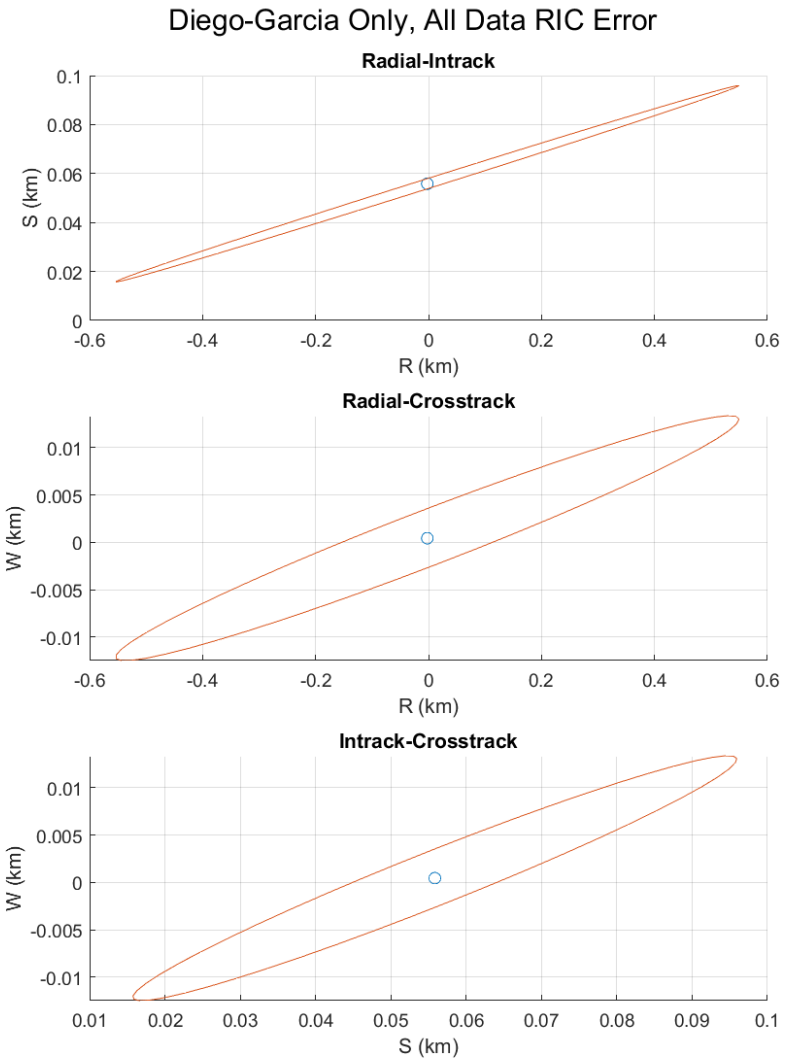


Fig. 12 Case E: Arecibo Only RIC Error

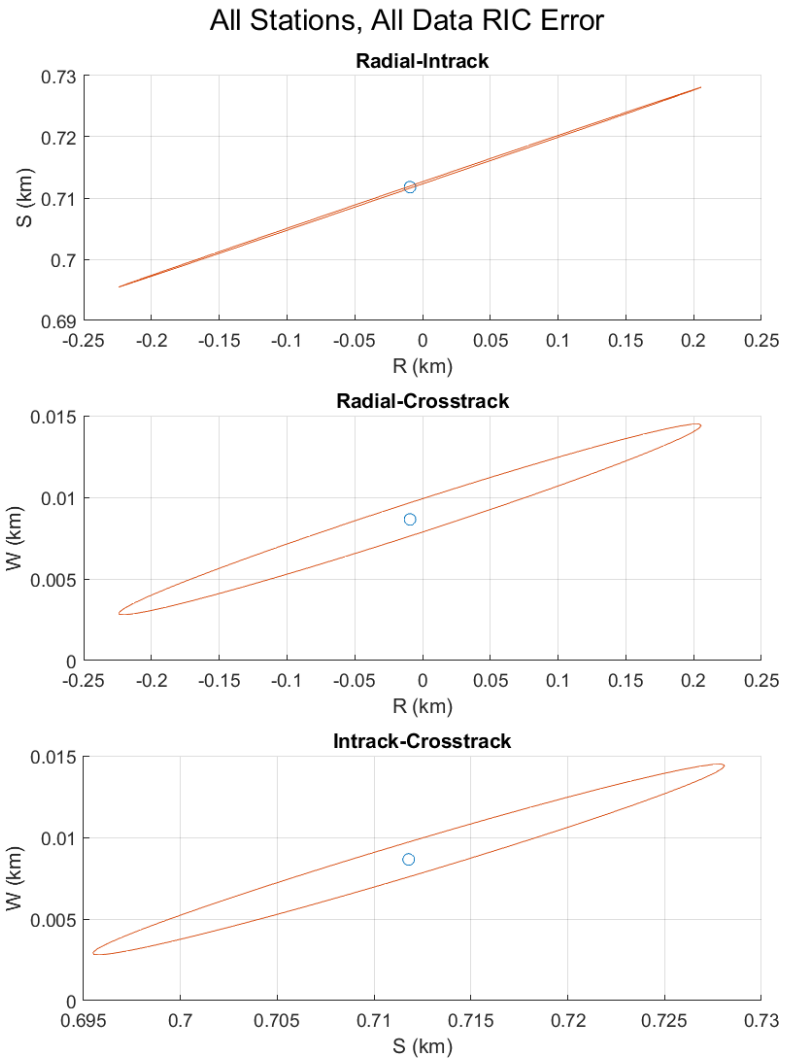


Fig. 13 Case F: Long-Arc Only RIC Error

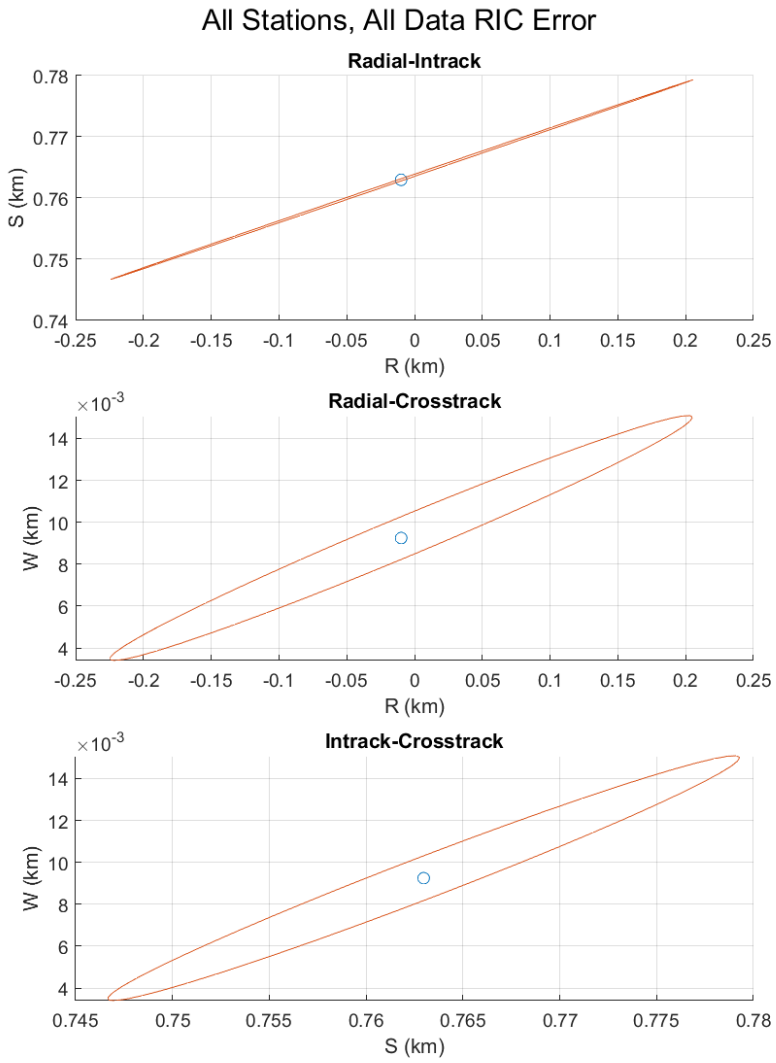


Fig. 14 Case G: Short-Arc Only RIC Error

Analysis

The final propagated states to the time of ΔV_1 for each case are given below:

Case	x (km)	y (km)	z (km)
Case A	445.6184	-7.1002e3	-183.8598
Case B	**445.7151**	**-7.1002e3**	**-183.8514**
Case C	445.3296	-7.1002e3	-183.8615
Case D	445.6596	-7.1002e3	-183.8532
Case E	444.9273	-7.1002e3	-183.8699
Case F	445.0057	-7.1002e3	-183.8771
Case G	444.9547	-7.1002e3	-183.8789

Table 1 Delivery Position Estimates in ECI frame

The best estimate which should be used for the ΔV_1 maneuver is Case B.

Case	Position Covariance (km)
Case A	$\begin{bmatrix} 0.20514 & 0.01521 & 0.005179 \\ 0.01521 & 0.0011318 & 0.00038245 \\ 0.005179 & 0.00038245 & 0.00013953 \end{bmatrix}$
Case B	$\begin{bmatrix} 0.046275 & 0.0035185 & 0.0012418 \\ 0.0035185 & 0.00026756 & 9.4283e-05 \\ 0.0012419 & 9.4283e-05 & 3.442e-05 \end{bmatrix}$
Case C	$\begin{bmatrix} 0.092019 & 0.0068838 & 0.0024976 \\ 0.0068838 & 0.00051547 & 0.00018697 \\ 0.0024976 & 0.00018697 & 7.4071e-05 \end{bmatrix}$
Case D	$\begin{bmatrix} 0.30446 & 0.022101 & 0.0069232 \\ 0.022101 & 0.0016086 & 0.00050364 \\ 0.0069234 & 0.00050366 & 0.00016723 \end{bmatrix}$
Case E	$\begin{bmatrix} 1.1775 & 0.079641 & 0.027798 \\ 0.079639 & 0.0054069 & 0.0018739 \\ 0.0278 & 0.0018741 & 0.00067882 \end{bmatrix}$
Case F	$\begin{bmatrix} 0.046158 & 0.0035054 & 0.0012366 \\ 0.0035054 & 0.00026624 & 9.3775e-05 \\ 0.0012366 & 9.3775e-05 & 3.4173e-05 \end{bmatrix}$
Case F	$\begin{bmatrix} 0.046175 & 0.0035063 & 0.001237 \\ 0.0035063 & 0.00026628 & 9.3798e-05 \\ 0.001237 & 9.3798e-05 & 3.4184e-05 \end{bmatrix}$

Table 2 Delivery Position Covariances in ECI frame

A. Dynamic Model

The spacecraft dynamics model incorporated perturbations due to spherical harmonics, lunisolar effects, solar radiation pressure, and atmospheric drag.

1. Gravity Model

The spherical harmonics model used was GGM03S, which is derived purely on 4 years of highly accurate data from the GRACE satellite (from January 2003 to December 2006), truncated to 20 degree and order. EGM2008, the spherical harmonic model released by the National Geospatial-Intelligence Agency in 2008, combines the long wavelength GRACE data with global 5 arc-minute equiangular grid of area-mean free-air gravity anomalies.

The gravitational coefficients for the gravity spherical harmonics model were taken from GGM03S, which is available on the GRACE website [1]. The gravitational potential function in the form derived by Levinson is [2]:

$$V = \frac{GM}{r}\left\{1 + \sum_{n_max}^{n=2}\left[\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \bar{P}_{n,m}(sin\beta)(\bar{C}_{n,m}cosm\lambda + \bar{S}_{n,m}sinm\lambda)\right]\right\} \tag{1}$$

where G is the universal gravitational constant, M is the mass of the Earth, n is the order, m is the degree, β is the longitude, λ is the latitude, $\bar{P}$ is a normalized associated Legendre function based on the degree and order, and $\bar{C}$ and $\bar{S}$ are normalized gravitational coefficients from GGM03S. To produce the spacecraft accelerations ΔV, Equation .A.1 was differentiated with respect to r, λ, and β and substituted into the following equation, where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the orthogonal basis unit vectors for the ECI frame:

$$\Delta V = \left(\frac{\delta V}{\delta r}cos\beta cos\lambda - \frac{\delta V}{\delta \lambda}\frac{sin\lambda}{rcos\beta} - \frac{\delta V}{\delta \beta}\frac{sin\beta cos\lambda}{r}\right)\hat{x} + \left(\frac{\delta V}{\delta r}cos\beta sin\lambda - \frac{\delta V}{\delta \lambda}\frac{cos\lambda}{rcos\beta} - \frac{\delta V}{\delta \beta}\frac{sin\beta sin\lambda}{r}\right)\hat{y} + \left(\frac{\delta V}{\delta r}sin\beta + \frac{\delta V}{\delta \beta}\frac{cos\beta}{r}\right)\hat{z} \tag{2}$$

To compute the A matrix of partial derivatives, only the zonal terms J2, J3, and J4 were considered, in which n = 2, 3, 4; and m = 0.

2. Lunisolar Effects

Lunisolar effects were modeled as third-body gravitational perturbations. The perturbing force contribution according to Born is [3]:

$$f_{3B} = \sum_{j=1}^{n_p} \mu_j \left(\frac{\boldsymbol{\Delta}_j}{\Delta_j^3} - \frac{\boldsymbol{r}_j}{r_j^3}\right) \tag{3}$$

where n_p is the number of additional bodies, j represents a specific body such as the sun or the moon, μ_j is the gravitational parameter of the specific body, $\boldsymbol{\Delta}_j$ is the position vector of the body with respect to the satellite, and $\boldsymbol{r}_j$ is the position vector of the body with respect to the Earth.

SPICE was used to find the position vectors of the third bodies in the J2000 frame using `naif0011.tls`, `de421.bsp`, and `pck00010.tpc`, all of which are available to download on the SPICE website `https://naif.jpl.nasa.gov/naif/toolkit.html`.

3. Solar Radiation Pressure

The method for computing solar radiation pressure (SRP) was taken from [4]. The SIGHT algorithm was used to check whether the Sun is lighting the satellite (pg. 308), which uses line-of-sight (LOS) geometry to check if the sum of the angles of the Earth to satellite and the Earth to Sun is larger than a requisite minimum angle for LOS.

If LOS exists, then the equation to find solar radiation pressure is calculated as:

$$a_{srp} = -\sum_{i=1} \frac{p_{srp}A_i cos(\theta_{inc_i})}{m}\left\{2\left(\frac{c_{Rd_i}}{3} + c_{Rsi}cos(\theta_{inc_i})\right)\hat{n} + (1 - c_{Rs_i})\hat{s}\right\} \tag{4}$$

where p_{srp} is the incoming solar pressure, θ_{inc} is the angle between the surface normal and incoming radiation (in direction of sun vector), m is the satellite mass, c_{Rd} is the diffuse reflectivity, c_{Rs} is the specular reflectivity, $\hat{n}$ is the surface normal vector, and $\hat{s}$ is the sun vector from the Earth to the sun direction.

The SRP due to different coatings on each face was modeled. The surface normals of the local spacecraft x, y, and z axes were calculated in the ECI frame, and the specific specular and diffuse reflectivity applied for whether it was the positive or negative face illuminated.

If LOS does not exist, then the SRP is zero.

4. Atmospheric Drag

The drag model is relatively simple compared to the other modeled perturbation effects and was taken from Born [3]:

$$a_{drag} = -1/2\rho\left(\frac{C_D A}{m}\right)V_r \boldsymbol{V}_r \tag{5}$$

where ρ is the atmospheric density, C_D is the drag coefficient, $\boldsymbol{V}_r$ is the vehicle velocity relative to the atmosphere, A is the cross-sectional area, and m is the mass. Only the drag on the solar panel was accounted for in the drag model.

B. Estimation Process

The measurement data was smoothed using a batch processor and Extended Kalman Filter. The batch processor was used to improve the approximated initial state of the satellite and the Extended Kalman Filter (EKF) was used to sequentially process the measurement data up to the final day of dynamic propagation for ΔV_1. The EKF was chosen due to its flexibility with nonlinear systems and computational efficiency. The EKF is also aided by having a better initial state which is provided by the batch processor.

Both filtering methods were taken from Born [3]. All position and range units are in kilometers and all velocity and range-rate units are in kilometers/second.

To set up both filters, the following measurement covariances were used for each station:

$$R_{KJL} = \begin{bmatrix} (10e-3)^2 & 0 \\ 0 & (0.5e-6)^2 \end{bmatrix} \tag{6}$$

$$R_{DGO} = \begin{bmatrix} (5e-3)^2 & 0 \\ 0 & (1e-6)^2 \end{bmatrix} \tag{7}$$

$$R_{ACB} = \begin{bmatrix} (10e-3)^2 & 0 \\ 0 & (0.5e-6)^2 \end{bmatrix} \tag{8}$$

in which KJL is Kwajalein, DGO is Diego Garcia, and ACB is Arecibo. The state covariance was initialized for position error for 10 km and velocity error of 10 m/s:

$$P_0 = \begin{bmatrix} 10^2 I_{3x3} & 0_{3x3} \\ 0_{3x3} & (10^{-3})^2 I_{3x3} \end{bmatrix} \tag{9}$$

The state for filters was the same as the state for the dynamic equations of motion, which is a [6x1] vector containing the satellite position ([3x1] km) and velocity ([3x1] km/s) in ECI frame.

1. Batch Processor

The batch processor filtered through the first 28 measurements to arrive at the following initial state. As stated above, the algorithm was taken from Born (pg. 196):

$$X_0 = \begin{bmatrix} 6978.25613502827 \\ 1616.30084111326 \\ 19.7187226552466 \\ -1.66208631750821 \\ 7.26104887569659 \\ 0.270612922289392 \end{bmatrix} \tag{10}$$

2. Extended Kalman Filter

The EKF was used to produce the delivery estimates for each case. For cases using data from only one station, the covariances for unused stations were blown up by 1e10. As stated in the previous section, the algorithm was taken from Born[3] (pg. 212).

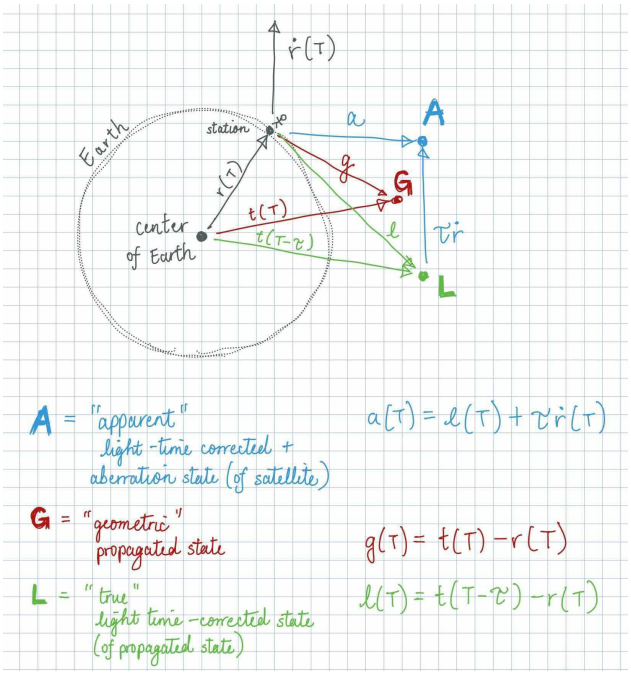
Process noise was used to compensate for unmodeled dynamics. The process noise Q used in the EKF for each measurement update was:

$$Q = \begin{bmatrix} (10e-10)^2 & 0 & 0 \\ 0 & (10e-10)^2 & 0 \\ 0 & 0 & (10e-10)^2 \end{bmatrix} \tag{11}$$

Experimenting with increasing and lowering the process noise and how it affected the delivery accuracy led to the above values. Some amount of process noise was applied to the RIC frame for the **final propagation process** with the below process noise, but it did not affect the final position delivery estimate, only increasing the uncertainty covariance. I was wary of adding too much artificial process noise and used a delta-time of 60 seconds for applying RIC process noise:

$$Q_{RIC} = \begin{bmatrix} (100e-10)^2 & 0 & 0 \\ 0 & (10e-10)^2 & 0 \\ 0 & 0 & (100e-10)^2 \end{bmatrix} \tag{12}$$

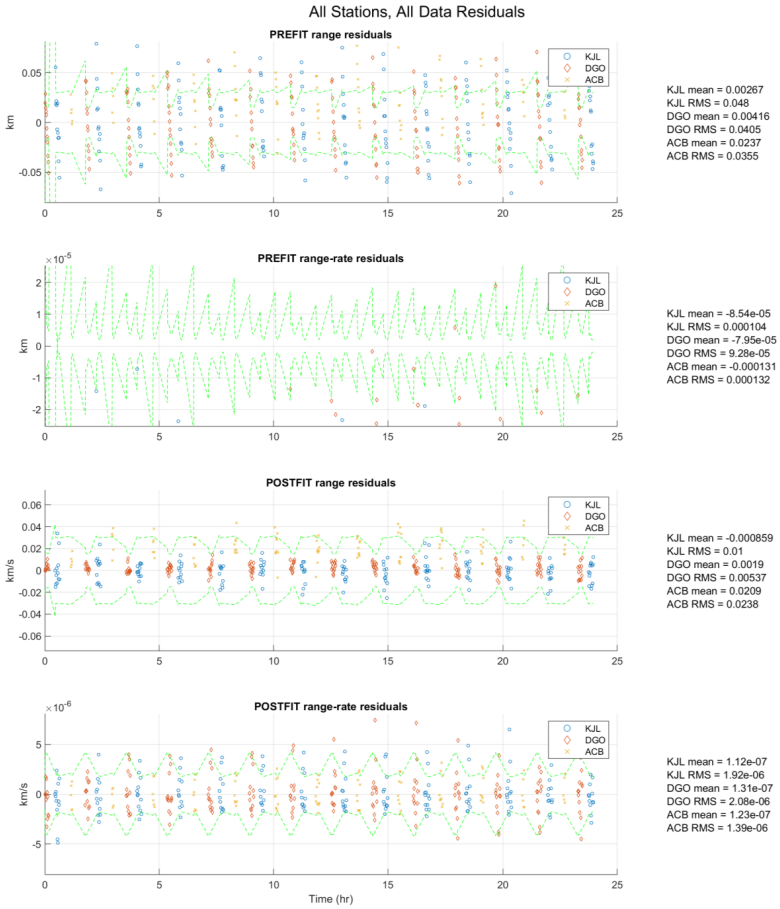
Lighttime and bias correction were used in the EKF. The following figure illustrates how lighttime and stellar aberration affect station measurements:



The light time correction was found by dividing the range measurement by the speed of light and subtracting the time of the propagated satellite state with the light time correction. The state was then back-propagated to the corrected time.

Aberration effect can be calculated by multiplying the light-time correction (a delta time) with the velocity of the station in ECI frame, and then adding the result to the back-propagated state. I tried incorporating aberration effects but it resulted in a delivery result that was more incorrect. It is possible that this is due to my ECEF to ECI function, whose implementation was taken from Vallado (the reference frames transformations methodology I used was explored in Homework 4 [**?**]). I was unable to debug this phenomenon in time and thus left aberration effects out of the EKF.

Bias was found by running the all stations, all data case and observing the postfit residual means. figure .B.2 shows a day's worth of filtering with no modeled bias effects (although at the time of creating this figure, no modeled light time or aberration effects either). The Kwajalein and Diego Garcia range residuals are almost zero-mean, but the Arecibo range residual mean is 1 to 2 orders of magnitude larger than the other stations. The Arecibo mean found through this analysis was 0.0209 km. Dr. Jah later revealed in the course that the Arecibo mean is actually 0.020 km. The prefit and postfit residuals plotted in the first section use the correct bias for Arecibo.



The measurement model was updated with bias for Arecibo, which included updating the G function and the H function, which is the partial derivative of the G function with respect to the satellite state. **The correct way to implement bias would be to add the station bias to the measurement model range calculation, however this led to more inaccurate results.** Instead, I was able to achieve better results by subtracting, not adding the bias, as shown below on line 17.

```
1   function y = G_bias_fn(X, XS, STA)
2   % X  = satellite coords (SAME COORD)
3   % XS = station coords (SAME COORD)
4
5   KJL_rbias = 0;
6   DGO_rbias = 0;
7   % ACB_rbias = 0;
8   ACB_rbias = 0.020;
9
10  if      STA == 1;    rbias = KJL_rbias;
```

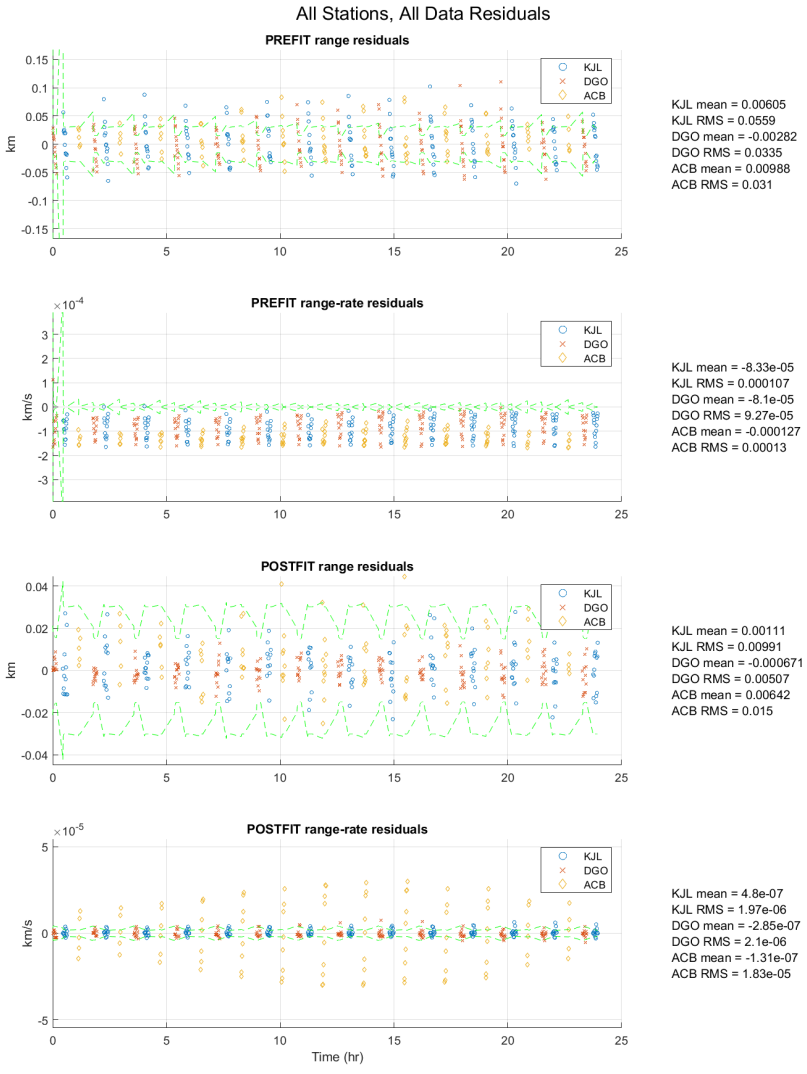
```
        elseif  STA == 2;    rbias = DGO_rbias;
        elseif  STA == 3;    rbias = ACB_rbias;
        end

        r_site = [X(1)-XS(1); X(2)-XS(2); X(3)-XS(3)];
        v_site = [X(4)-XS(4); X(5)-XS(5); X(6)-XS(6)];
        d      = norm(r_site) - rbias;
        v      = dot(v_site, r_site / d);

        y      = [d; v];

        end
```

Another consequence of updating the measurement model to include bias was that the RMS of the range-rate residuals for Arecibo became larger by an order of magnitude, as shown below. Compare the POSTFIT range-rate residuals in Figure .B.2 (below) with Figure .B.2:



I was also unable to debug this phenomenon, but I suspect that there is a bug in the measurement model which is related to the negative sign of the bias. I decided to leave the bias in the measurement model because although it made the postfit rate-rate residuals for Arecibo look worse, it improved the final delivery estimate for cases including Arecibo.

I. Final thoughts

The range-rate data proved to be more accurate than the range data and was my best estimate. That is because range-rate also encodes some range information, in addition to range-rate data. For example, the H matrix at the first measurement was calculated as follows:

$$H_{t0} = \begin{bmatrix} 0.66516 & -0.61572 & 0.42244 & 0 & 0 & 0 \\ 0.00095726 & 0.0018599 & 0.0012037 & 0.66516 & -0.61572 & 0.42245 \end{bmatrix} \tag{13}$$

The first row is the gradient of the range measurement with respect to the satellite position, and the second row is the gradient of the range-rate measurement with respect to the satellite position. Moreover, the standard deviation of the range data was larger than the standard deviation of the range-rate data. Kwajalein, for example, had a standard deviation of 10 m for range, but a 0.5 mm/s standard deivation for range-rate. More information and accuracy are retained in the range-rate calculation than the range calculation. Nevertheless, the range-only case was not bad, ending up 0.097 km from the best estimate.

Kwajalein and Diego-Garcia were the most accurate stations for me. Diego-Garcia was likely the best single station for me overall, as Diego-Garcia is within 0.055 km overall of the best estimate, with Kwajalein following at 0.386 km. The final delivery Arecibo position is 0.790 km within the best case.

I expected the short-arc case to have the best result. The state covariance was widened to 1 km in position, 1 m/s in velocity at the start of propagation, but Case G ended up as the 2nd-worst performing case at 0.763 km away from the best estimate. The long arc did not fare much better, at 0.711 km away from the best estimate. This is probably due to the inclusion of Arecibo range data.

Dynamical errors include the drag model only accounting for solar panel. I would have liked to implement EGM2008 as well to compare with just the GRACE gravity model but was not able to in time. Rather than using SPICE, it would have been interesting to use Vallado's routine to compute the sun vector and observe the differences. The ECEF to ECI function also may have errors, likely from Earth rotation and timing, which I did not change from Homework 4.

Overall, including as much as possible to produce finer resolution models and intelligently filtering (adjusting station covariances, process noise) would lead to better orbit determination and prediction, which is a science and, as this class has shown me, an art.

Appendix

Final Project MATLAB code

Note: The equations of motion and functions to transform from ECI to ECEF and back were submitted as part of Homework 4, and so are left out of this appendix.

1. finalproj_EOM_batch.m:

```
% HW 5
% Junette Hsin

% clear; clc
addpath(genpath('mice'));
addpath(genpath('spice_data'));

% Load SPICE kernel file
cspice_furnsh( 'spice_data/naif0011.tls' )
cspice_furnsh( 'spice_data/de421.bsp' )
cspice_furnsh( 'spice_data/pck00010.tpc ')

format long g

%% Parameters

global const
global muE RE muS AU muM eE wE J2 J3 J4 Cd Cs eop_data
global A m_SC p p0 r0_drag H CD

% initial state initial guess (M --> KM)
const.CD  = 1.88;
% X0 = [ 6984.45711518852
%         1612.2547582643
%         13.0925904314402
%        -1.67667852227336
%         7.26143715396544
%         0.259889857225218 ];

% ballpark right answer
X0  = [    6978.25613502827
1616.30084111326
19.7187226552466
-1.66208631750821
7.26104887569659
0.270612922289392 ];

nX  = length(X0);

% initialize STM
STM0 = eye(length(X0));
STM0 = reshape(STM0, [length(X0)^2  1]);
XSTM0 = [X0; STM0];

% Constants
const.muE = 398600.4415;          % Earth Gravitational Parameter (km^3/s^2)
const.RE  = 6378.1363;            % Earth Radius (km)
const.muS = 132712440018;         % Sun's Gravitational Parameter (km^3/s^2)
const.AU  = 149597870.7;          % 1 Astronomical Unit (km)
const.muM = 4902.800066;          % Moon's Gravitational Parameter (km^3/s^2)
const.eE  = 0.081819221456;       % Earth's eccentricity
const.wE  = 7.292115146706979e-5; % Earth's rotational velocity (rad/s)
const.m_SC = 2000;                 % satellite mass (kg)
const.Cd  = 0.04;                 % diffuse reflection
const.Cs  = 0.04;                 % specular reflection

% face areas
const.Ax = 6 / 1e6; % m^2 --> km^2
```

```
59          const.Ay = 8 / 1e6;
60          const.Az = 12 / 1e6;
61          const.Asp = 15 / 1e6;
62
63          % Cs and Cd for x, y, and z
64          const.MLI_Kapton_Cs = 0.59;
65          const.MLI_Kapton_Cd = 0.04;
66          const.White_Paint_Cs = 0.04;
67          const.White_Paint_Cd = 0.80;
68          const.Germanium_Kapton_Cs = 0.18;
69          const.Germanium_Kapton_Cd = 0.28;
70          const.Solar_Cells_Cs = 0.04;
71          const.Solar_Cells_Cd = 0.04;
72
73          const.p_srp      = 4.57e-6;      % solar pressure per unit area, in N/m^2
74          const.p_srp      = const.p_srp * 1e6; % N/m^2 --> N/km^2
75
76          const.J2 = 1.08262617385222e-3;
77          const.J3 = -2.53241051856772e-6;
78          const.J4 = -1.61989759991697e-6;
79
80          global r_KJL_ECEF r_DGO_ECEF r_ACB_ECEF
81
82          % Station coords. Convert M --> KM
83          r_KJL_ECEF = [-6143584  1364250  1033743]' / 1000;  % Kwajalein
84          r_DGO_ECEF = [ 1907295  6030810 -817119 ]' / 1000;  % Diego
85          r_ACB_ECEF = [ 2390310 -5564341  1994578]' / 1000;  % Arecibo
86
87          global LEO_DATA_Apparent
88          % Load observation data
89          % load('LEO_DATA_Apparent.mat')
90          load LEO_DATA_Apparent_3Days.mat
91          test = load('LEO_DATA_Apparent_Days4-6.mat');
92          days4to6 = test.LEO_DATA_Apparent;
93          days4to6(:,2) = days4to6(:,2) + 3*86400;
94          LEO_DATA_Apparent = [LEO_DATA_Apparent; days4to6];
95
96          % Station data
97          ID_STA    = 1;
98          i_STA     = find(LEO_DATA_Apparent(:, 1) == ID_STA);
99          Yobs_KJL  = LEO_DATA_Apparent(i_STA, :);
100         t_KWJ     = Yobs_KJL(:,2);
101
102         ID_STA    = 2;
103         i_STA     = find(LEO_DATA_Apparent(:, 1) == ID_STA);
104         Yobs_DGO  = LEO_DATA_Apparent(i_STA, :);
105         t_DGO     = Yobs_DGO(:,2);
106
107         ID_STA    = 3;
108         i_STA     = find(LEO_DATA_Apparent(:, 1) == ID_STA);
109         Yobs_ACB  = LEO_DATA_Apparent(i_STA, :);
110         t_ACB     = Yobs_DGO(:,2);
111
112         eop_data = load('finals_iau1980.txt');
113
114         % Atmospheric drag
115         r    = norm(X0(1:3));              % km
116         const.H    = 88667.0 / 1000;           % m --> km
117         const.r0_drag   = (700 + RE);          % m --> km
118         const.p0   = 3.614e-13 * 1e9;          % kg/m3 --> kg/km^3
119         const.p    = p0*exp( -(r-r0_drag)/H );
120         const.A    = 15 / 1e6;                 % m^2 --> km^2
121
122         global Cnm Snm
123
124         % Gravity
125         Cnm = zeros(181,181);
126         Snm = zeros(181,181);
```

```
fid = fopen('GGM03S.txt','r');
for n=0:180
for m=0:n
temp = fscanf(fid,'%d %d %f %f %f %f',[6 1]);
Cnm(n+1,m+1) = temp(3);
Snm(n+1,m+1) = temp(4);
end
end

%% Convert t0 to ET, i.e. seconds past J2000, the base time variable for SPICE. function
    calls.

%  Epoch for initial conditions
% epoch = 23 March 2018, 08:55:03 UTC ;
t0      = 'March 23, 2018, 08:55:03 UTC';
abcorr  = 'NONE';

%  Convert the epoch to ephemeris time.
et_t0   = cspice_str2et( t0 );

% extract observation epochs
epochs = LEO_DATA_Apparent(:,2);
epochs = et_t0 + epochs;


%% Derive A and H matrices

X  = sym('X', [length(X0) 1]);
dX = fn.EOM(et_t0, X);

% compute partials
Amat    = jacobian( dX, X );
Amat_fn = matlabFunction(Amat);

% DO EVERYTHING IN ECI FRAME

X  = sym('X', [nX; 1]);
XS = sym('XS', [nX; 1]);

r_site = [X(1)-XS(1); X(2)-XS(2); X(3)-XS(3)];
v_site = [X(4)-XS(4); X(5)-XS(5); X(6)-XS(6)];
d      = norm(r_site);
v      = dot(v_site, r_site/norm(r_site));

Htmat      = sym(zeros(2,nX));
Htmat(1,:) = simplify(gradient(d, X));
Htmat(2,:) = simplify(gradient(v, X));
Ht_fn      = matlabFunction(Htmat);

Ht_r_fn  = matlabFunction(Htmat(1,:));
Ht_rr_fn = matlabFunction(Htmat(2,:));

% Ht with G bias function
rbias = 0.02;

r_site = [X(1)-XS(1); X(2)-XS(2); X(3)-XS(3)];
v_site = [X(4)-XS(4); X(5)-XS(5); X(6)-XS(6)];
d      = norm(r_site) - rbias;
v      = dot(v_site, r_site / d);

Htmat      = sym(zeros(2,nX));
Htmat(1,:) = simplify(gradient(d, X));
Htmat(2,:) = simplify(gradient(v, X));
Ht_bias_fn = matlabFunction(Htmat);

%% integrate EOM

% set ode45 params
```

```
rel_tol = 1e-10;          % 1e-14 accurate; 1e-6 coarse
abs_tol = 1e-10;
options = odeset('reltol', rel_tol, 'abstol', abs_tol );

% Set run state
run_state = 1;
disp('Running sim ...')

if run_state == 1
[t, X] = ode45(@fn.EOM, [epochs(1) : 60 : epochs(end)], X0, options);
elseif run_state == 2
[t, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), [epochs(1) : 60 : epochs(
    end)], XSTM0, options);
X = XSTM(:, 1:6);
end
disp('Pos and Vel end: ')
disp(X(end, 1:6)');

XSTM_ref0   = XSTM;
t_XSTM_ref0 = t;


%% Setting up filters

% weighting matrices m --> km, mm --> km
global R_KJL R_DGO R_ACB
const.R_KJL = [(10e-3)^2 0; 0 (0.5e-6)^2];
const.R_DGO = [(5e-3)^2  0; 0 (1e-6)^2];
const.R_ACB = [(10e-3)^2 0; 0 (0.5e-6)^2];

% initial covariance error
% 10 km std - position
% 10 m/s    - velocity
P0 = [ 10^2*eye(3), zeros(3);
zeros(3),  (10e-3)^2*eye(3) ];
Lambda0 = inv(P0);
global Lambda_KJL0 Lambda_DGO0 Lambda_ACB0
Lambda_KJL0 = Lambda0;
Lambda_DGO0 = Lambda0;
Lambda_ACB0 = Lambda0;

global N_KJL0 N_DGO0 N_ACB0
N0     = Lambda0*X0;
N_KJL0 = N0;
N_DGO0 = N0;
N_ACB0 = N0;


%% Batch all stations to refine IC

Xstar  = 100*ones(1,3);
XSTM   = XSTM_ref0;
t_XSTM = t_XSTM_ref0;
XSTM0  = XSTM_ref0(1,:)';
iter   = 0;
N_prev = N0;
Lambda_prev = Lambda0;

% Batch first 28 measurements

tic
while norm(Xstar(1:3)) > 0.1

% keep track of iterations
iter = iter + 1;
sprintf('iter = %d', iter)

% Test - All stations
```

```
        [Ycalc_all, Lambda, N] = ...
        fn.batch_LSQ(LEO_DATA_Apparent, t_XSTM, XSTM, et_t0, Ht_fn, Lambda_prev, N_prev);

        %     % Test - Kwajalein
        %     [Ycalc_all, Lambda, N] = ...
        %         fn.batch_LSQ(Yobs_KJL, t_XSTM, XSTM, et_t0, Ht_fn, Lambda0, N0);

        % Solve normal equation
        Xstar = inv(Lambda) * N;

        % update covariance
        Lambda_prev = Lambda0;
        N_prev      = N0;

        % update initial conditions
        XSTM0(1:nX) = XSTM0(1:nX) + Xstar;
        [t, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), [epochs(1) : 60 : epochs(
            end)], XSTM0, options);

        disp('xhat pos'); Xstar(1:3)
        disp('xhat pos norm'); norm(Xstar(1:3))
        disp('x IC pos'); XSTM0(1:3)

        end
        toc

        Lambda_batch = Lambda;
        XSTM_batch   = XSTM;
        XSTM0_batch  = XSTM0;
```

2. finalproj_EKF.m:

```

% HW 5
% Junette Hsin

% finalproj_EOM_batch;
close all; format long g;
load WS_batch.mat

% Load SPICE stuff
addpath(genpath('mice'));
addpath(genpath('spice_data'));
cspice_furnsh( 'spice_data/naif0011.tls' )
cspice_furnsh( 'spice_data/de421.bsp' )
cspice_furnsh( 'spice_data/pck00010.tpc ' )

%%%

global eopdata const

SAT_Const

% read Earth orientation parameters
fid = fopen('eop19620101.txt','r');
%   ---------------------------------------------------------------------------------------------------

% |  Date      MJD      x         y        UT1-UTC      LOD       dPsi     dEpsilon     dX        dY
        DAT
% |(0h UTC)              "         "          s          s         "         "          "         "
        s
%   ---------------------------------------------------------------------------------------------------

eopdata = fscanf(fid,'%i %d %d %i %f %f %f %f %f %f %f %f %i',[13 inf]);
```

```
fclose(fid);

%%

% plot options
plot_orbit = 0;
plot_RSW   = 0;

% Selct sim params
run_STA  = 1 ;       % run STATIONS
run_DATA = 0 ;       % run DATA
save_nag = 0 ;       % save NAG
DAYS     = 1;        % PROPAGATE to 1 day, 3 days, or 7 days.

%% EKF - all observations

% loop through STATIONS
if run_STA == 1
DATA = 0;        % 0 = all data
for STATIONS = 3    % 0 = all stations, 1 = KJL, 2 = DGO, 3 = ACB
EKF_STA_DATA;
end
end

% loop through DATA
if run_DATA == 1
STATIONS = 0;        % 0 = all stations
for DATA = 1:2       % 0 = all data, 1 = range, 2 = range-rate, 3 = short-arc
EKF_STA_DATA;
end
end

X_sol1 = [ -6330.16736001325
3306.81591178162
127.736863438565
-3.43787907681733
-6.63350511630163
-0.235613730204275 ];


%% save NAG

% Save in NAG format
if save_nag == 1

load WS_R_7.mat      % case A
save_case(Xi, P_bar, 'A');
load WS_RR_7.mat     % case B
save_case(Xi, P_bar, 'B');
load WS_KJL_7.mat    % case C
save_case(Xi, P_bar, 'C');
load WS_DGO_7.mat    % case D
save_case(Xi, P_bar, 'D');
load WS_ACB_7.mat    % case E
save_case(Xi, P_bar, 'E');
load WS_ALL_7.mat    % case F (LONG arc now)
save_case(Xi, P_bar, 'F');
load WS_ALL_7_short_arc.mat   % case F (short arc now)
save_case(Xi, P_bar, 'G');

save('hsinj.mat' ...
, 'hsinj_pos_caseA', 'hsinj_poscov_caseA' ...   % Range only
, 'hsinj_pos_caseB', 'hsinj_poscov_caseB' ...   % Range-rate only
, 'hsinj_pos_caseC', 'hsinj_poscov_caseC' ...   % Kwajalein only
, 'hsinj_pos_caseD', 'hsinj_poscov_caseD' ...   % Diego-Garcia only
, 'hsinj_pos_caseE', 'hsinj_poscov_caseE' ...   % Arecibo only
, 'hsinj_pos_caseF', 'hsinj_poscov_caseF' ...   % All stations, all data
, 'hsinj_pos_caseG', 'hsinj_poscov_caseG' ...
```

```
);

end

%% subfunctions

function save_case(Xi, P_bar, case_str)

pos_str = ['hsinj_pos_case' case_str];
poscov_str = ['hsinj_poscov_case' case_str];

evalin('base', [pos_str, ' = Xi(1:3)'])      % Not recommended but oh well
evalin('base', [poscov_str, ' = P_bar(1:3,1:3)'])      % Not recommended but oh well

end
```

batch_LSQ.m:

```
function [Ycalc_STA, Lambda, N] = ...
batch_LSQ(Yobs_STA, t_XSTM, XSTM, et_t0, Ht_fn, Lambda0, N0)

global wE R_KJL R_DGO R_ACB
global r_KJL_ECEF r_DGO_ECEF r_ACB_ECEF

% Initialize calculated Y
Ycalc_STA = zeros(size(Yobs_STA));
Ycalc_STA(:, 1:2) = Yobs_STA(:, 1:2);

% Set up covariance
nX      = length(N0);
Lambda = Lambda0;
N       = N0;

% for i = 1:length(Yobs_STA)
for i = 1:28

% find index for same time state and observation
Yi    = Yobs_STA(i, :);
ti    = Yi(2) + et_t0;
i_X   = find(t_XSTM == ti);

% get JD time
JD_UTC = cspice_et2utc(ti, 'J', 10);
JD_UTC = str2num(extractAfter(JD_UTC, 'JD '));

% observation covariance
if Yi(1) == 1
R = R_KJL;
r_STA_ECEF = r_KJL_ECEF;
elseif Yi(1) == 2
R = R_DGO;
r_STA_ECEF = r_DGO_ECEF;
else
R = R_ACB;
r_STA_ECEF = r_ACB_ECEF;
end

% Convert station to ECI frame
r_STA_ECI  = fn.ECEFtoECI(JD_UTC, r_STA_ECEF);
v_KJL_ECEF = [0; 0; 0];
a_ECEF     = v_KJL_ECEF + cross([ 0 0 wE ]', r_STA_ECEF);
v_STA_ECI  = fn.ECEFtoECI(JD_UTC, a_ECEF); % Technically wrong. Look in Vallado
XSi        = [r_STA_ECI; v_STA_ECI];

% Extract states (all in ECI)
Xi    = XSTM( i_X, 1:nX)';
STMi = XSTM( i_X, nX+1 : nX+nX^2 );
STMi = reshape(STMi, [nX nX]);

```

```
% compute H [2x7]
Hi = Ht_fn(Xi(1), Xi(2), Xi(3), Xi(4), Xi(5), Xi(6), XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi
    (6)) * STMi;

% Accumulate observation
Ycalc_STA(i,3:4) = Hi * Xi;

% Obtain y difference
yi = Yobs_STA(i,3:4)' - fn.G_fn(Xi, XSi);

% Accumulate covariance
Lambda = Lambda + Hi' * inv(R) * Hi;
N      = N + Hi' * inv(R) * yi;

end

end
```

3. EKF_STA_DATA.m

```
        % HW 5
        % Junette Hsin
        % TEST EKF SCRIPT

        % weighting matrices m --> km, mm --> km
        global R_KJL R_DGO R_ACB
        R_KJL = [(10e-3)^2  0; 0 (0.5e-6)^2];
        R_DGO = [(5e-3)^2   0; 0 (1e-6)^2];
        R_ACB = [(10e-3)^2  0; 0 (0.5e-6)^2];

        % try smaller P0
        P_prev     = [ diag([0.1  0.1  0.1]).^2, zeros(3); zeros(3), diag([0.1e-3  0.1e-3  0.1e-3])
            .^2 ];

        if DAYS == 1        % 1 day
        load LEO_DATA_Apparent.mat
        hrs = 24 * 1;
        elseif DAYS == 3    % 3 days
        load('LEO_DATA_Apparent_3Days.mat')
        hrs = 24 * 7;
        else                % 7 days
        load LEO_DATA_Apparent_3Days.mat
        temp = load('LEO_DATA_Apparent_Days4-6.mat');
        days4to6 = temp.LEO_DATA_Apparent;
        days4to6(:,2) = days4to6(:,2) + 3*86400;
        LEO_DATA_Apparent = [LEO_DATA_Apparent; days4to6];
        hrs = 24 * 7;
        end

        Yobs_STA    = LEO_DATA_Apparent;
        et_obs      = Yobs_STA(:,2) + et_t0;
        XSTM_prev   = XSTM0_batch;
        iter        = 0;

        % SHORT ARC
        if DATA == 3

        load WS_ALL_3.mat

        temp = load('LEO_DATA_Apparent_Days4-6.mat');
        days4to6 = temp.LEO_DATA_Apparent;
        days4to6(:,2) = days4to6(:,2) + 3*86400;
        LEO_DATA_Apparent = days4to6;
        % propagate to last day, which is now the 4th day. hrs = 24*4 = 96
        hrs = 24*4;

        % try smaller P0 ... but wider than other cases for short arc
```

```
P_prev      = [ diag([1 1 1]).^2, zeros(3); zeros(3), diag([1e-3 1e-3 1e-3]).^2 ];

X_EKF0      = X_EKF(end,:)';
XSTM_prev   = [X_EKF0; STM0];
t_EKF0      = t_X_EKF(end);
et_t0       = t_EKF0;
end

if STATIONS == 0    % use all station data
elseif STATIONS == 1 % Kwajalein
R_DGO = R_DGO * 1e10;
R_ACB = R_ACB * 1e10;
elseif STATIONS == 2 % Diego
R_KJL = R_KJL * 1e10;
R_ACB = R_ACB * 1e10;
elseif STATIONS == 3 % STATIONS == 3 % Arecibo
R_KJL = R_KJL * 1e10;
R_DGO = R_DGO * 1e10;
elseif STATIONS == 4 % Kwajalein and Diego - NO ARECIBO
R_ACB = R_ACB * 1e10;
elseif STATIONS == 5 % Kwajalein and Arecibo - NO DIEGO
R_DGO = R_DGO * 1e10;
else % STATIONS == 6. Diego and Arecibo - NO KWAJALEIN
R_KJL = R_KJL * 1e10;
end

X_EKF       = [];     t_X_EKF     = [];     Y_prefit    = [];     Y_postfit   = [];
Lpre_mat    = [];     Lpost_mat   = [];     sigma3_pre  = [];     sigma3_post = [];

% EKF
tic
for i = 1:length(et_obs)

% keep track of iterations
iter = iter + 1;
sprintf('iter = %d', iter)

% Propagate state
if      i == 1 && et_obs(1) == et_t0; t_prop = et_obs(i);
elseif i == 1;                        t_prop = [et_t0 : 60 : et_obs(1) ];
else
%         t_prop = [et_obs(i-1) : 60 : et_obs(i)];
t_prop = [t_lt et_obs(i)];
end

% EKF. All data, range, or range-only
[t_XSTM, XSTM, Xstar, Y_pre, Y_post, P, L_pre, L_post, t_lt] = fn.EKF(Yobs_STA, XSTM_prev
    , nX, ...
epochs(1), t_prop, options, Amat_fn, Ht_fn, Ht_bias_fn, Ht_r_fn, Ht_rr_fn, P_prev, DATA);

% save states from current iteration
if i == 1 && et_obs(1) == et_t0; X_EKF = XSTM(1:nX)';
else;                            X_EKF = [X_EKF; XSTM(:, 1:nX)];
end

t_X_EKF    = [t_X_EKF; t_XSTM];
Y_prefit   = [Y_prefit; Y_pre];
Y_postfit  = [Y_postfit; Y_post];

% update measurement for next iteration
XSTM_prev = [Xstar; STM0];
P_prev    = P;

% innovations covariance
Lpre_mat  = [Lpre_mat; L_pre];
Lpost_mat = [Lpost_mat; L_post];

% 3-sigma STD
```

```
        if DATA == 0
        sigma3_pre  = [sigma3_pre; sqrt(L_pre(1,1))*3, sqrt(L_pre(2,2))*3];
        sigma3_post = [sigma3_post; sqrt(L_post(1,1))*3, sqrt(L_post(2,2))*3];
        else
        sigma3_pre  = [sigma3_pre; sqrt(L_pre(1,1))*3];
        sigma3_post = [sigma3_post; sqrt(L_post(1,1))*3];
        end

        end
        toc

        %% Propagate to last period of time

        t_prop = [ et_obs(end) : 60 : et_t0 + 60*60*hrs ];
        [t_XSTM, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), t_prop, XSTM_prev,
            options);
        Xi    = XSTM(end,1:nX)';
        STMi = XSTM(end,nX+1:end);
        STMi = reshape(STMi, [nX nX]);

        T_ECI2RSW = fn.ECItoRSW_T(Xi);
        T_RSW2ECI = T_ECI2RSW';

        % Time update + process noise
        % dt        = t_prop(end) - t_prop(1);
        dt        = 60;
        Q         = diag( (100e-10)^2 * [1 1 1] );
        Gamma     = [diag( dt^2/2 * [1 1 1] ); diag([dt dt dt])];
        P_noise   = Gamma * Q * Gamma';

        RSW_Q     = diag( [1e-10^2 1000e-10^2 1e-10^2] );
        RSW_noise = Gamma * RSW_Q * Gamma';

        P_bar     = STMi * P_prev * STMi' + P_noise +  RSW_noise;

        % Save propagated states
        X_prop    = XSTM(:,1:nX);
        t_X_prop = t_XSTM;

        % PLOT RESIDUALS
        EKF_res;

        %% Save data

        if STATIONS == 0        % all stations
        if DATA == 0        % all data
        ws_mat          = 'WS_ALL';
        elseif DATA == 1    % range only
        ws_mat          = 'WS_R';
        elseif DATA == 2    % range-rate only
        ws_mat          = 'WS_RR';
        else                % DATA == 3, short arc
        ws_mat          = 'WS_ALL_short';
        end
        elseif STATIONS == 1    % Kwajalein only
        ws_mat              = 'WS_KJL';
        elseif STATIONS == 2    % Diego-Gardcia only
        ws_mat              = 'WS_DGO';
        elseif STATIONS == 3    % Arecibo only
        ws_mat              = 'WS_ACB';
        elseif STATIONS == 4;    ws_mat = 'WS_KJL_DGO';
        elseif STATIONS == 5;    ws_mat = 'WS_KJL_ACB';
        else;                    ws_mat = 'WS_DGO_ACB'; % STATIONS == 6
        end

        if DAYS == 1;            ws_mat = [ ws_mat '_1.mat' ];    % DAYS = 1
        elseif DAYS == 3;        ws_mat = [ ws_mat '_3.mat' ];    % DAYS = 3
        else;                    ws_mat = [ ws_mat '_7.mat' ];    % DAYS = 7
```

```
181         end
182
183         save(ws_mat);
184
185
186         %%% radial-intrack-cross-track frame transformation for best estimate
187         plot_RSW = 1;
188         if plot_RSW == 1
189
190         % RANGE-RATE
191         X_best = [ 445.715135753907
192         -7100.16248363109
193         -183.851374717676
194         7.48601245362928
195         0.469793893676187
196         0.177543134574878 ];
197
198         T_best = fn.ECItoRSW_T(X_best);
199
200         dr_ECI = X_best(1:3) - Xi(1:3);
201
202         % transform all measurements
203         dr_RSW = T_best * dr_ECI;
204
205         % Plot radial-intrack-crosstrack
206         %      ftitle = 'Radial-Intrack-Crosstrack';
207         RICtitle = strrep(ftitle, 'Residuals', 'RIC Error');
208         figh = figure('name', ftitle, 'position', [100 100 600 800]);
209         subplot(3,1,1)
210         scatter(dr_RSW(1), dr_RSW(2)); hold on;
211         P = P_bar(1:2, 1:2);
212         h3 = fn.plot_gaussian_ellipsoid([dr_RSW(1) dr_RSW(2)], P);
213         xlabel('R (km)')
214         ylabel('S (km)')
215         title('Radial-Intrack')
216         subplot(3,1,2)
217         scatter(dr_RSW(1), dr_RSW(3)); hold on;
218         P = [P_bar(1), P_bar(1,3); P_bar(3,1), P_bar(3,3)];
219         h3 = fn.plot_gaussian_ellipsoid([dr_RSW(1) dr_RSW(3)], P);
220         xlabel('R (km)')
221         ylabel('W (km)')
222         title('Radial-Crosstrack')
223         subplot(3,1,3)
224         scatter(dr_RSW(2), dr_RSW(3)); hold on;
225         P = P_bar(2:3, 2:3);
226         h3 = fn.plot_gaussian_ellipsoid([dr_RSW(2) dr_RSW(3)], P);
227         xlabel('S (km)')
228         ylabel('W (km)')
229         title('Intrack-Crosstrack')
230
231         sgtitle(RICtitle);
232
233         % Vallado ed 4 p. 229
234         end
235
236
237         %%% Plot satellite position
238
239         if plot_orbit == 1
240         ftitle = 'JahSat Orbit';
241         figure('name', ftitle);
242         plot3(XSTM_ref0(:,1), XSTM_ref0(:,2), XSTM_ref0(:,3)); hold on; grid on;
243         plot3(XSTM_batch(:,1), XSTM_batch(:,2), XSTM_batch(:,3));
244         plot3(X_EKF(:,1), X_EKF(:,2), X_EKF(:,3));
245         plot3(X_prop(:,1), X_prop(:,2), X_prop(:,3))
246         xlabel('x (km)'); ylabel('y (km)'); zlabel('z (km)');
247         legend('initial', 'batch', 'EKF', 'prop')
248         title(ftitle)
```

```
249         end
```

EKF.m:

```
1           function [t_XSTM, XSTM, X_update, Y_prefit, Y_postfit, P_update, L_pre, L_post, t_lt] =
                ...
2           EKF(Yobs_STA, XSTM_prev, nX, et_t0, t_prop, options, Amat_fn, Ht_fn, Ht_bias_fn, Ht_r_fn,
                 Ht_rr_fn, P_prev, DATA)
3
4           global R_KJL R_DGO R_ACB
5           global r_KJL_ECEF r_DGO_ECEF r_ACB_ECEF
6
7           % Integrate ref trajectory and STM from t = i-1 (prev) to t = i (curr)
8           if length(t_prop) > 1
9           [t_XSTM, XSTM] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), t_prop, XSTM_prev,
                options);
10          Xi    = XSTM(end,1:nX)';
11          STMi = XSTM(end,nX+1:end);
12          else
13          t_XSTM = t_prop;
14          XSTM = XSTM_prev;
15          Xi    = XSTM(1:nX);
16          STMi = XSTM(nX+1:end);
17          end
18          STMi = reshape(STMi, [nX nX]);
19
20          % find index for same time state and observation
21          t_Y    = Yobs_STA(:,2) + et_t0; % time after initial epoch
22          ti_X   = t_XSTM(end);
23          i_Y    = find(t_Y == ti_X);
24          Yi     = Yobs_STA(i_Y, :);
25
26          % observation covariance
27          if Yi(1) == 1
28          R = R_KJL;
29          r_STA_ECEF = r_KJL_ECEF;
30          elseif Yi(1) == 2
31          R = R_DGO;
32          r_STA_ECEF = r_DGO_ECEF;
33          else
34          R = R_ACB;
35          r_STA_ECEF = r_ACB_ECEF;
36          end
37
38          % get JD time
39          JD_UTC = cspice_et2utc(t_XSTM(end), 'J', 10);
40          JD_UTC = str2num(extractAfter(JD_UTC, 'JD '));
41
42          % Convert station to ECI frame
43          XSi_OG = rv_ECEFtoECI(JD_UTC, r_STA_ECEF, [0; 0; 0]);
44          MJD_UTC = JD_UTC - 2400000.5;
45          XSi = ECEF2ECI( MJD_UTC, [r_STA_ECEF; 0; 0; 0] );
46
47          % Time update + process noise
48          dt     = t_prop(end) - t_prop(1);
49          Q      = diag( (10e-10)^2 * [1 1 1] );
50          Gamma = [diag( dt^2/2 * [1 1 1] ); diag([dt dt dt])];
51          P_noise = Gamma * Q * Gamma';
52          Ppre_bar = STMi * P_prev * STMi' + P_noise;
53
54          % Y prefit
55          Y_prefit(1:2) = Yi(1:2);
56          Y_prefit(3:4) = fn.G_fn(Xi, XSi)';
57
58          % PREFIT Observation-state matrix
59          if DATA == 0 || DATA == 3
60          Hti_pre = Ht_fn(Xi(1), Xi(2), Xi(3), Xi(4), Xi(5), Xi(6), XSi(1), XSi(2), XSi(3), XSi(4),
                 XSi(5), XSi(6));
61          elseif DATA == 1
```

```
62          R = R(1,1);
63          Hti_pre = Ht_r_fn(Xi(1), Xi(2), Xi(3), XSi(1), XSi(2), XSi(3));
64          else % DATA == 2
65          R = R(2,2);
66          Hti_pre = Ht_rr_fn(Xi(1), Xi(2), Xi(3), Xi(4), Xi(5), Xi(6), XSi(1), XSi(2), XSi(3), XSi
                (4), XSi(5), XSi(6));
67          end
68
69          % PREFIT Innovation (information) covariance
70          L_pre = (Hti_pre * Ppre_bar * Hti_pre' + R);
71
72          % CORRECT + UPDATE. Light time correction
73          c      = 299792.458; % km/s
74          lt     = Y_prefit(3) / c;  % range / c = delta time (s)
75          t_lt   = ti_X - lt;
76          t_back = [ti_X, t_lt];
77          if length(t_prop) > 1
78          XSTM_end = XSTM(end,:);
79          else
80          XSTM_end = XSTM;
81          end
82          nX = length(Xi);
83          [~, XSTM_lt] = ode45(@(t, XSTM) fn.EOM_STM(t, XSTM, Amat_fn, nX), t_back, XSTM_end,
                options);
84          Xi_lt          = XSTM_lt(end,1:nX)';
85
86          % Satellite ECI coords with aberration and light time corrections. IF
87          % RANGE-RATE ONLY, TURN OFF ABERRATION
88          v_STA_ECI       = XSi(4:6);
89          Xi_lt_abr       = Xi_lt;
90          % if ~isequal(DATA, 2)
91          %     Xi_lt_abr(1:3) = Xi_lt(1:3) + lt * v_STA_ECI;
92          % end
93
94          if DATA == 0 || DATA == 3
95
96          % Compute Hti again, with satellite ECI lt and abr corrected state, and same station ECI
                state
97          Hti_lt_abr = Ht_bias_fn(Xi_lt_abr(1), Xi_lt_abr(2), Xi_lt_abr(3), Xi_lt_abr(4), Xi_lt_abr
                (5), Xi(6), ...
98          XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi(6));
99          %     Hti_lt_abr = Ht_fn(Xi_lt_abr(1), Xi_lt_abr(2), Xi_lt_abr(3), Xi_lt_abr(4),
                Xi_lt_abr(5), Xi(6), ...
100         %         XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi(6));
101
102         % Gain matrix
103         Ki = Ppre_bar * Hti_lt_abr' * inv( Hti_lt_abr * Ppre_bar * Hti_lt_abr' + R );
104
105         % Obtain y difference
106         yi = Yi(3:4)' - fn.G_bias_fn(Xi_lt_abr, XSi, Yi(1));
107         %     yi = Yi(3:4)' - fn.G_fn(Xi_lt_abr, XSi);
108         %     yi(1) = yi(1) + rbias;
109
110         elseif DATA == 1
111
112         % Compute Hti again, with satellite ECI lt and abr corrected state, and same station ECI
                state
113         Hti_lt_abr = Ht_r_fn(Xi_lt_abr(1), Xi_lt_abr(2), Xi_lt_abr(3), XSi(1), XSi(2), XSi(3));
114
115         % Gain matrix
116         Ki = Ppre_bar * Hti_lt_abr' * inv( Hti_lt_abr * Ppre_bar * Hti_lt_abr' + R);
117
118         % Obtain y difference (range only)
119         yi = Yi(3:4)' - fn.G_fn(Xi_lt_abr, XSi);
120         %     yi = Yi(3:4)' - fn.G_bias_fn(Xi_lt_abr, XSi, Yi(2));
121         yi = yi(1);
122
123         elseif DATA == 2
```

```
124
125         % Compute Hti again, with satellite ECI lt and abr corrected state, and same station ECI
                state
126         Hti_lt_abr = Ht_rr_fn(Xi_lt_abr(1), Xi_lt_abr(2), Xi_lt_abr(3), Xi_lt_abr(4), Xi_lt_abr
                (5), Xi_lt_abr(6), ...
127         XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi(6));
128
129         % Gain matrix
130         Ki = Ppre_bar * Hti_lt_abr' * inv( Hti_lt_abr * Ppre_bar * Hti_lt_abr' + R);
131
132         % Obtain y difference (range-rate only)
133         yi = Yi(3:4)' - fn.G_fn(Xi_lt_abr, XSi);
134         yi = yi(2);
135
136         end
137
138         % Measurement and reference orbit update. Add xhat to dynamically propagated satellite
                ECI state
139         xhat      = Ki * yi;
140         if norm(xhat(1:3)) > 50
141         disp('xhat pos > 50'); return
142         end
143         X_update = Xi_lt + xhat;
144         nX        = length(Xi);
145         P_update = ( eye(nX) - Ki * Hti_pre ) * Ppre_bar;
146
147         % POSTFIT Observation-state matrix <-- FIX, USE XSi
148         if DATA == 0 || DATA == 3
149         Hti_post = Ht_fn(X_update(1), X_update(2), X_update(3), X_update(4), X_update(5),
                X_update(6), ...
150         XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi(6));
151         elseif DATA == 1
152         Hti_post = Ht_r_fn(X_update(1), X_update(2), X_update(3), XSi(1), XSi(2), XSi(3));
153         else
154         Hti_post = Ht_rr_fn(X_update(1), X_update(2), X_update(3), X_update(4), X_update(5),
                X_update(6), ...
155         XSi(1), XSi(2), XSi(3), XSi(4), XSi(5), XSi(6));
156         end
157
158         % POSTFIT Innovation (information) covariance
159
160         L_post = (Hti_post * P_update * Hti_post' + R);
161
162         % Y postfit
163         Y_postfit(1:2) = Yi(1:2);
164         Y_postfit(3:4) = fn.G_fn(X_update, XSi)';
165
166
167         end
168
169         %% subfunctions
170
171         function XSi = rv_ECEFtoECI(JD_UTC, r_STA_ECEF, v_STA_ECEF)
172
173         global wE
174
175         r_STA_ECI  = fn.ECEFtoECI(JD_UTC, r_STA_ECEF);
176         v_STA_ECI  = v_STA_ECEF + cross([ 0 0 wE ]', r_STA_ECEF);
177         v_STA_ECI  = fn.ECEFtoECI(JD_UTC, v_STA_ECI); % Technically wrong. Look in Vallado p. 228
178         XSi        = [r_STA_ECI; v_STA_ECI];
179
180         end
```

4. EKF_res.m

```
1
2
```

```
3
4           %% Calculate residuals
5
6           Ypre_KJL    = [];     Ypre_DGO    = [];     Ypre_ACB    = [];
7           Ypost_KJL   = [];     Ypost_DGO   = [];     Ypost_ACB   = [];
8           sigma3_KJL  = [];     sigma3_DGO  = [];     sigma3_ACB  = [];
9
10          % Extract states that correspond with station measurements
11          for i = 1:length(Y_postfit)
12
13          ti  = Y_postfit(i, 2);
14          ti  = ti + et_t0;
15          i_X = find(t_X_EKF == ti);
16          i_STA = Y_postfit(i, 1);
17
18          if i_STA == 1
19          Ypre_KJL    = [Ypre_KJL; Y_prefit(i,:)];
20          Ypost_KJL   = [Ypost_KJL; Y_postfit(i,:)];
21          elseif i_STA == 2
22          Ypre_DGO    = [Ypre_DGO; Y_prefit(i,:)];
23          Ypost_DGO   = [Ypost_DGO; Y_postfit(i,:)];
24          else
25          Ypre_ACB    = [Ypre_ACB; Y_prefit(i,:)];
26          Ypost_ACB   = [Ypost_ACB; Y_postfit(i,:)];
27          end
28
29          end
30
31
32          % Station data
33          i_STA     = find(LEO_DATA_Apparent(:, 1) == 1);
34          Yobs_KJL = LEO_DATA_Apparent(i_STA, :);
35          i_STA     = find(LEO_DATA_Apparent(:, 1) == 2);
36          Yobs_DGO = LEO_DATA_Apparent(i_STA, :);
37          i_STA     = find(LEO_DATA_Apparent(:, 1) == 3);
38          Yobs_ACB = LEO_DATA_Apparent(i_STA, :);
39          [dpre_err_KJL, dpre_rms_KJL, vpre_err_KJL, vpre_rms_KJL] = calc_res_all(Yobs_KJL,
                Ypre_KJL, DATA);
40          [dpre_err_DGO, dpre_rms_DGO, vpre_err_DGO, vpre_rms_DGO] = calc_res_all(Yobs_DGO,
                Ypre_DGO, DATA);
41          [dpre_err_ACB, dpre_rms_ACB, vpre_err_ACB, vpre_rms_ACB] = calc_res_all(Yobs_ACB,
                Ypre_ACB, DATA);
42          [dpost_err_KJL, dpost_rms_KJL, vpost_err_KJL, vpost_rms_KJL] = calc_res_all(Yobs_KJL,
                Ypost_KJL, DATA);
43          [dpost_err_DGO, dpost_rms_DGO, vpost_err_DGO, vpost_rms_DGO] = calc_res_all(Yobs_DGO,
                Ypost_DGO, DATA);
44          [dpost_err_ACB, dpost_rms_ACB, vpost_err_ACB, vpost_rms_ACB] = calc_res_all(Yobs_ACB,
                Ypost_ACB, DATA);
45          t_KJL = Yobs_KJL(:,2) / 3600;
46          t_DGO = Yobs_DGO(:,2) / 3600;
47          t_ACB = Yobs_ACB(:,2) / 3600;
48          t_ALL = Y_prefit(:,2) / 3600 ;
49
50          if DATA == 0
51
52          if STATIONS == 0;        ftitle = 'All Stations, All Data Residuals';
53          elseif STATIONS == 1;    ftitle = 'Kwajalein Only, All Data Residuals';
54          elseif STATIONS == 2;    ftitle = 'Diego-Garcia Only, All Data Residuals';
55          elseif STATIONS == 3;    ftitle = 'Arecibo Only, All Data Residuals';
56          elseif STATIONS == 4;    ftitle = 'Kwajalein and Diego-Garcia, NO Arecibo, All Data
                Residuals';
57          elseif STATIONS == 5;    ftitle = 'Kwajalein and Arecibo, NO Diego-Garcia, All Data
                Residuals';
58          else;                    ftitle = 'Diego-Garcia and Arecibo, NO Kwajalein, All Data
                Residuals'; % STATIONS == 6
59          end
60
61          % 1 2 3 4
```

```
62          % 5  6  7  8
63          % 9  10  11  12
64          % 13  14  15  16
65          figure('name', ftitle);
66          % first row: subplot(4,4,1:4)
67          plot_res(4, 4, 1:4, 'PREFIT range residuals', t_ALL, sigma3_pre(:,1), t_KJL, t_DGO, t_ACB
                , dpre_err_KJL, ...
68          dpre_err_DGO, dpre_err_ACB, dpre_rms_KJL, dpre_rms_DGO, dpre_rms_ACB, 'km')
69          % second row: subplot(4,4,5:8)
70          plot_res(4, 4, 5:8, 'PREFIT range-rate residuals', t_ALL, sigma3_pre(:,2), t_KJL, t_DGO,
                t_ACB, vpre_err_KJL, ...
71          vpre_err_DGO, vpre_err_ACB, vpre_rms_KJL, vpre_rms_DGO, vpre_rms_ACB, 'km/s')
72          % third row: subplot(4,4,9:12)
73          plot_res(4, 4, 9:12, 'POSTFIT range residuals', t_ALL, sigma3_post(:,1), t_KJL, t_DGO,
                t_ACB, dpost_err_KJL, ...
74          dpost_err_DGO, dpost_err_ACB, dpost_rms_KJL, dpost_rms_DGO, dpost_rms_ACB, 'km')
75          % fourth row: subplot(4,4,13:16)
76          plot_res(4, 4, 13:16, 'POSTFIT range-rate residuals', t_ALL, sigma3_post(:,2), t_KJL,
                t_DGO, t_ACB, vpost_err_KJL, ...
77          vpost_err_DGO, vpost_err_ACB, vpost_rms_KJL, vpost_rms_DGO, vpost_rms_ACB, 'km/s')
78
79          xlabel('Time (hr)')
80          sgtitle(ftitle);
81
82          elseif DATA == 1
83
84          ftitle = 'All Stations, Range Only Residuals';
85          figure('name', ftitle);
86          % first row: subplot(4,4,1:4)
87          plot_res(4, 4, 1:4, 'PREFIT range residuals', t_ALL, sigma3_pre, t_KJL, t_DGO, t_ACB,
                dpre_err_KJL, ...
88          dpre_err_DGO, dpre_err_ACB, dpre_rms_KJL, dpre_rms_DGO, dpre_rms_ACB, 'km')
89          % second row: subplot(4,4,5:8)
90          plot_res(4, 4, 5:8, 'PREFIT range-rate residuals', t_ALL, NaN * ones(size(t_ALL)), t_KJL,
                t_DGO, t_ACB, vpre_err_KJL, ...
91          vpre_err_DGO, vpre_err_ACB, vpre_rms_KJL, vpre_rms_DGO, vpre_rms_ACB, 'km/s')
92          % third row: subplot(4,4,9:12)
93          plot_res(4, 4, 9:12, 'POSTFIT range residuals', t_ALL, sigma3_post, t_KJL, t_DGO, t_ACB,
                dpost_err_KJL, ...
94          dpost_err_DGO, dpost_err_ACB, dpost_rms_KJL, dpost_rms_DGO, dpost_rms_ACB, 'km')
95          % fourth row: subplot(4,4,13:16)
96          plot_res(4, 4, 13:16, 'POSTFIT range-rate residuals', t_ALL, NaN * ones(size(t_ALL)),
                t_KJL, t_DGO, t_ACB, vpost_err_KJL, ...
97          vpost_err_DGO, vpost_err_ACB, vpost_rms_KJL, vpost_rms_DGO, vpost_rms_ACB, 'km/s')
98
99          xlabel('Time (hr)')
100         sgtitle(ftitle);
101
102         else
103
104         ftitle = 'All Stations, Range-Rate Only Residuals';
105         figure('name', ftitle);
106         % first row: subplot(4,4,1:4)
107         plot_res(4, 4, 1:4, 'PREFIT range residuals', t_ALL, NaN * ones(size(t_ALL)), t_KJL,
                t_DGO, t_ACB, dpre_err_KJL, ...
108         dpre_err_DGO, dpre_err_ACB, dpre_rms_KJL, dpre_rms_DGO, dpre_rms_ACB, 'km')
109         % second row: subplot(4,4,5:8)
110         plot_res(4, 4, 5:8, 'PREFIT range-rate residuals', t_ALL, sigma3_pre, t_KJL, t_DGO, t_ACB
                , vpre_err_KJL, ...
111         vpre_err_DGO, vpre_err_ACB, vpre_rms_KJL, vpre_rms_DGO, vpre_rms_ACB, 'km/s')
112         % third row: subplot(4,4,9:12)
113         plot_res(4, 4, 9:12, 'POSTFIT range residuals', t_ALL, NaN * ones(size(t_ALL)), t_KJL,
                t_DGO, t_ACB, dpost_err_KJL, ...
114         dpost_err_DGO, dpost_err_ACB, dpost_rms_KJL, dpost_rms_DGO, dpost_rms_ACB, 'km')
115         % fourth row: subplot(4,4,13:16)
116         plot_res(4, 4, 13:16, 'POSTFIT range-rate residuals', t_ALL, sigma3_post, t_KJL, t_DGO,
                t_ACB, vpost_err_KJL, ...
117         vpost_err_DGO, vpost_err_ACB, vpost_rms_KJL, vpost_rms_DGO, vpost_rms_ACB, 'km/s')
```

```
118
119         xlabel('Time (hr)')
120         sgtitle(ftitle);
121
122         end
123
124
125
126         %% Subfunctions
127
128         function plot_res(m, n, ivec, ftitle, t_ALL, sigma3, t_KJL, t_DGO, t_ACB, pre_err_KJL,
                ...
129         pre_err_DGO, pre_err_ACB, pre_rms_KJL, pre_rms_DGO, pre_rms_ACB, yl)
130
131         if ~isnan(sigma3)
132         %         yrange = 3 * mean(real(sigma3));
133         yrange = 3 * max( [ max(abs( pre_rms_DGO )) max(abs( pre_rms_KJL )) max(abs( pre_rms_ACB
                )) ] );
134
135         end
136
137         subplot(m, n, ivec(end) )
138         stext = { sprintf('KJL mean = %.3g', mean(pre_err_KJL)); sprintf('KJL RMS = %.3g',
                pre_rms_KJL);
139             sprintf('DGO mean = %.3g', mean(pre_err_DGO)); sprintf('DGO RMS = %.3g',
                    pre_rms_DGO);
140             sprintf('ACB mean = %.3g', mean(pre_err_ACB)); sprintf('ACB RMS = %.3g',
                    pre_rms_ACB)};
141             text(0, 0.5, stext); axis off
142             subplot(m, n, ivec(1:end-1) )
143             scatter(t_KJL, pre_err_KJL, 8 ); hold on; grid on;
144             scatter(t_DGO, pre_err_DGO, 8, 'x');
145             scatter(t_ACB, pre_err_ACB, 8, 'd');
146             plot(t_ALL, sigma3, 'g--');
147             plot(t_ALL, -sigma3, 'g--');
148             if ~isnan(sigma3)
149             ylim( [ -yrange, yrange ] );
150             end
151             title(ftitle)
152             ylabel(yl)
153             legend('KJL', 'DGO', 'ACB')
154
155             end
156
157             function [d_err_STA, d_rms_STA, v_err_STA, v_rms_STA] = calc_res_all(Yobs_STA,
                    Ycalc_STA, DATA)
158
159             % Calculate residuals
160             d_err_STA = Yobs_STA(:,3) - Ycalc_STA(:,3);
161             d_rms_STA = rms(d_err_STA);
162             v_err_STA = Yobs_STA(:,4) - Ycalc_STA(:,4);
163             v_rms_STA = rms(v_err_STA);
164
165             end
```

5. ECItoRSW_t.m:

```
1           function T = ECItoRSW_T(X_ECI)
2
3           r_ECI = X_ECI(1:3);
4           v_ECI = X_ECI(4:6);
5
6           R = r_ECI / norm(r_ECI);
7           W = cross(r_ECI, v_ECI) / norm(cross(r_ECI, v_ECI));
8           S = cross(W, R);
9
10          T = [R'; S'; W'];
```

```
11
12          end
```

References

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[2] Levinson, D. A., "Gravitational Acceleration Formula with Earth's Oblateness Taken into Account," , 2012.

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[4] Vallado, D. A., and McClain, W. D., *Fundamentals of Astrodynamics and Applications*, 4^{th} ed., Microcosm Press, 2013.