# ASE 381P Final Exam

Posting Date: December 14, 2021

**Exam Rules:** Do all problems. Do problems on standard 8 1/2 by 11 inch paper. Hand in the completed exam to Dr. Humphreys in his office by 9:30 A.M. on Wednesday, December 15, 2021. No collaboration or consultation is allowed with any other person besides Dr. Humphreys. He is willing to discuss problems if he is available. You may use non-human outside sources (e.g., books); if you use such sources, please list them.

**Give all Matlab-produced values to 14 significant figures (i.e., use Matlab's `format long`).**

1. [**10 points**] Problem Set 6, Number 3, except use the matrices $Q(k) = [200, 10; 10, 20]$ and $R(k) = 3$. Hand in plots of your filtered and smoothed time histories, and hand in numerical values for $\hat{\boldsymbol{x}}(10)$, $\boldsymbol{x}^*(10)$, $P(10)$, and $P^*(10)$. Choose an appropriate test to determine whether $P(10) \leq P^*(10)$. Comment on the qualitative smoothness of $\boldsymbol{x}^*(k)$ vs. $\hat{\boldsymbol{x}}(10)$.

2. [**15 points**] Problem Set 7, Number 4. Describe your solution in a step-by-step formulation in terms of square-root information equations (data equations) as was done in the lecture notes for the SRIF.

3. [**10 points**] Bar Shalom Problem 10-10.

4. [**20 points**] Problem Set 8, Number 1, except use the measurement time history `ztruekhist` and control time history `utruekhist` from the file `mmExampleModeSwitchingData.mat`. The file also contains the truth state time history `xtruekhist` so that you can compare the errors in the estimated state with the estimate error covariance predicted by the multiple-model filter. Note that the first valid measurement is stored in `ztruekhist(2)`. Let the initial state estimate be $\hat{\boldsymbol{x}}(0) = [0; 0.1]$. Leave all other parameters as described in the original problem.

   The simulation used to generate the data in `mmExampleModeSwitchingData.mat` simulates the mode-switching scenario described in the last paragraph of the original problem. Accommodate the mode-switching behavior using the approach described in that paragraph. Initialize the mode probabilities as equally probable among the three possible modes. Choose a lower bound for the mode probabilities that yields sensible results.

   Hand in your code. Plot the time history of the mode probabilities. On a separate figure, plot the state errors and $\pm\sigma$ bounds for both state components. For each plot, note the value you chose for the mode probability lower bound $\mu_{\min}$. Answer the following questions:

   (a) Choose a sensible metric for filter consistency. What effect does increasing and decreasing the mode probability lower bound have on this metric? A qualitative answer is acceptable.

   (b) Develop an algorithm for finding the optimum value of $\mu_{\min}$ in the sense that it maximizes consistency for the truth and measurement data provided. Give a step-by-step description of your algorithm and also give the resulting optimum value of $\mu_{\min}$. Then find the value of $\mu_{\min}$ that minimizes the diagonal elements of the empirical estimation error covariance matrix $E[\tilde{x}\tilde{x}^T]$ for the truth and measurement data provided. Discuss why the consistency-maximizing and the covariance-minimizing $\mu_{\min}$ values differ.

(c) What system properties should be taken into account in the selection of the mode probability lower bound?

(d) What is the apparent mode time history (i.e., what modes were in effect and over what intervals)?

(e) Suppose it were known that the faulty boom system tended to slip preferentially into one mode. How could the filter be modified to account for this?

5. [**10 points**] Do the unscented transform problem (Problem Set 8, Number II). Hand in your values for the means and the covariances in the following format. Comment on your results.

```
Case i:
Linearized mean and covariance
   [2-by-1 mean]
   [2-by-2 cov]
Unscented mean and covariance
   [2-by-1 mean]
   [2-by-2 cov]
True mean and covariance
   [2-by-1 mean]
   [2-by-2 cov]

Case ii:
Linearized mean and covariance
   [2-by-1 mean]
   [2-by-2 cov]
Unscented mean and covariance
   [2-by-1 mean]
   [2-by-2 cov]
True mean and covariance
   [2-by-1 mean]
   [2-by-2 cov]
```

6. [**15 points**] Do the sigma point filtering problem (Problem Set 8, Number III). Answer all questions posed in Section (b) of the problem. Hand in time histories of all six estimated state components. Overlay on each component time history the corresponding $\pm\sigma$ bounds and the truth time history. Instead of the original measurement data, use the data in the file `problem3dataMod.mat`, whose range and bearing measurements are slightly noisier. Retain the same tuning parameters and assumed measurement noise covariance as in the original problem.

7. [**15 points**] Do the particle filtering problem (Problem Set 8, Number IV). Answer all questions posed in Section (b) of the problem. Hand in time histories of all three estimated state components. Overlay on each component time history the corresponding $\pm\sigma$ bounds and the truth time history. Use the original data files.

8. [**5 points**] Problem Set 1, Number 11. Fully explain your reasoning.