## Contents



```
% Final prob 1: Problem Set 6, Number 3, except use the matrices Q(k) = [200, 10; 10, 20] and
% R(k) = 3. Hand in plots of your filtered and smoothed time histories, and hand in numerical
% values for xˆ(10), x∗(10), P(10), and P∗(10). Choose an appropriate test to determine whether
% P(10) ≤ P∗(10). Comment on the qualitative smoothness of x∗(k) vs. xˆ(10).

% set 6 prob 3: Calculate the smoothed estimates for the problem in kf example03a.m. Compare xˆ(10)
% with x∗(10) and compare P(10) with P∗(10). Is P∗(10) ≤ P(10)? Do the smoothed state
% time history estimate plots look “smoother” than the filtered state time history estimate
% plots?
```

## SRIF (forward dynamics)

```
clear; clc

disp('EXAMPLE 03A')
kf_example03a;

Qk = [200, 10; 10, 20];
Rk = 3;

uk = [0; 0];
nx = length(xhat0);
nv = length(Qk);
[xhat_arr_srif, Rxx_arr, zx_arr, zv_bar_arr, Rvv_bar_cell, Rvx_bar_cell] = ...
    srif( xhat0, P0, zhist, Fk, Gammak, Qk, Hk, Rk, uk, nx, nv);
```

```
EXAMPLE 03A
```

## smoother (backward dynamics)

```
zx_star = zx_arr(end,:)';
Rxx_star = Rxx_arr{end};
% wx_star = wx;

% START AT k = N
x_star = inv(Rxx_star) * zx_star;
P_star = inv(Rxx_star) * inv(Rxx_star)';
Rvv = chol(inv(Qk));
zv  = zeros(nv, 1);

% initialize
N = length(zhist);
x_star_arr = zeros(N, nx);
P_star_cell = cell(N,1);

x_star_arr(N,:) = x_star';
P_star_cell{N} = P_star';

% smoother filter
for k = N-1 : -1 : 1

    zx_star = zx_arr(k+1, :)';
    Rxx_star = Rxx_arr{k+1};

    Rvv_bar = Rvv_bar_cell{k+1};
    Rvx_bar = Rvx_bar_cell{k+1};

    A = [ Rvv_bar + Rvx_bar * Gammak,   Rvx_bar * Fk;
          Rxx_star * Gammak,            Rxx_star * Fk ];
    [QA, RA] = qr(A);

    R_QR = QA' * A;
    Rxx_star = R_QR(nv+1:end, nv+1:end);

    zv_bar = zv_bar_arr(k+1,:)';
    z_star = QA' * [ zv_bar; zx_star ];
    zx_star = z_star(nv+1:end);

    % extract state and covariance
    x_star = inv(Rxx_star) * zx_star;
```

```
    P_star = inv(Rxx_star) * inv(Rxx_star)';

    % save outputs
    x_star_arr(k,:) = x_star';
    P_star_cell{k} = P_star;

end
```

## plot

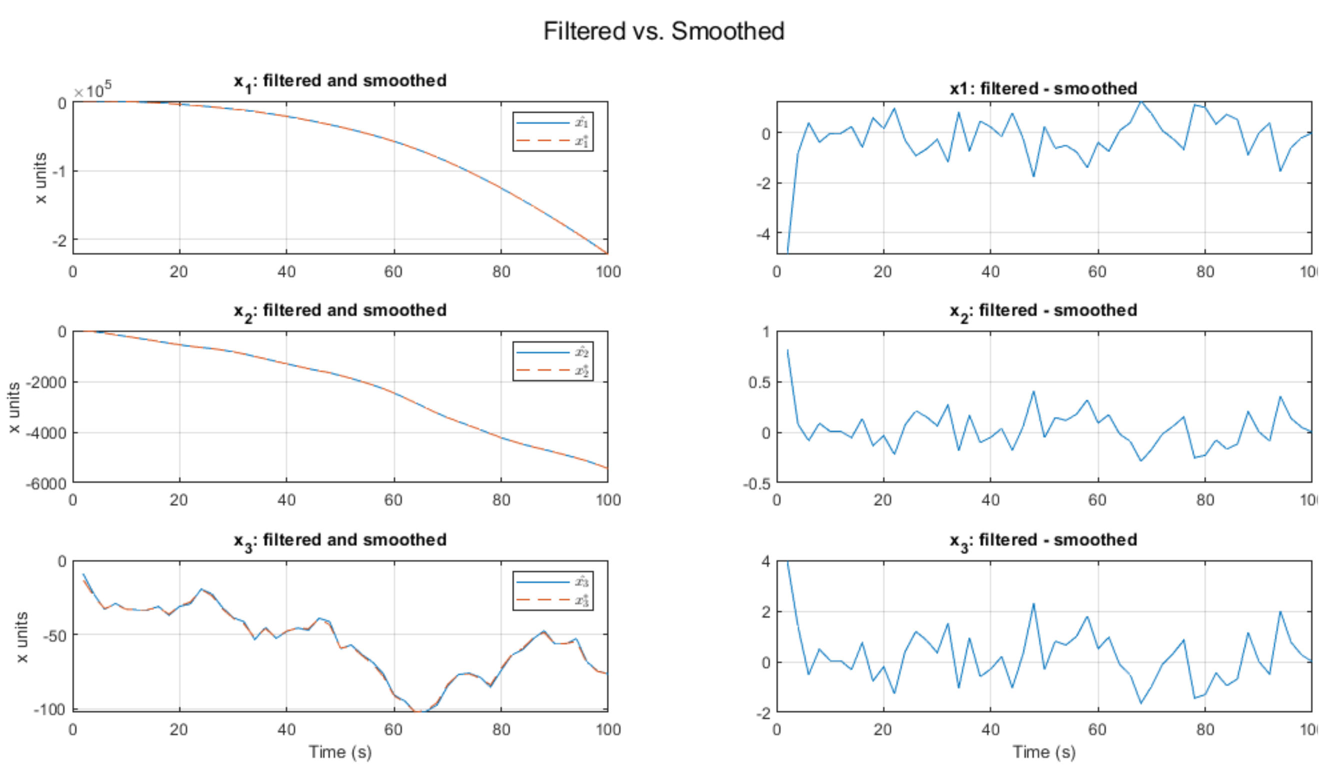
```
fname = 'Filtered vs. Smoothed';
figure('name', fname, 'position', [100 100 1200 600]);
n = 3; p = 2;

subplot(n,p,1)
    plot(thist, xhat_arr_srif(:,1)); hold on; grid on;
    plot(thist, x_star_arr(:,1), '--');
    title('x_1: filtered and smoothed');
    legend('$\hat{x_1}$', '$x^*_1$', 'Interpreter', 'latex')
    ylabel('x units')
subplot(n,p,2)
    plot(thist, xhat_arr_srif(:,1) - x_star_arr(:,1));
    title('x1: filtered - smoothed');

subplot(n,p,3)
    plot(thist, xhat_arr_srif(:,2)); hold on; grid on;
    plot(thist, x_star_arr(:,2), '--');
    title('x_2: filtered and smoothed');
    legend('$\hat{x_2}$', '$x^*_2$', 'Interpreter', 'latex')
    ylabel('x units')
subplot(n,p,4)
    plot(thist, xhat_arr_srif(:,2) - x_star_arr(:,2));
    title('x_2: filtered - smoothed');

subplot(n,p,5)
    plot(thist, xhat_arr_srif(:,3)); hold on; grid on;
    plot(thist, x_star_arr(:,3), '--');
    title('x_3: filtered and smoothed');
    xlabel('Time (s)')
    legend('$\hat{x_3}$', '$x^*_3$', 'Interpreter', 'latex')
    ylabel('x units')
subplot(n,p,6)
    plot(thist, xhat_arr_srif(:,3) - x_star_arr(:,3));
    title('x_3: filtered - smoothed');
    xlabel('Time (s)')

sgtitle(fname);
```



**periodogram**

```
% [pxx_hat, fs_hat]   = periodogram(xhat_arr_srif);
% [pss_star, fs_star] = periodogram(x_star_arr);
%
% fname = 'Filtered vs. Smoothed PSD';
% figure('name', fname)
%
% subplot(3,1,1)
%     semilogy(fs_hat, pxx_hat(:,1)); hold on;
%     semilogy(fs_star, pss_star(:,1), '--');
%
% subplot(3,1,2)
%     semilogy(fs_hat, pxx_hat(:,2)); hold on;
%     semilogy(fs_star, pss_star(:,2), '--');
%
% subplot(3,1,3)
%     semilogy(fs_hat, pxx_hat(:,3)); hold on;
%     semilogy(fs_star, pss_star(:,3), '--');
```

**hand in numerical values for xˆ(10), x∗(10), P(10), and P∗(10). Choose**

an appropriate test to determine whether P(10) ≤ P∗(10). Comment on the qualitative smoothness of x∗(k) vs. xˆ(10).

```
disp('x^(10) = ')
disp(xhat_arr_srif(10,:) )

disp('x*(10) = ')
disp(x_star_arr(10,:))

disp('x^(10) - x*(10) = ')
disp(xhat_arr_srif(10,:) - x_star_arr(10,:) )

disp('P(10) = ')
Rxx10 = Rxx_arr{10}; Pxx10 = inv(Rxx10) * inv(Rxx10)';
disp(Pxx10 )

disp('P*(10) = ')
disp(P_star_cell{10} )

disp('It can be shown that P*(10) < P(10):')

disp('Norm(P(10)) = ')
disp(norm(Pxx10) )

disp('Norm(P*(10)) = ')
disp(norm(P_star_cell{10}))

txt = [ 'x* is slightly smoother than x^. This is a good thing as we wish ', ...
    'to retain as much valuable information in the data while reducing as ', ...
    'much noise as possible without curve fitting to a particular function.', ...
    ] ;
disp(txt)

% In statistics and image processing, to smooth a data set is to create an
% approximating function that attempts to capture important patterns in the
% data, while leaving out noise or other fine-scale structures/rapid phenomena.
%
% In smoothing, the data points of a signal are modified so individual points
% higher than the adjacent points (presumably because of noise) are reduced,
% and points that are lower than the adjacent points are increased leading to
% a smoother signal. Smoothing may be used in two important ways that can aid
% in data analysis (1) by being able to extract more information from the
% data as long as the assumption of smoothing is reasonable and (2) by being
% able to provide analyses that are both flexible and robust.[1] Many
% different algorithms are used in smoothing.
%
% Smoothing may be distinguished from the related and partially overlapping
% concept of curve fitting in the following ways:
%
% curve fitting often involves the use of an explicit function form for the
% result, whereas the immediate results from smoothing are the "smoothed"
% values with no later use made of a functional form if there is one;
% the aim of smoothing is to give a general idea of relatively slow changes
% of value with little attention paid to the close matching of data values,
% while curve fitting concentrates on achieving as close a match as possible.
```

```
x^(10) =
         -3454.14901159571         -560.052811977033         -31.0412760936411

x*(10) =
         -3454.30365843153         -560.017490062307         -30.8407969018893

x^(10) - x*(10) =
```

```
        0.154646835822859        -0.035321914725273        -0.200479191751747

P(10) =
         13.5520894202657         -1.70208065918152         -7.34473971512019
        -1.70208065918152          2.31626499888182         0.377097282076377
        -7.34473971512019         0.377097282076377          9.10633440247233

P*(10) =
         12.6385332698102         -1.49342102335987         -6.16043485888851
        -1.49342102335987          2.26860635896187         0.106597630927508
        -6.16043485888851         0.106597630927508          7.57103961868193

It can be shown that P*(10) < P(10):
Norm(P(10)) =
         19.1536883016155

Norm(P*(10)) =
         16.8819091503244

x* is slightly smoother than x^. This is a good thing as we wish to retain as much valuable information in the data while reducing as much noise as possible without
```

### subfunctions

```matlab
function [xhat_arr, Rxx_cell, zx_arr, zv_bar_arr, Rvv_bar_cell, Rvx_bar_cell] = ...
    srif( xhat0, P0, zhist, Fk, Gk, Qk, Hk, Rk, uk, nx, nv)

% Initialize
xhat_arr = [];
Rxx_cell = {};
zx_arr = [];
zv_bar_arr = [];
Rvv_bar_cell = {};
Rvx_bar_cell = {};

% START AT k = 0:
I = inv(P0);
Rxx = chol(I);
Rvv = chol(inv(Qk));
zv = zeros(nv, 1);
zx = Rxx * xhat0;

for k = 1:length(zhist)

    % PROPAGATION STEP
    % a) QR factorize
    A = [Rvv, zeros(2,3); -Rxx * inv(Fk) * Gk, Rxx * inv(Fk)];
    [QA, RA] = qr(A);

    % [ Rvv_bar(k) [2x2], Rvx_bar(k+1) [2x3];
    %   0 [3x2]         , Rxx_bar(k+1) [3x3] ] = RA

    % b) orthonormal transformation
    B = [ zv; zx + Rxx * inv(Fk) * Gk * uk ];
    [zv_zx_bar] = QA' * B;

    % c) extract Rxx_bar(k+1) and zx_bar(k+1)
    zv_bar = zv_zx_bar(1:nv);
    zx_bar = zv_zx_bar(nv+1:end);
    Rxx_bar = RA(nv+1:end, nv+1:end);

    % extract Rvv and Rvx bars
    Rvv_bar = RA(1:nv, 1:nv);
    Rvx_bar = RA(1:nv, nv+1:end);

    % MEASUREMENT UPDATE:
    % a) Cholesky factorize R
    Ra = chol(Rk);

    % b) Transform z(k+1) and H(k+1)
    za = inv(Ra)' * zhist(k);
    Ha = inv(Ra)' * Hk;

    % c) perform another QR factorization:
    [QB, RB] = qr([Rxx_bar; Ha]);

    % d) transform as
    [zx_zr] = QB' * [zx_bar; za];

    % e) extract Rxx_bar(k+1) and zx(k+1)
    zx = zx_zr(1:nx);
    Rxx = RB(1:nx, :);

    xhat = inv(Rxx) * zx;

    zx_arr = [zx_arr; zx'];
    zv_bar_arr = [zv_bar_arr; zv_bar'];
```

```
    xhat_arr = [xhat_arr; xhat'];
    Rxx_cell{k} = Rxx;
    Rvv_bar_cell{k} = Rvv_bar;
    Rvx_bar_cell{k} = Rvx_bar;

end

end
```

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