(Preprint) AAS 19-801

SUN-AVOIDANCE SLEW PLANNING ALGORITHM WITH POINTING AND ACTUATOR CONSTRAINTS

Mohammad Ayoubi*and Junette Hsin†

This paper presents a geometric approach for a sun (or any bright object) avoidance slew maneuver with pointing and actuator constraints. We assume spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints. Furthermore, we assume the initial and final attitudes, instrument boresight vector, and sun vector are known. Then we use Pontryagin's minimum principle (PMP) and derive the desired or target-frame quaternions, angular velocity and acceleration. In the end, a Monte Carlo simulation is performed to show the viability of the proposed algorithm with control-torque and angular momentum constraints.

INTRODUCTION

Large-angle slew maneuvers are required during any Earth-pointing or interplanetary missions. In many space missions, and for safety consideration, a sensitive payload such as imaging camera or telescope needs to be retargeted while avoiding the sun vector or other bright objects in the sky.

The attitude reorientation problem in the presence of attitude constrained zones has been studied in the last three decades. McInnes[1] addressed this problem via an artificial potential function. He proposed an entirely analytical guidance law which was suitable for onboard implementation. However, he used Euler angles, which are singular for large slew angles.

A geometric approach was proposed by Spindle,[2] Hablani,[3] and Biggs and Colley[4] where a feasible attitude maneuver, or a guidance law, is precomputed based on the attitude-avoidance-zone constraints. Another approach for addressing this problem used randomized algorithms.[5] However, depending on the number of constraints and initial and final attitudes, this approach can be computationally expensive and not suitable for onboard implementation.

Another approach for solving the time optimal reorientation maneuver subject to boundaries and path constraints was proposed by Spiller et al.[6] They used the particle swarm optimization (PSO) technique to find a sub-optimal solution with keep-out constraints. Another approach casted the problem as a convex optimization problem and used semi-definite programming (SDP) or quadratically constrained quadratic programming (QCQP) in its solution (see for instance Kim and Mesbahi,[7] Kim et al.,[8] Sun and Dai,[9] and Lee and Mesbahi[10]). Recently, Ramos and Schaub[11] proposed

*Associate Professor, Department of Mechanical Engineering, Santa Clara University, 500 El Camino Real, Santa Clara, CA 95053 U.S.A. AIAA senior member, AAS senior member.

†Engineer, Dynamics and Control Analysis Group, Maxar Space Solutions (formerly Space Systems/Loral), 3825 Fabian Way, Palo Alto, CA 94303 U.S.A.

a method based on the Lyapunov stability theorem and logarithmic barrier potential function to derive a steering law for attitude control of a spacecraft subject to conically constrained inclusion and exclusion regions. They also considered the control-torque constraint in their algorithm.

In this paper, we present a novel geometric approach for large-angle slew planning with pointing and actuator constraints. We assume that the spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints. Furthermore, we assume that the initial and final attitudes, instrument boresight vector, and sun vector are known. Then, we derive the desired or target-frame quaternions, angular velocities, and angular accelerations based on the Pontryagin's minimum principle (PMP) for the proposed maneuver.

The proposed algorithm in this paper is intuitive, deterministic, easy to implement, and includes the control-torque and reaction wheels' angular momentum constraints. The main drawback of the proposed algorithm is its limitation for a single sensitive-payload. A Monte Carlo simulation is performed to show the viability of the proposed algorithm with control-torque and angular momentum constraints.

PROBLEM FORMULATION

Consider a gyrostat (a rigid body with reaction wheels) and let us define a newtonian frame, N, and a gyrostat-centered unit sphere frame G with a center G^* at the center-of-mass of the gyrostat as shown in Fig. 1. The sun or bright-object avoidance planning problem can be stated as follows:

Assume that the initial state, $x_i = \left[{}_{\mathcal{N}}\hat{P}_i, {}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_i), {}^{\mathcal{N}}q^{\mathcal{G}}(t_i)\right] \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{SO}(3)$, final state, $x_f = \left[{}_{\mathcal{N}}\hat{P}_f, {}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_f), {}^{\mathcal{N}}q^{\mathcal{G}}(t_f)\right] \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{SO}(3)$, the sun unit vector in the inertial frame,${}_{\mathcal{N}}\hat{S} \in \mathbb{R}^3$, the sensitive instrument boresight unit vector in the body-fixed frame, ${}_{\mathcal{G}}\hat{P} \in \mathbb{R}^3$, and the half-cone angle $\epsilon_p \in \mathbb{R}$ are given.

Find: A sequence of slew maneuvers such that the sun vector does not enter into the on-board sensitive instrument forbidden cone for all times $t \in [t_i, t_f]$ subject to actuator constraints.

SUN-AVOIDANCE SLEW (SAS) ALGORITHM DESCRIPTION

The first step is to determine if there is the sun vector intrusion. To this end we check the angular separation, α, between the sun unit vector, ${}_{\mathcal{N}}\hat{S}$, and the $\hat{P}_i - \hat{P}_f$ plane or "slew plane."

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot \hat{e}) \tag{1}$$

where the eigenaxis unit vector is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \tag{2}$$

If $|\alpha| >= \epsilon_p$ then the sun vector intrusion has not happened. Otherwise, we need to perform sun-avoidance slew maneuver which is explained in the next section.

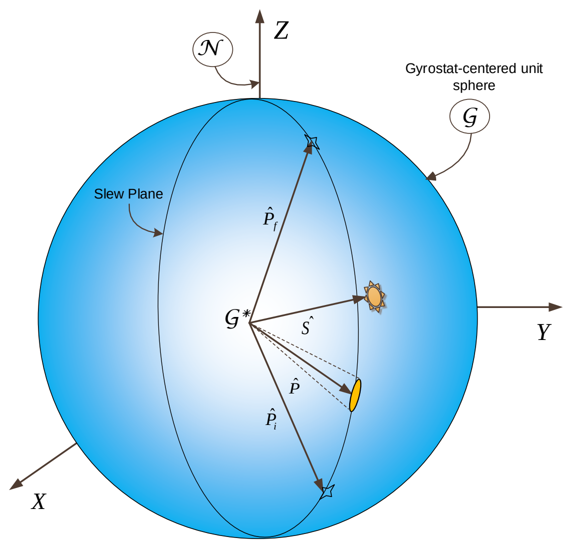


Figure 1. Gyrostat-centered unit sphere centered at point $\mathcal{G}^*$.

NUMERICAL SIMULATION

The proposed algorithm was examined by running cases in which the initial, final, and sun position vectors were randomized. Consider a spacecraft as a rigid body with one sensitive instrument. The orientation of the boresight of the instrument to the spacecraft body axes, the current and final (desired) positions with respect to the inertial frame, and the location of the sun vector are known. Only one exclusion zone around a bright object (the sun) is considered. The initial and final positions are outside of the exclusion zone.

SAS Algorithm Pseudocode

SAS Algorithm Pseudocode:

1. Find: sequence of slew maneuvers to avoid sun vector
   (a) Check the sun vector intrusion
      i. Find eigenaxis $\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|}$
      ii. Compute $\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot_{\mathcal{N}} \hat{e})$
      iii. IF $|\alpha| < \epsilon_p$, THEN find $\vec{S}_{||} = \hat{S}\cos\alpha$
   (b) Compute ϕ_1: $\phi_1 = \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p$
   (c) Compute ϕ_2:
      i. IF $\alpha \neq 0$, THEN $\phi_2 = 2\sin^{-1}\left(\frac{\sin\epsilon_p}{\sin\theta}\right),\ \theta = \cos^{-1}(\hat{P}_1 \cdot \hat{S})$
      ii. IF $\alpha = 0$, THEN $\phi_2 = \pi$
   (d) Compute ϕ_3: $\phi_3 = \cos^{-1}({}_{\mathcal{G}}\hat{P}_f.\hat{P}_2)$
2. Find: commanded angular velocity, angular acceleration, and quaternion profiles
   (a) Compute $\phi_{tt} = \frac{\dot{\phi}^2_{max}}{\ddot{\phi}_{max}}$
   (b) Compute t_1, t_2, and t_f.
   IF $\phi > \phi_{tt}$, THEN :
   $t_1 = t_0 + \frac{\dot{\phi}_{max} - \dot{\phi}_0}{\ddot{\phi}_{max}}$
   $t_2 = t_1 + \frac{1}{\dot{\phi}_max}\left[\phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2}\ddot{\phi}_{max}(t_f 1 - t_0)^2 - \right.$
   $\left.\frac{\dot{\phi}_{max}(\dot{\phi}_{max} - \dot{\phi}_f)}{\ddot{\phi}_{max}} + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}}\right]$
   $t_f = t_1 + \frac{1}{\dot{\phi}_{max}}\left[\phi_f - \dot{\phi}_0(t_1 - t_0) - \frac{1}{2}\ddot{\phi}_{max}(t_1 - t_0)^2 + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}}\right]$.
   ELSE:
   $t_f = \sqrt{\frac{\dot{\phi}^2_{max}}{\ddot{\phi}_{max}}}$
   $t_2 = t_f/2$
   $t_1 = t_2$
   (c) Find ${}^D R^N$: ${}^D R^N = \left[(cos\phi)I_{3x3} + (1 - cos\phi)\hat{e}\hat{e}^T - (sin\alpha)E^x\right]$
   (d) Find ${}_{\mathcal{B}}\dot{\omega}^D$: ${}_{\mathcal{B}}\dot{\omega}^D = {}^D R^N \ddot{\phi}_{max} \cdot_{\mathcal{N}} \hat{e}$

(e) Solve for control torque, u: $J \cdot_{\mathcal{B}} \dot{\omega}^D = u -_{\mathcal{B}} \omega^C \times J \cdot_{\mathcal{B}} \omega^C$

(f) FOR each ϕ between switching times, propagate ω and q between switching times by solving above eqn and $\dot{q} = \frac{1}{2}\Omega q$

where

$$\Omega = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & -\omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & \omega_2 & -\omega_3 & 0 \end{bmatrix}$$

with correct u for each switching time interval.

As can be seen in the results, the angular velocity and acceleration never exceeded the velocity and acceleration constraints for any axis. There is no noise modeled in the actuator system, as the purpose of this simulation was to validate the slewing maneuvers described by the algorithm.

Two cases are shown here - one in which the sun angle is greater than 0 from the slew plane. The other case is one in which the sun vector lies directly on the slew plane, so that $\phi_2 = 180$ degrees.

Case I: $\alpha > 0$

The parameters for a test case in which the sun vector did not lie directly on the slew plane are shown in Table 1. The calculated slew angles are shown in Table 2.

The constraints for maximum angular velocity and acceleration were chosen to demonstrate that the angular velocity and acceleration profiles would

Table 1. Initial, Final, and Sun Positions in Inertial Frame and Constraints (Sample Inputs)

Unit Vector	Comp
P_i	[-0.50, 0.57, 0.65]
P_f	[0.76, -0.48, -0.44]
S	[0.30, -0.50, -0.81]
Parameter	**Constraint**
α_{max}	$1\ rad/s^2$
ω_{max}	$1\ rad/s$

Table 2. Slew Angles ϕ_1, ϕ_2, and ϕ_3

ϕ	deg
1	32.08
2	102.56
3	17.76

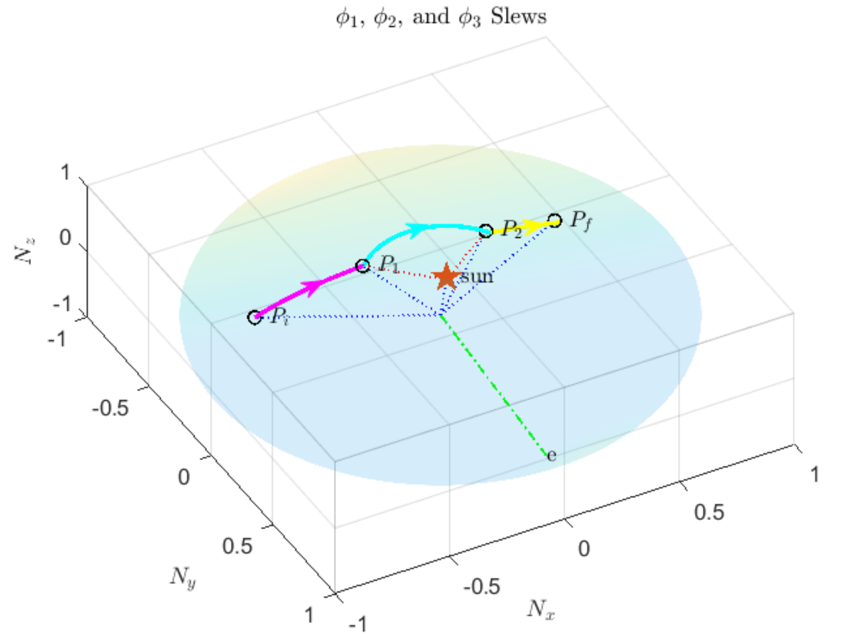


Figure 2. Attitude Profile of the Entire Slew.

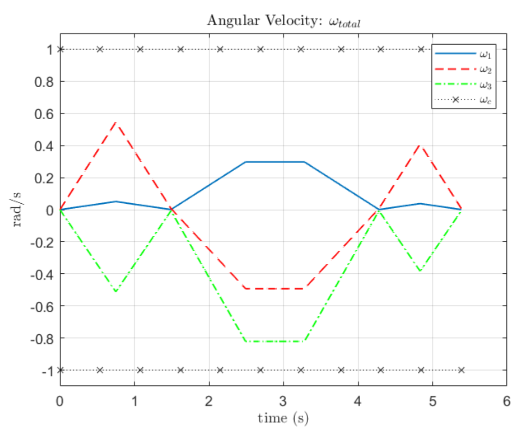


Figure 3. Angular Velocity when $\alpha > 0$.

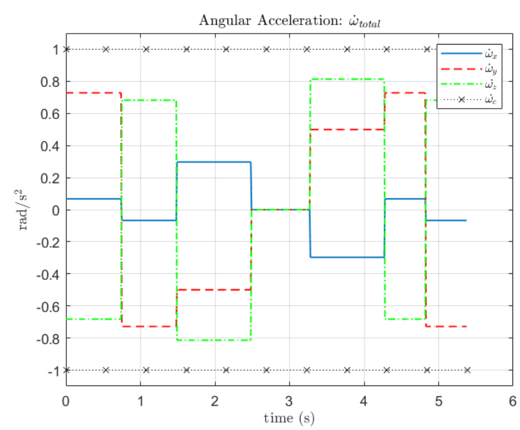
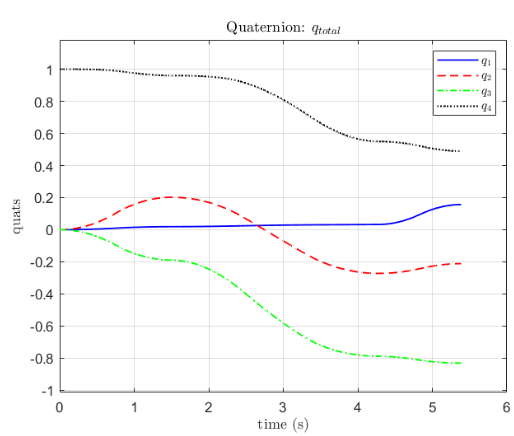


Figure 4. Angular Acceleration when $\alpha > 0$.



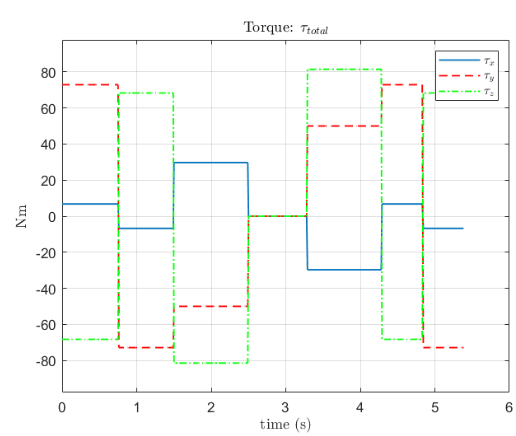


Figure 5. Quaternions when $\alpha > 0$.

Figure 6. Torque when $\alpha > 0$.

Case II: $\alpha = 0$

For the case in which the sun vector lies directly on the slew plane, ϕ_2 = 180 degrees. For this case, P_2 and P_f were almost superimposed; therefore, ϕ_3, the angle between the two vectors appears 0.

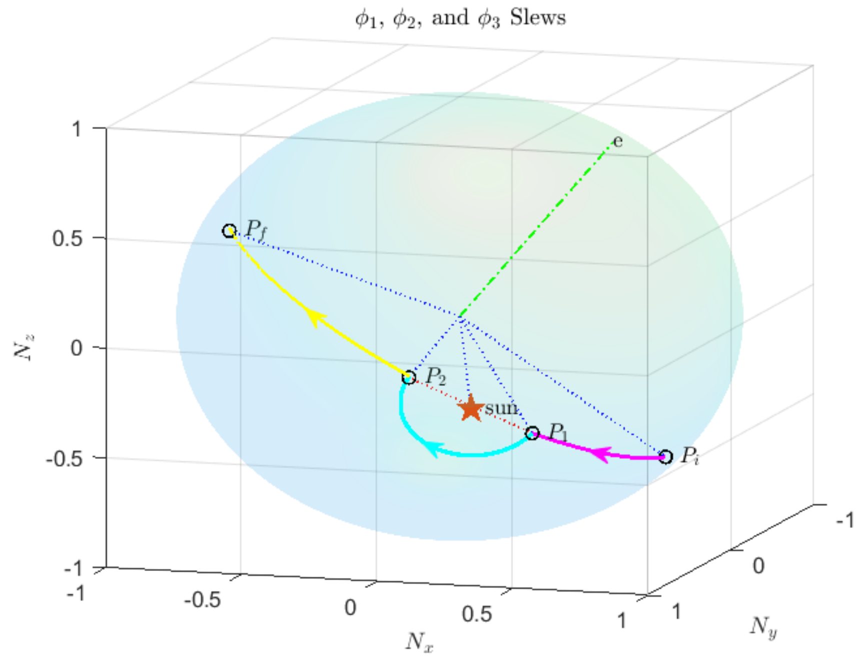


Figure 7. Attitude Profile of the Entire Slew when $\alpha = 0$.

The initial, final, and sun positions in the inertial frame and the constraits are in Table 3.

Table 3. Initial, Final, and Sun Positions in Inertial Frame and Constraints (inputs)

Unit Vector	Comp
P_i	[0.65, -0.35, -0.67]
P_f	[-0.93, -0.25, 0.28]
S	[-0.20, -0.78, -0.59]
Parameter	**Constraint**
α_{max}	0.02 rad/s^2
ω_{max}	0.01 rad/s

Table 4. Slew Angles ϕ_1, ϕ_2, and ϕ_3

ϕ	deg
1	41.82
2	180.00
3	62.45

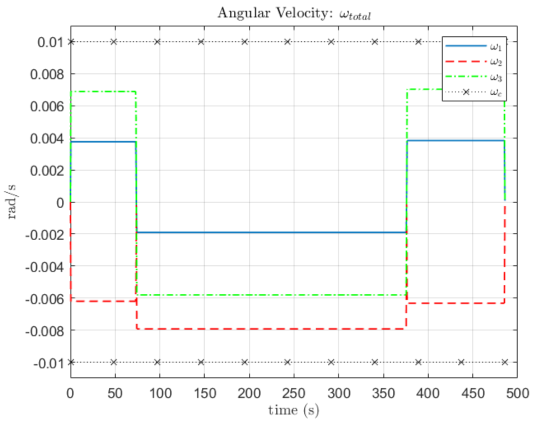


Figure 8. Angular Velocity when $\alpha = 0$.

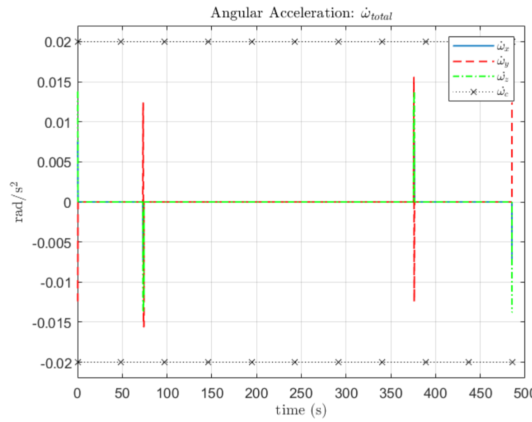


Figure 9. Angular Acceleration when $\alpha = 0$.

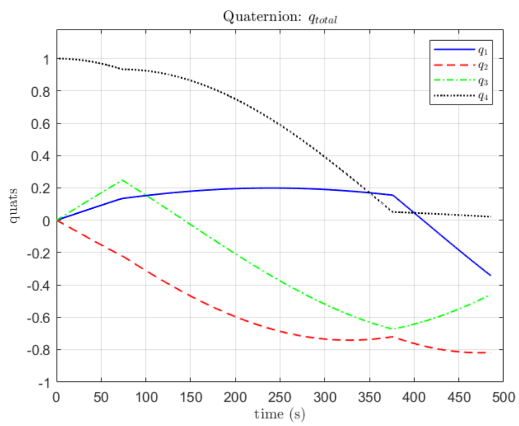


Figure 10. Quaternions when $\alpha = 0$.

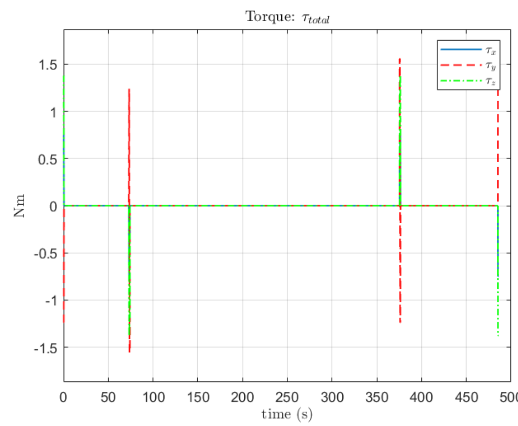


Figure 11. Applied Torque when $\alpha = 0$.

CONCLUSION

A new geometric approach for large-angle slew planning with pointing and actuator constraints is presented. The spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints. Furthermore, we assume that the initial and final attitudes, instrument line-of-sight vector, and sun vector are known. Then the desired or target-frame quaternions, angular velocities, and angular accelerations are derived based on the PMP. The proposed algorithm is intuitive, deterministic, and easy to implement. The main drawback of the proposed algorithm is its limitation for a single sensitive-payload. The feasibility of the proposed algorithm is demonstrated for two arbitrary cases and it has been investigated via extensive numerical simulations.

ACKNOWLEDGMENT

The research of the authors has been supported by Maxar Space Solutions (Formerly Space Systems / Loral). The second author would like to acknowledge Luke DeGalan for his useful comments.

NOTATION

$\mathcal{G}$-frame	gyrostat body-fixed frame
${}^{\mathcal{N}}H^{\mathcal{G}/\mathcal{G}*}$	the total angular momentum of the gyrostat with respect to its center of mass
$I^{\mathcal{G}/\mathcal{G}^*}$	the mass-moment-of-inertia of the gyrostat
$I^{w/w*}$	the mass-moment-of-inertia of reaction wheels with respect to their center of masses
M_{max}	the maximum available torque along the eigenaxis
$\mathcal{N}$-frame	the Newtonian frame
${}_{\mathcal{G}}\hat{P}$	unit vector along the bore sight of payload in the $\mathcal{G}$-frame
${}_{\mathcal{N}}\hat{P}_i$	unit vector of the initial point in the $\mathcal{N}$-frame
${}_{\mathcal{N}}\hat{P}_f$	unit vector of the final point in the $\mathcal{N}$-frame
${}^{\mathcal{N}}q^{\mathcal{T}}$	quaternion of the $\mathcal{T}$-frame in the $\mathcal{N}$-frame
${}_{\mathcal{N}}\hat{S}$	unit vector of the sun vector in the $\mathcal{N}$-frame
$\mathcal{T}$-frame	the target frame
ϵ_p	Payload half-cone angle
${}^{\mathcal{N}}\alpha^{\mathcal{T}}$	angular acceleration of the $\mathcal{T}$-frame in the $\mathcal{N}$-frame
${}^{\mathcal{N}}\omega^{\mathcal{T}}$	angular velocity of the $\mathcal{T}$-frame in the $\mathcal{N}$-frame

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