Sun-Avoidance Slew (SAS) Maneuver with Single Payload

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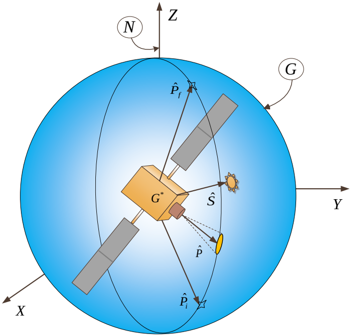
Sun-Avoidance Slew (SAS) Maneuver

Problem Statement:
Given: ${}_N\hat{P}_i$, ${}_N\hat{P}_f$, ${}_N\hat{S}$, ${}_G\hat{P}$, ϵ_p, ${}^Nq^G$, ${}^N_G\omega^G(t_i)$, and ${}^N_G\omega^G(t_f)$.
Find:

1. A sequence of slew maneuvers to avoid sun vector.
2. the commanded angular velocity, angular acceleration, and quaternion profiles.



Sun-Avoidance Slew (SAS) Maneuver

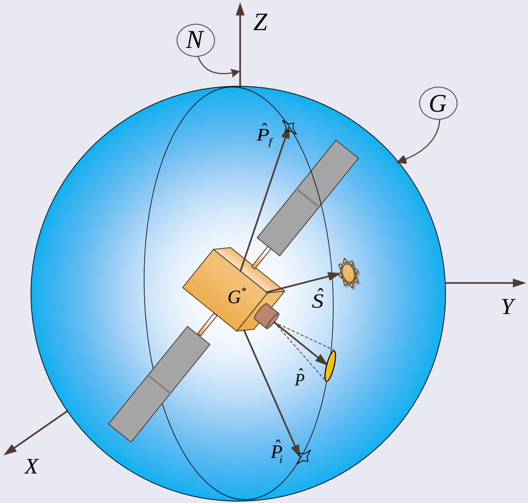


Figure: The spacecraft is not drawn to scale.

Nomenclature

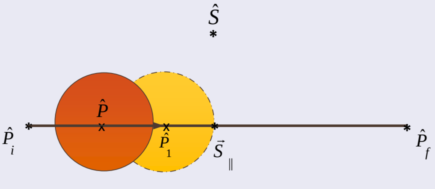
- G frame: Unit sphere attached to the gyrostat.
- N: frame: The Newtonian frame fixed in the inertial space.
- ${}_G\hat{P}$: Unit vector along the bore sight of payload in the G frame.
- ${}_N\hat{P}_i$: Unit vector of the initial point in the G frame.
- ${}_N\hat{P}_f$: Unit vector of the final point in the G frame.
- ${}_N\hat{S}$: Unit vector of the sun vector in the N frame.
- ϵ_p: Payload half-cone angle.

Check the Sun Vector Intrusion

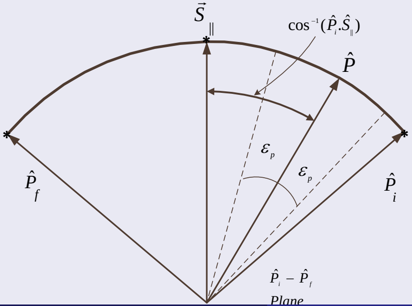
1. Check the angular separation between the sun vector, $\hat{S}$, and the $\hat{P}_i - \hat{P}_f$ plane.
$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S}.\hat{e}) \quad (1)$$
where the eigenaxis is determined by
$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \quad (2)$$
2. IF $|\alpha| < \epsilon_p$,THEN determine the projection of the sun vector into the $\hat{P}_i - \hat{P}_f$ plane.
$$\vec{S}_{||} = \hat{S}\cos\alpha \quad (3)$$

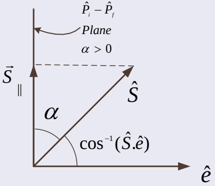
1. The 1^{st} slew around the eigenaxis, $\hat{e}$, through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p & when \;\; \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p - 2\pi & when \;\; \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p > \pi \end{cases} \quad (4)$$



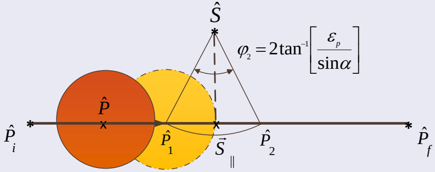
Top View



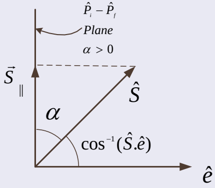


Side View

2. The 2^{nd} slew around the unit sun vector, $\hat{S}$, via ϕ_2.
   a. when $\alpha \neq 0$



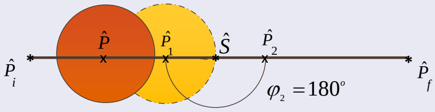
Top View



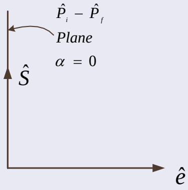
Side View

Slew Maneuvers

2. The 2^{nd} slew around the unit sun vector, $\hat{S}$, via $\phi_2 = 180°$.
   b) when $\alpha = 0$



Top View

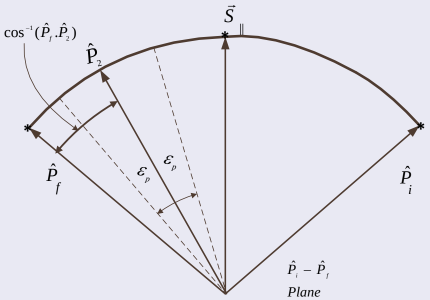


Side View

Slew Maneuvers

3. The 3^{rd} slew about the $\hat{e}$ through angle:

$$\phi_3 = \begin{cases} \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) & when\ {}_G\hat{P}_f.\hat{P}_2 \geq 0 \\ \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) - 2\pi & when\ {}_G\hat{P}_f.\hat{P}_2 < 0 \end{cases} \tag{5}$$



4. The final slew is about the instrument boresight axis to go to the final attitude.

Summary of the Algorithm

1. Slew around the eigenaxis, $\hat{e}$, through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p & when \;\; \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p - 2\pi & when \;\; \cos^{-1}(\hat{P}._G\hat{S}_{||}) - \epsilon_p > \pi \end{cases} \tag{6}$$

2. Slew around the $\hat{S}$ via:

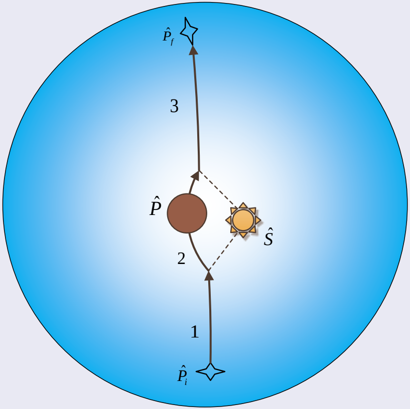
$$\phi_2 = \begin{cases} 2\tan^{-1}(\frac{\epsilon_p}{\sin\alpha}) & when \;\; \alpha \neq 0 \\ \pi & when \;\; \alpha = 0 \end{cases} \tag{7}$$

3. Slew about the $\hat{e}$ through angle:

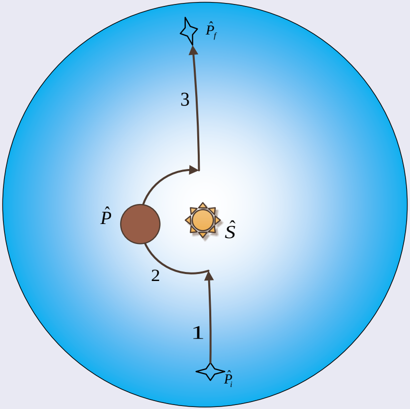
$$\phi_3 = \begin{cases} \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) & when \;\; {}_G\hat{P}_f.\hat{P}_2 \geq 0 \\ \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) - 2\pi & when \;\; {}_G\hat{P}_f.\hat{P}_2 < 0 \end{cases} \tag{8}$$

4. Perform the final rotation, ϕ_4, about the instrument boresight axis to adjust the attitude.

Summary of the Algorithm



$\alpha > 0$



$\alpha = 0$

Computing the Steering Profiles

- Case 1) Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints.

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

Problem Statement:

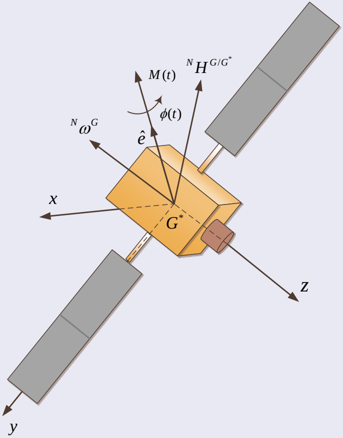
Consider the motion of a rigid spacecraft around a given inertially-fixed axis, ${}_G\hat{e} = [e_x, e_y, e_z]^T$. The problem of minimum-time slew maneuver around the $\hat{e}$ axis can be formulated as

$$\underset{u}{Minimze}\ J[x(.), u(.), t_f] = \int_{t0}^{t_f} dt, \quad (9)$$

subject to the following dynamic constraint

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M/I_{\hat{e}}^{G/G*} = u, \end{cases} \quad (10)$$

where $x_1 \triangleq \phi$ and $x_2 = \dot{\phi}$.



Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

The boundary conditions are

$$BCs : \begin{cases} \phi(t_0) = 0, \phi(t_f) = \phi_f, \\ \dot{\phi}(t_0) = \dot{\phi}_0, \dot{\phi}(t_f) = \dot{\phi}_f, \end{cases} \tag{11}$$

and velocity (state) and acceleration (control) constraints are

$$C_1 : \begin{cases} |x_2 = \dot{\phi}| \leq \dot{\phi}_{max}, \\ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}, \end{cases} \tag{12}$$

in which

$$\dot{\phi}_{max} = [I^{w/w^*}]^{-1}[{}^N H^{G/G*} - (I^{G/G*} + I^{w/w*})^N \omega^G]/(e_x + e_y + e_z), \tag{13}$$

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

and

$$\ddot{\phi}_{max} =_B \hat{e}^T \ {}_B M_{max} / I_{\hat{e}}^{G/G*}, \tag{14}$$

where ${}^N H^{G/G*}$ is the total angular momentum of the gyrostat with respect to its center of mass, G^*, in the N-frame. I^{G/G^*} and $I^{w/w*}$ represent the inertia dyadic of the gyrostat and reaction wheel with respect to their center of masses, respectively. ${}_B M_{max}$ is the maximum generated torque along the body-axes in the body frame.

Find: $\phi(t)$, $\dot{\phi}(t)$, and $\ddot{\phi}(t)$.

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

Using the Pontryagin's minimum principle (PMP), we derive the necessary conditions for the optimal solution as follows:

1. State Eqs.:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \\ \dot{x}_3 = (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}), \end{cases} \tag{15}$$

where the unit step function, $\mathbb{U}$, is defined as

$$\mathbb{U}(X) = \begin{cases} 1, & X > 0, \\ 0, & X \leq 0. \end{cases} \tag{16}$$

Note: $(x_3(t_0) = x_3(t_f) = 0 \ \& \ x_3(t) \geq 0\ ) \rightarrow x_3(t) = 0, t \in [t_0, t_f]$.

2. Hamltonian:

$$\begin{aligned} \mathscr{H} =& 1 + \lambda_1 x_2 + \lambda_2 u + \lambda_3 \Big[(x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ & (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}) \Big] \end{aligned} \tag{17}$$

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

3. Costate Eqs.:

$$\begin{cases} \dot{\lambda}_1 = -\frac{\partial \mathscr{H}}{\partial x_1} = 0, \\ \dot{\lambda}_2 = -\frac{\partial \mathscr{H}}{\partial x_2} = -\lambda_1 - 2\lambda_3(x_2 + \dot{\phi}_{max})\mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ +2\lambda_3(\dot{\phi}_{max} - x_2)\mathbb{U}(x_2 - \dot{\phi}_{max}), \\ \dot{\lambda}_3 = -\frac{\partial \mathscr{H}}{\partial x_3} = 0. \end{cases} \tag{18}$$

4. Applying the Pontryagin's minimum principle (PMP),

$$u^* = arg \min_{u \in \mathcal{U}} \mathscr{H}, \tag{19}$$

where $\mathcal{U}$ defines the domain of feasible controls. The optimal control can be determined as

$$u^*(t) = \begin{cases} \ddot{\phi}_{max} & \lambda_2 < 0, \\ ? & \lambda_2 = 0, \\ -\ddot{\phi}_{max} & \lambda_2 > 0. \end{cases} \tag{20}$$

This is a *singular arc* optimal control problem.

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

5. Determining the optimal control in the singular arc:

$$\frac{d^2}{dt^2}\left(\frac{\partial \mathscr{H}}{\partial u}\right) = \ddot{\lambda}_2 = 0 \rightarrow \dot{x}_2 = 0 \rightarrow u^* = 0 \tag{21}$$

6. Checking the Generalized Legendre-Clebsch condition for optimality:

$$(-1)^2 \frac{\partial}{\partial u}\left[\frac{d^2}{dt^2}\left(\frac{\partial \mathscr{H}}{\partial u}\right)\right] = 1 \geq 0 \tag{22}$$

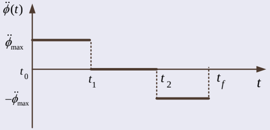
7. The transversality condition:

$$\mathscr{H}|_{(*,t_f)} = 0 \text{ and } \mathscr{H} \neq \mathscr{H}(t) \rightarrow \mathscr{H} = 0, \forall t \in [t_0, t_f]. \tag{23}$$

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- Angular acceleration profile (bang-off-bang):

$$\ddot{\phi}(t) = u = \begin{cases} \ddot{\phi}_{max} & when\ t_0 \le t \le t_1, \\ 0 & when\ t_1 \le t \le t_2, \\ -\ddot{\phi}_{max} & when\ t_2 \le t \le t_f. \end{cases} \tag{24}$$



- Angular velocity profile:

$$\dot{\phi}(t) = \begin{cases} \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0) & when\ t_0 \le t \le t_1, \\ \dot{\phi}_{max} & when\ t_1 \le t \le t_2, \\ \dot{\phi}_{max} - \ddot{\phi}_{max}(t - t_2) & when\ t_2 \le t \le t_f. \end{cases} \tag{25}$$

- Angular position profile:

$$\phi(t) = \begin{cases} \dot{\phi}_0(t - t_0) + \frac{1}{2}\ddot{\phi}_{max}(t - t_0)^2 & when\ t_0 \le t \le t_1, \\ \phi(t_1) + \dot{\phi}_{max}(t - t_1) & when\ t_1 \le t \le t_2, \\ \phi(t_2) + \dot{\phi}_{max}(t - t_2) - \frac{1}{2}\ddot{\phi}_{max}(t - t_2)^2 & when\ t_2 \le t \le t_f. \end{cases} \tag{26}$$

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- Using the conditions, $\dot{\phi}(t_1) = \dot{\phi}_{max}$, $\dot{\phi}(t_f) = \dot{\phi}_f$, $\phi(t_f) = \phi_f$, we can determine switching times t_1, t_2, and final time t_f as:

$$t_1 = t_0 + \frac{\dot{\phi}_{max} - \dot{\phi}_0}{\ddot{\phi}_{max}}, \tag{27}$$

$$\begin{aligned} t_2 = & t_1 + \frac{1}{\dot{\phi}_{max}}\left[\phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2}\ddot{\phi}_{max}(t_1 - t_0)^2 \right. \\ & \left. - \frac{\dot{\phi}_{max}(\dot{\phi}_{max} - \dot{\phi}_f)}{\ddot{\phi}_{max}} + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}}\right], \end{aligned} \tag{28}$$

and

$$t_f = t_1 + \frac{1}{\dot{\phi}_{max}}\left[\phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2}\ddot{\phi}_{max}(t_1 - t_0)^2 + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}}\right]. \tag{29}$$

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints

- Steering profiles:

$$ {}^{N}q^{D}(t) = [e_x \sin\frac{\phi(t)}{2}, e_y \sin\frac{\phi(t)}{2}, e_z \sin\frac{\phi(t)}{2}, \cos\frac{\phi(t)}{2}]^T \tag{30} $$

$$ {}^{N}_{G}\omega^{D}(t) = \dot{\phi}(t)_{G}\hat{e} \tag{31} $$

$$ {}^{N}_{G}\alpha^{D}(t) = \ddot{\phi}(t)_{G}\hat{e} \tag{32} $$

Computing the Steering Profiles

- Case 2) Single-Axis, Agile Slew Maneuver with Acceleration Constraint.

Single-Axis, Agile Slew Maneuver with Acceleration Constraint

Problem Statement:

Consider the optimal control problem described by Eqs.(9), (10), (11), and subject to control constraint

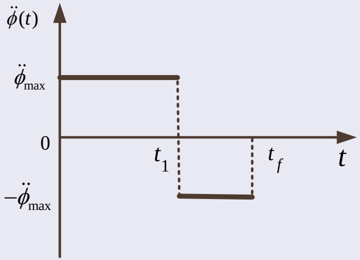
$$C_2: \ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}. \tag{33}$$

Find: $\phi(t)$, $\dot{\phi}(t)$, and $\ddot{\phi}(t)$.

Single-Axis, Agile Slew Maneuver with Acceleration Constraint

- Angular acceleration about the $\hat{e}$ axis:

$$\ddot{\phi}(t) = \ddot{\phi}_{max}\mathbb{U}(t_0) - 2\ddot{\phi}_{max}\mathbb{U}(t - t_1) \tag{34}$$



where the switching and the final times are given by

$$t_1 = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_f + \dot{\phi}_f^2 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \tag{35}$$

Single-Axis, Agile Slew Maneuver with Acceleration Constraint

and

$$t_f = t_0 - \frac{\dot{\phi}_f + \dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2}\sqrt{\ddot{\phi}^2_{max}(2\ddot{\phi}_{max}\phi_f + \dot{\phi}^2_{ef} + \dot{\phi}^2_0)}}{\ddot{\phi}^2_{max}} \quad (36)$$

- Angular velocity about the $\hat{e}$ axis:

$$\dot{\phi}(t) = \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0)\mathbb{U}(t_0) - 2\ddot{\phi}_{max}(t - t_1)\mathbb{U}(t - t_1) \quad (37)$$

- Angular position about the $\hat{e}$ axis:

$$\begin{aligned} \phi(t) = \dot{\phi}_0(t - t_0) + \ddot{\phi}_{max}\frac{(t - t_0)^2}{2}\mathbb{U}(t_0) \\ - 2\ddot{\phi}_{max}\frac{(t - t_1)^2}{2}\mathbb{U}(t - t_1) \end{aligned} \quad (38)$$

The Agile Sun-Avoidance Slew Maneuver

The First Slew Maneuver:
A single-axis nonrest-to-rest maneuver around the $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = \dot{\phi}_0, \phi(t_0) = 0, \dot{\phi}(t_{f1}) = 0, \phi(t_{f1}) = \phi_1. \tag{39}$$

The switching time, t_{11}, and minimum-time, t_{f1}, are

$$t_{11} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \tag{40}$$

$$t_{f1} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \tag{41}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (39) and t_{11} and t_{f1} in to Eqs. (34), (37), and (38), respectively.

The Agile Sun-Avoidance Slew Maneuver

The Second Slew Maneuver: A rest-to-rest maneuver around the sun vector

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f2}) = 0, \phi(t_{f2}) = \phi_2. \tag{42}$$

The switching time, t_{12}, and the minimum-time, t_{f2}, are

$$t_{12} = t_0 - \frac{\sqrt{\phi_2}}{\ddot{\phi}_{max}} \tag{43}$$

$$t_{f2} = t_0 - \frac{2\sqrt{\phi_2}}{\ddot{\phi}_{max}} \tag{44}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (42) and t_{12} and t_{f2} in to Eqs. (34), (37), and (38), respectively.

The Agile Sun-Avoidance Slew Maneuver

The Third Slew Maneuver: A single-axis rest-to-nonrest maneuver around the $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f3}) = \dot{\phi}_f, \phi(t_{f3}) = \phi_3. \tag{45}$$

The switching time, t_{13}, and the minimum-time, t_{f3}, are

$$t_{13} = t_0 + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \tag{46}$$

$$t_{f3} = t_0 - \frac{\dot{\phi}_f}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\ddot{\phi}_{max}^2} \tag{47}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (45) and t_{13} and t_{f3} in to Eqs. (34), (37), and (38), respectively.