VI. Conclusion

This paper shows that there is a chance to find analytical or almost analytical feedback solutions for simplified models by searching for "integrals of motion." The feedback control obtained gives insight into the structure of the turning problem and may be used as a basis for the construction of more realistic controls.

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Large Angle Slew Maneuvers with Autonomous Sun Vector Avoidance

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Introduction

FREQUENT, large angle attitude slew maneuvers are required for certain space science missions for retargeting of payload instrumentation. For missions with sensitive payloads, such as cryogenically cooled infrared telescopes, the slew maneuver must be achieved without directing the payload along the Sun vector or at other infrared bright regions of the sky.[1] The planning of such constrained maneuvers can be costly in terms of increased ground segment work load and lack of system flexibility.

A methodology has been developed which allows large angle slew maneuvers to be achieved with autonomous avoidance of the Sun vector or other undesired orientations. The method is an extension of previous studies of large angle slews using the second method of Lyapunov.[2,3] The method is generalized by using artificial potential functions, where the local topology of the potential guides the satellite attitude during the slew maneuver. Undesired attitudes are then avoided by superimposing regions of high potential about these orientations.

The second method of Lyapunov allows expressions for the required control torques to be obtained analytically in closed form. Therefore, attitude control commands may be generated in real time, so that the method may be suitable for autonomous, onboard operations. Retargeting of payload instrumentation may then take place autonomously from a list of target objects to be viewed, thus reducing the ground segment loads and subsequent costs.

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System Dynamics and Control

A general, rigid satellite will be considered rotating under the influence of body-fixed torquing devices only. The attitude motion may then be described by the standard Euler equations, viz.

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1 \tag{1a}$$

$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2 \tag{1b}$$

$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3 \tag{1c}$$

where ω_i, I_i, and T_i ($i = 1, \ldots, 3$) are the satellite body rate, moment of inertia, and control torque about the ith principal axis. The body rates are related to the standard Euler angles through the kinematic relations

$$\dot{\theta}_i = \sum_{j=1}^{3} G_{ij}\omega_j,$$

$$\{G_{ij}\} = \begin{Bmatrix} 1 & \sin\theta_1\tan\theta_2 & \cos\theta_1\tan\theta_2 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1\sec\theta_2 & \cos\theta_1\sec\theta_2 \end{Bmatrix} \tag{2}$$

A scalar potential function V will be defined to be positive definite everywhere, except at the target point of the system state space where it will vanish. If an admissible control is then determined such that $\dot{V}$ is negative definite, Lyapunov's theorem guarantees that the satellite will slew to the target attitude from any initial orientation.

From Eq. (1), it may easily be verified that for open-loop, torque-free motion the total rotational kinetic energy of the system is conserved. Motivated by this, an initial potential function is chosen as

$$V = \frac{1}{2}\sum_{i=1}^{3} I_i\omega_i^2 + \frac{1}{2}\sum_{i=1}^{3}\lambda_i(\theta_i - \tilde{\theta}_i)^2 \tag{3}$$

where the second term represents an artificial potential energy possessed by the system relative to the target attitude ($\tilde{\theta}_1$, $\tilde{\theta}_2$, $\tilde{\theta}_3$). To obtain the required controls, a set of nonconservative torques are sought which render $\dot{V}$ negative definite. Differentiating the potential and substituting from Eq. (1), it is found that

$$\dot{V} = \sum_{i=1}^{3}\omega_i T_i + \sum_{i=1}^{3}\lambda_i\dot{\theta}_i(\theta_i - \tilde{\theta}_i) \tag{4}$$

where $\dot{\theta}_i$ ($i = 1, \ldots, 3$) are related to the body rates by Eq. (2). The choice of control is obviously non-unique; however, the simplest control is selected as

$$T_i = -\kappa\omega_i - \sum_{j=1}^{3}\{G_{ij}\}^T\lambda_j(\theta_j - \tilde{\theta}_j) \tag{5}$$

where the gain constants κ and λ_i ($i = 1, \ldots, 3$) will be chosen to shape the maneuver. Using these control torques, the rate of descent of the potential function then becomes

$$\dot{V} = -\kappa\sum_{i=1}^{3}\omega_i^2 \tag{6}$$

which is negative semidefinite. However, by substituting the controls of Eq. (5) into the Euler equations, it may be verified that the target attitude is in fact the only attractive equilibrium point for the system.

Sun Vector Avoidance

The methodology of the previous section will now be extended to include potential shaping for avoiding undesired attitudes. These undesired attitudes will be isolated by regions of high artificial potential (Fig. 1). This method has been successfully applied to path constrained rendezvous.[4]

In this analysis, Gaussian functions are used to represent the regions of high potential so that there are no singularities in the potential and the controls are bounded. The Gaussian functions also decay rapidly so that the attitude motion is undisturbed away from the undesired attitudes. The Sun avoidance potential will be centered on the Sun vector and defined as

$$\Phi = \alpha \exp\left\{-\beta \sum_{i=1}^{3} (\theta_i - \hat{\theta}_i)^2\right\} \tag{7}$$

where in practice the third Euler angle will be absent due to the symmetry of satellite rotations about this principal axis.

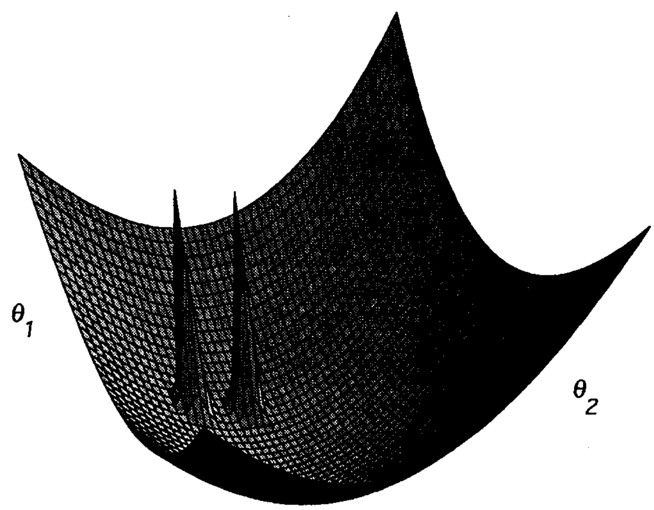

Fig. 1 θ_1–θ_2 projection of the artificial potential function.

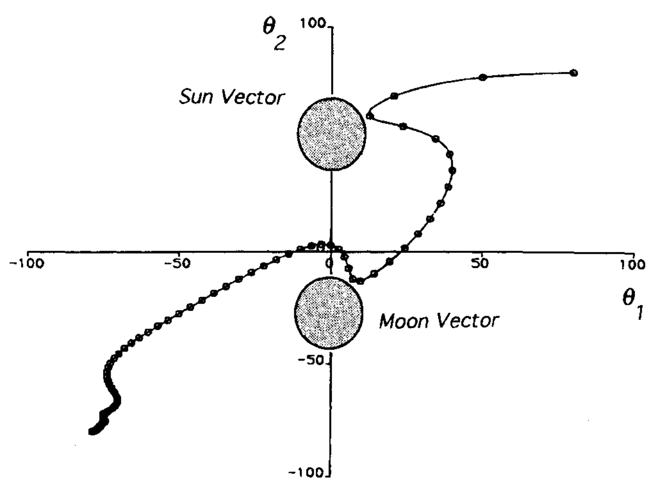

Fig. 2 Large angle slew from (80 deg, 80 deg) to (−80 deg, −80 deg) with λ = 200, α = 20, β = 10, I_1 = 10,000 kg-m², I_2 = 9,000 kg-m², and I_3 = 12,000 kg-m².

The Sun vector, or other undesired attitude, is located at ($\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$) where the parameters α and β shape the height and width of the potential.

The full potential is now defined by superimposing Eq. (7) on the quadratic potential to obtain

$$V = \frac{1}{2}\sum_{i=1}^{3} I_i \omega_i^2 + \frac{1}{2}\sum_{i=1}^{3} \lambda_i(\theta_i - \tilde{\theta}_i)^2 + \sum_{j=1}^{N} \Phi_j \tag{8}$$

where there may be N undesired attitudes to be avoided. In practice, local minima may be formed for certain superpositions of potential. This may lead to trapping in orientations other than the desired target attitude. However, for simple potentials, such as Sun vector avoidance, it is found that the local minima produced are unstable saddle points. Similarly, the Gaussian functions produce little displacement of the global minimum of the potential function.

The time derivative of the augmented potential may now be obtained using Eq. (2) as

$$\dot{V} = \sum_{i=1}^{3} \omega_i T_i + \sum_{i=1}^{3} \lambda_i \dot{\theta}_i (\theta_i - \tilde{\theta}_i)$$
$$-2\alpha\beta \sum_{i=1}^{3} \dot{\theta}_i(\theta_i - \hat{\theta}_i)\exp\left\{-\beta \sum_{i=1}^{3} (\theta_i - \hat{\theta}_i)^2\right\} \tag{9}$$

The control torques must now be chosen such that V is at least negative semidefinite. Again, the choice of control is nonunique; however, the follow nonlinear controls are chosen, viz.

$$T_i = -\kappa\omega_i - \sum_{j=1}^{3} \{G_{ij}\}^T \lambda_j (\theta_j - \tilde{\theta}_j)$$
$$+ 2\alpha\beta \sum_{j=1}^{3} \{G_{ij}\}^T (\theta_j - \hat{\theta}_j)\exp\left\{-\beta \sum_{i=1}^{3} (\theta_i - \hat{\theta}_i)^2\right\} \tag{10}$$

Using these controls, the rate of descent of the augmented potential is found to be

$$\dot{V} = -\kappa \sum_{i=1}^{3} \omega_i^2 \tag{11}$$

The rate of descent is therefore only negative semidefinite. However, for simple potential topologies there are no undesired local minima, as discussed.

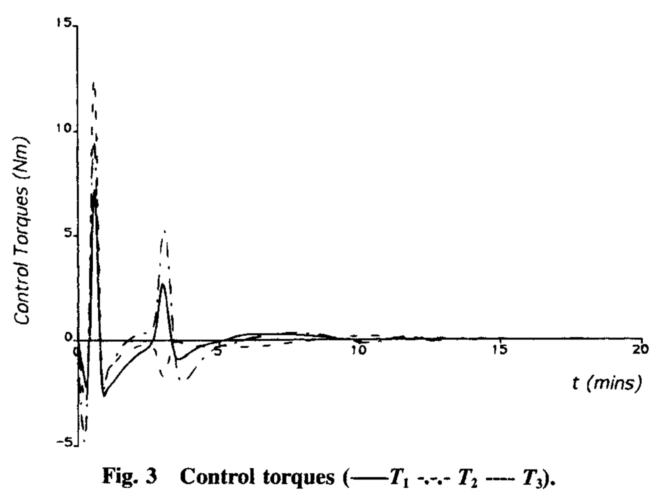

Fig. 3 Control torques (—— T_1 -.-.- T_2 ---- T_3).

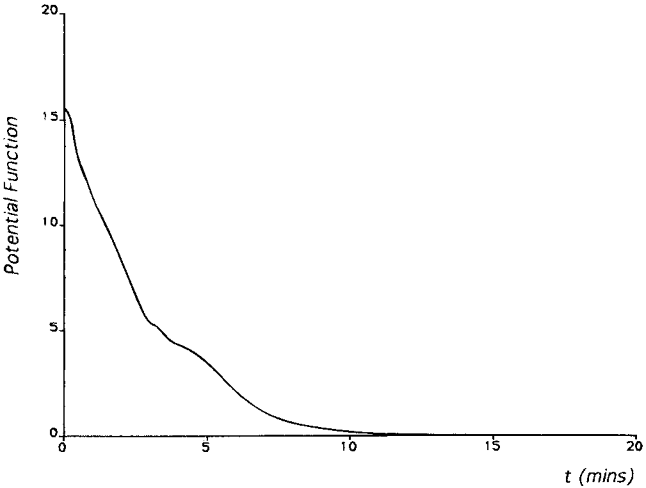

Fig. 4 Potential function.

Using the methodology described, a large angle slew will be performed with autonomous avoidance of undesired attitudes. The slew is carried out with both Sun and Moon vector avoidance (Figs. 1 and 2). The solar and lunar positions have been instantaneously fixed for ease of illustration. However, the motion of the Sun and Moon relative to the body axes may be incorporated in the potential function.

It can be seen from Fig. 3 that large control torque pulses are autonomously produced to guide the satellite around the Sun and Moon vectors. It is assumed that continuous control torques are produced using pulse-width, pulse-frequency modulation of cold gas thrusters.[3] Furthermore, as required by Lyapunov's theorem, the potential is monotonically decreasing (Fig. 4).

Conclusions

A methodology has been developed for controlling large angle satellite slew maneuvers with autonomous avoidance of undesired attitudes. It has been demonstrated that the local potential shape may be manipulated to guide the satellite attitude during the slew. Such guidance allows a globally stable, point-to-point constrained maneuver using analytic guidance commands. Since the method is entirely analytical, it may be implemented onboard in real time with a minimum of computational power. As such, it represents a possible method for autonomous large angle slew control with opportunities for a reduction in ground segment loads and subsequent costs.

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