ASE387P.2 Mission Analysis and Design
Homework 4: Orbit Design

Junette Hsin
Masters Student, Aerospace Engineering and Engineering Mechanics, University of Texas, Austin, TX 78712

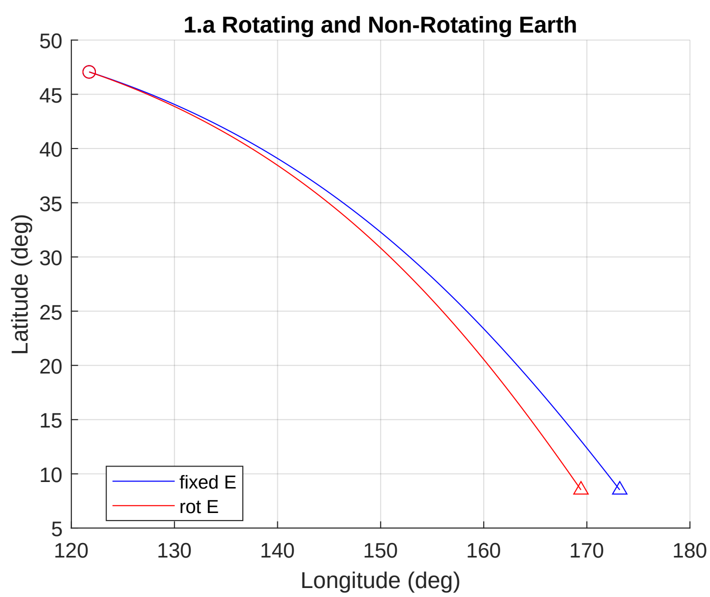
Problem 1: ISS Ground Tracks

A

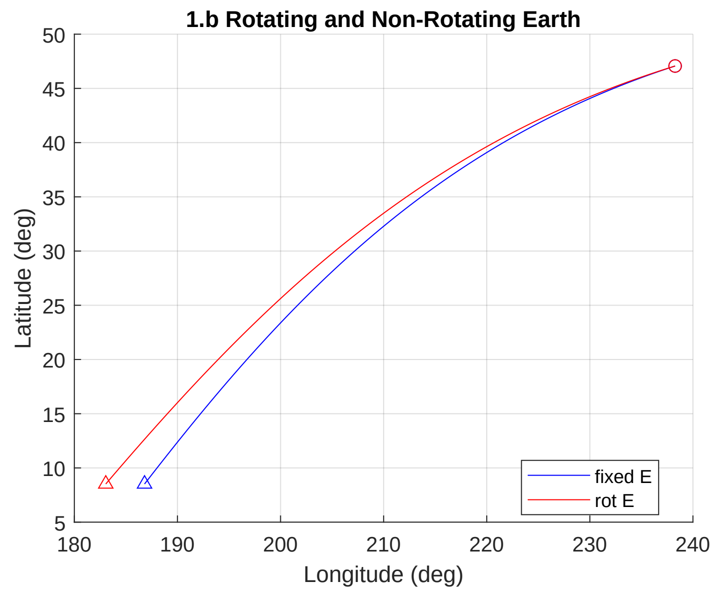
The following parameters were used to initialize the ISS orbit, which were taken from `https://in-the-sky.org/spacecraft_elements.php?id=25544` on April 19:

- a = 6794.588 km
- e = 0.00049
- i = 51.627 deg
- ω = 40.1116 deg
- Ω = 0 deg
- M = 70.88

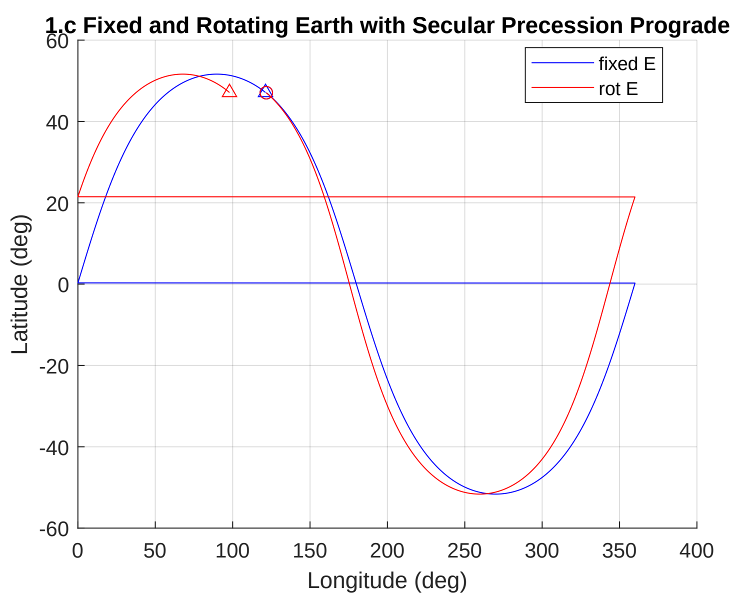
The right ascension of the ascending node, Ω, was set to 0 as stated in the problem statement.



B



C



To derive a formula for predicting the westward drift, first calculate the Draconitic period:

$$T_d = \frac{2\pi}{\dot{\omega} + \dot{M}} \tag{1}$$

Then calculate the rate of secular variation for the ascending node due to J2:

$$\dot{\Omega} = -\frac{3}{2}\bar{n}J_2(\frac{a_e}{\bar{a}})^2\frac{1}{(1-\bar{e}^2)^2}cos\bar{I} \tag{2}$$

$\bar{a}$, $\bar{e}$, and $\bar{I}$ can be taken from the initial ISS orbit parameters, and $\bar{n}$ is the mean motion of the orbit:

$$\bar{n} = \sqrt{\frac{\mu}{\bar{a}^3}} \tag{3}$$

Finally, the change in lambda was calculated as:

$$\Delta\lambda = -(w_E - \dot{\Omega})T_d \tag{4}$$

where w_E is the rotation rate of the Earth. $\Delta\lambda$ came out to **-22.870**°.

The calculated change in drift of the ascending node was taken from the difference between the initial longitude and the final longitude after one revolution, which came out to be **-22.790**°. The calculated drift was within 0.1° of the analytical prediction.

Problem 2: Orbit Design

Continuing from Equation 4, after m revolutions,

$$m\Delta\lambda = -(w_E - \dot{\Omega})T_d m \tag{5}$$

A condition for ground track repeat is that $m\Delta\lambda$ needs to be a multiple (let's say k) of 2π, or

$$k = \frac{m\Delta\lambda}{2\pi} = \frac{m(w_E - \dot{\bar{\Omega}})T_d}{2\pi} = m\frac{T_d}{D_n} \tag{6}$$

For a ground track repeat orbit, the conditions for which the sub-satellite point repeatedly passes a geographical location (ϕ, λ) at regular intervals are given by:

$$(\omega_e - \dot{\bar{\Omega}})D_n = k2\pi \tag{7}$$

$$(\dot{\bar{\omega}} + \dot{\bar{M}})T_d = m2\pi \tag{8}$$

As for frozen orbits, the largest perturbation on gravitational acceleration is due to J2, which is on the order of 10^{-3}. The next-largest perturbation is J3, which is on the order of 10^-6. Equation 2 gives the scalar secular variation on $\dot{\bar{\Omega}}$ due to J2 which grows indefinitely. Equations 9 and 10 give the secular variations due to J2 for $\dot{\bar{\omega}}$ and $\dot{\bar{M}}$:

$$\dot{\bar{\omega}} = -\frac{3}{4}\bar{n}J_2(\frac{a_e}{\bar{a}})^2\frac{1}{(1-\bar{e}^2)^2}(1-5cos^2\bar{I}) \tag{9}$$

$$\dot{\bar{M}} = \bar{n}\left[1 - \frac{3}{4}(\frac{a_e}{\bar{a}})^2 J_2\frac{1}{(1-\bar{e}^2)^{\frac{3}{2}}}(1-3cos^2\bar{I})\right] \tag{10}$$

J2 also leads to a short-period perturbation on all elements, but only semi-major axis is of interest:

$$\Delta a_{SP}(t) = \bar{a}J_2(\frac{a_e}{\bar{a}})^2\left[\left(1-\frac{3}{2}sin^2\bar{I}\right)\left((\frac{\bar{a}}{r})^3 - \frac{1}{(1-\bar{e}^2)^{\frac{3}{2}}}\right) + \frac{3}{2}(\frac{\bar{a}}{r})^3 sin^2\bar{I}cos2(\bar{w}+\bar{f})\right] \tag{11}$$

J3 exerts a long-period perturbation on all elements other than the semi-major axis, but only the eccentricity and perigee are of interest:

$$\Delta e_{LP}(t) = -\frac{1}{2}\frac{J_3}{J_2}\frac{a_e}{\bar{a}}sin\bar{I}sin\bar{w}(t) \tag{12}$$

$$\bar{e}\Delta\omega_{LP}(t) = -\frac{1}{2}\frac{J_3}{J_2}\frac{a_e}{\bar{a}}\frac{1}{(1-\bar{e}^2)}sin\bar{I}\ cos\bar{w}(t) \tag{13}$$

For frozen orbits, the average variation rates of e and w are set to 0. Thus, the mean argument of perigee should be around 90 °.

$$w = 90^\circ \tag{14}$$

An orbit is sun-synchronous when the precession rate equals the mean motion of the Earth around the Sun. the Ω nodal rate needs to match the average rate of the Sun's motion projected onto the Earth's equator:

$$\frac{d\Omega}{dt} = \dot{\bar{\Omega}} = \frac{360^\circ}{365.242\ days/year} = 0.9856\ ^\circ/day \tag{15}$$

The angular precession for an Earth orbiting satellite is given by Equation 2. One can reform Equation 2 as a formula for inclination:

$$\bar{I} = cos^{-1}\left[-\frac{2}{3}\frac{d\Omega}{dt}\frac{1}{J_2\bar{n}}\Big(\frac{\bar{a}(1-\bar{e}^2)}{R_E}\Big)^2 \right] \tag{16}$$

The process for determining the mean elements $\bar{a}$, $\bar{e}$, and $\bar{I}$ involves making initial guesses and then minimizing the misclosure rate through an optimization routine:

$$\epsilon = m(w_E - \dot{\bar{\Omega}}) - k(\dot{\bar{\omega}} - \dot{\bar{M}}) \tag{17}$$

Initial computed states for position and velocity will be integrated and iterated until a sun-synchronous, frozen, and repeated ground track orbit is found.

B

Problem 3

The calculations for this section were taken from the paper **Five Special Types of Orbits Around Mars** (2010 Liu, Baoyin, and Ma). A frozen orbit is possible on Mars. Mars' J2 = 1.95545e-3 and J3 = 3.14498e-5. In comparison, the Earth J2 = 1.08263e-3 and J3 = -2.53266e-6. The same positive sign between J2 and J3 must be accounted for to prevent the desired eccentricity from becoming negative. For Earth, ω is set to 90 °. For Mars, ω must be set to around 270 °.

The average variation rate of e from Equation 12 is given by:

$$\dot{\bar{e}} = \frac{3nJ_3R_p^{\bar{3}}sin\bar{I}}{4a^3(1-e^2)^2}\Big(\frac{5}{2}sin^2\bar{I} - 2\Big)cos\omega \tag{18}$$

And from Equation 13, the average variation rate of ω is given by:

$$\dot{\bar{\omega}} = \frac{3\bar{n}J_2R_p^2}{2a^2(1-e^2)^2}\left[\Big(2-\frac{5}{2}sin^2\bar{I}\Big)\Big(1+\frac{J_3R_p}{2J_2a(1-e^2)}\big(\frac{sin^2\bar{I}-\bar{e}^2cos^2\bar{I}}{sin\bar{I}}\big)\frac{sin\omega}{\bar{e}}\Big) + \frac{3J_2R_p^2}{2\bar{a}^2(1-e^2)^2}D\right] \tag{19}$$

where

$$D = (4+\frac{7}{12}\bar{e}^2 + 2\sqrt{1-e^2}) - sin^2\bar{I}(\frac{103}{102}+\frac{3}{8}\bar{e}^2+\frac{11}{2}\sqrt{1-\bar{e}^2}) + sin^4\bar{I}(\frac{215}{418}-\frac{15}{32}\bar{e}^2+\frac{15}{4}\sqrt{1-\bar{e}^2}) + ...H.O.T. \tag{20}$$

H.O.T. stands for higher order terms than J3. As stated in Problem 2, for frozen orbits, the average variation rates of e and ω are set to 0.

Solving the above equations yields the following for e and ω:

$$e = \frac{\frac{J_3R_p}{2J_2\bar{a}}sin\bar{I}sin\omega}{1-\frac{3J_2R_p^2E}{\bar{a}^2(5sin^2\bar{I}-4)}} \tag{21}$$

$$\omega = 270^\circ \tag{22}$$

Appendix

MATLAB code

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%% HW 4
% Junette Hsin
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