

# Sun-Avoidance Slew (SAS) Maneuver with Single Payload

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## Sun-Avoidance Slew (SAS) Maneuver

### Problem Statement:

Given:  ${}_{\mathcal{N}}\hat{P}_i$ ,  ${}_{\mathcal{N}}\hat{P}_f$ ,  ${}_{\mathcal{N}}\hat{S}$ ,  ${}_G\hat{P}$ ,  $\epsilon_p$ ,  ${}_{\mathcal{N}}q^G$ ,  ${}_{\mathcal{G}}\omega^G(t_i)$ , and  ${}_{\mathcal{G}}\omega^G(t_f)$  .

Find:

- 1 A sequence of slew maneuvers to avoid sun vector.
- 2 the commanded angular velocity, angular acceleration, and quaternion profiles.

## Sun-Avoidance Slew (SAS) Maneuver

Figure: The gyrostat-centered unit sphere.

## Nomenclature

- $\mathcal{G}$  frame: Unit sphere attached to the gyrostat.
- $\mathcal{N}$ : frame: The Newtonian frame fixed in the inertial space.
- ${}_G\hat{P}$ : Unit vector along the bore sight of payload in the  $\mathcal{G}$  frame.
- ${}_G\hat{P}_i$ : Unit vector of the initial point in the  $\mathcal{G}$  frame.
- ${}_G\hat{P}_f$ : Unit vector of the final point in the  $\mathcal{G}$  frame.
- ${}_N\hat{S}$ : Unit vector of the sun vector in the  $\mathcal{N}$  frame.
- $\epsilon_p$ : Payload half-cone angle.

## Check the Sun Vector Intrusion

- 1 Check the angular separation between the sun vector,  $\hat{S}$ , and the  $\hat{P}_i - \hat{P}_f$  or “slew” plane.

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot_{\mathcal{N}} \hat{e}) \quad (1)$$

where the eigenaxis is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \quad (2)$$

- 2 IF  $|\alpha| < \epsilon_p$ , THEN determine the projection of the sun vector into the slew plane.

$$\vec{S}_{||} = \hat{S} \cos \alpha \quad (3)$$

## Slew Maneuvers

- ① The 1<sup>st</sup> slew around the eigenaxis,  $\hat{e}$ , through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p & \text{when } \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p - 2\pi & \text{when } \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p > \pi \end{cases} \quad (4)$$

## Slew Maneuvers

2 The 2<sup>nd</sup> slew around the unit sun vector,  $\hat{S}$ , via  $\phi_2$ .

a when  $\alpha \neq 0$

$$\phi_2 = 2 \sin^{-1} \left( \frac{\sin \epsilon_p}{\sin \theta} \right), \quad \theta = \cos^{-1}(\hat{P}_1 \cdot \hat{S}) \quad (5)$$

## Slew Maneuvers

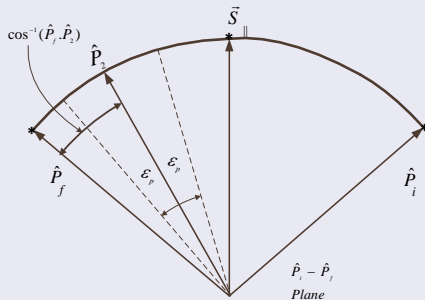
- 2 The 2<sup>nd</sup> slew around the unit sun vector,  $\hat{S}$ , via  $\phi_2 = 180^\circ$ .
  - b) when  $\alpha = 0$



## Slew Maneuvers

- 3 The 3<sup>rd</sup> slew about the  $\hat{e}$  through angle:

$$\phi_3 = \begin{cases} \cos^{-1}({}_G\hat{P}_f \cdot \hat{P}_2) & \text{when } {}_G\hat{P}_f \cdot \hat{P}_2 \geq 0 \\ \cos^{-1}({}_G\hat{P}_f \cdot \hat{P}_2) - 2\pi & \text{when } {}_G\hat{P}_f \cdot \hat{P}_2 < 0 \end{cases} \quad (6)$$



- 4 The final slew is about the instrument boresight axis to go to the final attitude.

## Summary of the Algorithm

- ① Slew around the eigenaxis,  $\hat{e}$ , through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p & \text{when } \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p - 2\pi & \text{when } \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p > \pi \end{cases} \quad (7)$$

- ② Slew around the  $\hat{S}$  via:

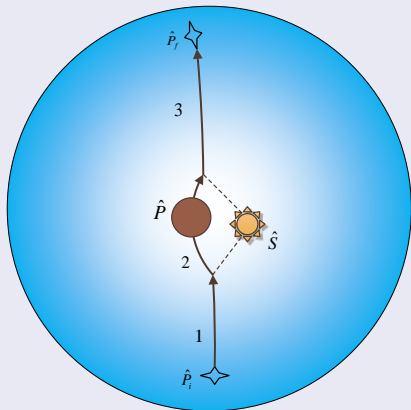
$$\phi_2 = \begin{cases} 2 \sin^{-1} \left( \frac{\sin \epsilon_p}{\sin \theta} \right), \quad \theta = \cos^{-1}(\hat{P}_1 \cdot \hat{S}) & \text{when } \alpha \neq 0 \\ \pi & \text{when } \alpha = 0 \end{cases} \quad (8)$$

- ③ Slew about the  $\hat{e}$  through angle:

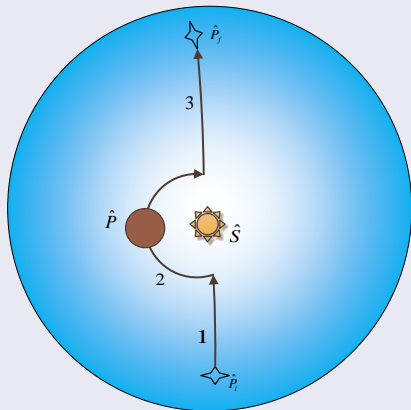
$$\phi_3 = \begin{cases} \cos^{-1}({}_{\mathcal{G}}\hat{P}_f \cdot \hat{P}_2) & \text{when } {}_{\mathcal{G}}\hat{P}_f \cdot \hat{P}_2 \geq 0 \\ \cos^{-1}({}_{\mathcal{G}}\hat{P}_f \cdot \hat{P}_2) - 2\pi & \text{when } {}_{\mathcal{G}}\hat{P}_f \cdot \hat{P}_2 < 0 \end{cases} \quad (9)$$

- ④ Perform the final rotation,  $\phi_4$ , about the instrument boresight axis to adjust the attitude.

## Summary of the Algorithm



$\alpha > 0$



$\alpha = 0$

# Computing the Steering Profiles

- Case 1) Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints.

# Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

## Problem Statement:

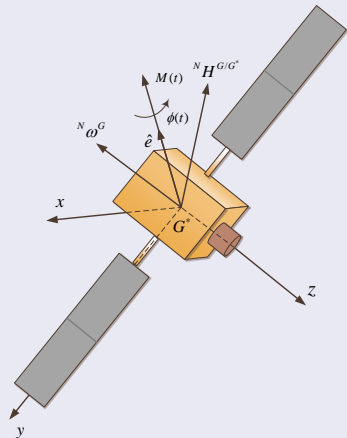
Consider the motion of a **rigid** spacecraft around a given inertially-fixed axis,  ${}^G\hat{e} = [e_x, e_y, e_z]^T$ . The problem of minimum-time slew maneuver around the  $\hat{e}$  axis can be formulated as

$$\underset{u}{\text{Minimize}} \ J[x(.), u(.), t_f] = \int_{t_0}^{t_f} dt, \quad (10)$$

subject to the following dynamic constraint

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M/I_{\hat{e}}^{G/G^*} = u, \end{cases} \quad (11)$$

where  $x_1 \triangleq \phi$  and  $x_2 = \dot{\phi}$ .



## Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

The boundary conditions are

$$BCs : \begin{cases} \phi(t_0) = 0, \phi(t_f) = \phi_f, \\ \dot{\phi}(t_0) = \dot{\phi}_0, \dot{\phi}(t_f) = \dot{\phi}_f, \end{cases} \quad (12)$$

and velocity (state) and acceleration (control) constraints are

$$C_1 : \begin{cases} |x_2 = \dot{\phi}| \leq \dot{\phi}_{max}, \\ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}, \end{cases} \quad (13)$$

in which

$$\dot{\phi}_{max} = [I^{w/w*}]^{-1} [{}^N H^{G/G*} - (I^{G/G*} + I^{w/w*}) {}^N \omega^G] / (e_x + e_y + e_z), \quad (14)$$

## Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

and

$$\ddot{\phi}_{max} = {}_B \hat{e}^T {}_B M_{max} / I_{\hat{e}}^{G/G^*}, \quad (15)$$

where  ${}^N H^{G/G^*}$  is the total angular momentum of the gyrostat with respect to its center of mass,  $G^*$ , in the  $N$ -frame.  $I^{G/G^*}$  and  $I^{w/w^*}$  represent the inertia dyadic of the gyrostat and reaction wheel with respect to their center of masses, respectively.  ${}_B M_{max}$  is the maximum generated torque along the body-axes in the body frame.

**Find:**  $\phi(t)$ ,  $\dot{\phi}(t)$ , and  $\ddot{\phi}(t)$ .

# Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

Using the Pontryagin's minimum principle (PMP), we derive the necessary conditions for the optimal solution as follows:

## 1 State Eqs.:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \\ \dot{x}_3 = (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}), \end{cases} \quad (16)$$

where the unit step function,  $\mathbb{U}$ , is defined as

$$\mathbb{U}(X) = \begin{cases} 1, & X > 0, \\ 0, & X \leq 0. \end{cases} \quad (17)$$

Note:  $(x_3(t_0) = x_3(t_f) = 0 \ \& \ x_3(t) \geq 0) \rightarrow x_3(t) = 0, t \in [t_0, t_f]$ .

## 2 Hamiltionian:

$$\mathcal{H} = 1 + \lambda_1 x_2 + \lambda_2 u + \lambda_3 \left[ (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}) \right] \quad (18)$$



# Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

③ Costate Eqs.:

$$\begin{cases} \dot{\lambda}_1 = -\frac{\partial \mathcal{H}}{\partial x_1} = 0, \\ \dot{\lambda}_2 = -\frac{\partial \mathcal{H}}{\partial x_2} = -\lambda_1 - 2\lambda_3(x_2 + \dot{\phi}_{max})\mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ \quad + 2\lambda_3(\dot{\phi}_{max} - x_2)\mathbb{U}(x_2 - \dot{\phi}_{max}), \\ \dot{\lambda}_3 = -\frac{\partial \mathcal{H}}{\partial x_3} = 0. \end{cases} \quad (19)$$

④ Applying the Pontryagin's minimum principle (PMP),

$$u^* = \underset{u \in \mathcal{U}}{\operatorname{argmin}} \mathcal{H}, \quad (20)$$

where  $\mathcal{U}$  defines the domain of feasible controls. The optimal control can be determined as

$$u^*(t) = \begin{cases} \ddot{\phi}_{max} & \lambda_2 < 0, \\ ? & \lambda_2 = 0, \\ -\ddot{\phi}_{max} & \lambda_2 > 0. \end{cases} \quad (21)$$

This is a *singular arc* optimal control problem.

## Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- 5 Determining the optimal control in the singular arc:

$$\frac{d^2}{dt^2} \left( \frac{\partial \mathcal{H}}{\partial u} \right) = \ddot{\lambda}_2 = 0 \rightarrow \dot{x}_2 = 0 \rightarrow u^* = 0 \quad (22)$$

- 6 Checking the Generalized Legendre-Clebsch condition for optimality:

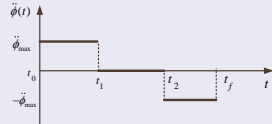
$$(-1)^2 \frac{\partial}{\partial u} \left[ \frac{d^2}{dt^2} \left( \frac{\partial \mathcal{H}}{\partial u} \right) \right] = 1 \geq 0 \quad (23)$$

- 7 The transversality condition:

$$\mathcal{H}|_{(*, t_f)} = 0 \text{ and } \mathcal{H} \neq \mathcal{H}(t) \rightarrow \mathcal{H} = 0, \forall t \in [t_0, t_f]. \quad (24)$$

# Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- Angular acceleration profile (bang-off-bang):

$$\ddot{\phi}(t) = u = \begin{cases} \ddot{\phi}_{max} & \text{when } t_0 \leq t \leq t_1, \\ 0 & \text{when } t_1 \leq t \leq t_2, \\ -\ddot{\phi}_{max} & \text{when } t_2 \leq t \leq t_f. \end{cases} \quad (25)$$


- Angular velocity profile:

$$\dot{\phi}(t) = \begin{cases} \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0) & \text{when } t_0 \leq t \leq t_1, \\ \dot{\phi}_{max} & \text{when } t_1 \leq t \leq t_2, \\ \dot{\phi}_{max} - \ddot{\phi}_{max}(t - t_2) & \text{when } t_2 \leq t \leq t_f. \end{cases} \quad (26)$$

- Angular position profile:

$$\phi(t) = \begin{cases} \dot{\phi}_0(t - t_0) + \frac{1}{2}\ddot{\phi}_{max}(t - t_0)^2 & \text{when } t_0 \leq t \leq t_1, \\ \phi(t_1) + \dot{\phi}_{max}(t - t_1) & \text{when } t_1 \leq t \leq t_2, \\ \phi(t_2) + \dot{\phi}_{max}(t - t_2) - \frac{1}{2}\ddot{\phi}_{max}(t - t_2)^2 & \text{when } t_2 \leq t \leq t_f. \end{cases} \quad (27)$$

## Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- Using the conditions,  $\dot{\phi}(t_1) = \dot{\phi}_{max}$ ,  $\dot{\phi}(t_f) = \dot{\phi}_f$ ,  $\phi(t_f) = \phi_f$ , we can determine switching times  $t_1$ ,  $t_2$ , and final time  $t_f$  as:

$$t_1 = t_0 + \frac{\dot{\phi}_{max} - \dot{\phi}_0}{\ddot{\phi}_{max}}, \quad (28)$$

$$t_2 = t_1 + \frac{1}{\dot{\phi}_{max}} \left[ \phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2} \ddot{\phi}_{max}(t_1 - t_0)^2 - \frac{\dot{\phi}_{max}(\dot{\phi}_{max} - \dot{\phi}_f)}{\ddot{\phi}_{max}} + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}} \right], \quad (29)$$

and

$$t_f = t_1 + \frac{1}{\dot{\phi}_{max}} \left[ \phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2} \ddot{\phi}_{max}(t_1 - t_0)^2 + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}} \right]. \quad (30)$$

## Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- Steering profiles:

$${}^N q^D(t) = [e_x \sin \frac{\phi(t)}{2}, e_y \sin \frac{\phi(t)}{2}, e_z \sin \frac{\phi(t)}{2}, \cos \frac{\phi(t)}{2}]^T \quad (31)$$

$${}^N_G \omega^D(t) = \dot{\phi}(t)_G \hat{e} \quad (32)$$

$${}^N_G \alpha^D(t) = \ddot{\phi}(t)_G \hat{e} \quad (33)$$

# Computing the Steering Profiles

- Case 2) Single-Axis, Agile Slew Maneuver with Acceleration Constraint.

## Single-Axis, Agile Slew Maneuver with Acceleration Constraint

### Problem Statement:

Consider the optimal control problem described by Eqs.(??), (??), (??), and subject to control constraint

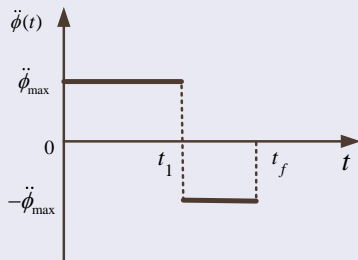
$$C_2 : |u = \ddot{\phi}| \leq \ddot{\phi}_{max}. \quad (34)$$

**Find:**  $\phi(t)$ ,  $\dot{\phi}(t)$ , and  $\ddot{\phi}(t)$ .

# Single-Axis, Agile Slew Maneuver with Acceleration Constraint

- Angular acceleration about the  $\hat{e}$  axis:

$$\ddot{\phi}(t) = \ddot{\phi}_{\max} \mathbb{U}(t_0) - 2\ddot{\phi}_{\max} \mathbb{U}(t - t_1) \quad (35)$$



where the switching and the final times are given by

$$t_1 = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{\max}} + \frac{\sqrt{\ddot{\phi}_{\max}^2 (2\ddot{\phi}_{\max}\phi_f + \dot{\phi}_f^2 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{\max}^2} \quad (36)$$



## Single-Axis, Agile Slew Maneuver with Acceleration Constraint

and

$$t_f = t_0 - \frac{\dot{\phi}_f + \dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2}\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_f + \dot{\phi}_{ef}^2 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \quad (37)$$

- Angular velocity about the  $\hat{e}$  axis:

$$\dot{\phi}(t) = \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0)\mathbb{U}(t_0) - 2\ddot{\phi}_{max}(t - t_1)\mathbb{U}(t - t_1) \quad (38)$$

- Angular position about the  $\hat{e}$  axis:

$$\begin{aligned} \phi(t) = & \dot{\phi}_0(t - t_0) + \ddot{\phi}_{max}\frac{(t - t_0)^2}{2}\mathbb{U}(t_0) \\ & - 2\ddot{\phi}_{max}\frac{(t - t_1)^2}{2}\mathbb{U}(t - t_1) \end{aligned} \quad (39)$$

# The Agile Sun-Avoidance Slew Maneuver

## The First Slew Maneuver:

A single-axis nonrest-to-rest maneuver around the  $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = \dot{\phi}_0, \phi(t_0) = 0, \dot{\phi}(t_{f1}) = 0, \phi(t_{f1}) = \phi_1. \quad (40)$$

The switching time,  $t_{11}$ , and minimum-time,  $t_{f1}$ , are

$$t_{11} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \quad (41)$$

$$t_{f1} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \quad (42)$$

The  $\ddot{\phi}(t)$ ,  $\dot{\phi}(t)$ , and  $\phi(t)$ , can be found by substituting the boundary conditions given by (??) and  $t_{11}$  and  $t_{f1}$  in to Eqs. (??), (??), and (??), respectively.

# The Agile Sun-Avoidance Slew Maneuver

## The Second Slew Maneuver: A rest-to-rest maneuver around the sun vector

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f2}) = 0, \phi(t_{f2}) = \phi_2. \quad (43)$$

The switching time,  $t_{12}$ , and the minimum-time,  $t_{f2}$ , are

$$t_{12} = t_0 - \frac{\sqrt{\phi_2}}{\ddot{\phi}_{max}} \quad (44)$$

$$t_{f2} = t_0 - \frac{2\sqrt{\phi_2}}{\ddot{\phi}_{max}} \quad (45)$$

The  $\ddot{\phi}(t)$ ,  $\dot{\phi}(t)$ , and  $\phi(t)$ , can be found by substituting the boundary conditions given by (??) and  $t_{12}$  and  $t_{f2}$  in to Eqs. (??), (??), and (??), respectively.

# The Agile Sun-Avoidance Slew Maneuver

The Third Slew Maneuver: A single-axis rest-to-nonrest maneuver around the  $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f3}) = \dot{\phi}_f, \phi(t_{f3}) = \phi_3. \quad (46)$$

The switching time,  $t_{13}$ , and the minimum-time,  $t_{f3}$ , are

$$t_{13} = t_0 + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\sqrt{2}\ddot{\phi}_{max}} \quad (47)$$

$$t_{f3} = t_0 - \frac{\dot{\phi}_f}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\ddot{\phi}_{max}} \quad (48)$$

The  $\ddot{\phi}(t)$ ,  $\dot{\phi}(t)$ , and  $\phi(t)$ , can be found by substituting the boundary conditions given by (??) and  $t_{13}$  and  $t_{f3}$  in to Eqs. (??), (??), and (??), respectively.