

# Sun-Avoidance Slew Planning Algorithm with Pointing and Actuator Constraints (AAS 19-801)

AIAA/AAS Astrodynamics Specialist Conference,  
Portland, ME, 11– 15 August 2019

Mohammad. A. Ayoubi<sup>1</sup>    Junette Hsin<sup>2</sup>

<sup>1</sup>Associate Professor, Department of Mechanical Engineering, Santa Clara University,

<sup>2</sup>Engineer, Dynamics and Control Analysis Group, Maxar Space Infrastructure  
(Formerly Space Systems/Loral)

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## Introduction

Large-angle slew maneuvers are required during any Earth-pointing or interplanetary missions.

In many space missions, and for safety consideration, a sensitive payload such as imaging camera or telescope needs to be retargeted while avoiding the sun vector or other bright objects in the sky.

## Previous Literature

The attitude reorientation problem in the presence of attitude constrained zones has been studied in the last three decades:

- ① McInnes (1994): entirely analytical guidance law which was suitable for onboard implementation.
  - Euler angles were used, which are singular for large slew angles.
- ② Spindle (1998), Hablani (1998), Biggs and Colley (2016), Frazzoli (2001): geometric approach where a feasible attitude maneuver, or a guidance law, is precomputed based on the attitude-avoidance-zone constraints or randomized algorithms.
  - Depending on the number of constraints and initial and final attitudes, this approach can be computationally expensive.

## Previous Literature

- 1 Spiller (2016): Particle swarm optimization (PSO) technique to find a sub-optimal solution with keep-out constraints.
- 2 Another approach casted the problem as a convex optimization problem and used semi-definite programming (SDP) or quadratically constrained quadratic programming (QCQP) in its solution (see for instance Kim and Mesbahi, Kim et al., Sun and Dai, and Lee and Mesbahi).
- 3 Recently, Ramos and Schaub proposed a method based on the Lyapunov stability theorem and logarithmic barrier potential function to derive a steering law for attitude control of a spacecraft subject to conically constrained inclusion and exclusion regions. They also considered the control-torque constraint in their algorithm.

## SAS Algorithm

The Sun Avoidance Slew (SAS) algorithm is a simple geometric approach for a sun (or any bright object) avoidance slew maneuver with pointing and actuator constraints.

The proposed algorithm in this paper is intuitive, deterministic, easy to implement, and includes the control-torque and reaction wheels' angular momentum constraints.

## SAS Algorithm

Assumptions in this algorithm:

- 1 Spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints
- 2 The initial and final attitudes, instrument boresight vector, and sun vector are known.

## SAS Algorithm

Pontryagin's minimum principle (PMP) is used to derive the desired or target-frame quaternions, angular velocity and acceleration for two cases:

- 1 with control-torque and reaction wheels' angular momentum constraints
- 2 with control-torque constraints

Numerical simulation is performed to show the viability of the proposed algorithm with control-torque and angular momentum constraints.



# Sun-Avoidance Slew (SAS) Algorithm

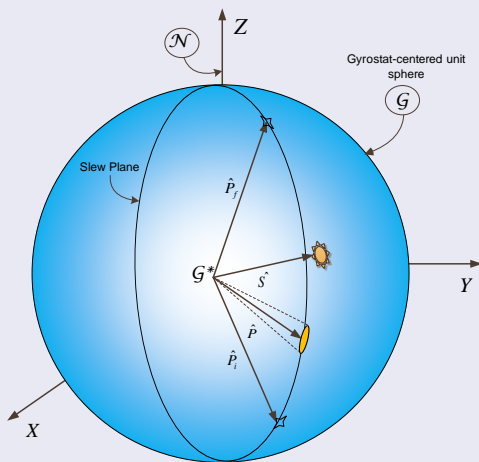


Figure: The gyrostat-centered unit sphere.

# Sun-Avoidance Slew (SAS) Algorithm

## Problem Statement:

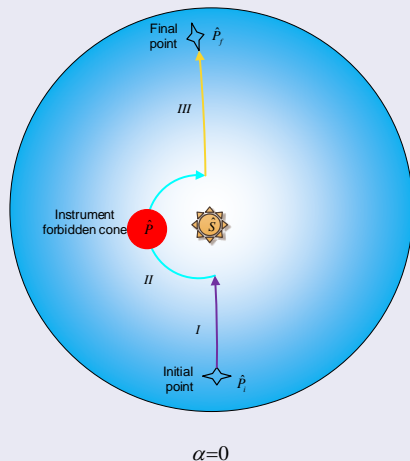
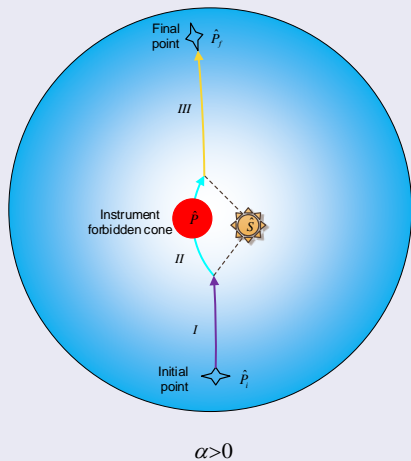
Given:  ${}^N\hat{P}_i, {}^N\hat{P}_f, {}^N\hat{S}, {}^g\hat{P}, \epsilon_p, {}^Nq^g, {}^N\omega^g(t_i)$ , and  ${}^N\omega^g(t_f)$ .

Assumption: The spacecraft is rigid.

Find:

- 1 A sequence of slew maneuvers to avoid sun vector.
- 2 the commanded angular velocity, angular acceleration, and quaternion profiles.

# Summary of the Algorithm



# Sun-Avoidance Slew (SAS) Algorithm

## Check the Sun Vector Intrusion

- 1 Check the angular separation,  $\alpha$ , between the sun vector,  $\hat{S}$ , and the  $\hat{P}_i - \hat{P}_f$  or “slew” plane.

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot_{\mathcal{N}} \hat{e}) \quad (1)$$

where the eigenaxis is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \quad (2)$$

- 2 IF  $|\alpha| < \epsilon_p$ , THEN determine the projection of the sun vector into the slew plane.

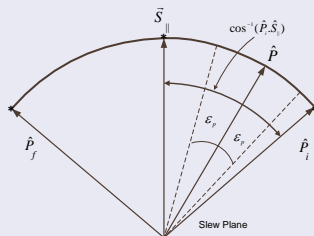
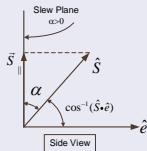
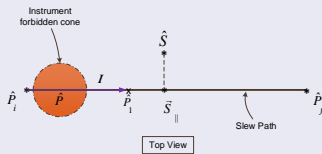
$$\vec{S}_{||} = \hat{S} \cos \alpha \quad (3)$$

# Sun-Avoidance Slew (SAS) Algorithm

## Slew Maneuvers

- The 1<sup>st</sup> slew around the eigenaxis,  $\hat{e}$ , through angle:

$$\phi_1 = \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \quad \text{when } \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \leq \pi \quad (4)$$

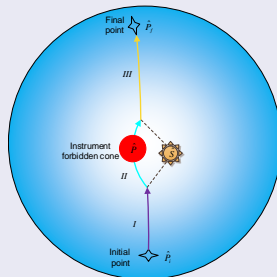
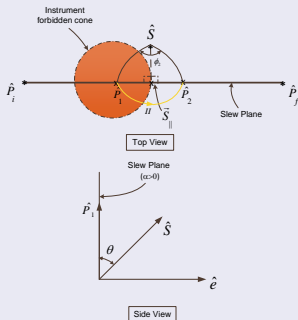


# Sun-Avoidance Slew (SAS) Algorithm

## Slew Maneuvers

- 2 The 2<sup>nd</sup> slew around the unit sun vector,  $\hat{S}$ , via  $\phi_2$ .  
 when  $\alpha \neq 0$

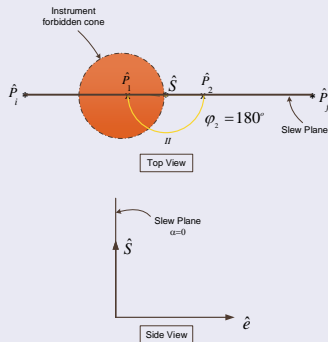
$$\phi_2 = 2 \tan^{-1} \left[ \frac{\hat{S} \cdot (\hat{P}_1 \times \hat{S}_{||})}{(\hat{P}_1 \cdot \hat{S}_{||}) - (\hat{S} \cdot \hat{P}_1)(\hat{S} \cdot \hat{S}_{||})} \right], \quad (5)$$



# Sun-Avoidance Slew (SAS) Algorithm

## Slew Maneuvers

- 2 The 2<sup>nd</sup> slew around the unit sun vector,  $\hat{S}$ , via  $\phi_2 = 180^\circ$ .  
b) when  $\alpha = 0$

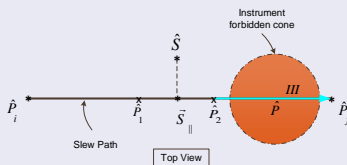
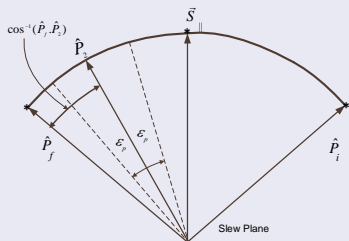


# Sun-Avoidance Slew (SAS) Algorithm

## Slew Maneuvers

- ③ The 3<sup>rd</sup> slew about the  $\hat{e}$  through angle:

$$\phi_3 = \cos^{-1}({}_G\hat{P}_f \cdot \hat{P}_2) \quad \text{when } {}_G\hat{P}_f \cdot \hat{P}_2 \geq 0 \quad (6)$$



- ④ Optional: The final slew is about the instrument boresight axis to rotate into final attitude.



# Summary of the Algorithm

- 1 Slew around the eigenaxis,  $\hat{e}$ , through angle:

$$\phi_1 = \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \quad \text{when } \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \leq \pi \quad (7)$$

- 2 Slew around the  $\hat{S}$  via:

$$\phi_2 = \begin{cases} \phi_2 = 2 \tan^{-1} \left[ \frac{\hat{S} \cdot (\hat{P}_1 \times \hat{S}_{||})}{(\hat{P}_1 \cdot \hat{S}_{||}) - (\hat{S} \cdot \hat{P}_1)(\hat{S} \cdot \hat{S}_{||})} \right], & \text{when } \alpha \neq 0 \\ \pi & \text{when } \alpha = 0 \end{cases} \quad (8)$$

- 3 Slew about the  $\hat{e}$  through angle:

$$\phi_3 = \cos^{-1}({}_G \hat{P}_f \cdot \hat{P}_2) \quad \text{when } {}_G \hat{P}_f \cdot \hat{P}_2 \geq 0 \quad (9)$$

- 4 Optional: perform the final rotation,  $\phi_4$ , about the instrument boresight axis to adjust the attitude.

# Computing the Steering Profiles

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints.

# Computing Steering Profiles

## Problem Statement

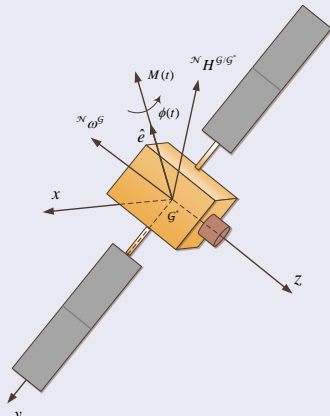
Consider the motion of a **rigid** spacecraft around a given inertially-fixed axis,  ${}^G\hat{e} = [e_x, e_y, e_z]^T$ . The problem of minimum-time slew maneuver around the  $\hat{e}$  axis can be formulated as

$$\underset{u}{\text{Minimize}} J[x(.), u(.), t_f] = \int_{t_0}^{t_f} dt, \quad (10)$$

subject to the following dynamic constraint

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M/I_{\hat{e}}^{G/G^*} = u, \end{cases} \quad (11)$$

where  $x_1 \triangleq \phi$  and  $x_2 = \dot{\phi}$ .



# Computing Steering Profiles

## Problem Statement Continued

The boundary conditions are

$$BCs : \begin{cases} \phi(t_0) = 0, \phi(t_f) = \phi_f, \\ \dot{\phi}(t_0) = \dot{\phi}_0, \dot{\phi}(t_f) = \dot{\phi}_f, \end{cases} \quad (12)$$

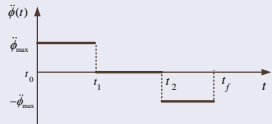
and velocity (state) and acceleration (control) constraints are

$$C_1 : \begin{cases} |x_2 = \dot{\phi}| \leq \dot{\phi}_{max}, \\ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}, \end{cases} \quad (13)$$

**Find:**  $\phi(t)$ ,  $\dot{\phi}(t)$ , and  $\ddot{\phi}(t)$ .

# Computing Steering Profiles

- Angular acceleration profile (bang-off-bang):

$$\ddot{\phi}(t) = u = \begin{cases} \ddot{\phi}_{max} & \text{when } t_0 \leq t \leq t_1, \\ 0 & \text{when } t_1 \leq t \leq t_2, \\ -\ddot{\phi}_{max} & \text{when } t_2 \leq t \leq t_f. \end{cases} \quad (14)$$


- Angular velocity profile:

$$\dot{\phi}(t) = \begin{cases} \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0) & \text{when } t_0 \leq t \leq t_1, \\ \dot{\phi}_{max} & \text{when } t_1 \leq t \leq t_2, \\ \dot{\phi}_{max} - \ddot{\phi}_{max}(t - t_2) & \text{when } t_2 \leq t \leq t_f. \end{cases} \quad (15)$$

- Angular position profile:

$$\phi(t) = \begin{cases} \dot{\phi}_0(t - t_0) + \frac{1}{2}\ddot{\phi}_{max}(t - t_0)^2 & \text{when } t_0 \leq t \leq t_1, \\ \phi(t_1) + \dot{\phi}_{max}(t - t_1) & \text{when } t_1 \leq t \leq t_2, \\ \phi(t_2) + \dot{\phi}_{max}(t - t_2) - \frac{1}{2}\ddot{\phi}_{max}(t - t_2)^2 & \text{when } t_2 \leq t \leq t_f. \end{cases} \quad (16)$$

# Computing Steering Profiles

- Using the conditions,  $\dot{\phi}(t_1) = \dot{\phi}_{max}$ ,  $\dot{\phi}(t_f) = \dot{\phi}_f$ ,  $\phi(t_f) = \phi_f$ , we can determine switching times  $t_1$ ,  $t_2$ , and final time  $t_f$  as:

$$t_1 = t_0 + \frac{\dot{\phi}_{max} - \dot{\phi}_0}{\ddot{\phi}_{max}}, \quad (17)$$

$$t_2 = t_1 + \frac{1}{\dot{\phi}_{max}} \left[ \phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2} \ddot{\phi}_{max}(t_1 - t_0)^2 - \frac{\dot{\phi}_{max}(\dot{\phi}_{max} - \dot{\phi}_f)}{\ddot{\phi}_{max}} + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}} \right], \quad (18)$$

and

$$t_f = t_1 + \frac{1}{\dot{\phi}_{max}} \left[ \phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2} \ddot{\phi}_{max}(t_1 - t_0)^2 + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}} \right]. \quad (19)$$

# Computing Steering Profiles

Desired acceleration,  ${}_{\mathcal{N}}\dot{\omega}^D$ , in the inertial frame:

$${}_{\mathcal{N}}\dot{\omega}^D = \ddot{\phi}_{max} \cdot {}_{\mathcal{N}} \hat{e} \quad (20)$$

Desired acceleration,  ${}_B\dot{\omega}^D$ , in the body frame:

$${}_B\dot{\omega}^D = {}^D R^N \ddot{\phi}_{max} \cdot {}_{\mathcal{N}} \hat{e} \quad (21)$$

Find the rotation from the inertial to desired body frame:

$${}^D R^N = [(\cos\phi)I_{3 \times 3} + (1 - \cos\phi)\hat{e}\hat{e}^T - (\sin\phi)E^X] \quad (22)$$

$E^X$  is the skew matrix of the eigenaxis vector:

$$E^X = [Eskewmatrix] \quad (23)$$

# Computing Steering Profiles

Solve for desired control torque,  $u$ , with current velocity,  ${}_B\omega^C$ , and Euler's equations for rigid body dynamics:

$$J \cdot {}_B\dot{\omega}^D = u - {}_B\omega^C \times J \cdot {}_B\omega^C \quad (24)$$

Now, with the actuator system, apply the desired control torque and solve the dynamical equations and kinematic equations of motion to propagate angular velocity and attitude:

$$J \cdot {}_B\dot{\omega}^C = u - {}_B\omega^C \times J \cdot {}_B\omega^C \quad (25)$$

$${}_B\dot{q}^C = \frac{1}{2}\Omega({}_B\omega^B){}_Bq^C \quad (26)$$



# Numerical Simulations

## Introduction

- 1 Numerical simulations were run examine the proposed algorithm.
- 2 The initial, final, and sun position vectors were randomized for each run.
- 3 Two cases shown in these slides - one in which the sun angle is greater than 0 from the slew plane, the other in which the sun vector lies directly on the slew plane.

# Numerical Simulations

$\alpha > 0$

**Table:** Initial, Final, and Sun Positions in Inertial Frame and Constraints (inputs)

Unit Vector	Comp
$P_i$	(-0.50, 0.57, 0.65)
$P_f$	(0.76, -0.48, -0.44)
$S$	(0.30, -0.50, -0.81)
Parameter	Constraint
$\alpha_{max}$	1 rad/s <sup>2</sup>
$\omega_{max}$	1 rad/s

**Table:** Slew Angles  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$

$\phi$	rad
1	0.56
2	1.79
3	0.31

# Numerical Simulations

$$\alpha > 0$$

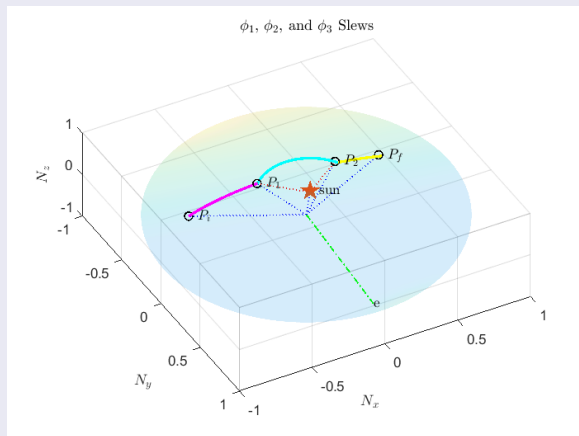


Figure: Attitude Profile of the Entire Slew

# Numerical Simulations

$$\alpha > 0$$

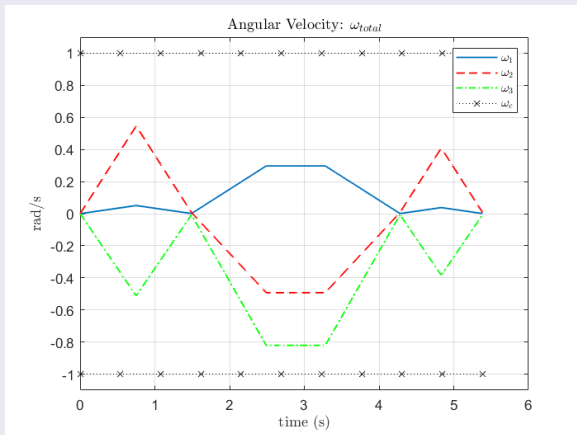


Figure: Angular Velocity in Spacecraft Frame

# Numerical Simulations

$$\alpha > 0$$

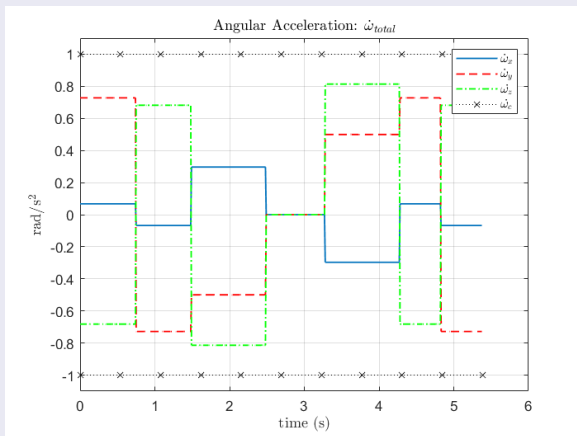


Figure: Angular Acceleration

# Numerical Simulations

$$\alpha > 0$$

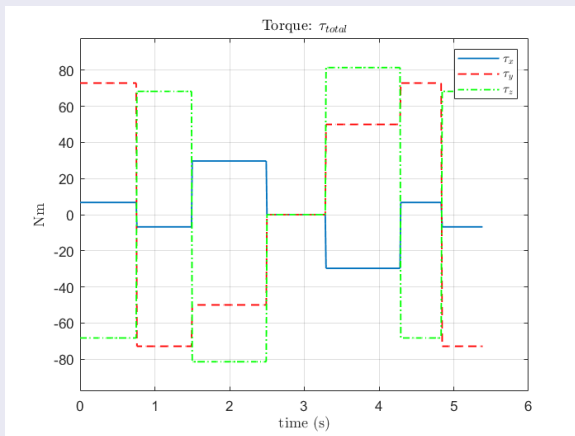


Figure: Torque Applied from Actuator System

# Numerical Simulations

$$\alpha > 0$$

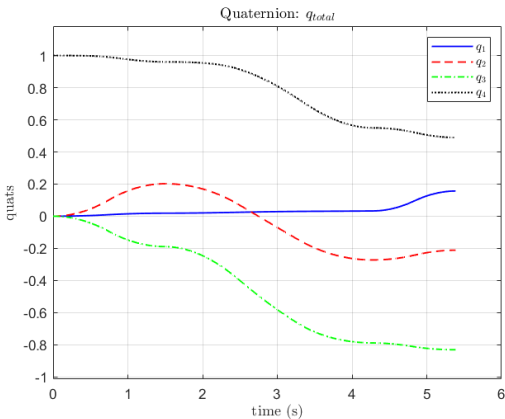


Figure: Quaternion Attitude

# Summary and Conclusion

- 1 Geometric approach for large-angle slew planning with pointing and actuator constraints
- 2 Assumed that initial and final attitudes, instrument boresight, and sun vector are known
- 3 Target-frame quaternions, angular velocities, and angular accelerations are derived base on the PMP
- 4 Limitation is for single sensitive-payload



# Acknowledgments

The research has been supported by Maxar Space Infrastructure (formerly Space Systems/Loral). We would also like to acknowledge Luke DeGalan for his useful comments.

# Q & A

# Back-up Slides

# Computing the Steering Profiles

Case II: Single-Axis, Agile Slew Maneuver with Acceleration Constraint.

## Single-Axis, Agile Slew Maneuver with Acceleration Constraint

### Problem Statement:

Consider the optimal control problem described by Eqs.(10), (11), (12), and subject to control constraint

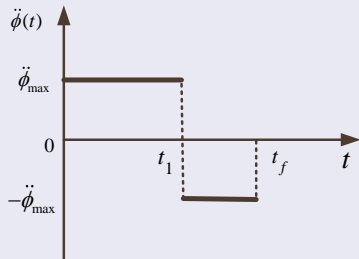
$$C_2 : |u = \ddot{\phi}| \leq \ddot{\phi}_{max}. \quad (27)$$

**Find:**  $\phi(t)$ ,  $\dot{\phi}(t)$ , and  $\ddot{\phi}(t)$ .

# Single-Axis, Agile Slew Maneuver with Acceleration Constraint

- Angular acceleration about the  $\hat{e}$  axis:

$$\ddot{\phi}(t) = \ddot{\phi}_{\max} \mathbb{U}(t_0) - 2\ddot{\phi}_{\max} \mathbb{U}(t - t_1) \quad (28)$$



where the switching and the final times are given by

$$t_1 = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{\max}} + \frac{\sqrt{\ddot{\phi}_{\max}^2(2\ddot{\phi}_{\max}\phi_f + \dot{\phi}_f^2 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{\max}^2} \quad (29)$$

## Single-Axis, Agile Slew Maneuver with Acceleration Constraint

and

$$t_f = t_0 - \frac{\dot{\phi}_f + \dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2}\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_f + \dot{\phi}_{ef}^2 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \quad (30)$$

- Angular velocity about the  $\hat{e}$  axis:

$$\dot{\phi}(t) = \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0)\mathbb{U}(t_0) - 2\ddot{\phi}_{max}(t - t_1)\mathbb{U}(t - t_1) \quad (31)$$

- Angular position about the  $\hat{e}$  axis:

$$\begin{aligned} \phi(t) = & \dot{\phi}_0(t - t_0) + \ddot{\phi}_{max}\frac{(t - t_0)^2}{2}\mathbb{U}(t_0) \\ & - 2\ddot{\phi}_{max}\frac{(t - t_1)^2}{2}\mathbb{U}(t - t_1) \end{aligned} \quad (32)$$

# The Agile Sun-Avoidance Slew Maneuver

## The First Slew Maneuver:

A single-axis nonrest-to-rest maneuver around the  $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = \dot{\phi}_0, \phi(t_0) = 0, \dot{\phi}(t_{f1}) = 0, \phi(t_{f1}) = \phi_1. \quad (33)$$

The switching time,  $t_{11}$ , and minimum-time,  $t_{f1}$ , are

$$t_{11} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \quad (34)$$

$$t_{f1} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \quad (35)$$

The  $\ddot{\phi}(t)$ ,  $\dot{\phi}(t)$ , and  $\phi(t)$ , can be found by substituting the boundary conditions given by (33) and  $t_{11}$  and  $t_{f1}$  in to Eqs. (28), (31), and (32), respectively.



# The Agile Sun-Avoidance Slew Maneuver

## The Second Slew Maneuver: A rest-to-rest maneuver around the sun vector

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f2}) = 0, \phi(t_{f2}) = \phi_2. \quad (36)$$

The switching time,  $t_{12}$ , and the minimum-time,  $t_{f2}$ , are

$$t_{12} = t_0 - \frac{\sqrt{\phi_2}}{\ddot{\phi}_{max}} \quad (37)$$

$$t_{f2} = t_0 - \frac{2\sqrt{\phi_2}}{\ddot{\phi}_{max}} \quad (38)$$

The  $\ddot{\phi}(t)$ ,  $\dot{\phi}(t)$ , and  $\phi(t)$ , can be found by substituting the boundary conditions given by (36) and  $t_{12}$  and  $t_{f2}$  in to Eqs. (28), (31), and (32), respectively.

# The Agile Sun-Avoidance Slew Maneuver

## The Third Slew Maneuver: A single-axis rest-to-nonrest maneuver around the $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f3}) = \dot{\phi}_f, \phi(t_{f3}) = \phi_3. \quad (39)$$

The switching time,  $t_{13}$ , and the minimum-time,  $t_{f3}$ , are

$$t_{13} = t_0 + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \quad (40)$$

$$t_{f3} = t_0 - \frac{\dot{\phi}_f}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\ddot{\phi}_{max}^2} \quad (41)$$

The  $\ddot{\phi}(t)$ ,  $\dot{\phi}(t)$ , and  $\phi(t)$ , can be found by substituting the boundary conditions given by (39) and  $t_{13}$  and  $t_{f3}$  in to Eqs. (28), (31), and (32), respectively.

# Computing Steering Profiles

Using the Pontryagin's minimum principle (PMP), we derive the necessary conditions for the optimal solution as follows:

① State Eqs.:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \\ \dot{x}_3 = (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}), \end{cases} \quad (42)$$

where the unit step function,  $\mathbb{U}$ , is defined as

$$\mathbb{U}(X) = \begin{cases} 1, X > 0, \\ 0, X \leq 0. \end{cases} \quad (43)$$

Note:  $(x_3(t_0) = x_3(t_f) = 0 \ \& \ x_3(t) \geq 0) \rightarrow x_3(t) = 0, \forall t \in [t_0, t_f]$ .

② Hamiltonian:

$$\begin{aligned} \mathcal{H} = & 1 + \lambda_1 x_2 + \lambda_2 u + \lambda_3 \left[ (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) \right. \\ & \left. (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}) \right] \end{aligned} \quad (44)$$

# Computing Steering Profiles

- ③ Determining the optimal control in the singular arc:

$$\frac{d^2}{dt^2} \left( \frac{\partial \mathcal{H}}{\partial u} \right) = \ddot{\lambda}_2 = 0 \rightarrow \dot{x}_2 = 0 \rightarrow u^* = 0 \quad (45)$$

- ④ Checking the Generalized Legendre-Clebsch condition for optimality:

$$(-1)^2 \frac{\partial}{\partial u} \left[ \frac{d^2}{dt^2} \left( \frac{\partial \mathcal{H}}{\partial u} \right) \right] = 1 \geq 0 \quad (46)$$

- ⑤ The transversality condition:

$$\mathcal{H}|_{(*, t_f)} = 0 \text{ and } \mathcal{H} \neq \mathcal{H}(t) \rightarrow \mathcal{H} = 0, \forall t \in [t_0, t_f]. \quad (47)$$

# Numerical Simulations

$$\alpha = 0$$

**Table:** Initial, Final, and Sun Positions in Inertial Frame and Constraints (inputs)

Unit Vector	Comp
$P_i$	(0.65, -0.35, -0.67)
$P_f$	(-0.93, -0.25, 0.28)
$S$	(-0.20, -0.78, -0.59)
Parameter	Constraint
$\alpha_{max}$	0.02 $rad/s^2$
$\omega_{max}$	0.01 $rad/s$

**Table:** Slew Angles  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$

$\phi$	rad
1	0.73
2	3.14
3	1.09

# Numerical Simulations

$$\alpha = 0$$

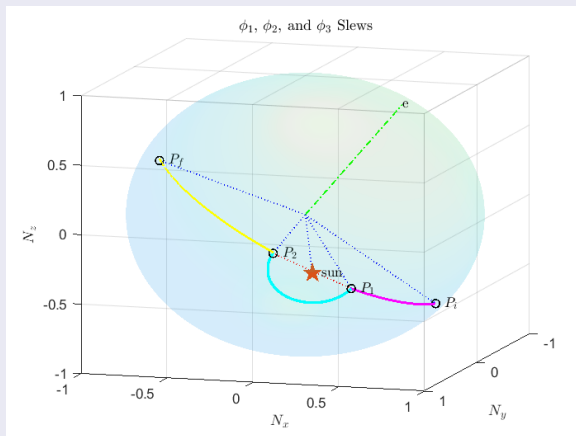


Figure: Attitude Profile of the Entire Slew

# Numerical Simulations

$$\alpha = 0$$

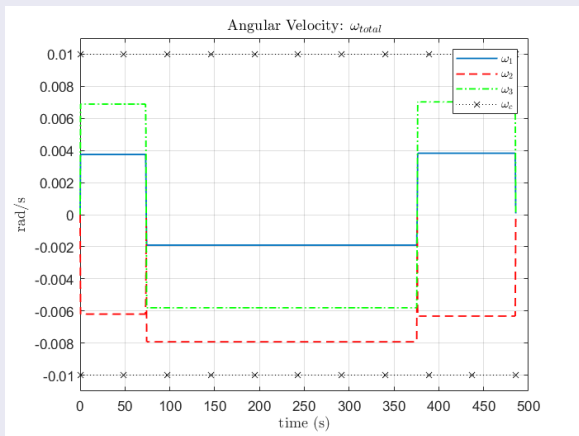


Figure: Angular Velocity in Spacecraft Frame

# Numerical Simulations

$$\alpha = 0$$

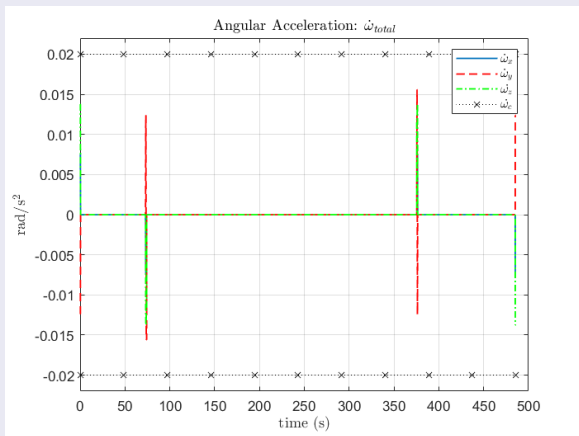


Figure: Angular Acceleration



# Numerical Simulations

$$\alpha = 0$$

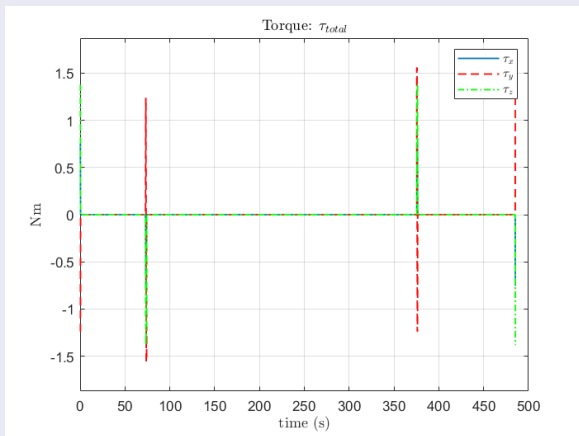


Figure: Torque Applied from Actuator System

# Numerical Simulations

$$\alpha = 0$$

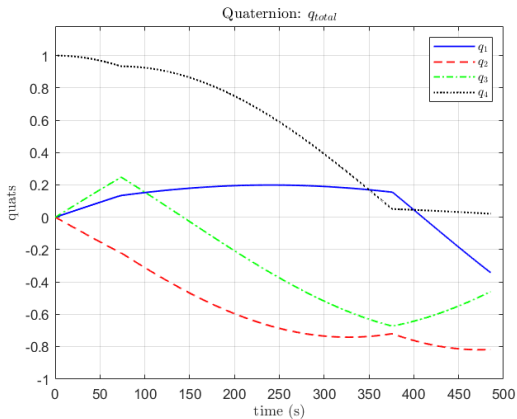


Figure: Quaternion Attitude