Sun-Avoidance Slew Planning Algorithm with Pointing and Actuator Constraints (AAS 19-801)

AIAA/AAS Astrodynamics Specialist Conference, Portland, ME, 11 – 15 August 2019

Mohammad. A. Ayoubi[1] Junette Hsin[2]

[1]Associate Professor, Department of Mechanical Engineering, Santa Clara University,
[2]Engineer, Dynamics and Control Analysis Group, Maxar Space Infrastructure (Formerly Space Systems/Loral)

Outline

Introduction

- Large-angle slew maneuvers are required during any Earth-pointing or interplanetary missions.
- For safety in many space missions, a sensitive payload such as an imaging camera or telescope needs to be retargeted while avoiding the sun or other bright objects in the sky.

Previous Literature

The attitude reorientation problem in the presence of attitude constrained zones has been studied in the last three decades:

- *McInnes (1994)*: artificial potential function in the form of an analytical guidance law suitable for onboard implementation.
  - Euler angles were used, which are singular for large slew angles.
- *Spindle (1998), Hablani (1998), Biggs and Colley (2016), Frazzoli (2001)*: geometric approach where a feasible attitude maneuver is precomputed based on avoidance zone constraints for attitude.
  - Depending on the number of constraints and initial and final attitudes, this approach can be computationally expensive.
- *Spiller (2016)*: Particle swarm optimization (PSO) technique to find a sub-optimal solution with keep-out constraints.

SAS Algorithm

- The proposed Sun Avoidance Slew (SAS) algorithm is an intuitive geometric approach for a sun (or any bright object) avoidance slew maneuver with pointing and actuator constraints.
- The algorithm is deterministic, easy to implement, and includes the control-torque and reaction wheels' angular momentum constraints.

SAS Algorithm

Assumptions in this algorithm:

- Spacecraft has a single light-sensitive payload with control-torque and reaction wheels' angular momentum constraints.
- The initial and final attitudes, instrument boresight vector, and sun vector are known.

SAS Algorithm

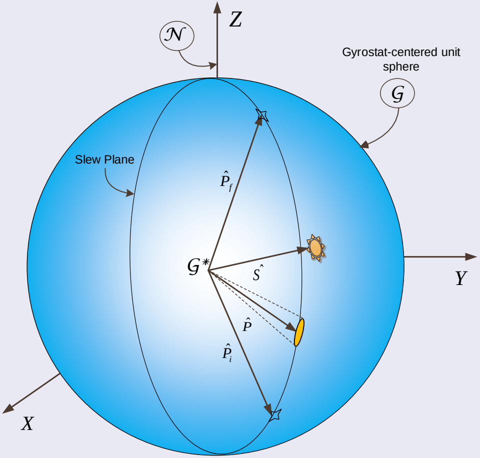


Figure: The gyrostat-centered unit sphere.

SAS Algorithm

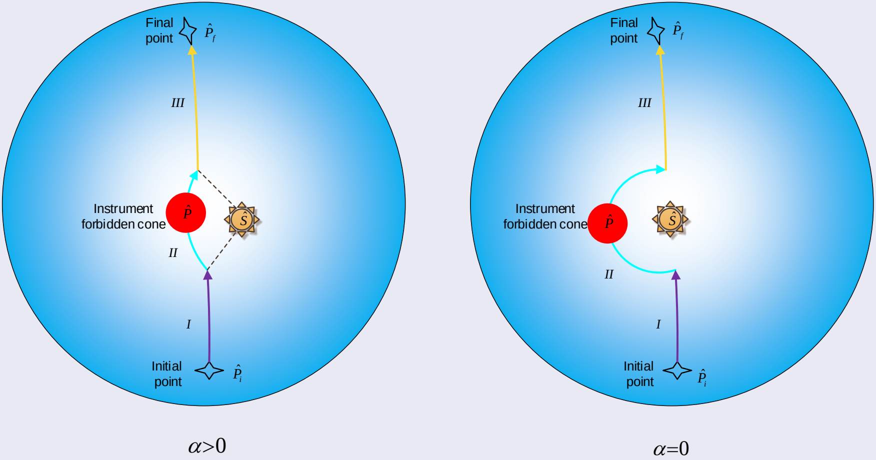
Problem Statement:

Given: ${}_{\mathcal{N}}\hat{P}_i$, ${}_{\mathcal{N}}\hat{P}_f$, ${}_{\mathcal{N}}\hat{S}$, ${}_{\mathcal{G}}\hat{P}$, ϵ_p, ${}^{\mathcal{N}}q^{\mathcal{G}}$, ${}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_i)$, and ${}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_f)$.
Find:

1. A sequence of slew maneuvers to avoid sun vector.
2. The commanded angular velocity, angular acceleration, and quaternion profiles.

SAS Algorithm

Summary



SAS Algorithm

Check the Sun Vector Intrusion

- Check the angular separation, α, between the sun vector, $\hat{S}$, and the $\hat{P}_i - \hat{P}_f$ or "slew" plane.

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot_{\mathcal{N}} \hat{e}) \tag{1}$$

where the eigenaxis is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \tag{2}$$

- IF $|\alpha| < \epsilon_p$,THEN determine the projection of the sun vector into the slew plane.

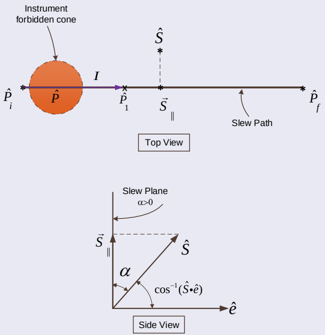
$$\vec{S}_{||} = \hat{S} \cos\alpha \tag{3}$$

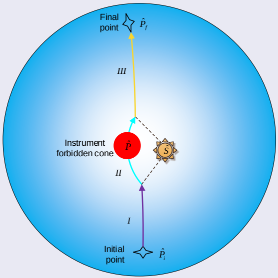
SAS Algorithm

Slew Maneuvers

1. The 1^{st} slew around the eigenaxis, $\hat{e}$, through angle:

$$\phi_1 = \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \quad when \quad \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \leq \pi \tag{4}$$



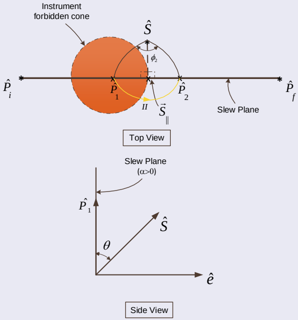


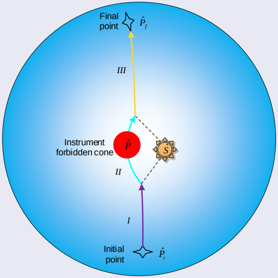
SAS Algorithm

Slew Maneuvers

- II The 2^{nd} slew around the unit sun vector, $\hat{S}$, via ϕ_2.
  - a when $\alpha \neq 0$

$$\phi_2 = 2\tan^{-1}\left[\frac{\hat{S}\cdot(\hat{P}_1 \times \hat{S}_{||})}{(\hat{P}_1\cdot\hat{S}_{||}) - (\hat{S}\cdot\hat{P}_1)(\hat{S}\cdot\hat{S}_{||})}\right], \quad (5)$$

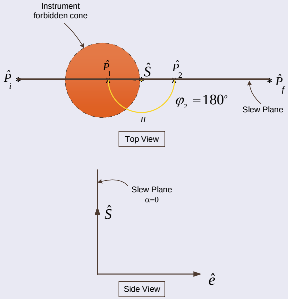


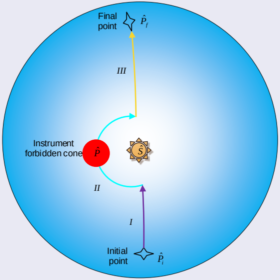


SAS Algorithm

Slew Maneuvers

- The 2^{nd} slew around the unit sun vector, $\hat{S}$, via $\phi_2 = 180°$.
  - b) when $\alpha = 0$



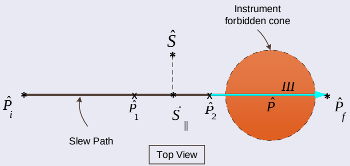


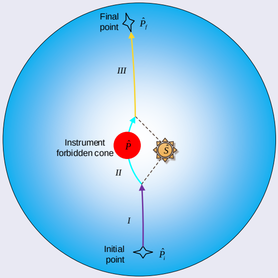
SAS Algorithm

Slew Maneuvers

- III The 3^{rd} slew about the ê through angle:

$$\phi_3 = \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) \quad \textit{when}\ {}_G\hat{P}_f.\hat{P}_2 \geq 0 \tag{6}$$





- IV Optional: The final slew is about the instrument boresight axis to rotate into final attitude.

Summary of SAS Algorithm

1. Slew around the eigenaxis, $\hat{e}$, through angle:

$$\phi_1 = \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \quad \textit{when} \ \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \leq \pi \tag{7}$$

2. Slew around the $\hat{S}$ via:

$$\phi_2 = \begin{cases} \phi_2 = 2\tan^{-1}\left[\frac{\hat{S}\cdot(\hat{P}_1 \times \hat{S}_{||})}{(\hat{P}_1\cdot\hat{S}_{||})-(\hat{S}\cdot\hat{P}_1)(\hat{S}\cdot\hat{S}_{||})}\right], & \textit{when} \ \alpha \neq 0 \\ \pi & \textit{when} \ \alpha = 0 \end{cases} \tag{8}$$

3. Slew about the ê through angle:

$$\phi_3 = \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) \quad \textit{when} \ {}_G\hat{P}_f.\hat{P}_2 \geq 0 \tag{9}$$

4. Optional: perform the final rotation, ϕ_4, about the instrument boresight axis to adjust the attitude.

Computing the Steering Profiles

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints.

Computing Steering Profiles

Problem Statement

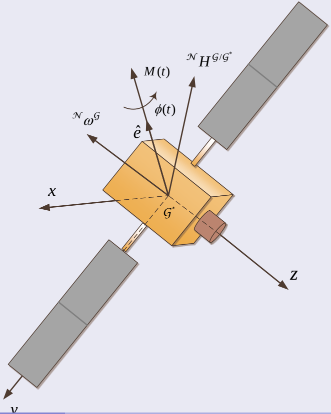
Consider the motion of a rigid spacecraft around a given inertially-fixed axis, ${}_G\hat{e} = [e_x, e_y, e_z]^T$. The problem of minimum-time slew maneuver around the $\hat{e}$ axis can be formulated as

$$\underset{u}{Minimze}\; J[x(.), u(.), t_f] = \int_{t0}^{t_f} dt, \tag{10}$$

subject to the following dynamic constraint

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M/I_{\hat{e}}^{G/G*} = u, \end{cases} \tag{11}$$

where $x_1 \triangleq \phi$ and $x_2 = \dot{\phi}$.



Computing Steering Profiles

Problem Statement Continued

The boundary conditions are

$$BCs : \begin{cases} \phi(t_0) = 0, \phi(t_f) = \phi_f, \\ \dot{\phi}(t_0) = \dot{\phi}_0, \dot{\phi}(t_f) = \dot{\phi}_f, \end{cases} \tag{12}$$

and velocity (state) and acceleration (control) constraints are

$$C_1 : \begin{cases} |x_2 = \dot{\phi}| \leq \dot{\phi}_{max}, \\ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}, \end{cases} \tag{13}$$

Find: $\phi^*(t)$, $\dot{\phi}^*(t)$, and $\ddot{\phi}^*(t)$.

Computing Steering Profiles

SAS Algorithm

Pontryagin's minimum principle (PMP) is used to derive the desired or target-frame quaternions, angular velocity and acceleration for two cases:

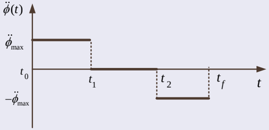
1. with control-torque and reaction wheels' angular momentum constraints
2. with control-torque constraints

Numerical simulation is performed to show the viability of the proposed algorithm with control-torque and angular momentum constraints.

Computing Steering Profiles

- Angular acceleration profile (bang-off-bang):

$$\ddot{\phi}(t) = u = \begin{cases} \ddot{\phi}_{max} & \text{when } t_0 \le t \le t_1, \\ 0 & \text{when } t_1 \le t \le t_2, \\ -\ddot{\phi}_{max} & \text{when } t_2 \le t \le t_f. \end{cases} \tag{14}$$



- Angular velocity profile:

$$\dot{\phi}(t) = \begin{cases} \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0) & \text{when } t_0 \le t \le t_1, \\ \dot{\phi}_{max} & \text{when } t_1 \le t \le t_2, \\ \dot{\phi}_{max} - \ddot{\phi}_{max}(t - t_2) & \text{when } t_2 \le t \le t_f. \end{cases} \tag{15}$$

- Angular position profile:

$$\phi(t) = \begin{cases} \dot{\phi}_0(t - t_0) + \frac{1}{2}\ddot{\phi}_{max}(t - t_0)^2 & \text{when } t_0 \le t \le t_1, \\ \phi(t_1) + \dot{\phi}_{max}(t - t_1) & \text{when } t_1 \le t \le t_2, \\ \phi(t_2) + \dot{\phi}_{max}(t - t_2) - \frac{1}{2}\ddot{\phi}_{max}(t - t_2)^2 & \text{when } t_2 \le t \le t_f. \end{cases} \tag{16}$$

Computing Steering Profiles

- Using the conditions, $\dot{\phi}(t_1) = \dot{\phi}_{max}$, $\dot{\phi}(t_f) = \dot{\phi}_f$, $\phi(t_f) = \phi_f$, we can determine switching times t_1, t_2, and final time t_f as:

$$t_1 = t_0 + \frac{\dot{\phi}_{max} - \dot{\phi}_0}{\ddot{\phi}_{max}}, \tag{17}$$

$$\begin{aligned} t_2 = & t_1 + \frac{1}{\dot{\phi}_{max}} \Bigg[\phi_f - \dot{\phi}_0 (t_1 - t_0) - \frac{1}{2} \ddot{\phi}_{max} (t_1 - t_0)^2 \\ & - \frac{\dot{\phi}_{max} (\dot{\phi}_{max} - \dot{\phi}_f)}{\ddot{\phi}_{max}} + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}} \Bigg], \end{aligned} \tag{18}$$

and

$$t_f = t_1 + \frac{1}{\dot{\phi}_{max}} \left[\phi_f - \dot{\phi}_0 (t_1 - t_0) - \frac{1}{2} \ddot{\phi}_{max} (t_1 - t_0)^2 + \frac{(\dot{\phi}_{max} - \dot{\phi}_f)^2}{2\ddot{\phi}_{max}} \right]. \tag{19}$$

Computing Steering Profiles

- Desired acceleration, ${}_{\mathcal{B}}\dot{\omega}^{D}$, in the body frame:

$$ {}_{\mathcal{B}}\dot{\omega}^{D} = {}^{D}R^{N}\ddot{\phi}_{max} \cdot_{\mathcal{N}} \hat{e} \tag{20} $$

- Find rotation from inertial to desired body frame, where E^{x} is the skew-symmetric matrix form of the cross product with ${}_{\mathcal{N}}\hat{e}$:

$$ {}^{D}R^{N} = \left[(cos\phi) I_{3x3} + (1 - cos\phi)\hat{e}\hat{e}^{T} - (sin\alpha)E^{x} \right] \tag{21} $$

- Solve for desired control torque, u, with current velocity, ${}_{\mathcal{B}}\omega^{C}$, and Euler's equations for rigid body dynamics:

$$ J \cdot_{\mathcal{B}} \dot{\omega}^{D} = u -_{\mathcal{B}} \omega^{C} \times J \cdot_{\mathcal{B}} \omega^{C} \tag{22} $$

- Apply the desired control torque and solve the dynamical equations and kinematic equations of motion to propagate angular velocity and attitude.

Numerical Simulations

Introduction

- Numerical simulations were run examine the proposed algorithm.
- The initial, final, and sun position vectors were randomized for each run.
- Two cases shown in these slides - one in which the sun angle, α, is greater than 0 from the slew plane, the other in which the sun vector lies directly on the slew plane.
  1. $\alpha > 0$
  2. $\alpha = 0$

Numerical Simulations

$\alpha > 0$

Table: Initial, Final, and Sun Positions in Inertial Frame and Constraints (Sample Inputs)

Unit Vector	Comp
P_i	(-0.50, 0.57, 0.65)
P_f	(0.76, -0.48, -0.44)
S	(0.30, -0.50, -0.81)
Parameter	**Constraint**
α_{max}	$1\ rad/s^2$
ω_{max}	$1\ rad/s$

Table: Slew Angles ϕ_1, ϕ_2, and ϕ_3

ϕ	deg
1	32.08
2	102.56
3	17.76

Numerical Simulations

$\alpha > 0$

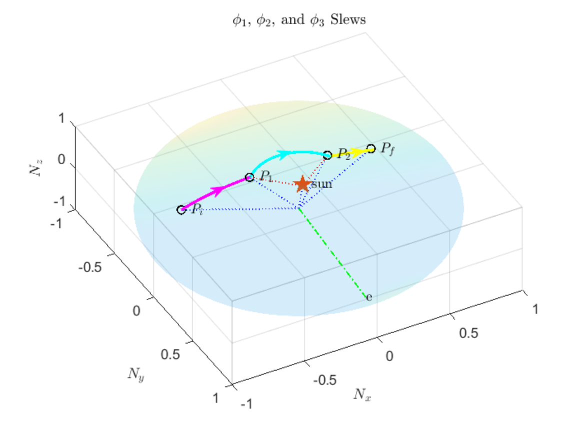


Figure: Attitude Profile of the Entire Slew

Numerical Simulations

$\alpha > 0$

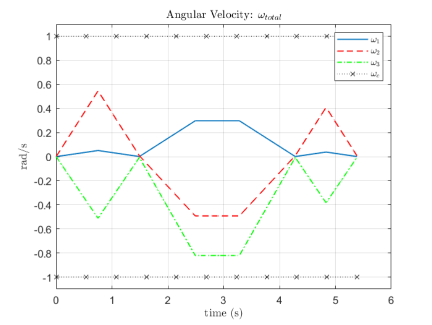


Figure: Angular Velocity. ω_c is the velocity constraint

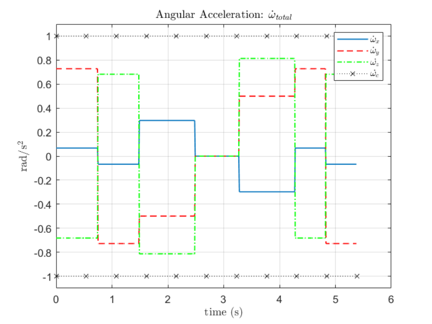


Figure: Angular Acceleration. $\dot{\omega}_c$ is the acceleration constraint

Numerical Simulations

$\alpha > 0$

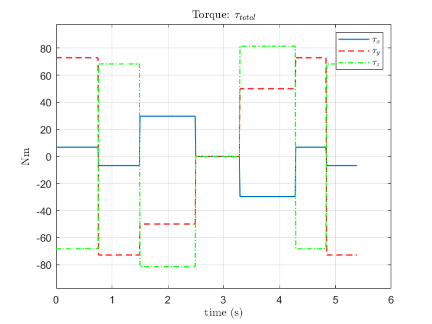


Figure: Torque

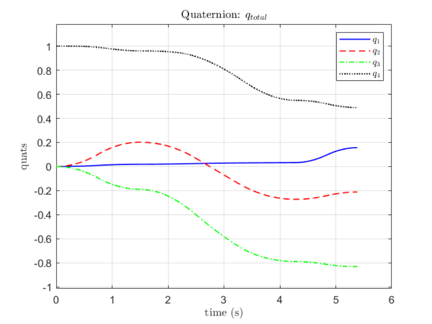


Figure: Quaternion Attitude

Summary and Conclusion

- Developed intuitive, geometric approach for large-angle sun (or any bright object) avoidance slew with pointing and actuator constraints.
- SAS algorithm is deterministic, easy to implement, and includes the control-torque and reaction wheels' angular momentum constraints.
- Assumed that initial and final attitudes, instrument boresight, and sun vector are known.
- Target-frame quaternions, angular velocities, and angular accelerations are derived based on the PMP.
- Limitation is for single sensitive-payload.

Acknowledgments

The research has been supported by Maxar Space Infrastructure (formerly Space Systems/Loral). We would also like to acknowledge Luke DeGalan for his useful comments.

Q & A

Back-up Slides

Computing the Steering Profiles

Case II: Single-Axis, Agile Slew Maneuver with Acceleration Constraint.

Single-Axis, Agile Slew Maneuver with Acceleration Constraint

Problem Statement:

Consider the optimal control problem described by Eqs.(**??**), (**??**), (**??**), and subject to control constraint

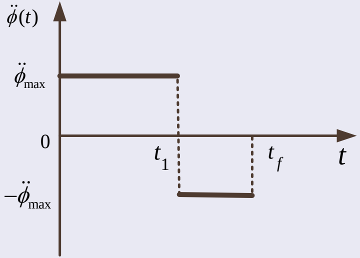
$$C_2 : \ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}. \quad (23)$$

Find: $\phi(t)$, $\dot{\phi}(t)$, and $\ddot{\phi}(t)$.

Single-Axis, Agile Slew Maneuver with Acceleration Constraint

- Angular acceleration about the $\hat{e}$ axis:

$$\ddot{\phi}(t) = \ddot{\phi}_{max}\mathbb{U}(t_0) - 2\ddot{\phi}_{max}\mathbb{U}(t - t_1) \tag{24}$$



where the switching and the final times are given by

$$t_1 = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}^2_{max}(2\ddot{\phi}_{max}\phi_f + \dot{\phi}^2_f + \dot{\phi}^2_0)}}{\sqrt{2}\ddot{\phi}^2_{max}} \tag{25}$$

Single-Axis, Agile Slew Maneuver with Acceleration Constraint

and

$$t_f = t_0 - \frac{\dot{\phi}_f + \dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2}\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_f + \dot{\phi}_{ef}^2 + \dot{\phi}_0^2)}}{\ddot{\phi}_{max}^2} \quad (26)$$

- Angular velocity about the $\hat{e}$ axis:

$$\dot{\phi}(t) = \dot{\phi}_0 + \ddot{\phi}_{max}(t - t_0)\mathbb{U}(t_0) - 2\ddot{\phi}_{max}(t - t_1)\mathbb{U}(t - t_1) \quad (27)$$

- Angular position about the $\hat{e}$ axis:

$$\begin{aligned}\phi(t) = \dot{\phi}_0(t - t_0) + \ddot{\phi}_{max}\frac{(t - t_0)^2}{2}\mathbb{U}(t_0) \\ - 2\ddot{\phi}_{max}\frac{(t - t_1)^2}{2}\mathbb{U}(t - t_1)\end{aligned} \quad (28)$$

The Agile Sun-Avoidance Slew Maneuver

The First Slew Maneuver: A single-axis nonrest-to-rest maneuver around the ê

- The BCs:

$$\dot{\phi}(t_0) = \dot{\phi}_0, \phi(t_0) = 0, \dot{\phi}(t_{f1}) = 0, \phi(t_{f1}) = \phi_1. \tag{29}$$

The switching time, t_{11}, and minimum-time, t_{f1}, are

$$t_{11} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{\ddot{\phi}^2_{max}(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}^2_0)}}{\sqrt{2}\ddot{\phi}^2_{max}} \tag{30}$$

$$t_{f1} = t_0 - \frac{\dot{\phi}_0}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}^2_{max}(2\ddot{\phi}_{max}\phi_1 + \dot{\phi}^2_0)}}{\ddot{\phi}^2_{max}} \tag{31}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (**??**) and t_{11} and t_{f1} in to Eqs. (**??**), (**??**), and (**??**), respectively.

The Agile Sun-Avoidance Slew Maneuver

The Second Slew Maneuver: A rest-to-rest maneuver around the sun vector

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f2}) = 0, \phi(t_{f2}) = \phi_2. \tag{32}$$

The switching time, t_{12}, and the minimum-time, t_{f2}, are

$$t_{12} = t_0 - \frac{\sqrt{\phi_2}}{\ddot{\phi}_{max}} \tag{33}$$

$$t_{f2} = t_0 - \frac{2\sqrt{\phi_2}}{\ddot{\phi}_{max}} \tag{34}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (**??**) and t_{12} and t_{f2} in to Eqs. (**??**), (**??**), and (**??**), respectively.

The Agile Sun-Avoidance Slew Maneuver

The Third Slew Maneuver: A single-axis rest-to-nonrest maneuver around the $\hat{e}$

- The BCs:

$$\dot{\phi}(t_0) = 0, \phi(t_0) = 0, \dot{\phi}(t_{f3}) = \dot{\phi}_f, \phi(t_{f3}) = \phi_3. \tag{35}$$

The switching time, t_{13}, and the minimum-time, t_{f3}, are

$$t_{13} = t_0 + \frac{\sqrt{\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\sqrt{2}\ddot{\phi}_{max}^2} \tag{36}$$

$$t_{f3} = t_0 - \frac{\dot{\phi}_f}{\ddot{\phi}_{max}} + \frac{\sqrt{2\ddot{\phi}_{max}^2(2\ddot{\phi}_{max}\phi_3 + \dot{\phi}_f^2)}}{\ddot{\phi}_{max}^2} \tag{37}$$

The $\ddot{\phi}(t)$, $\dot{\phi}(t)$, and $\phi(t)$, can be found by substituting the boundary conditions given by (**??**) and t_{13} and t_{f3} in to Eqs. (**??**), (**??**), and (**??**), respectively.

Computing Steering Profiles

Using the Pontryagin's minimum principle (PMP), we derive the necessary conditions for the optimal solution as follows:

1. State Eqs.:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \\ \dot{x}_3 = (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}), \end{cases} \tag{38}$$

   where the unit step function, $\mathbb{U}$, is defined as

$$\mathbb{U}(X) = \begin{cases} 1, X > 0, \\ 0, X \leq 0. \end{cases} \tag{39}$$

   Note: $(x_3(t_0) = x_3(t_f) = 0 \ \& \ x_3(t) \geq 0\ ) \rightarrow x_3(t) = 0, \forall t \in [t_0, t_f]$.

2. Hamiltonian:

$$\begin{aligned} \mathscr{H} =& 1 + \lambda_1 x_2 + \lambda_2 u + \lambda_3 \Big[(x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ & (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}) \Big] \end{aligned} \tag{40}$$

Computing Steering Profiles

3. Determining the optimal control in the singular arc:

$$\frac{d^2}{dt^2}\left(\frac{\partial \mathscr{H}}{\partial u}\right) = \ddot{\lambda}_2 = 0 \rightarrow \dot{x}_2 = 0 \rightarrow u^* = 0 \tag{41}$$

4. Checking the Generalized Legendre-Clebsch condition for optimality:

$$(-1)^2 \frac{\partial}{\partial u}\left[\frac{d^2}{dt^2}\left(\frac{\partial \mathscr{H}}{\partial u}\right)\right] = 1 \geq 0 \tag{42}$$

5. The transversality condition:

$$\mathscr{H}|_{(*,t_f)} = 0 \text{ and } \mathscr{H} \neq \mathscr{H}(t) \rightarrow \mathscr{H} = 0, \forall t \in [t_0, t_f]. \tag{43}$$

Numerical Simulations

$\alpha = 0$

Table: Initial, Final, and Sun Positions in Inertial Frame and Constraints (inputs)

Unit Vector	Comp
P_i	(0.65, -0.35, -0.67)
P_f	(-0.93, -0.25, 0.28)
S	(-0.20, -0.78, -0.59)
Parameter	**Constraint**
α_{max}	0.02 rad/s^2
ω_{max}	0.01 rad/s

Table: Slew Angles ϕ_1, ϕ_2, and ϕ_3

ϕ	deg
1	41.82
2	180
3	62.45

Numerical Simulations

$\alpha = 0$

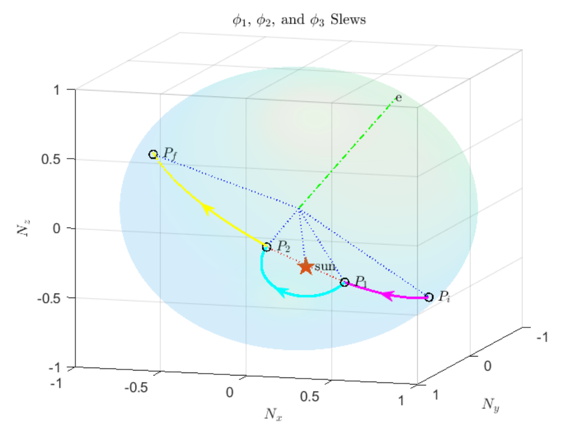


Figure: Attitude Profile of the Entire Slew

Numerical Simulations

$\alpha = 0$

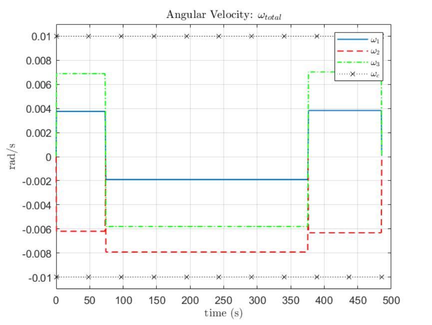


Figure: Angular Velocity. ω_c is the velocity constraint

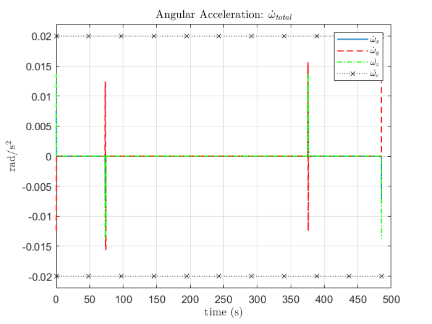


Figure: Angular Acceleration. $\dot{\omega_c}$ is the acceleration constraint

Numerical Simulations

$\alpha = 0$

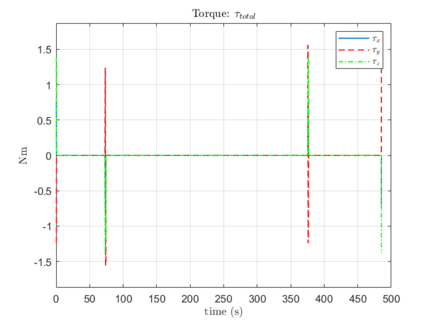


Figure: Torque

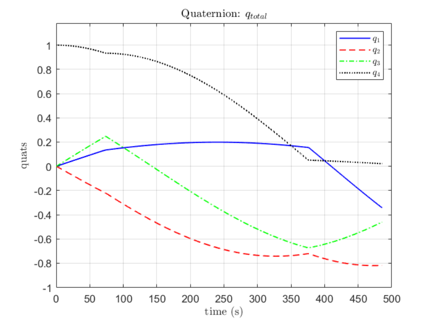


Figure: Quaternion Attitude