Contents

Problem Set 4, Number 1

```
% set4_prob1
```

Exam 2 version

```
clear; clc; close all

J_fn = @(x) ...
    [ 1+x(2),   1+x(1)      ;
      2*x(1),   2-2*x(2)    ];

fn = @(x) ...
    [ x(1) + x(2) + x(1)*x(2) + 5   ;
      x(1)^2 + 2*x(2) - x(2)^2 - 2  ];

% initialize
xg_arr = [];
f_arr = [];

for i = -10 : 10
    for j = -10 : 10

        xg = [i; j];

        k = 0;
        % first 6 iterates
        while norm(fn(xg)) > 0.00000001

            k = k + 1;
            xg = xg - inv(J_fn(xg)) * fn(xg);

        end

        xg_arr = [xg_arr; i, j, xg(1), xg(2)];
        f = fn(xg);
        f_arr  = [f_arr; i, j, f(1), f(2)];

    end
end
```

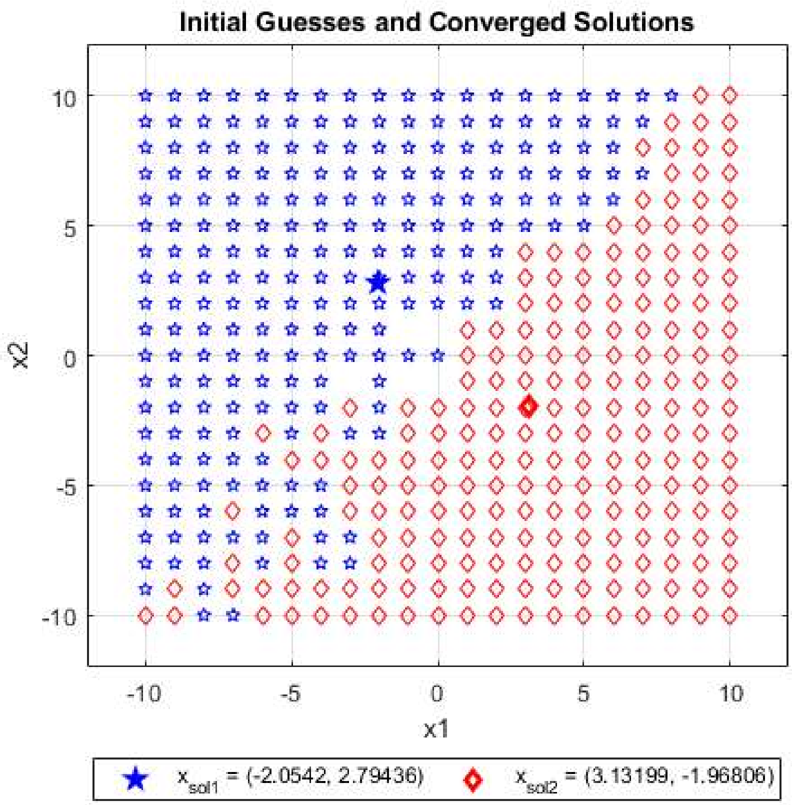
```
Warning: Matrix is singular to working precision.
Warning: Matrix is singular to working precision.
Warning: Matrix is singular to working precision.
Warning: Matrix is singular to working precision.
Warning: Matrix is singular to working precision.
Warning: Matrix is close to singular or badly scaled. Results may be
inaccurate. RCOND =  1.110223e-16.
```

unique converged values

```
tol   = 1e-5;
xg_u  = rmmissing(xg_arr);
xg_u(:,3) = round(xg_u(:,3), 5);
xg_u(:,4) = round(xg_u(:,4), 5);

% unique x1
row_u = unique(xg_u(:,3));

for j = 1:length(row_u)
    for i = 1:length(xg_u)
        if xg_u(i,3) == row_u(j)
            xg_u(i,5) = j;
        end
    end
end

% first unique points
temp = xg_u(:,3) == row_u(1);
idx  = find(temp, 1, 'first');
u1   = xg_u(idx, 3:4);

% second unique points
temp = xg_u(:,3) == row_u(2);
idx  = find(temp, 1, 'first');
u2   = xg_u(idx, 3:4);
```

plot

```
ftitle = 'Initial Guesses and Converged Solutions';
figure('name', ftitle, 'position', [100 100 500 500])

    % plot unique values
    plot(u1(1), u1(2), 'bp', 'linewidth', 2); grid on; hold on;
    plot(u2(1), u2(2), 'rd', 'linewidth', 2);

    % plot initial guesses
    for i = 1:length(xg_u)

        if xg_u(i, 5) == 1
            plot(xg_u(i,1), xg_u(i,2), 'bp');
        elseif xg_u(i, 5) == 2
            plot(xg_u(i,1), xg_u(i,2), 'rd');
        end

    end

    legend(sprintf('x_{sol1} = (%g, %g)', u1(1), u1(2)), ...
        sprintf('x_{sol2} = (%g, %g)', u2(1), u2(2)), ...
        'location', 'southoutside', 'orientation', 'horizontal')

    xlim([-10 10])
    ylim([-10 10])
    bigger_ylim
    bigger_xlim
    ylabel('x2');
    xlabel('x1');

    title(ftitle);
```

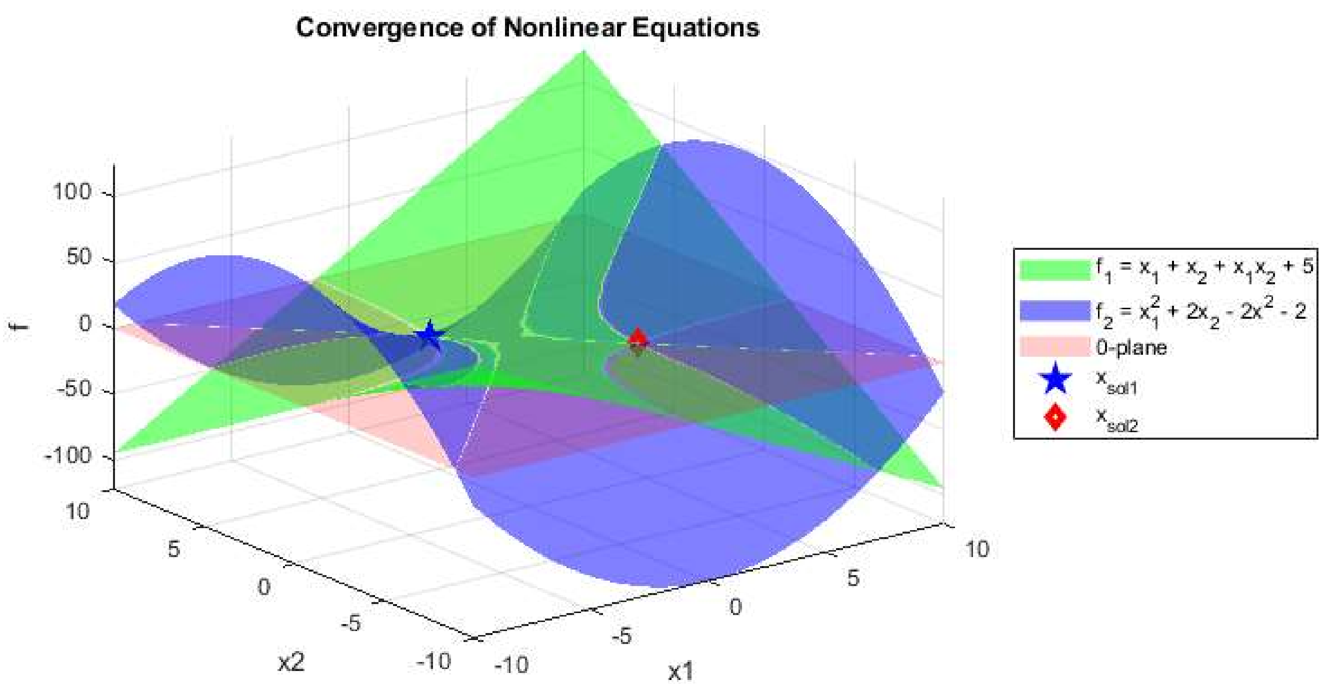


Convergence points

There are 2 convergence points (labelled in the legend). The plot above shows that initial guesses that lie closer to either solution will converge towards that respective solution. There is a zone in between the convergence points where Newton's method fails to converge; at those points, the calculated Jacobian is singular, and thus further computations to find a solution fail. The figure below shows how the nonlinear equations intersect with each other and the 0-plane.

```
x = [-10:1:10];

[X1,X2] = meshgrid(x);
F1 = X1 + X2 + X1.*X2 + 5;
F2 = X1.^2 + 2*X2 - X2.^2 - 2;

% sanity check
for i = 1:length(x)
    for j = 1:length(x)
        temp = fn([x(i),x(j)]);
        F1_test(i,j) = temp(1);
        F2_test(i,j) = temp(2);
    end
end

ftitle = 'Convergence of Nonlinear Equations';
figure('name', ftitle, 'position', [100 100 800 400])
    surf(X1,X2,F1, 'FaceColor','g', 'FaceAlpha',0.5, 'EdgeColor','none');
    hold on; grid on;
    surf(X1,X2,F2, 'FaceColor','b', 'FaceAlpha',0.5, 'EdgeColor','none');
    surf(X1,X2,zeros(size(F2)), 'FaceColor','r', 'FaceAlpha',0.2, 'EdgeColor','none');

    plot3(u1(1), u1(2), 0, 'bp', 'linewidth', 3)
    plot3(u2(1), u2(2), 0, 'rd', 'linewidth', 3)

    xlabel('x1');
    ylabel('x2');
```

```
zlabel('f');

legend('f_1 = x_1 + x_2 + x_1x_2 + 5', 'f_2 = x_1^2 + 2x_2 - 2x^2 - 2', '0-plane', ...
    sprintf('x_{sol1}', u1(1), u1(2)), ...
    sprintf('x_{sol2}', u2(1), u2(2)), ...
'location', 'eastoutside');

title(ftitle);
```



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