Sun-Avoidance Slew Planning Algorithm with Pointing and Actuator Constraints
AAS 10-801

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Outlines

Introduction

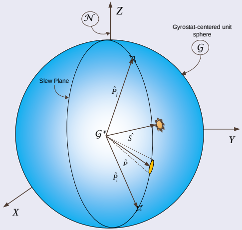
1. Geometric approach for large-angle slew maneuvers with pointing and actuator constraints
2. Assumed single light-sensitive payload with control-torque and reaction wheel's angular momentum constraints
3. Assumed initial and final attitudes, instrument boresight vector, and sun vector are known
4.

Sun-Avoidance Slew (SAS) Algorithm

Problem Statement:

Given:${}_{\mathcal{N}}\hat{P}_i$, ${}_{\mathcal{N}}\hat{P}_f$, ${}_{\mathcal{N}}\hat{S}$, ${}_{\mathcal{G}}\hat{P}$, ϵ_p, ${}^{\mathcal{N}}q^{\mathcal{G}}$, ${}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_i)$, and ${}^{\mathcal{N}}_{\mathcal{G}}\omega^{\mathcal{G}}(t_f)$.
Assumption: The spacecraft is rigid.
Find:

1. A sequence of slew maneuvers to avoid sun vector.
2. the commanded angular velocity, angular acceleration, and quaternion profiles.



Sun-Avoidance Slew (SAS) Algorithm

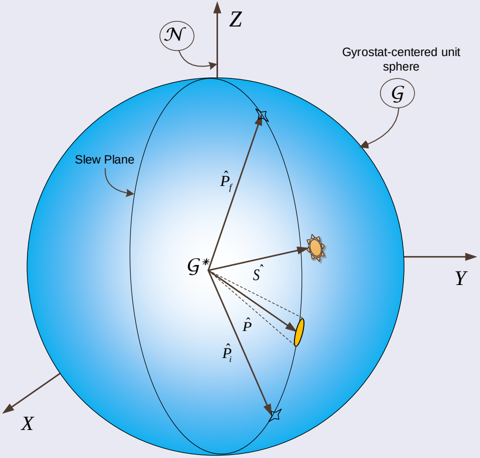


Figure: The gyrostat-centered unit sphere.

Sun-Avoidance Slew (SAS) Algorithm

Nomenclature

- $\mathcal{G}$ frame: Unit sphere attached to the gyrostat.
- $\mathcal{N}$: frame: The Newtonian frame fixed in the inertial space.
- ${}_{\mathcal{G}}\hat{P}$: Unit vector along the bore sight of payload in the $\mathcal{G}$ frame.
- ${}_{\mathcal{G}}\hat{P}_i$: Unit vector of the initial point in the $\mathcal{G}$ frame.
- ${}_{\mathcal{G}}\hat{P}_f$: Unit vector of the final point in the $\mathcal{G}$ frame.
- ${}_{\mathcal{N}}\hat{S}$: Unit vector of the sun vector in the $\mathcal{N}$ frame.
- ϵ_p: Payload half-cone angle.

Sun-Avoidance Slew (SAS) Algorithm

Check the Sun Vector Intrusion

1. Check the angular separation between the sun vector, $\hat{S}$, and the $\hat{P}_i - \hat{P}_f$ or "slew" plane.

$$\alpha = \frac{\pi}{2} - \cos^{-1}(\hat{S} \cdot_{\mathcal{N}} \hat{e}) \tag{1}$$

   where the eigenaxis is determined by

$$\hat{e} = \frac{\hat{P}_i \times \hat{P}_f}{|\hat{P}_i \times \hat{P}_f|} \tag{2}$$

2. IF $|\alpha| < \epsilon_p$,THEN determine the projection of the sun vector into the slew plane.

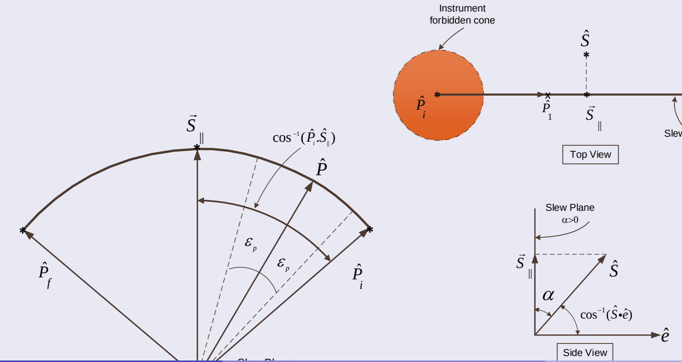
$$\vec{S}_{||} = \hat{S} \cos\alpha \tag{3}$$

Sun-Avoidance Slew (SAS) Algorithm

Slew Maneuvers

1. The 1^{st} slew around the eigenaxis, $\hat{e}$, through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p & \mathit{when}\ \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p - 2\pi & \mathit{when}\ \cos^{-1}(\hat{P}_i \cdot_G \hat{S}_{||}) - \epsilon_p > \pi \end{cases} \tag{4}$$

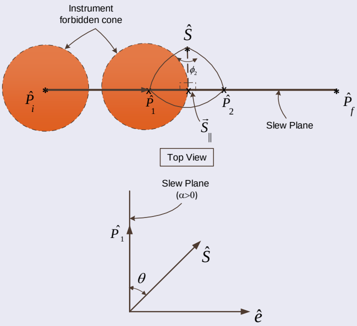


Sun-Avoidance Slew (SAS) Algorithm

Slew Maneuvers

2. The 2^{nd} slew around the unit sun vector, $\hat{S}$, via ϕ_2.
   a. when $\alpha \neq 0$

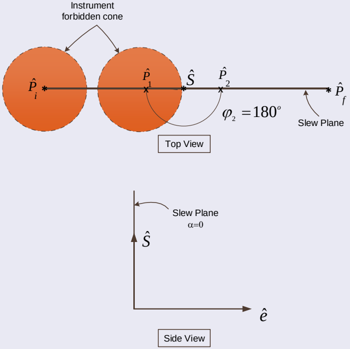
$$\phi_2 = \phi_2 = 2\tan^{-1}\left[\frac{\hat{S}\cdot(\hat{P}_1 \times \hat{S}_{||})}{(\hat{P}_1\cdot\hat{S}_{||}) - (\hat{S}\cdot\hat{P}_1)(\hat{S}\cdot\hat{S}_{||})}\right], \qquad (5)$$



Sun-Avoidance Slew (SAS) Algorithm

Slew Maneuvers

2. The 2^{nd} slew around the unit sun vector, $\hat{S}$, via $\phi_2 = 180°$.
   - b) when $\alpha = 0$

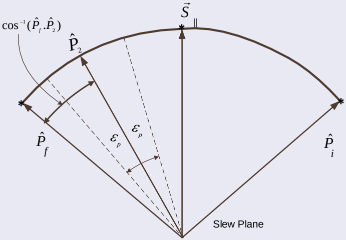


Sun-Avoidance Slew (SAS) Algorithm

Slew Maneuvers

3. The 3^{rd} slew about the $\hat{e}$ through angle:

$$\phi_3 = \begin{cases} \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) & when\ {}_G\hat{P}_f.\hat{P}_2 \geq 0 \\ \cos^{-1}({}_G\hat{P}_f.\hat{P}_2) - 2\pi & when\ {}_G\hat{P}_f.\hat{P}_2 < 0 \end{cases} \quad (6)$$



4. The final slew is about the instrument boresight axis to go to the final attitude.

Summary of the Algorithm

1. Slew around the eigenaxis,$\hat{e}$, through angle:

$$\phi_1 = \begin{cases} \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p & when \ \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p \leq \pi \\ \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p - 2\pi & when \ \cos^{-1}(\hat{P}_i \cdot_{\mathcal{G}} \hat{S}_{||}) - \epsilon_p > \pi \end{cases} \tag{7}$$

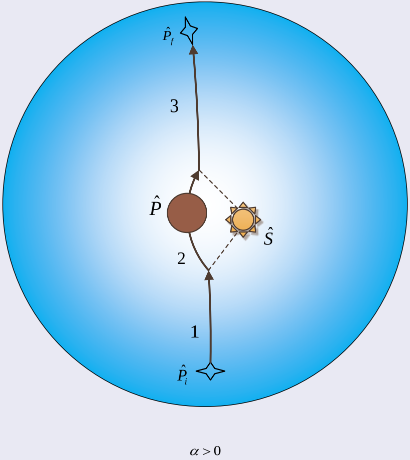
2. Slew around the $\hat{S}$ via:

$$\phi_2 = \begin{cases} \phi_2 = 2\tan^{-1}\left[\frac{\hat{S}\cdot(\hat{P}_1 \times \hat{S}_{||})}{(\hat{P}_1\cdot\hat{S}_{||})-(\hat{S}\cdot\hat{P}_1)(\hat{S}\cdot\hat{S}_{||})}\right], & when \ \alpha \neq 0 \\ \pi & when \ \alpha = 0 \end{cases} \tag{8}$$

3. Slew about the $\hat{e}$ through angle:

$$\phi_3 = \begin{cases} \cos^{-1}({}_{\mathcal{G}}\hat{P}_f.\hat{P}_2) & when \ {}_{\mathcal{G}}\hat{P}_f.\hat{P}_2 \geq 0 \\ \cos^{-1}({}_{\mathcal{G}}\hat{P}_f.\hat{P}_2) - 2\pi & when \ {}_{\mathcal{G}}\hat{P}_f.\hat{P}_2 < 0 \end{cases} \tag{9}$$

Summary of the Algorithm



Computing the Steering Profiles

Single-Axis, Agile Slew Maneuver with Velocity and Acceleration Constraints.

Computing Steering Profiles

Problem Statement

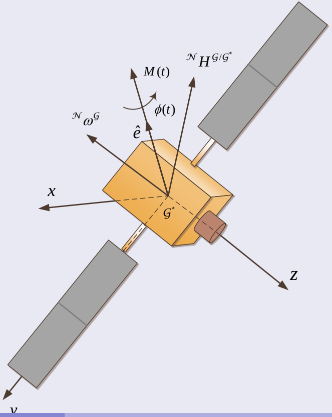
Consider the motion of a rigid spacecraft around a given inertially-fixed axis, ${}_G\hat{e} = [e_x, e_y, e_z]^T$. The problem of minimum-time slew maneuver around the $\hat{e}$ axis can be formulated as

$$\underset{u}{Minimze}\ J[x(.), u(.), t_f] = \int_{t0}^{t_f} dt, \tag{10}$$

subject to the following dynamic constraint

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M/I_{\hat{e}}^{G/G*} = u, \end{cases} \tag{11}$$

where $x_1 \triangleq \phi$ and $x_2 = \dot{\phi}$.



Computing Steering Profiles

Problem Statement Continued

The boundary conditions are

$$BCs : \begin{cases} \phi(t_0) = 0, \phi(t_f) = \phi_f, \\ \dot{\phi}(t_0) = \dot{\phi}_0, \dot{\phi}(t_f) = \dot{\phi}_f, \end{cases} \tag{12}$$

and velocity (state) and acceleration (control) constraints are

$$C_1 : \begin{cases} |x_2 = \dot{\phi}| \leq \dot{\phi}_{max}, \\ |u = \ddot{\phi}| \leq \ddot{\phi}_{max}, \end{cases} \tag{13}$$

in which

$$\dot{\phi}_{max} = [I^{w/w^*}]^{-1}[{}^N H^{G/G*} - (I^{G/G*} + I^{w/w*})^N \omega^G]/(e_x + e_y + e_z), \tag{14}$$

Computing Steering Profiles

Problem Statement Continued

and

$$\ddot{\phi}_{max} = {}_B\,\hat{e}^T\ {}_B M_{max}/I_{\hat{e}}^{G/G*}, \tag{15}$$

where ${}^N H^{G/G*}$ is the total angular momentum of the gyrostat with respect to its center of mass, G^*, in the N-frame. I^{G/G^*} and $I^{w/w*}$ represent the inertia dyadic of the gyrostat and reaction wheel with respect to their center of masses, respectively. ${}_B M_{max}$ is the maximum generated torque along the body-axes in the body frame.

Find: $\phi(t)$, $\dot{\phi}(t)$, and $\ddot{\phi}(t)$.

Computing Steering Profiles

Using the Pontryagin's minimum principle (PMP), we derive the necessary conditions for the optimal solution as follows:

1. State Eqs.:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \\ \dot{x}_3 = (x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) + (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}), \end{cases} \tag{16}$$

   where the unit step function, $\mathbb{U}$, is defined as

$$\mathbb{U}(X) = \begin{cases} 1, & X > 0, \\ 0, & X \leq 0. \end{cases} \tag{17}$$

   Note: $(x_3(t_0) = x_3(t_f) = 0 \;\&\; x_3(t) \geq 0\,) \rightarrow x_3(t) = 0, \forall t \in [t_0, t_f]$.

2. Hamiltonian:

$$\begin{aligned} \mathscr{H} =& 1 + \lambda_1 x_2 + \lambda_2 u + \lambda_3 \Big[(x_2 + \dot{\phi}_{max})^2 \mathbb{U}(-x_2 - \dot{\phi}_{max}) \\ & (\dot{\phi}_{max} - x_2)^2 \mathbb{U}(x_2 - \dot{\phi}_{max}) \Big] \end{aligned} \tag{18}$$

Computing Steering Profiles

3. Costate Eqs.:
4. Applying the Pontryagin's minimum principle (PMP),

$$u^* = \underset{u \in \mathcal{U}}{argmin} \mathscr{H}, \tag{19}$$

where $\mathcal{U}$ defines the domain of feasible controls. The optimal control can be determined as

$$u^*(t) = \begin{cases} \ddot{\phi}_{max} \lambda_2 < 0, \\ ?\lambda_2 = 0, \\ -\ddot{\phi}_{max} \lambda_2 > 0. \end{cases} \tag{20}$$