Contents

Preface *page* ix
Common Optimization Techniques, Equations, Symbols, and Acronyms xiii

Part I Dimensionality Reduction and Transforms 1

1 Singular Value Decomposition (SVD) 3
1.1 Overview 3
1.2 Matrix Approximation 7
1.3 Mathematical Properties and Manipulations 10
1.4 Pseudo-Inverse, Least-Squares, and Regression 15
1.5 Principal Component Analysis (PCA) 21
1.6 Eigenfaces Example 25
1.7 Truncation and Alignment 30
1.8 Randomized Singular Value Decomposition 37
1.9 Tensor Decompositions and N-Way Data Arrays 41

2 Fourier and Wavelet Transforms 47
2.1 Fourier Series and Fourier Transforms 47
2.2 Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT) 56
2.3 Transforming Partial Differential Equations 63
2.4 Gabor Transform and the Spectrogram 69
2.5 Wavelets and Multi-Resolution Analysis 75
2.6 2D Transforms and Image Processing 77

3 Sparsity and Compressed Sensing 84
3.1 Sparsity and Compression 84
3.2 Compressed Sensing 88
3.3 Compressed Sensing Examples 92
3.4 The Geometry of Compression 95
3.5 Sparse Regression 98
3.6 Sparse Representation 103
3.7 Robust Principal Component Analysis (RPCA) 107
3.8 Sparse Sensor Placement 110

Part II Machine Learning and Data Analysis 115

4 Regression and Model Selection 117
4.1 Classic Curve Fitting 118
4.2 Nonlinear Regression and Gradient Descent 123
4.3 Regression and $\mathbf{Ax} = \mathbf{b}$: Over- and Under-Determined Systems 130
4.4 Optimization as the Cornerstone of Regression 136
4.5 The Pareto Front and *Lex Parsimoniae* 140
4.6 Model Selection: Cross-Validation 143
4.7 Model Selection: Information Criteria 148

5 Clustering and Classification 154
5.1 Feature Selection and Data Mining 155
5.2 Supervised versus Unsupervised Learning 160
5.3 Unsupervised Learning: *k*-means Clustering 164
5.4 Unsupervised Hierarchical Clustering: Dendrogram 168
5.5 Mixture Models and the Expectation-Maximization Algorithm 172
5.6 Supervised Learning and Linear Discriminants 176
5.7 Support Vector Machines (SVM) 180
5.8 Classification Trees and Random Forest 185
5.9 Top 10 Algorithms in Data Mining 2008 190

6 Neural Networks and Deep Learning 195
6.1 Neural Networks: 1-Layer Networks 196
6.2 Multi-Layer Networks and Activation Functions 199
6.3 The Backpropagation Algorithm 204
6.4 The Stochastic Gradient Descent Algorithm 209
6.5 Deep Convolutional Neural Networks 212
6.6 Neural Networks for Dynamical Systems 216
6.7 The Diversity of Neural Networks 220

Part III Dynamics and Control 227

7 Data-Driven Dynamical Systems 229
7.1 Overview, Motivations, and Challenges 230
7.2 Dynamic Mode Decomposition (DMD) 235
7.3 Sparse Identification of Nonlinear Dynamics (SINDy) 247
7.4 Koopman Operator Theory 257
7.5 Data-Driven Koopman Analysis 266

8 Linear Control Theory 276
8.1 Closed-Loop Feedback Control 277
8.2 Linear Time-Invariant Systems 281
8.3 Controllability and Observability 287
8.4 Optimal Full-State Control: Linear Quadratic Regulator (LQR) 292

8.5 Optimal Full-State Estimation: The Kalman Filter 296
8.6 Optimal Sensor-Based Control: Linear Quadratic Gaussian (LQG) 299
8.7 Case Study: Inverted Pendulum on a Cart 300
8.8 Robust Control and Frequency Domain Techniques 308

9 Balanced Models for Control 321
9.1 Model Reduction and System Identification 321
9.2 Balanced Model Reduction 322
9.3 System identification 336

10 Data-Driven Control 345
10.1 Nonlinear System Identification for Control 346
10.2 Machine Learning Control 352
10.3 Adaptive Extremum-Seeking Control 362

Part IV Reduced Order Models 373

11 Reduced Order Models (ROMs) 375
11.1 POD for Partial Differential Equations 375
11.2 Optimal Basis Elements: The POD Expansion 381
11.3 POD and Soliton Dynamics 387
11.4 Continuous Formulation of POD 391
11.5 POD with Symmetries: Rotations and Translations 396

12 Interpolation for Parametric ROMs 403
12.1 Gappy POD 403
12.2 Error and Convergence of Gappy POD 409
12.3 Gappy Measurements: Minimize Condition Number 413
12.4 Gappy Measurements: Maximal Variance 418
12.5 POD and the Discrete Empirical Interpolation Method (DEIM) 423
12.6 DEIM Algorithm Implementation 426
12.7 Machine Learning ROMs 429

Glossary 436
Bibliography 443
Index 471