(Preprint) AAS 19-801

SUN-AVOIDANCE SLEW PLANNING ALGORITHM WITH POINTING AND ACTUATOR CONSTRAINTS

Mohammad Ayoubi* and Junette Hsin†

This paper presents a geometric approach for a sun (or any bright object) avoidance slew maneuver with pointing and actuator constraints. We assume spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints. Furthermore, we assume the initial and final attitudes, instrument boresight vector, and sun vector are known. Then we use Pontryagin's minimum principle (PMP) and derive the desired or target-frame quaternions, angular velocity and acceleration. In the end, a Monte Carlo simulation is performed to show the viability of the proposed algorithm with control-torque and angular momentum constraints.

INTRODUCTION

Large-angle slew maneuvers are required during any Earth-pointing or interplanetary missions. In many space missions, and for safety consideration, a sensitive payload such as imaging camera or telescope needs to be retargeted while avoiding the sun vector or other bright objects in the sky.

The attitude reorientation problem in the presence of attitude constrained zones has been studied in the last three decades. McInnes[1] addressed this problem via an artificial potential function. He proposed an entirely analytical guidance law which was suitable for onboard implementation. However, he used Euler angles, which are singular for large slew angles.

A geometric approach was proposed by Spindle,[2] Hablani,[3] and Biggs and Colley[4] where a feasible attitude maneuver, or a guidance law, is precomputed based on the attitude-avoidance-zone constraints. Another approach for addressing this problem used randomized algorithms.[5] However, depending on the number of constraints and initial and final attitudes, this approach can be computationally expensive and not suitable for onboard implementation.

Another approach for solving the time optimal reorientation maneuver subject to boundaries and path constraints was proposed by Spiller et al.[6] They used the particle swarm optimization (PSO) technique to find a sub-optimal solution with keep-out constraints. Another approach casted the problem as a convex optimization problem and used semi-definite programming (SDP) or quadratically constrained quadratic programming (QCQP) in its solution (see for instance Kim and Mesbahi,[7] Kim et al.,[8] Sun and Dai,[9] and Lee and Mesbahi[10]). Recently, Ramos and Schaub[11] proposed

*Associate Professor, Department of Mechanical Engineering, Santa Clara University, 500 El Camino Real, Santa Clara, CA 95053 U.S.A. AIAA senior member, AAS senior member.
†Engineer, Dynamics and Control Analysis Group, Maxar Space Solutions (formerly Space Systems/Loral), 3825 Fabian Way, Palo Alto, CA 94303 U.S.A.

a method based on the Lyapunov stability theorem and logarithmic barrier potential function to derive a steering law for attitude control of a spacecraft subject to conically constrained inclusion and exclusion regions. They also considered the control-torque constraint in their algorithm.

In this paper, we present a novel geometric approach for large-angle slew planning with pointing and actuator constraints. We assume that the spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints. Furthermore, we assume that the initial and final attitudes, instrument boresight vector, and sun vector are known. Then, we derive the desired or target-frame quaternions, angular velocities, and angular accelerations based on the Pontryagin's minimum principle (PMP) for the proposed maneuver.

The proposed algorithm in this paper is intuitive, deterministic, easy to implement, and includes the control-torque and reaction wheels' angular momentum constraints. The main drawback of the proposed algorithm is its limitation for a single sensitive-payload. A Monte Carlo simulation is performed to show the viability of the proposed algorithm with control-torque and angular momentum constraints.

PROBLEM FORMULATION

Consider a gyrostat (a rigid body with reaction wheels) and let us define a newtonian frame, N, and a gyrostat-centered unit sphere frame G with a center G^* at the center-of-mass of the gyrostat as shown in Fig. 1. The sun or bright-object avoidance planning problem can be stated as follows:

Assume that the initial state, $x_i = \left[{}_{\mathcal{N}}\hat{P}_i, {}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_i), {}^{\mathcal{N}}q^{\mathcal{G}}(t_i)\right] \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{SO}(3)$, final state, $x_f = \left[{}_{\mathcal{N}}\hat{P}_f, {}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_f), {}^{\mathcal{N}}q^{\mathcal{G}}(t_f)\right] \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{SO}(3)$, the sun unit vector in the inertial frame,${}_{\mathcal{N}}\hat{S} \in \mathbb{R}^3$, the sensitive instrument boresight unit vector in the body-fixed frame, ${}_{\mathcal{G}}\hat{P} \in \mathbb{R}^3$, and the half-cone angle $\epsilon_p \in \mathbb{R}$ are given.

Find: A sequence of slew maneuvers such that the sun vector does not enter into the on-board sensitive instrument forbidden cone for all times $t \in [t_i, t_f]$ subject to actuator constraints.

SUN-AVOIDANCE SLEW (SAS) ALGORITHM DESCRIPTION

The first step is to determine if there is the sun vector intrusion. To this end we check the angular separation, α, between the sun unit vector, ${}_{\mathcal{N}}\hat{S}$, and the $\hat{P}_i - \hat{P}_f$ plane or "slew plane."

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot \hat{e}) \tag{1}$$

where the eigenaxis unit vector is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \tag{2}$$

If $|\alpha| >= \epsilon_p$ then the sun vector intrusion has not happened. Otherwise, we need to perform sun-avoidance slew maneuver which is explained in the next section.

NUMERICAL SIMULATION

We use MATLAB to numerically simulate and examine the proposed algorithm. The results A simulation is shown in the following figures. The initial, final, and sun position vectors were randomized for each run. Two cases are shown here - one in which the sun angle is greater than 0

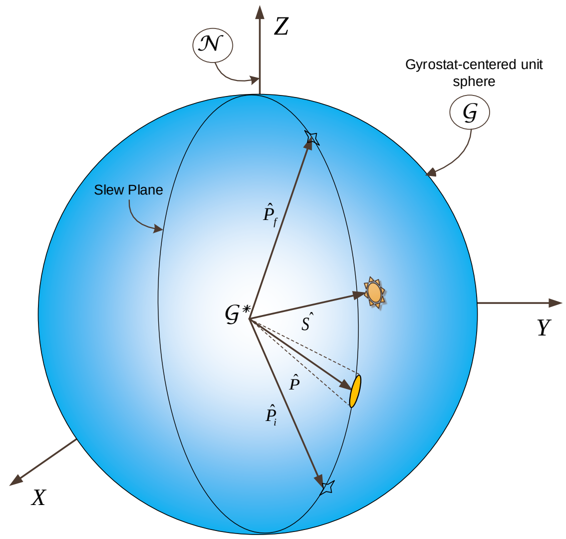

Figure 1. Gyrostat-centered unit sphere centered at point $\mathcal{G}^*$.

from the slew plane. The other case is one in which the sun vector lies directly on the slew plane, so that $\phi_2 = 180$ degrees.

Case I: $\alpha > 0$

ϕ_1, ϕ_2, and ϕ_3 were found using the methods discussed in the description of the algorithm. Several intermediate frames had to be calculated for the simulation to run. In addition to the slew plane, a sun-to-position frame was constructed in order to calculate the path that the spacecraft takes around the sun vector. SP This path had to be traced by rotating the vector that connects the sun and P_1 vectors, which is the green line on the right in the figure below and will be called V. V was rotated from P_1 to P_2 by angle ϕ_2.

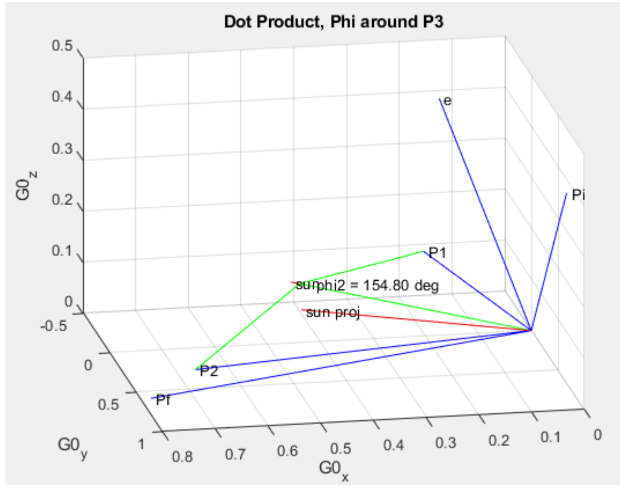

Figure 2. Chord geometry for finding ϕ_2.

For the first phase of the slew, the spacecraft was rotated around the eigenaxis of the slew plane by ϕ_1. For the second phase of the slew, the spacecraft was rotated around the sun vector fixed in inertial space by ϕ_2. For the third phase of the slew, the spacecraft was rotated around the eigenaxis of the slew plane by ϕ_3. The attitude generated would look like the profile in Figure .

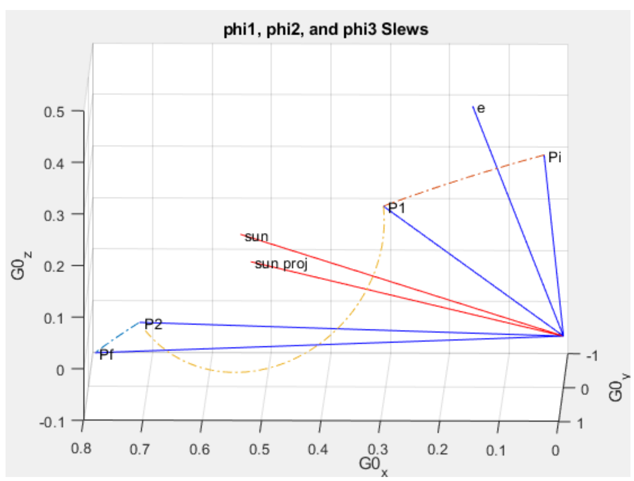

Figure 3. Attitude Profile of the Entire Slew.

From randomizing the initial, final, and sun position vectors for this simulation, the values of ϕ were found and listed in the table below. The angular velocity and acceleration never exceeded the velocity and acceleration constraints for any axis. There is no noise modeled in the actuator system, as the purpose of this simulation was to validate the slewing maneuvers described by the algorithm.

Table 1. Slew Angles ϕ_1, ϕ_2, and ϕ_3.

ϕ	1	2	3
Angle (rad)	0.29	2.70	0.13
Angle (deg)	16.61	154.80	7.33

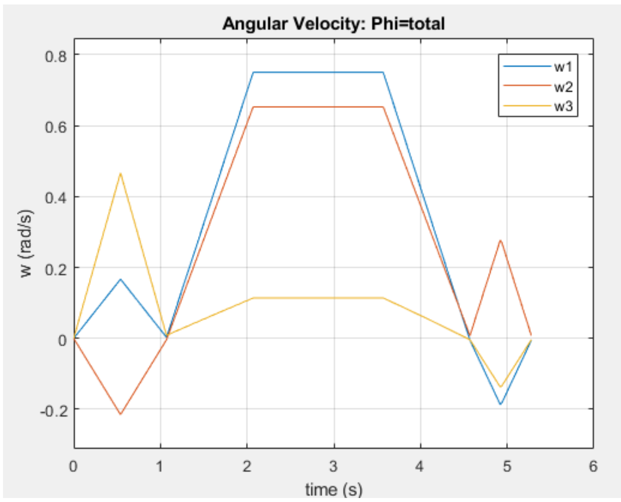

Figure 4. Time History of Angular Velocity when $\alpha > 0$.

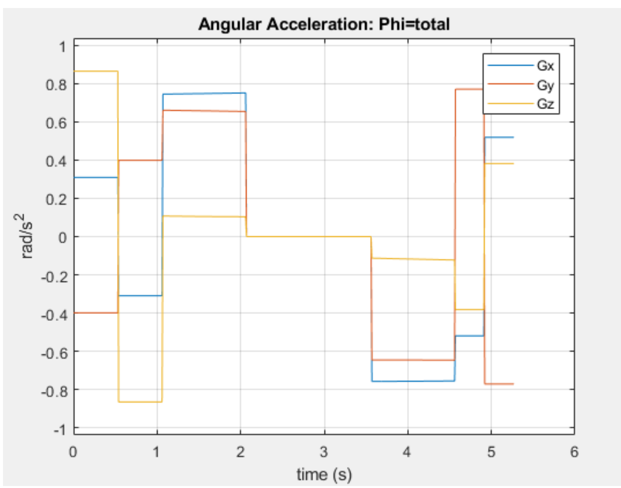

Figure 5. Time History of Angular Acceleration when $\alpha > 0$.

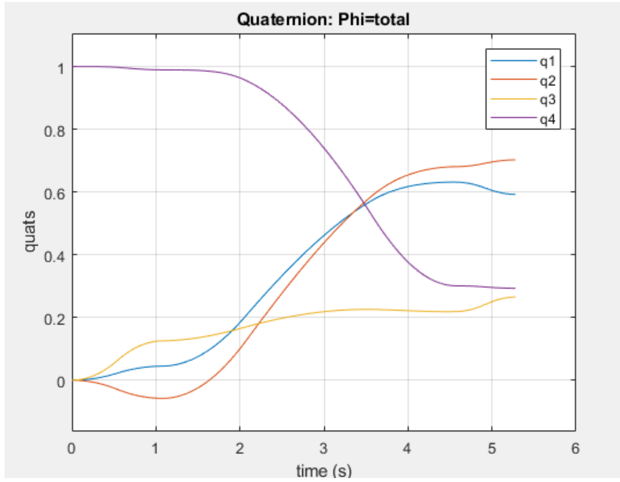

Figure 6. Time History of Quaternions when $\alpha > 0$.

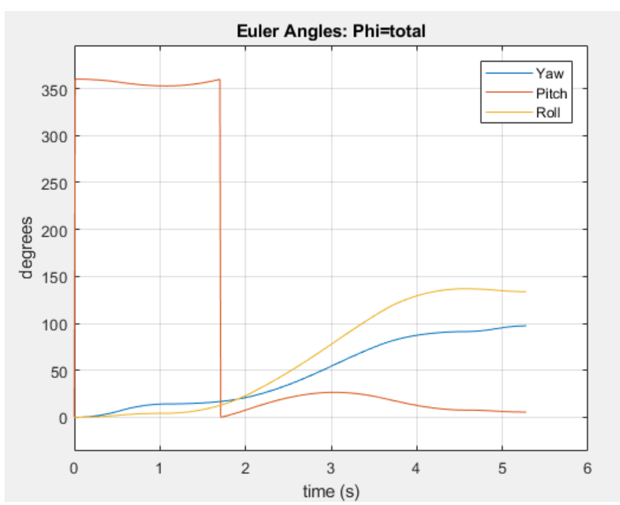

Figure 7. Time History of Euler Angles when $\alpha > 0$.

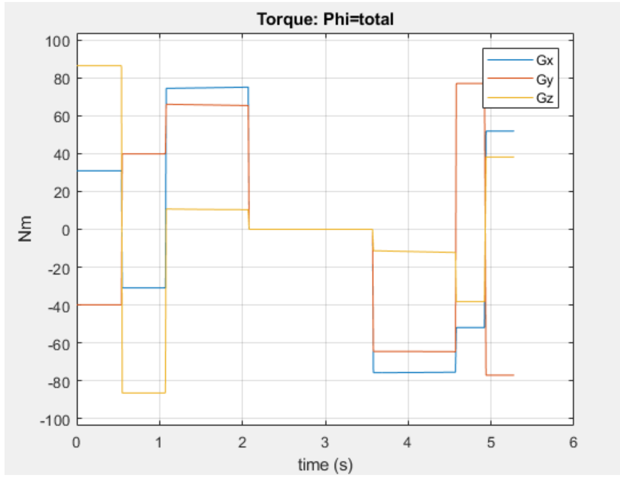

Figure 8. Applied Torque when $\alpha > 0$.

Case II: $\alpha = 0$

For the case in which the sun vector lies directly on the slew plane, ϕ_2 = 180 degrees. For this case, P_2 and P_f were almost superimposed; therefore, ϕ_3, the angle between the two vectors appears 0.

Table 2. Slew Angles ϕ_1, ϕ_2, and ϕ_3

ϕ	1	2	3
Angle (rad)	0.02	3.14	0.00
Angle (deg)	10.80	180	0.00

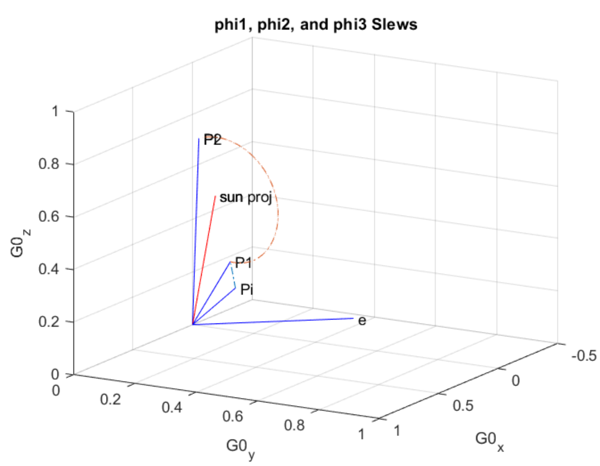

Figure 9. Attitude Profile of the Entire Slew when $\alpha = 0$.

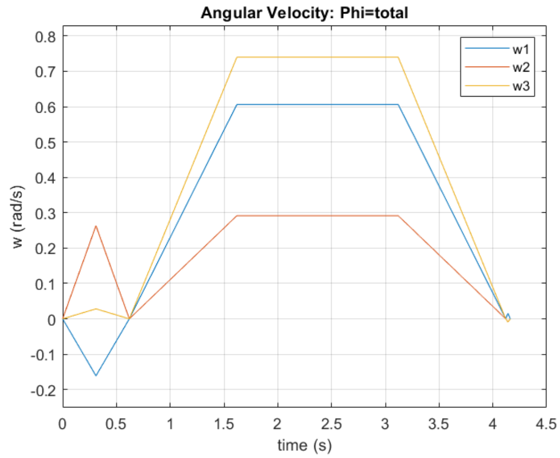

Figure 10. Time History of Angular Velocity when $\alpha = 0$.

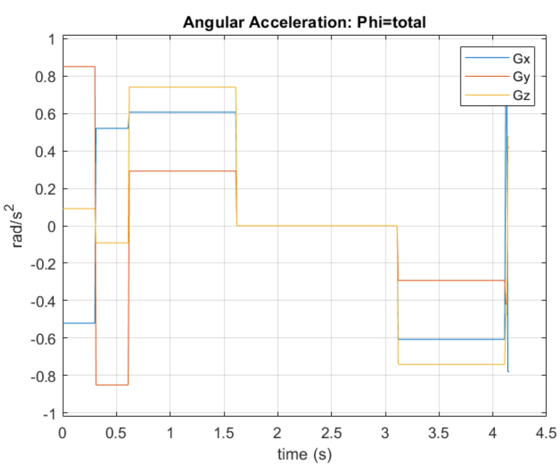

Figure 11. Time History of Angular Acceleration when $\alpha = 0$.

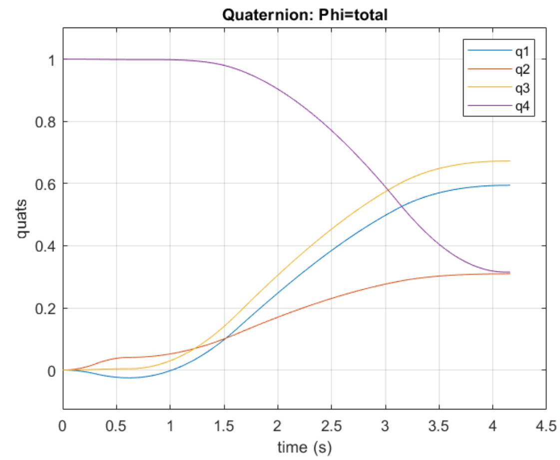

Figure 12. Time History of Quaternions when $\alpha = 0$.

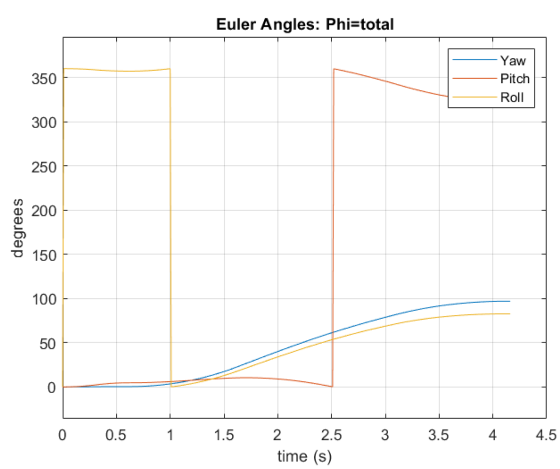

Figure 13. Time History of Euler Angles when $\alpha = 0$.

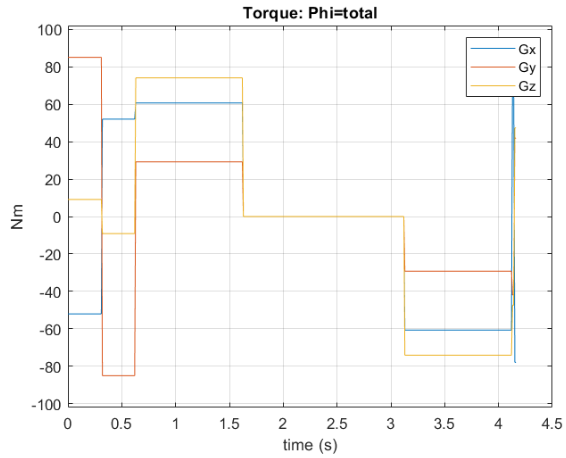

Figure 14. Applied Torque when $\alpha = 0$.

CONCLUSION

A new geometric approach for large-angle slew planning with pointing and actuator constraints is presented. The spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints. Furthermore, we assume that the initial and final attitudes, instrument line-of-sight vector, and sun vector are known. Then the desired or target-frame quaternions, angular velocities, and angular accelerations are derived based on the PMP. The proposed algorithm is intuitive, deterministic, and easy to implement. The main drawback of the proposed algorithm is its limitation for a single sensitive-payload. The feasibility of the proposed algorithm is demonstrated for two arbitrary cases and it has been investigated via extensive numerical simulations.

ACKNOWLEDGMENT

The research of the authors has been supported by Maxar Space Solutions (Formerly Space Systems / Loral). The second author would like to acknowledge Luke DeGalan for his useful comments.

NOTATION

$\mathcal{G}$-frame	gyrostat body-fixed frame
${}^{\mathcal{N}}H^{\mathcal{G}/\mathcal{G}*}$	the total angular momentum of the gyrostat with respect to its center of mass
$I^{\mathcal{G}/\mathcal{G}^*}$	the mass-moment-of-inertia of the gyrostat
$I^{w/w*}$	the mass-moment-of-inertia of reaction wheels with respect to their center of masses
M_{max}	the maximum available torque along the eigenaxis
$\mathcal{N}$-frame	the Newtonian frame
${}_{\mathcal{G}}\hat{P}$	unit vector along the bore sight of payload in the $\mathcal{G}$-frame
${}_{\mathcal{N}}\hat{P}_i$	unit vector of the initial point in the $\mathcal{N}$-frame
${}_{\mathcal{N}}\hat{P}_f$	unit vector of the final point in the $\mathcal{N}$-frame
${}^{\mathcal{N}}q^{\mathcal{T}}$	quaternion of the $\mathcal{T}$-frame in the $\mathcal{N}$-frame
${}_{\mathcal{N}}\hat{S}$	unit vector of the sun vector in the $\mathcal{N}$-frame
$\mathcal{T}$-frame	the target frame
ϵ_p	Payload half-cone angle
${}^{\mathcal{N}}\alpha^{\mathcal{T}}$	angular acceleration of the $\mathcal{T}$-frame in the $\mathcal{N}$-frame
${}^{\mathcal{N}}\omega^{\mathcal{T}}$	angular velocity of the $\mathcal{T}$-frame in the $\mathcal{N}$-frame

REFERENCES

[1] C. R. McInnes, "Large angle slew maneuvers with autonomous sun vector avoidance," *Journal of Guidance Control Dynamics*, Vol. 17, 06 1994, pp. 875–877, 10.2514/3.21283.

[2] K. Spindler, "New Methods in On-Board Attitude Control (AAS 98-308)," Vol. 100, 01 1998.

[3] H. B. Hablani, "Attitude commands avoiding bright objects and maintaining communication with ground station," *Advances in the Astronautical Sciences*, 1998, 10.2514/2.4469.

[4] J. D. Biggs and L. Colley, "Geometric Attitude Motion Planning for Spacecraft with Pointing and Actuator Constraints," *Journal of Guidance, Control, and Dynamics*, 2016, 10.2514/1.G001514.

[5] E. Frazzoli, M. A. Dahleh, E. Feron, R. P. Kornfeld, and R. P. Kornfeld, "A Randomized Attitude Slew Planning Algorithm for Autonomous Spacecraft," *In AIAA Guidance, Navigation, and Control Conference*, 2001.

[6] D. Spiller, L. Ansalone, and F. Curti, "Particle Swarm Optimization for Time-Optimal Spacecraft Reorientation with Keep-Out Cones," *Journal of Guidance, Control, and Dynamics*, 2016, 10.2514/1.G001228.

[7] Y. Kim and M. Mesbahi, "Quadratically constrained attitude control via semidefinite programming," *IEEE Transactions on Automatic Control*, 2004, 10.1109/TAC.2004.825959.

[8] Y. Kim, M. Mesbahi, G. Singh, and F. Y. Hadaegh, "On the convex parameterization of constrained spacecraft reorientation," *IEEE Transactions on Aerospace and Electronic Systems*, 2010, 10.1109/TAES.2010.5545176.

[9] C. Sun and R. Dai, "Spacecraft Attitude Control under Constrained Zones via Quadratically Constrained Quadratic Programming," 2015, 10.2514/6.2015-2010.

[10] U. Lee and M. Mesbahi, "Quaternion based optimal spacecraft reorientation under complex attitude constrained zones," *Advances in the Astronautical Sciences*, 2014.

[11] M. Diaz Ramos and H. Schaub, "Kinematic Steering Law for Conically Constrained Torque-Limited Spacecraft Attitude Control," *Journal of Guidance, Control, and Dynamics*, Vol. 41, jul 2018, pp. 1990–2001, 10.2514/1.G002873.