Contents


```
% close all;
clear;
% clc

% sim dt
dt = 0.1;

% process noise covariance
Q = diag([0.25 0.25 3 40*pi/180 0.1 0.1])^2 / dt;

% measurement vector:
% z = [b_min b_max r_min]';
%   b_min = minimum bearing (rad)
%   b_max = maximum bearing (rad)
%   r_min = minimum range (m)

% measurement covariance
R = diag( [2*pi/180, 2*pi/180, 0.1] )^2;

% initial state est and covar
x0 = [90; 4.25; 13; pi; 5; 2];
P0 = diag([2 5 1 pi/4 4 2])^2;

x_hat = x0;
P     = P0;

% for lambda
a = 10^-3;
b = 2;
k = 0;
nx = 6;
nz = 3;
nv = 6;

% load LIDAR data
% load problem3data.mat
load problem3dataMod.mat
N = length(lidar);

% initialize output arrays
x_hat_arr = [];
% P_cell = ;

% % create plot
% fname = 'Car Plot';
% h = figure('name', fname);
%     xlim([0 100]); ylim([-50 50]);
%     plotcar(x_hat, '-', h, 0)
```

UNSCENTED KALMAN FILTER

```
% iterate through measurements
for j = 1 : N
```

```
% augmented state and covariance
xa = [ x_hat; zeros(6,1) ];
Pa = [ P, zeros(6); zeros(6), Q ];
```

DYNAMICS PROPAGATION

```
% cholesky factorize Pa
Sx = chol(Pa)';

% build sigma points
lambda_xv = a^2 * (nx + nv + k) - (nx + nv);
XX = build_SP(xa, Pa, nx, nv, lambda_xv);

% propagate sigma points
XX_prop = [];
for i = 1:length(XX)

    [tvec, XX_i_prop] = ode45(@dyn_car, [0 dt], XX(i,:));
    XX_i_prop = XX_i_prop(end,:);
    XX_prop   = [ XX_prop; XX_i_prop ];

end

% combine sigma points
[x_bar, P_bar] = combine_SP(nx, nv, lambda_xv, a, b, XX_prop);
```

MEASUREMENT UPDATE

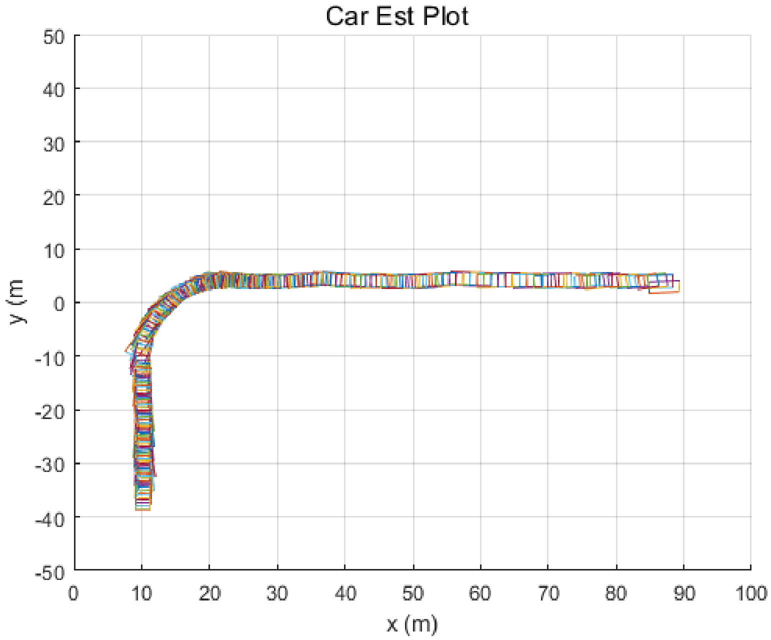
```
% push sigma points through measurement model. Use min bearing, max
% bearing, and min range
zj   = lidar(j).z;
z_rj = zj(:,1);     % range
z_bj = zj(:,2);     % bearing
z    = [ min(z_bj); max(z_bj); min(z_rj) ];

% stack states and covariance
xa_bar = [x_bar; zeros(nz,1)];
Pa_bar = [P_bar, zeros(nx, nz); zeros(nz, nx), R];

% build sigma points
lambda_xz = a^2 * (nx + nz + k) - (nx + nz);
XX_bar = build_SP(xa_bar, Pa_bar, nz, nx, lambda_xz);

% push sigma points through measurement model
ZZ_bar = [];
for i = 1:length(XX_bar)
    ZZ_i_bar = h_car(XX_bar(i,:));
    ZZ_bar   = [ ZZ_bar; ZZ_i_bar' ];
end

% combine sigma points
[z_bar, Pzz] = combine_SP(nz, nx, lambda_xz, a, b, ZZ_bar);

% calculate Pxz
Pxz = calc_Pxz(nx, nz, lambda_xz, a, b, XX_bar, x_bar, ZZ_bar, z_bar);

% LMMSE update!!!
x_hat = x_bar + Pxz * Pzz^(-1) * [z - z_bar];
P     = P_bar - Pxz * Pzz^(-1) * Pxz';

% save state and covariance
x_hat_arr = [x_hat_arr; x_hat'];
% P_cell{j} = {P};
P_arr(:,:,j) = P;

% update plot
% plotcar(x_hat, '-', h, lidar(j).t)
```

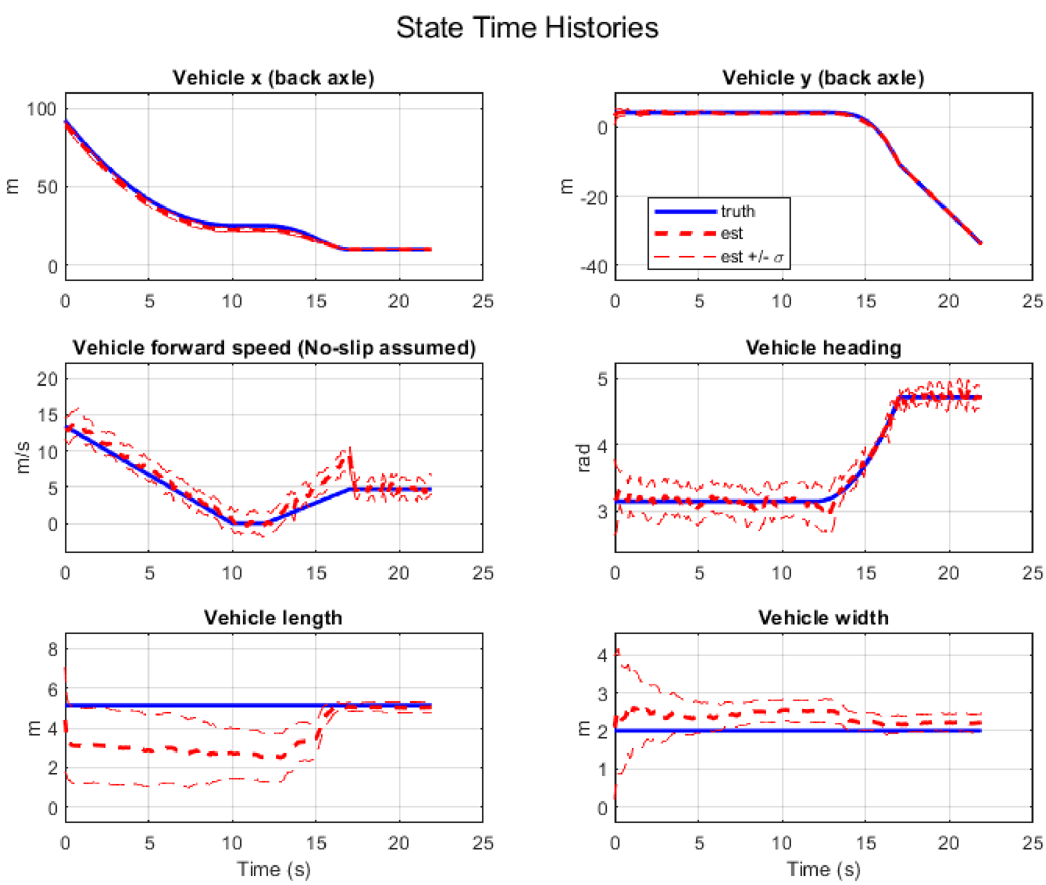
```
end
```

ANALYSIS

```
%   x - state of the rectangular car.  State elements are:
%       x - vehicle x-position (back axle, m)
```

```
%       y - vehicle y-position (back axle, m)
%       s - vehicle forward speed (no-slip assumed)
%       t - vehicle heading (rad.)
%       l - vehicle length (m)
%       w - vehicle width (m)

h = figure;
xlim([0 100]); ylim([-50 50]);
xlabel('x (m)')
ylabel('y (m' )
for i = 1:length(x_hat_arr)
    hold on;
    plotcar(x_hat_arr(i,:)', '-', h, lidar(i).t);
end

load problem3truth.mat
for i = 1:length(car)
    x_truth(i,:) = car(i).x';
    t(i,:)       = car(i).t;
end

sigma_arr = [];
for i = 1:6
    sigma_arr(:,i) = sqrt(squeeze(P_arr(i,i,:)));
end

titles = {'Vehicle x (back axle)';
    'Vehicle y (back axle)';
    'Vehicle forward speed (No-slip assumed)';
    'Vehicle heading';
    'Vehicle length';
    'Vehicle width'};
units = {'m', 'm', 'm/s', 'rad', 'm', 'm'};

fname = 'State Time Histories';
n = 3; p = 2;
figure('name', fname, 'position', [100 100 800 600]);

for i = 1:6

    subplot(n,p,i)
        plot(t, x_truth(:,i), 'b', 'linewidth', 2); hold on; grid on;
        plot(t, x_hat_arr(:,i), 'r--', 'linewidth', 2);
        plot(t, x_hat_arr(:,i) + sigma_arr(:,i), 'r--');
        plot(t, x_hat_arr(:,i) - sigma_arr(:,i), 'r--');
        title(titles{i});
        ylabel(units{i});
        bigger_ylim

    if i == 5 || i == 6
        xlabel('Time (s)')
    elseif i == 2
        legend('truth', 'est', 'est +/- \sigma', 'location', 'best');
    end

end

    sgtitle(fname)
```





PART B questions

```
% i. What effects, if any, do the values of a, b, and k have on the final
% state estimate?
%   A: The spread of the sigma points around the mean state value is controlled
% by two parameters α and κ. A third parameter, β, impacts the weights of the
% transformed points during state and measurement covariance calculations.
% α — Determines the spread of the sigma points around the mean state value.
% It is usually a small positive value. The spread of sigma points is proportional to α.
% Smaller values correspond to sigma points closer to the mean state.
```

```
% κ — A second scaling parameter that is usually set to 0. Smaller values
% correspond to sigma points closer to the mean state. The spread is proportional
% to the square-root of κ.
% β — Incorporates prior knowledge of the distribution of the state. For
% Gaussian distributions, β = 2 is optimal.

disp('Part b questions');

fprintf('\n')

disp('Question i: ')
disp('What effects, if any, do the values of a, b, and k have on the final state estimate? ');

fprintf('\n')

disp('Answer:')
disp('Alpha and kappa control the spread of the sigma points around the mean state value.')
disp('The spread of sigma points is proportional to alpha. A larger alpha means a larger spread of sigma points.')
disp('Kappa is a scaling parameter usually set to 0. A larger kappa also means a larger spread of sigma points.')
disp('The square root of kappa is proportional to the spread.')
disp('Beta impacts the weights of the transformed points during state and measurement calculations.')
disp('Beta incorporates a priori knowlege of the state distribution; beta = 2 is optimal for Gaussian distributions.')

fprintf('\n')
% ii. What do you notice about the state estimates, in particular length
% and width, as the target car performs its different maneuvers?
%   A: The length and width vary slightly in estimate due to the
%   orientation of the car relative to the LIDAR sensors. There are times
%   while the car is turning where the LIDAR is only able to sense
%   certain parameters past an inflection point of the car's motion.

disp('Question ii')
disp('What do you notice about the state estimates, in particular length and width, as the target car performs its different maneuvers? ');

fprintf('\n')

disp('Answer:')
disp('The length and width vary in estimate due to the orientation of the car relative to the LIDAR sensors.')
disp('The LIDAR is only able to see certain parts of the car past an inflection point of the motion of the car.')

fprintf('\n')

% iii. Why would it be difficult to implement this as an extended Kalman
% filter?
%   A: Nonlinearities would lead to degeneracy in the EKF.
% The UKF does not require computing Jacobians, can be used with
% discontinuous transformation, and is, most importantly, more accurate
% than EKF for highly nonlinear transformations.

disp('Question iii')
disp('Why would it be difficult to implement this as an extended Kalman filter? ')

fprintf('\n')

disp('Answer:')
disp('The EKF linearizes a nonlinear function about a single point. ')
disp('As can be seen from the state time histories plots, the truth states have high nonlinearities.')
disp('Computing the Jacobian would be difficult, and linearization would still be less accurate than the unscented transform.')

fprintf('\n')

% iv. Why did we choose to use the bearing-bearing-range measurement instead
% of some other measurement vector?
%   A: Using only bearing data would make the state weakly observable (weak
%   notion of location). Two bearing measurements and distance (range) are
%   the minimal set required for determining the length, width, location,
%   and orientation of the car.

disp('Question iv')
disp('Why did we choose to use the bearing-bearing-range measurement instead of some other measurement vector? ')

fprintf('\n')
disp('Answer:')
disp('Two bearing measurements and a range measurement are the minimal set required for determing the state.')
```

```
disp('At least two bearing measurements are required for determining the length and width of the car.')
disp('Using only bearing data would make the state weakly observable with an inaccurate notion of location.')
```

```
Part b questions

Question i:
What effects, if any, do the values of a, b, and k have on the final state estimate?

Answer:
Alpha and kappa control the spread of the sigma points around the mean state value.
The spread of sigma points is proportional to alpha. A larger alpha means a larger spread of sigma points.
Kappa is a scaling parameter usually set to 0. A larger kappa also means a larger spread of sigma points.
The square root of kappa is proportional to the spread.
Beta impacts the weights of the transformed points during state and measurement calculations.
Beta incorporates a priori knowlege of the state distribution; beta = 2 is optimal for Gaussian distributions.

Question ii
What do you notice about the state estimates, in particular length and width, as the target car performs its different maneuvers?

Answer:
The length and width vary in estimate due to the orientation of the car relative to the LIDAR sensors.
The LIDAR is only able to see certain parts of the car past an inflection point of the motion of the car.

Question iii
Why would it be difficult to implement this as an extended Kalman filter?

Answer:
The EKF linearizes a nonlinear function about a single point.
As can be seen from the state time histories plots, the truth states have high nonlinearities.
Computing the Jacobian would be difficult, and linearization would still be less accurate than the unscented transform.

Question iv
Why did we choose to use the bearing-bearing-range measurement instead of some other measurement vector?

Answer:
Two bearing measurements and a range measurement are the minimal set required for determing the state.
At least two bearing measurements are required for determining the length and width of the car.
Using only bearing data would make the state weakly observable with an inaccurate notion of location.
```

subfunctions

```
function XX = build_SP(xa, Pa, nx, nv, lambda)

    % cholesky factorize Pa
    Sx = chol(Pa)';

    % build sigma points. REMEMBER: there will be 2*(nx+nv)+1 sigma points
    XX(1,:) = xa';
    for i = 1 : nx + nv

        XXi = xa' + sqrt( nx + nv + lambda ) * Sx(:,i)';
        XX  = [ XX; XXi ];

    end
    for i = nx + nv + 1 : 2*(nx + nv)

        XXi = xa' - sqrt( nx + nv + lambda) * Sx(:, i - nx - nv)';
        XX  = [ XX; XXi ];

    end

end

function Pxz = calc_Pxz(nx, nz, lambda_xz, a, b, XX_bar, x_bar, ZZ_bar, z_bar)

    % determine weights
    w_0m = lambda_xz / (nx + nz + lambda_xz);
    w_im = 1 / (2*(nx + nz + lambda_xz));
    w_0c = lambda_xz / (nx + nz + lambda_xz) + 1 - a^2 + b;
    w_ic = w_im;

    Pxz = zeros(size(nx, nz));
```

```
    N_SP = 2*(nx + nz) + 1;
    for i = 1 : N_SP

        if i == 1;      wP = w_0c;
        else;           wP = w_ic;
        end

        % build Pa_bar
        xtilde = [ XX_bar(i, 1:nx)' - x_bar ];
        ztilde = [ ZZ_bar(i, 1:nz)' - z_bar ];

        Pi  = wP * (xtilde) * (ztilde)';
        Pxz = Pxz + Pi;

    end

end

function [x_bar, Pxx_bar] = combine_SP(nx, nv, lambda, a, b, XX_prop)

    % determine weights
    w_0m = lambda / (nx + nv + lambda);
    w_im = 1 / (2*(nx + nv + lambda));
    w_0c = lambda / (nx + nv + lambda) + 1 - a^2 + b;
    w_ic = w_im;

    N_SP = 2*(nx+nv) + 1;

    % use predicted sigma points to calculate predicted state x_bar
    x_bar = zeros(1, nx);
    for i = 1 : N_SP

        if i == 1;      wx = w_0m;
        else;           wx = w_im;
        end

        % build xa_bar
        xi_bar = wx * XX_prop(i, 1:nx);
        x_bar = x_bar + xi_bar;

    end

    % predict Pa_bar
    Pxx_bar = zeros(nx);
    for i = 1 : N_SP

        if i == 1;      wP = w_0c;
        else;           wP = w_ic;
        end

        % build Pa_bar
        xtilde = [ XX_prop(i, 1:nx) - x_bar ]';
        Pi_bar = wP * (xtilde) * (xtilde)';
        Pxx_bar = Pxx_bar + Pi_bar;

    end

    x_bar = x_bar';

end
```

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