

Contents

- [KALMAN FILTER](#)
- [Decide which is the best value in the following way: Calculate \$\text{err}\(v\(k\)\)\$](#)
- [Compute the RMS value of the difference time history](#)
- [subfunctions KALMAN FILTER](#)

```
% Repeat Problem 3, except use the problem matrices and measurement data
% that are defined by the Matlab script kf example02b.m. Notice that the R
% and Q values are different for this problem and that there is a different
% measurement time history.
```

```
% Run your Kalman filter two additional times using the two alternate Q
% values that are mentioned in the comments in the file kf example02b.m.
% It is uncertain which is the correct Q value.
```

```
clear; clc;
close all;
```

```
kf_example02b
```

```
% Qk_a = Qk
% Qk_b = first alternate Qk
% Qk_c = second alternate Qk
Gk = Gammak;
```

KALMAN FILTER

```
% Qk_a
[xhat_a, P11_a, P22_a, P_a, xbar_a, Pbar_a, nu_a, S_a] = kf( ...
    xhat0, P0, zhist, Fk, Gk, Qk_a, Hk, Rk );
e_a = xbar_a - xhat_a;
e_a_mean = mean(e_a);
```

```
% Qk_b
[xhat_b, P11_b, P22_b, P_b, xbar_b, Pbar_b, nu_b, S_b] = kf( ...
    xhat0, P0, zhist, Fk, Gk, Qk_b, Hk, Rk );
```

```
% Qk_c
[xhat_c, P11_c, P22_c, P_c, xbar_c, Pbar_c, nu_c, S_c] = kf( ...
    xhat0, P0, zhist, Fk, Gk, Qk_c, Hk, Rk );
```

Decide which is the best value in the following way: Calculate $\text{err}(v(k))$

for $k = 1, 2, \dots, 50$ for each of your runs. Compute the average of these 50 values. This average times 50, i.e., $\{\text{err}_v(1) + \text{err}_v(2) + \dots + \text{err}_v(50)\}$, will be a sample of chi-square distribution of degree 50 if the filter model is correct. Develop upper and lower limits between which the average $\{\text{err}_v(1) + \text{err}_v(2) + \dots + \text{err}_v(50)\}/50$ must lie 99% of the time if the Kalman filter model is correct, and test your averages for each of the three candidate Q values. Which is the most reasonable? Look at the state estimate differences between the best filter and the other two filters.

```
% err_nu_k = 1/2 * [ nu_k ]' * inv(S_k) * nu_k
for i = 1:length(nu_a)

    err_nu_a(i) = nu_a(i) * inv(S_a(i)) * nu_a(i);
    err_nu_b(i) = nu_b(i) * inv(S_b(i)) * nu_b(i);
    err_nu_c(i) = nu_c(i) * inv(S_c(i)) * nu_c(i);

end
err_nu_a_mean = mean(err_nu_a);
err_nu_b_mean = mean(err_nu_b);
err_nu_c_mean = mean(err_nu_c);

N = 50;
Nx = 2;
Nz = 1;

% NEED STATISTICS TOOLBOX
a = .01;
r1 = chi2inv( a/2, N * Nz ) / N;
r2 = chi2inv( 1 - a/2, N * Nz ) / N;

figure
```

```

subplot(3,1,1)
plot( thist, err_nu_a, 'b' ); hold on; grid on;
yline(err_nu_a_mean, 'b--');
yline(r1, 'g-.');
yline(r2, 'g-.');
bigger_ylim
title('First Qk');
legend('$\epsilon_{\nu}$', '$\bar{\epsilon}_{\nu}$', 'r1', 'r2', 'interpreter', 'latex')
subplot(3,1,2)
plot( thist, err_nu_b, 'r');
yline(err_nu_b_mean, 'r--');
yline(r1, 'g-.');
yline(r2, 'g-.');
bigger_ylim
title('Second Qk');
subplot(3,1,3)
plot( thist, err_nu_c, 'k');
yline(err_nu_c_mean, 'k');
yline(r1, 'g--');
yline(r2, 'g--');
title('Third Qk')
bigger_ylim
xlabel('Time')

if err_nu_a_mean > r1 && err_nu_a_mean < r2
    sprintf('Qk = %g is reasonable', Qk_a)
else
    sprintf('Qk = %g is NOT reasonable', Qk_a)
end

if err_nu_b_mean > r1 && err_nu_b_mean < r2
    sprintf('Qk = %g is reasonable', Qk_b)
else
    sprintf('Qk = %g is NOT reasonable', Qk_b)
end

if err_nu_c_mean > r1 && err_nu_c_mean < r2
    sprintf('Qk = %g is reasonable', Qk_c)
else
    sprintf('Qk = %g is NOT reasonable', Qk_c)
end

% possibly use Nz? Otherwise Q is not within bounds

```

```

ans =

    'Qk = 40 is NOT reasonable'

```

```

ans =

    'Qk = 0.4 is reasonable'

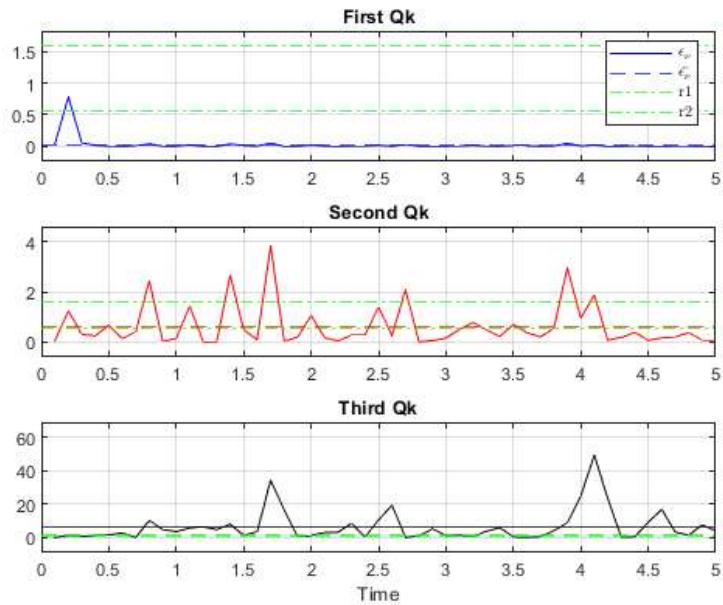
```

```

ans =

    'Qk = 0.004 is NOT reasonable'

```



Compute the RMS value of the difference time history

for each state vector element. Do the averaging over the last 40 points.

```
x1_rms_ab = rms( xhat_a(11:end,1) - xhat_b(11:end,1) );
x2_rms_ab = rms( xhat_a(11:end,2) - xhat_b(11:end,2) );

x1_rms_bc = rms( xhat_b(11:end,1) - xhat_c(11:end,1) );
x2_rms_bc = rms( xhat_b(11:end,2) - xhat_c(11:end,2) );

% Are these differences significant compared to the computed state
% estimation error standard deviations for the best filter?
x1_rms_ab / P11_b(end)
x2_rms_ab / P22_b(end)

x1_rms_bc / P11_b(end)
x2_rms_bc / P22_b(end)

disp('The second filter is the best filter. The first filter is closer to the second filter than the third filter is to the second filter');
```

```
ans =

    68.0758727260931
```

```
ans =

    15.8780431653489
```

```
ans =

    1211.25919352871
```

```
ans =

    86.8008602066569
```

The second filter is the best filter. The first filter is closer to the second filter than the third filter is to the second filter

subfunctions KALMAN FILTER

```
function [xhat_arr, Pxx_arr, Pzz_arr, P_cell, xbar_arr, Pbar_cell, nu_arr, S_arr] ...
    = kf( xhat0, P0, zhist, Fk, Gk, Qk, Hk, Rk )

% initialize for k = 0
```

```

xhat = xhat0;
P     = P0;

% Initialize saved output arrays
xbar_arr = [xhat'];
Pbar_cell = {P};
xhat_arr = [xhat'];
P_cell    = {P};
Pxx_arr   = [P(1,1)];
Pzz_arr   = [P(2,2)];
nu_arr    = [];
S_arr     = [];

% Propagate and filter through all measurements
for k = 0 : length(zhist)-1

    % propagate state and covar
    xbar = Fk * xhat;           % a priori state est
    Pbar = Fk * P * Fk' + Gk * Qk * Gk'; % a priori covar est

    % update
    nu = zhist(k+1) - Hk * xbar; % innovation
    S  = Hk * Pbar * Hk' + Rk;    % innovation covariance
    W  = Pbar * Hk' * inv(S);     % Kalman gain
    xhat = xbar + W * nu;         % a posteriori state est
    P    = Pbar - W * S * W';     % a posteriori covar est

    % next step
    k = k + 1;

    % save states and covariances
    xbar_arr = [xbar_arr; xbar'];
    Pbar_cell = {Pbar_cell; Pbar};
    xhat_arr = [xhat_arr; xhat'];
    P_cell    = {P_cell; P};
    Pxx_arr   = [Pxx_arr; P(1,1)];
    Pzz_arr   = [Pzz_arr; P(2,2)];
    nu_arr    = [nu_arr; nu];
    S_arr     = [S_arr; S];

end

end

```