ASE 381P Problem Set 6

You need not hand in anything. Instead, be prepared to answer any of these problems on an upcoming take-home exam. You may discuss your solutions with classmates up until the time that the exam becomes available, but do not swap work.

1. This problem refers back to Problems 5 & 6 of Problem Set 5. Re-do Problem 6 using the Kalman filter problem matrices of Problem 5. Note that in Problem 5 there are 3 possible Q values. Run your truth-model simulation using the largest of the three Q values, but run your Kalman filter using the smallest of the three Q values. What does this do to your consistency evaluation? Is this what you expect?

2. There are two Kalman filtering problems defined by the two MATLAB scripts `kf_example03a.m` and `kf_example03b.m`. Solve each of these Kalman filtering problems in two ways. First, use a standard Kalman filter. Second, use a square-root information filter (SRIF). The two filters should work about equally well for the problem in `kf_example03a.m`, but the Kalman filter will not work as well as the SRIF for the problem in `kf_example03b.m`. This is true because of the small $R(k)$ value, which causes the computed covariance matrix to be ill-conditioned. Compare the state estimates and the covariances for the two filters at the terminal time. Is there a significant difference for the second filtering problem but not for the first?

   Note: this is a relatively benign case. The improvement due to use of the SRIF is not extremely significant. There are, however, known practical situations where a Kalman filter completely breaks down while an SRIF functions well.

3. Calculate the smoothed estimates for the problem in `kf_example03a.m`. Compare $\hat{\boldsymbol{x}}(10)$ with $\boldsymbol{x}^*(10)$ and compare $P(10)$ with $P^*(10)$. Is $P^*(10) \leq P(10)$? Do the smoothed state time history estimate plots look "smoother" than the filtered state time history estimate plots?

4. Derive a Kalman filter for the following system, which effectively has correlated process noise and measurement noise:

$$\boldsymbol{x}(k+1) = F(k)\boldsymbol{x}(k) + G(k)\boldsymbol{u}(k) + \Gamma(k)\boldsymbol{v}(k)$$

$$\boldsymbol{z}(k) = H(k)\boldsymbol{x}(k) + \Lambda(k)\boldsymbol{v}(k) + \boldsymbol{w}(k)$$

   where $\boldsymbol{v}(k) \sim N[0, Q(k)]$, $\boldsymbol{w}(k) \sim N[0, R(k)]$, $E[\boldsymbol{v}(k)\boldsymbol{w}^T(k)] = 0$, $E[\boldsymbol{v}(k)\boldsymbol{v}^T(j)] = 0$ if $k \neq j$, and $E[\boldsymbol{w}(k)\boldsymbol{w}^T(j)] = 0$ if $k \neq j$. This problem can be solved by using the techniques of Bar-Shalom Section 8.3.1 if one makes appropriate re-definitions of the process noise and the measurement noise, but do not solve the problem in this way. Instead, create augmented vectors

$$\boldsymbol{x}_a(k) = \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{v}(k) \end{bmatrix}$$

and

$$\boldsymbol{x}_a(k+1) = \begin{bmatrix} \boldsymbol{x}(k+1) \\ \boldsymbol{v}(k+1) \end{bmatrix}$$

and work with expectation values, covariance matrices, and updates for these vectors. Express the expectation values and covariance matrices of these vectors in block form so that it is obvious how to determine each component of the augmented vectors and matrices.

Hints: You will have to derive the measurement update based on the principles that are outlined in Bar-Shalom Section 3.2.1 and that are used in Bar-Shalom Section 5.2.2 in order to derive the measurement update for the normal Kalman filter. In order to do this, you will have to figure out the expectation values of $\boldsymbol{x}_a(k+1)$ and $\boldsymbol{z}(k+1)$ given the data up through time k. You will also have to determine these vectors' error covariances and their cross-correlation. In order to do this, you will need to use the dynamics model, the measurement model, the joint a posteriori state and process noise statistics at time k, the a priori process noise statistics at time $k+1$, and the a priori measurement error statistics at time $k+1$.

Also do the following problem from Bar Shalom: 8-5