(Preprint) AAS XX-XXX

MANUSCRIPT TITLE (UP TO 6 INCHES IN WIDTH AND CENTERED, 14 POINT BOLD FONT, MAJUSCULE)

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A sun avoidance slew maneuver is described in this paper. The algorithm finds the angular velocity, angular acceleration, and quaternion profiles needed to avoid the sun vector that lies near the plane of a sensor's FOV onboard a spacecraft.

INTRODUCTION

Problem Justification

Given: ${}_N\hat{P}_i$, ${}_N\hat{P}_f$, ${}_N\hat{S}$, ${}_G\hat{P}$, ϵ_p, ${}^Nq^G$, ${}^N_G\omega^G(t_i)$, and ${}^N_G\omega^G(t_f)$.

Find:

1. A sequence of slew maneuvers to avoid sun vector.
2. the commanded angular velocity, angular acceleration, and quaternion profiles.

Nomenclature:

- G frame: Unit sphere attached to the gyrostat.
- N: frame: The Newtonian frame fixed in the inertial space.
- ${}_G\hat{P}$: Unit vector along the bore sight of payload in the G frame.
- ${}_N\hat{P}_i$: Unit vector of the initial point in the G frame.
- ${}_N\hat{P}_f$: Unit vector of the final point in the G frame.
- ${}_N\hat{S}$: Unit vector of the sun vector in the N frame.
- ϵ_p: Payload half-cone angle.

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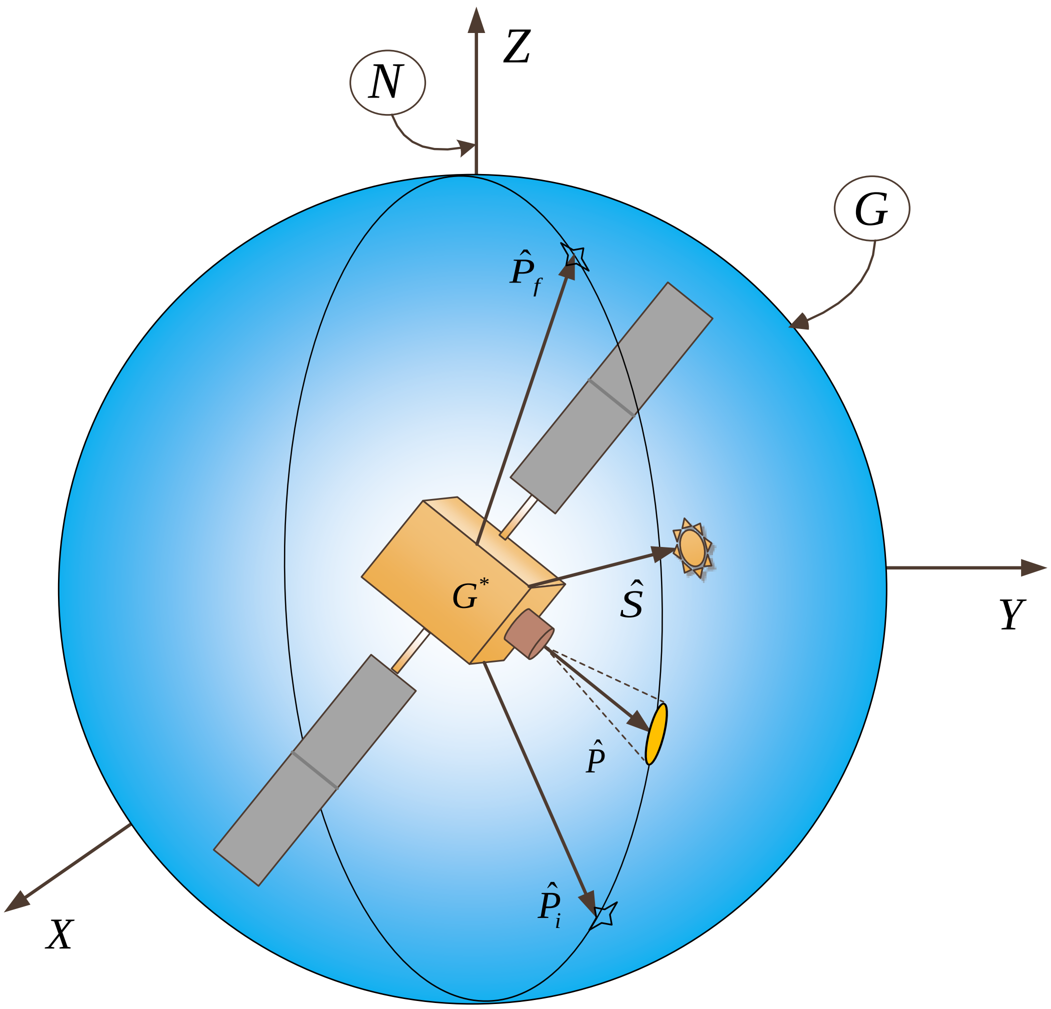


Figure 1. The spacecraft is not drawn to scale.

Literature surveyed

This is the literature surveyed.

Contribution of work

This is the contribution of work.

ALGORITHM DESCRIPTION

Slew Maneuver

Check the Sun Vector Intrusion

1. Check the angular separation between the sun vector, $\hat{S}$, and the $\hat{P}_i - \hat{P}_f$ plane.

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S}.\hat{e}) \tag{1}$$

   where the eigenaxis is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \tag{2}$$

2. IF $|\alpha| < \epsilon_p$,THEN determine the projection of the sun vector into the $\hat{P}_i - \hat{P}_f$ plane.

$$\vec{S}_{||} = \hat{S}\cos\alpha \tag{3}$$

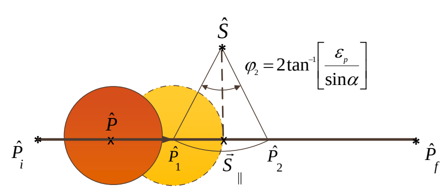
Slew Maneuvers

1. The 1^{st} slew around the eigenaxis,$\hat{e}$, through angle:

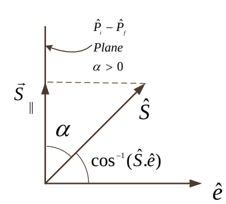
$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p & when\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p - 2\pi & when\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p > \pi \end{cases} \tag{4}$$

2. The 2^{nd} slew around the unit sun vector, $\hat{S}$, via ϕ_2.

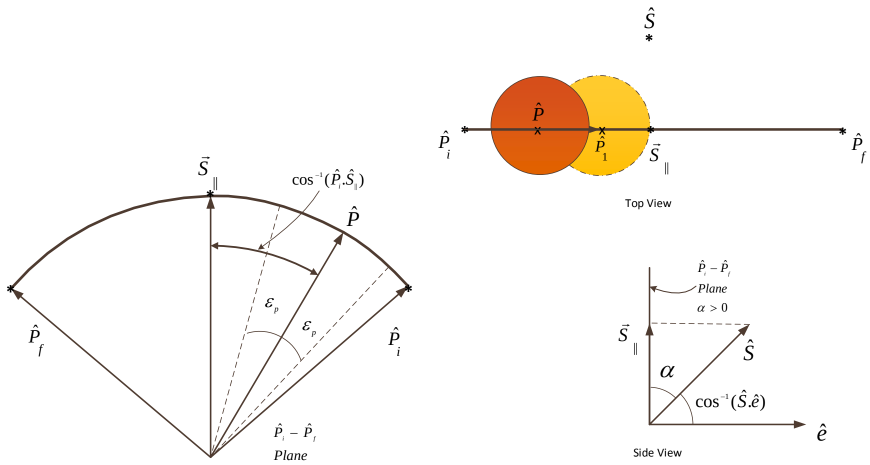
   (a) when $\alpha \neq 0$



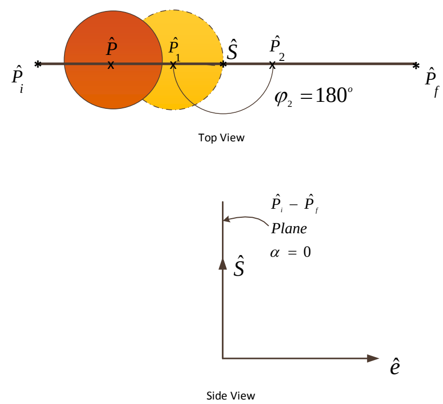
Top View



Side View

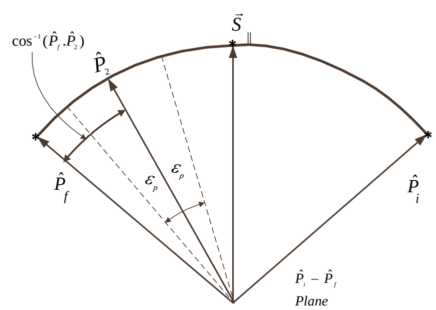


(b) when $\alpha = 0$



3. The 3^{rd} slew about the $\hat{e}$ through angle:

$$\phi_3 = \begin{cases} \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) & when\ {}_G\hat{P}_f.\hat{P}_2 \geq 0 \\ \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) - 2\pi & when\ {}_G\hat{P}_f.\hat{P}_2 < 0 \end{cases} \tag{5}$$



4. The final slew is about the instrument boresight axis to go to the final attitude.

Summary of Algorithm

1. Slew around the eigenaxis,$\hat{e}$, through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p & when\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p - 2\pi & when\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p > \pi \end{cases} \tag{6}$$

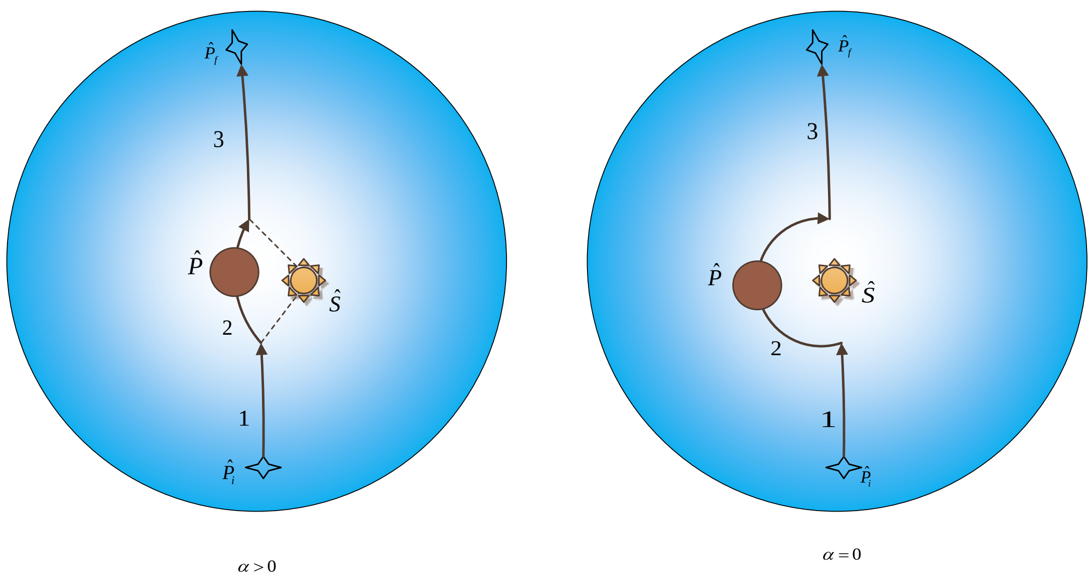
2. Slew around the $\hat{S}$ via:

$$\phi_2 = \begin{cases} 2\tan^{-1}(\frac{\epsilon_p}{\sin\alpha}) & when\ \alpha \neq 0 \\ \pi & when\ \alpha = 0 \end{cases} \tag{7}$$

3. Slew about the $\hat{e}$ through angle:

$$\phi_3 = \begin{cases} \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) & when\ {}_G\hat{P}_f.\hat{P}_2 \geq 0 \\ \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) - 2\pi & when\ {}_G\hat{P}_f.\hat{P}_2 < 0 \end{cases} \tag{8}$$

4. Perform the final rotation, ϕ_4, about the instrument boresight axis to adjust the attitude.



STEERING LAWS

Case 1: Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints.

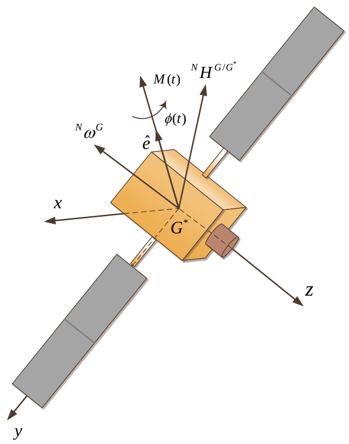
Problem Statement Consider the motion of a rigid spacecraft around a given inertially-fixed axis, ${}_G\hat{e} = [e_x, e_y, e_z]^T$. The problem of minimum-time slew maneuver around the $\hat{e}$ axis can be formulated as

$$\underset{u}{Minimze}\ J[x(.),u(.),t_f] = \int_{t0}^{t_f} dt, \tag{9}$$

subject to the following dynamic constraint

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M/I_{\hat{e}}^{G/G*} = u, \end{cases} \tag{10}$$

where $x_1 \triangleq \phi$ and $x_2 = \dot{\phi}$.



Velocity and Acceleration Constraints The boundary conditions are

$$BCs : \begin{cases} \phi(t_0) = 0, \phi(t_f) = \phi_f, \\ \dot{\phi}(t_0) = \dot{\phi}_0, \dot{\phi}(t_f) = \dot{\phi}_f, \end{cases} \tag{11}$$

and velocity (state) and acceleration (control) constraints are

$$C_1 : \begin{cases} |x_2 = \dot{\phi}| \leq \dot{\phi}_{max}, \\ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}, \end{cases} \tag{12}$$

in which

$$\dot{\phi}_{max} = [I^{w/w^*}]^{-1}[{}^N H^{G/G*} - (I^{G/G*} + I^{w/w*})^N\omega^G]/(e_x + e_y + e_z), \tag{13}$$

and

$$\ddot{\phi}_{max} =_B \hat{e}^T\ {}_B M_{max}/I_{\hat{e}}^{G/G*}, \tag{14}$$

where ${}^N H^{G/G*}$ is the total angular momentum of the gyrostat with respect to its center of mass, G^*, in the N-frame. I^{G/G^*} and $I^{w/w*}$ represent the inertia dyadic of the gyrostat and reaction wheel with respect to their center of masses, respectively. ${}_B M_{max}$ is the maximum generated torque along the body-axes in the body frame.

Find: $\phi(t)$, $\dot{\phi}(t)$, and $\ddot{\phi}(t)$.

Using the Pontryagin's minimum principle (PMP), we derive the necessary conditions for the optimal solution as follows:

1. State Eqs.:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \\ \dot{x}_3 = (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}), \end{cases} \tag{15}$$

   where the unit step function, $\mathbb{U}$, is defined as

$$\mathbb{U}(X) = \begin{cases} 1, X > 0, \\ 0, X \leq 0. \end{cases} \tag{16}$$

   Note: $(x_3(t_0) = x_3(t_f) = 0$ & $x_3(t) \geq 0\ ) \rightarrow x_3(t) = 0, t \in [t_0, t_f]$.

2. Hamiltonian:

$$\begin{aligned} H =& 1 + \lambda_1 x_2 + \lambda_2 u + \lambda_3 \Big[(x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ & (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}) \Big] \end{aligned} \tag{17}$$

Costate Eqs.:

$$\begin{cases} \dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = 0, \\ \dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = -\lambda_1 - 2\lambda_3 (x_2 + \dot{\phi}_{max}) \mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ +2\lambda_3 (\dot{\phi}_{max} - x_2) \mathbb{U}(x_2 - \dot{\phi}_{max}), \\ \dot{\lambda}_3 = -\frac{\partial H}{\partial x_3} = 0. \end{cases} \tag{18}$$

Applying the Pontryagin's minimum principle (PMP),

$$u^* = \underset{u \in \mathcal{U}}{argmin} H, \tag{19}$$

where $\mathcal{U}$ defines the domain of feasible controls. The optimal control can be determined as

$$u^*(t) = \begin{cases} \ddot{\phi}_{max} & \lambda_2 < 0, \\ ? & \lambda_2 = 0, \\ -\ddot{\phi}_{max} & \lambda_2 > 0. \end{cases} \tag{20}$$

This is a *singular arc* optimal control problem.

Determining the optimal control in the singular arc:

$$\frac{d^2}{dt^2}\Big(\frac{\partial H}{\partial u}\Big) = \ddot{\lambda}_2 = 0 \rightarrow \dot{x}_2 = 0 \rightarrow u^* = 0 \tag{21}$$

Checking the Generalized Legendre-Clebsch condition for optimality:

$$(-1)^2 \frac{\partial}{\partial u}\Big[\frac{d^2}{dt^2}\Big(\frac{\partial H}{\partial u}\Big)\Big] = 1 \geq 0 \tag{22}$$

The transversality condition:

$$H|_{(*,t_f)} = 0 \text{ and } H \neq H(t) \rightarrow H = 0, \forall t \in [t_0, t_f]. \tag{23}$$

Steering Profiles

- Angular acceleration profile (bang-off-bang):

$$\ddot{\phi}(t) = u = \begin{cases} \ddot{\phi}_{max} & when\ t_0 \le t \le t_1, \\ 0 & when\ t_1 \le t \le t_2, \\ -\ddot{\phi}_{max} & when\ t_2 \le t \le t_f. \end{cases} \tag{24}$$

- Angular velocity profile:

$$\dot{\phi}(t) = \begin{cases} \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0) & when\ t_0 \le t \le t_1, \\ \dot{\phi}_{max} & when\ t_1 \le t \le t_2, \\ \dot{\phi}_{max} - \ddot{\phi}_{max}(t - t_2) & when\ t_2 \le t \le t_f. \end{cases} \tag{25}$$

- Angular position profile:

$$\phi(t) = \begin{cases} \dot{\phi}_0(t - t_0) + \frac{1}{2}\ddot{\phi}_{max}(t - t_0)^2 & when\ t_0 \le t \le t_1, \\ \phi(t_1) + \dot{\phi}_{max}(t - t_1) & when\ t_1 \le t \le t_2, \\ \phi(t_2) + \dot{\phi}_{max}(t - t_2) - \frac{1}{2}\ddot{\phi}_{max}(t - t_2)^2 & when\ t_2 \le t \le t_f. \end{cases} \tag{26}$$

- Using the conditions, $\dot{\phi}(t_1) = \dot{\phi}_{max}$, $\dot{\phi}(t_f) = \dot{\phi}_f$, $\phi(t_f) = \phi_f$, we can determine switching times t_1, t_2, and final time t_f as:

$$t_1 = t_0 + \frac{\dot{\phi}_{max} - \dot{\phi}_0}{\ddot{\phi}_{max}}, \tag{27}$$

$$\begin{aligned} t_2 =& t_1 + \frac{1}{\dot{\phi}_{max}}\Big[\phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2}\ddot{\phi}_{max}(t_1 - t_0)^2 \\ &- \frac{\dot{\phi}_{max}(\dot{\phi}_{max} - \dot{\phi}_f)}{\ddot{\phi}_{max}} + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}}\Big], \end{aligned} \tag{28}$$

and

$$t_f = t_1 + \frac{1}{\dot{\phi}_{max}}\Big[\phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2}\ddot{\phi}_{max}(t_1 - t_0)^2 + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}}\Big]. \tag{29}$$

- Steering profiles:

$${}^N q^D(t) = [e_x \sin\frac{\phi(t)}{2}, e_y \sin\frac{\phi(t)}{2}, e_z \sin\frac{\phi(t)}{2}, \cos\frac{\phi(t)}{2}]^T \tag{30}$$

$${}^N_G\omega^D(t) = \dot{\phi}(t)_G\hat{e} \tag{31}$$

$${}^N_G\alpha^D(t) = \ddot{\phi}(t)_G\hat{e} \tag{32}$$

Case 2: Single-Axis, Agile Slew Maneuver with Acceleration Constraint

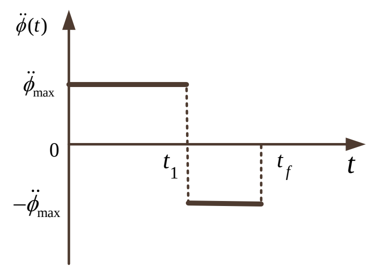
Problem Statement Consider the optimal control problem described by Eqs.(9), (10), (11), and subject to control constraint

$$C_2: \ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}. \tag{33}$$

Find: $\phi(t)$, $\dot{\phi}(t)$, and $\ddot{\phi}(t)$.

- Angular acceleration about the $\hat{e}$ axis:

$$\ddot{\phi}(t) = \ddot{\phi}_{max}\mathbb{U}(t_0) - 2\ddot{\phi}_{max}\mathbb{U}(t - t_1) \tag{34}$$



where the switching and the final times are given by

$$t_1 = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}^2_{max}(2\ddot{\phi}_{max}\phi_f + \dot{\phi}^2_f + \dot{\phi}^2_0)}}{\sqrt{2}\ddot{\phi}^2_{max}} \tag{35}$$

and

$$t_f = t_0 - \frac{\dot{\phi}_f + \dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2}\sqrt{\ddot{\phi}^2_{max}(2\ddot{\phi}_{max}\phi_f + \dot{\phi}^2_{ef} + \dot{\phi}^2_0)}}{\ddot{\phi}^2_{max}} \tag{36}$$

- Angular velocity about the $\hat{e}$ axis:

$$\dot{\phi}(t) = \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0)\mathbb{U}(t_0) - 2\ddot{\phi}_{max}(t - t_1)\mathbb{U}(t - t_1) \tag{37}$$

- Angular position about the $\hat{e}$ axis:

$$\begin{aligned} \phi(t) = \dot{\phi}_0(t - t_0) + \ddot{\phi}_{max}\frac{(t - t_0)^2}{2}\mathbb{U}(t_0) \\ - 2\ddot{\phi}_{max}\frac{(t - t_1)^2}{2}\mathbb{U}(t - t_1) \end{aligned} \tag{38}$$

The First Slew Maneuver: A single-axis nonrest-to-rest maneuver around the $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = \dot{\phi}_0, \phi(t_0) = 0, \dot{\phi}(t_{f1}) = 0, \phi(t_{f1}) = \phi_1. \tag{39}$$

The switching time, t_{11}, and minimum-time, t_{f1}, are

$$t_{11} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \tag{40}$$

$$t_{f1} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \tag{41}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (36) and t_{11} and t_{f1} in to Eqs. (31), (34), and (35), respectively.

The Second Slew Maneuver: A rest-to-rest maneuver around the sun vector

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f2}) = 0, \phi(t_{f2}) = \phi_2. \tag{42}$$

The switching time, t_{12}, and the minimum-time, t_{f2}, are

$$t_{12} = t_0 - \frac{\sqrt{\phi_2}}{\ddot{\phi}_{max}} \tag{43}$$

$$t_{f2} = t_0 - \frac{2\sqrt{\phi_2}}{\ddot{\phi}_{max}} \tag{44}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (39) and t_{12} and t_{f2} in to Eqs. (31), (34), and (35), respectively.

The Third Slew Maneuver: A single-axis rest-to-nonrest maneuver around the $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f3}) = \dot{\phi}_f, \phi(t_{f3}) = \phi_3. \tag{45}$$

The switching time, t_{13}, and the minimum-time, t_{f3}, are

$$t_{13} = t_0 + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \tag{46}$$

$$t_{f3} = t_0 - \frac{\dot{\phi}_f}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\ddot{\phi}_{max}^2} \tag{47}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (42) and t_{13} and t_{f3} in to Eqs. (31), (34), and (35), respectively.

NUMERICAL SIMULATION

CONCLUSION

ACKNOWLEDGMENT

APPENDIX: TITLE HERE

Miscellaneous Physical Dimensions

REFERENCES