Contents

set 4, prob 4

```
clear; clc

% loads rhoahist, rhobhist, and thist
% load radarmeasdata_missle.mat
load radarmeasdata_missile_new.mat

global la lb
global rhoahist thetaahist rhobhist thetabhist
la = 3.5e5;
lb = 4.0e5;

% meas error covariances
sigma_rhoa = 10;
sigma_rhob = 30;
sigma_thetaa = 0.01;
sigma_thetab = 0.03;
```

```
Warning: The value of local variables may have been changed to match the
globals.  Future versions of MATLAB will require that you declare a variable to
be global before you use that variable.
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globals.  Future versions of MATLAB will require that you declare a variable to
be global before you use that variable.
```

Initial condition guessing

```
xg0_arr = [];
% First guess
for i = 1:5
    for f = 23:28
        xg0 = find_xg0(rhoahist, rhobhist, thist, i, f);
        xg0_arr = [xg0_arr; xg0'];
    end
end

y = [xg0_arr(:,1), xg0_arr(:,3)];
ftitle = 'IC guessing: first 5 and last 5 range meas';
figure('name', ftitle);
    subplot(2,1,1)
        plot(y(:,1), y(:,2),'.')
        grid on; hold on;
        yline(0, 'r')
        xlabel('y1'); ylabel('y2');
        bigger_ylim; bigger_xlim
        title('Distance')
    subplot(2,1,2)
        plot(xg0_arr(:,3), xg0_arr(:,4), '.');
        grid on; hold on;
        xlabel('v1'); ylabel('v2');
        bigger_ylim; bigger_xlim
        title('Velocity')
    sgtitle(ftitle);
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored.
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```

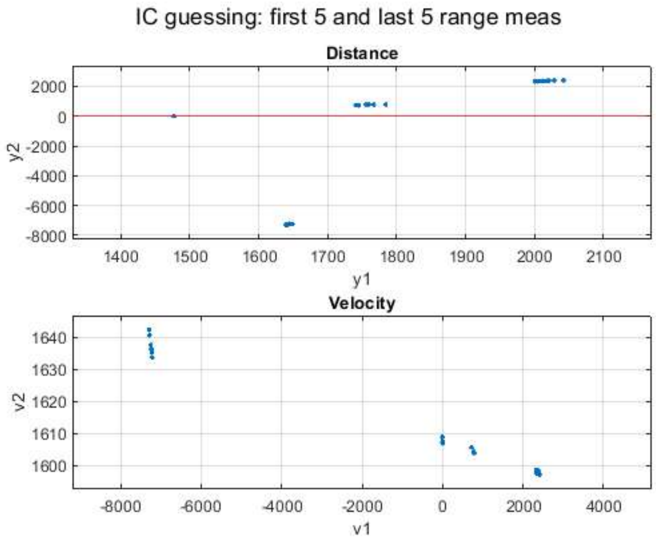

this one looks good

```
xg0_OG = find_xg0(rhoahist, rhobhist, thist, 3, 25)
xg0 = xg0_OG;
```

```
xg0_OG =

          2019.29620390645
           899.18143927834
          2349.46128871635
          1597.49910053977
```

Gauss-Newton method

```
% Just rho measurements
disp('Part b: Using just rho data from both radars: ')
meas_data = 'rho';
[Ra, zhist] = build_Ra_z(meas_data, thist, sigma_rhoa, sigma_rhob, sigma_thetaa, sigma_thetab);
[xg0_soln_rho, Pxx_rho] = GN(xg0_OG, thist, zhist, Ra, meas_data)

% rho AND theta measurements
disp('Part c: Using both rho and theta data from both radars: ')
meas_data = 'both';
[Ra, zhist] = build_Ra_z(meas_data, thist, sigma_rhoa, sigma_rhob, sigma_thetaa, sigma_thetab);
[xg0_soln_both, Pxx_both] = GN(xg0_OG, thist, zhist, Ra, meas_data)

% Compare just rho vs. rho and theta
disp('The bearing data made the error covariance slightly smaller (slightly improved) for the Gauss-Newton approximated solution. ')
disp('Norm of error covariance using just range data: ')
disp(norm(Pxx_rho))
disp('Norm of error covariance using range and bearing data: ')
disp(norm(Pxx_both))
disp('Worth noting is the difference between the range-only and the range-and-bearing solution is: ')
disp(xg0_soln_both - xg0_soln_rho)
disp('The largest difference, 1.62 meters, is not extremely large but can make a difference especially when trying to predict the dynamics of the missile.')

disp(' ')

% just theta_a measurements
disp('Part d: Using just theta from radar A: ')
meas_data = 'theta_a';
[Ra, zhist] = build_Ra_z(meas_data, thist, sigma_rhoa, sigma_rhob, sigma_thetaa, sigma_thetab);
[xg0_soln_theta_a, Pxx_theta_a] = GN(xg0_soln_both, thist, zhist, Ra, meas_data)

% Jacobian H
x = sym('x', [4 1]);
syms la_sym lb_sym tj g

y1 = x(1) + x(2)*tj;
dy_1a = la_sym - y1;
dy_1b = lb_sym - y1;
dy_2 = x(3) + tj * x(4) - 4.9*tj^2;

h_rhoa   = sqrt( dy_1a^2 + dy_2^2 );
h_rhob   = sqrt( dy_1b^2 + dy_2^2 );
h_thetaa = atan2( dy_2, dy_1a );
h_thetab = atan2( dy_2, dy_1b );

Hhist_j = matlabFunction( [ jacobian(h_thetaa, x) ] );
```

```
H = [];
for j = 1:length(thist)
    H = [ H; Hhist_j(la, thist(j), xg0_soln_both(1), xg0_soln_both(2), xg0_soln_both(3), xg0_soln_both(4)) ];
end
disp('Bearing-only jacobian (H) using "best" estimate from part c:')
H
fprintf('The rank of H from bearing-only from radar A data = %d \n', rank(H))
disp('Technically according to Matlab output, H is full rank which implies that the initial position and velocity estimate are observable.')
disp('However, the elements of H that were derived with respect to position are nearly 0. You can get velocity but not position from bearing (angle) measurements.')
disp('If the position differential elements of H were to go to 0, then the rank of H would be 2.')
disp(' ')

% theta_a estimation error covariance
disp('Part e: bearing-only estimate error covariance')
disp('The estimation of the solution using just theta from radar A is useless.')
disp('The diagonal elements of the estimation error covariance are much larger than the diagonal elements of the measurement covariance.')
disp('See the ratios of the diagonal elements below: ')
Pxx_theta_a(1,1) / sigma_rhoa^2
Pxx_theta_a(2,2) / sigma_thetaa^2
Pxx_theta_a(3,3) / sigma_rhob^2
Pxx_theta_a(4,4) / sigma_thetab^2
```

```
Part b: Using just rho data from both radars:
```

subfunctions

```
function h = h_NL(x, t, meas_data)
% Nonlinear measurement h

global la lb

    % Initialize h
    h = [];

    for i = 1:length(t)

        % y1 = y10 + v10*t = x1 + x2*t
        y1 = x(1) + x(2)*t(i);
        dy_1a = la - y1;
        dy_1b = lb - y1;

        % y2 = y20 + v20*t - 0.5 * 9.8 * t^2
        dy_2 = x(3) + x(4)*t(i) - 4.9*t(i)^2;

        h_rhoa   = sqrt( dy_1a^2 + dy_2^2 );
        h_rhob   = sqrt( dy_1b^2 + dy_2^2 );
        h_thetaa = atan2( dy_2, dy_1a );
        h_thetab = atan2( dy_2, dy_1b );

        % Build nonlinear h from guess
        switch meas_data

            case 'rho'
                h = [h; h_rhoa; h_rhob];

            case 'theta_a'
                h = [h; h_thetaa];

            case 'both'
                h = [h; h_rhoa; h_thetaa; h_rhob; h_thetab];

        end

    end

end

function H = Hhist(x, thist, Hhist_j, meas_data)
% Full jacobian of h

    global la lb

    H = [];
    for j = 1:length(thist)

        switch meas_data

            case 'rho'
                H = [ H; Hhist_j(la, lb, thist(j), x(1), x(2), x(3), x(4)) ];

            case 'theta_a'
                H = [ H; Hhist_j(la, thist(j), x(1), x(2), x(3), x(4)) ];

            case 'both'
                H = [ H; Hhist_j(la, lb, thist(j), x(1), x(2), x(3), x(4)) ];

        end
```

```matlab
    end

end

function [Jg, h, H, dx] = cost_fn(xg, thist, zhist, Ra, Hhist_j, meas_data)
% Cost function

    % Normalized NL at guess
    h = inv(Ra') * h_NL(xg, thist, meas_data);

    % Normalized jacobian at guess
    H = inv(Ra') * Hhist(xg, thist, Hhist_j, meas_data);

    % Normalized measurement
    z = inv(Ra') * zhist;

    % Gauss-Newton dx
    dx = inv((H' * H)) * H' * (z - h);

    % Cost function
    Jg = norm(z - h);

end

function xg0_OG = find_xg0(rhoahist, rhobhist, thist, i, f)
% Initial condition guessing (crude) using just range data

    % "initial" measurements
    p_ai = rhoahist(i);
    p_bi = rhobhist(i);

    global la lb

    y_1i = 1/( 2*lb - 2*la ) * ( p_ai^2 - la^2 - p_bi^2 + lb^2);
    y_2i = sqrt( p_ai^2 - ( la - y_1i )^2 );

    % last measurements
    p_af = rhoahist(f);
    p_bf = rhobhist(f);

    y_1f = 1/( 2*lb - 2*la ) * ( p_af^2 - la^2 - p_bf^2 + lb^2 );
    y_2f = sqrt( p_af^2 - ( la - y_1f )^2 );

    % guessing x1 (y10) and x2 (v10)
    % y_1s = (1)*y10 + (ts)*v10
    % y_1f = (1)*y10 + (tf)*v10
    ti = thist(i); tf = thist(f);
    x = pinv( [ 1 ti; 1 tf ] ) * [y_1i; y_1f];
    y_10 = x(1);
    v_10 = x(2);

    % guessing x3 (y20) and x4 (v20)
    % y_2s = (1)*y20 + (ts)*v20 - 4.9ts^2
    % y_2f = (1)*y20 + (tf)*v20 - 4.9tf^2
    x = pinv( [ 1 ti; 1 tf ] ) * ( [ y_2i; y_2f ] + 4.9 * [ ti^2; tf^2 ] );
    y_20 = x(1);
    v_20 = x(2);

    % SANITY CHECK linear algebra
    t = [ 0; -0.5 * 9.8 * ti^2; 0; -0.5 * 9.8 * tf^2 ];
    y = [y_1i; y_2i; y_1f; y_2f];
    A = [1, ti, 0, 0;
         0, 0, 1, ti;
         1, tf, 0, 0;
         0, 0, 1, tf ];
    x = pinv( A ) * (y - t);

    % First guess
    xg0_OG = [y_10; v_10; y_20; v_20];

end

function [Ra, zhist] = build_Ra_z(meas_data, thist, sigma_rhoa, sigma_rhob, sigma_thetaa, sigma_thetab)
% Build Ra and measurement vector for Gauss-Newton method

    global rhoahist thetaahist rhobhist thetabhist

    switch meas_data

        case 'rho'

            R_j = [sigma_rhoa^2 0; 0 sigma_rhob^2];

            % Build full R matrix
            R = zeros(length(thist));
            for j = 1:length(thist)
                R(2*j-1 : 2*j, 2*j-1 : 2*j) = R_j;
            end

            % build full zhist
```

```
            zhist = [];
            for j = 1:length(thist)
                zhist = [ zhist; rhoahist(j); rhobhist(j) ];
            end

        case 'theta_a'

            R_aj = zeros(1);
            R_aj(1,1) = sigma_thetaa^2;

            % Build full R matrix
            R = zeros(length(thist));
            for j = 1:length(thist)
                R(j,j) = R_aj;
            end

            % build full zhist
            zhist = [];
            for j = 1:length(thist)
                zhist = [ zhist; thetaahist(j) ];
            end

        case 'both'

            R_aj = zeros(4);
            R_aj(1,1) = sigma_rhoa^2;
            R_aj(2,2) = sigma_thetaa^2;
            R_aj(3,3) = sigma_rhob^2;
            R_aj(4,4) = sigma_thetab^2;

            % Build full R matrix
            R = zeros(length(thist));
            for j = 1:length(thist)
                R(4*j-3 : 4*j, 4*j-3 : 4*j) = R_aj;
            end
            % build full zhist
            zhist = [];
            for j = 1:length(thist)
                zhist = [ zhist; rhoahist(j); thetaahist(j); rhobhist(j); thetabhist(j) ];
            end

    end

    Ra = chol(R);

end

function [xg0_sol, Pxx] = GN(xg0_OG, thist, zhist, Ra, meas_data)
```

```
% GAUSS-NEWTON METHOD

    xg0 = xg0_OG;
```

Jacobian H

```
    x = sym('x', [4 1]);
    syms la_sym lb_sym tj g

    y1 = x(1) + x(2)*tj;
    dy_1a = la_sym - y1;
    dy_1b = lb_sym - y1;
    dy_2 = x(3) + tj * x(4) - 4.9*tj^2;

    h_rhoa   = sqrt( dy_1a^2 + dy_2^2 );
    h_rhob   = sqrt( dy_1b^2 + dy_2^2 );
    h_thetaa = atan2( dy_2, dy_1a );
    h_thetab = atan2( dy_2, dy_1b );

    % inputs: la, lb, tj, x1, x2, x3, x4

    switch meas_data

        case 'rho'
            Hhist_j = matlabFunction( [ jacobian(h_rhoa, x); jacobian(h_rhob, x) ] );

        case 'theta_a'
            Hhist_j = matlabFunction( [ jacobian(h_thetaa, x) ] );

        case 'both'
            Hhist_j = matlabFunction( [ jacobian(h_rhoa, x); jacobian(h_thetaa, x); jacobian(h_rhob, x); jacobian(h_thetab, x) ] );

    end
```

First cost function

```
    [Jg, h, H, dx] = cost_fn(xg0, thist, zhist, Ra, Hhist_j, meas_data);
```

```
    % first a step
    a = 1;

    % First step-size adjusted cost function
    xg = xg0 + a * dx;
    [Jgnew, h, H, ~] = cost_fn(xg, thist, zhist, Ra, Hhist_j, meas_data);

    % Gauss-Newton dx
    % dx = inv((H' * H)) * H' * (z - h);
```

The while loop: Jgnew > Jg

```
    Jg_i = [];
    while norm(dx) > 0.0000001
```

```
        while Jgnew >= Jg

            % Next a
            a = a/2;
            if a < 0.001
                break; end

            % Step size-adjusted guess and cost fn
            xg = xg0 + a * dx;
            [Jgnew, h, H, ~] = cost_fn(xg, thist, zhist, Ra, Hhist_j, meas_data);

        end
```

While loop: "New" first guess - saved from last iteration

```
    %     if a < eps
    %         break; end

        xg0 = xg;
        Jg  = Jgnew;

        % Gauss-Newton dx (H, z, and h saved from last iteration)
        z  = inv(Ra') * zhist;
        dx = inv((H' * H)) * H' * (z - h);

        % first a step
        a = 1;

        % "new" step-size adjusted guess
        xg = xg0 + a * dx;
        [Jgnew, h, H, ~] = cost_fn(xg, thist, zhist, Ra, Hhist_j, meas_data);

        Jg_i = [Jg_i; Jg];
```

```
    end

    % Gauss-Newton approximated solution
    xg0_sol = xg0;

    % covariance
    Pxx = inv(H' * H);
```

```
end
```

```
xg0_soln_rho =

          2009.30611040881
          899.941932686552
          2251.95108550068
           1598.8004967853


Pxx_rho =

  Columns 1 through 3

          31.9213706620175        -0.311059816746055          63.7995095606022
        -0.311059816746055        0.0214376222781052          2.20108246476619
          63.7995095606022          2.20108246476619           726.14823901803
        -0.522075164954113        0.0175144980469437        0.0469768130607384

  Column 4

        -0.522075164954113
        0.0175144980469437
        0.0469768130607384
        0.0213456658406535
```

```
Part c: Using both rho and theta data from both radars:

xg0_soln_both =

          2009.37317814612
          899.930080622043
          2250.33006742093
          1598.79191840315


Pxx_both =

  Columns 1 through 3

          31.8997213721097        -0.309085320893451          64.0179446060099
        -0.309085320893451        0.0211683191056671          2.16647104432628
          64.0179446060099          2.16647104432628          721.526969961692
        -0.520479035921291        0.0173132374010022         0.021697715758036

  Column 4

        -0.520479035921291
        0.0173132374010022
         0.021697715758036
        0.0211932737104801

The bearing data made the error covariance slightly smaller (slightly improved) for the Gauss-Newton approximated solution.
Norm of error covariance using just range data:
          731.969116912611

Norm of error covariance using range and bearing data:
          727.425626880099

Worth noting is the difference between the range-only and the range-and-bearing solution is:
        0.0670677373163926
       -0.0118520645090712
         -1.62101807974659
       -0.0085783821559744

The largest difference, 1.62 meters, is not extremely large but can make a difference especially when trying to predict the dynamics of the missile.

Part d: Using just theta from radar A:

xg0_soln_theta_a =

         -15643.3440454783
          816.482916600182
          1720.80391215354
          1718.72845003669


Pxx_theta_a =

  Columns 1 through 3

          112889548.992297          390681.183368644         -2020937.34404701
          390681.183368644          4992.32213325016          100080.423704409
         -2020937.34404701          100080.423704409          4722329.48740887
          -653907.31101803         -5418.66331307737         -88845.0074177702

  Column 4

          -653907.31101803
         -5418.66331307737
         -88845.0074177702
          6573.79125168794

Bearing-only jacobian (H) using "best" estimate from part c:

H =

  Columns 1 through 3

      1.85820219879722e-08                         0      2.87352045498948e-06
      1.54024393394754e-07      1.54024393394754e-06      2.94186387715783e-06
      2.93499988287863e-07      5.86999976575726e-06      3.00167823544675e-06
       4.3567197993976e-07      1.30701593981928e-05       3.0531671528681e-06
      5.79298385030704e-07      2.31719354012282e-05      3.09683124910884e-06
      7.23289275411382e-07      3.61644637705691e-05      3.13343768144897e-06
      8.66750452211393e-07      5.20050271326836e-05      3.16398016808037e-06
       1.0090123204329e-06       7.0630862430303e-05      3.18963511864583e-06
      1.14964463629202e-06      9.19715709033615e-05      3.21171924174916e-06
      1.28845928609277e-06      0.000115961335748349      3.23165317919826e-06
      1.42550416904209e-06       0.00014255041690421      3.25093463618417e-06
      1.56105161490053e-06      0.000171715677639058      3.27112342719334e-06
      1.69558468543654e-06      0.000203470162252385      3.29384006883134e-06
      1.82978434274381e-06      0.000237871964556696      3.32077917800977e-06
      1.96451996879856e-06      0.000275032795631799      3.35373907613714e-06
      2.10084520390324e-06      0.000315126780585486      3.39466973893544e-06
      2.24000059745302e-06      0.000358400095592482      3.44574268407477e-06
```

```
      2.38342412620231e-06      0.000405182101454393       3.5094487722963e-06
      2.53277013607455e-06      0.000455898624493419      3.58873362062604e-06
       2.6899364587433e-06      0.000511087927161227      3.68718613171573e-06
      2.85709781113222e-06      0.000571419562226445      3.80930485842071e-06
      3.03673995737753e-06       0.00063771539104928       3.9608819173186e-06
      3.23168097266706e-06      0.000710969813986754       4.1495692178123e-06
      3.44504756669811e-06      0.000792360940340566       4.3857347307427e-06
       3.6801323584654e-06      0.000883231766031697      4.68379196731837e-06
      3.93995949629449e-06      0.000984989874073621      5.06432098604006e-06
      4.22614790137994e-06       0.00109879845435878      5.55754364409298e-06
      4.53606261945704e-06        0.0012247369072534       6.2091504990381e-06

  Column 4

                         0
      2.94186387715783e-05
      6.00335647089351e-05
      9.15950145860431e-05
      0.000123873249964354
      0.000156671884072449
      0.000189838810084822
      0.000223274458305208
      0.000256937539339933
      0.000290848786127843
      0.000325093463618417
      0.000359823576991268
      0.000395260808259761
      0.000431701293141271
        0.0004695234706592
      0.000509200460840315
      0.000551318829451963
       0.00059660629129037
      0.000645972051712688
      0.000700565365025989
      0.000761860971684142
      0.000831785202636906
      0.000912905227918705
       0.00100871898807082
       0.00112411007215641
       0.00126608024651002
       0.00144496134746417
       0.00167647063474029
```

```
The rank of H from bearing-only from radar A data = 4
Technically according to Matlab output, H is full rank which implies that the initial position and velocity estimate are observable.
However, the elements of H that were derived with respect to position are nearly 0. You can get velocity but not position from bearing (angle) measurements.
If the position differential elements of H were to go to 0, then the rank of H would be 2.

Part e: bearing-only estimate error covariance
The estimation of the solution using just theta from radar A is useless.
The diagonal elements of the estimation error covariance are much larger than the diagonal elements of the measurement covariance.
See the ratios of the diagonal elements below:

ans =

          1128895.48992297


ans =

          49923221.3325016


ans =

          5247.03276378764


ans =

          7304212.50187549
```

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