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Research on Approaching Strategies of Non cooperative Targets in Space Based on Game Theory

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ABSTRACT

This paper studies the spacecraft short range pursuit game. Firstly, the scenario of spacecraft short range pursuit game is constructed, and the non-linear model of spacecraft attitude orbit integration is established. By designing the payment function, the pursuit game is transformed into a non-linear quadratic differential game problem. Then the optimal control rate based on state variables is designed, and the simulation is carried out. The results show that the spacecraft based on game strategy can better achieve high dynamic target tracking and acquisition under certain degree of maneuverability.

Keywords: Game theory; Spacecraft; Pursuit game

1. INTRODUCTION

With the development of science and technology, spacecraft's ability to obtain space information, orbit and attitude maneuverability are constantly improving, and intelligence and autonomy are constantly strengthening. Future spacecraft will have certain analysis and decision-making capabilities. At this time, the traditional non cooperative target approach methods can no longer meet the task requirements, and unilateral approach strategies are no longer applicable. In addition, high dynamic targets can take position maneuver or attitude maneuver and other measures to avoid control before being contacted. Orbit and attitude constraints need to be considered simultaneously in the tracking process, which brings greater challenges to the approaching control of non-cooperative targets. Through the application of game theory and optimal control method, the research on the approach strategy of spacecraft near non cooperative target based on game theory is an important research direction to improve the on-board service and operation capability of spacecraft.

There have been many studies on space non cooperative target approach at home and abroad, but there are few studies on close non cooperative target approach based on dynamic game, and the research scenarios are quite different. Taking space robots and runaway satellites as research objects, Li Zixing[2] carried out satellite game model building and solving, robot arm configuration optimization and adjustment for satellite approach, robot arm approach control under environmental constraints of arm movement, and satellite game strategy simulation experiments; Wang Chunbao[3] and others deduced the segmented game strategy of spacecraft for spacecraft interception, deduced the optimal game strategy between spacecraft and target based on zero control miss distance, and studied and simulated the game problem involving three parties; Sun Songtao[4] proposed an improved multi-objective genetic algorithm and multiple shooting method to solve the non-cooperative target approach game of spacecraft based on the assumption that both spacecraft in low Earth orbit are continuous and small thrust. The above documents are all studies on the game of spacecraft long distance orbit approach, and do not consider the near distance non cooperative target approach scenarios involving attitude maneuver. This paper will focus on this feature.

2. CONSTRUCTION OF NON-COOPERATIVE TARGET APPROACHING GAME SCENARIO

In order to study the approaching problem of two spacecraft non cooperative targets near a reference point in near Earth orbit, a game scenario of spacecraft non cooperative target approaching in near distance was constructed, and numerical algorithms were used to solve the approaching strategy of non-cooperative targets. Consider that the non-cooperative target approach of spacecraft is carried out near a large space station. A spacecraft (main spacecraft P) near the space station hopes to approach another spacecraft (target spacecraft E) within a fixed time, while the target spacecraft hopes to stay away from the main spacecraft. It is assumed that when two spacecraft are engaged in a close relative approach game, the control device of the main spacecraft can aim at the specific orientation of the target spacecraft, while the target spacecraft makes attitude adjustment to avoid the maneuver of the main spacecraft.

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The two spacecraft are equipped with fixed nozzles at the tail for maneuver, so the two sides are involved in the game of orbit and attitude. Therefore, a dynamic model of attitude orbit coupling is considered. If the terminal distance between the two spacecraft is a payoff function, the main spacecraft hopes to minimize the payoff function, while the target spacecraft hopes to maximize the payoff function.

Using the LVLH coordinate system, take the moving point o on a reference orbit near the main spacecraft and the target spacecraft as the origin of the moving coordinate, select the x-axis as the connecting direction between the reference point o and the earth center, and the y-axis as the direction of motion of the reference point o along the orbit and the The axis is vertical, and the z axis forms the right-hand rule with the x and y axes.

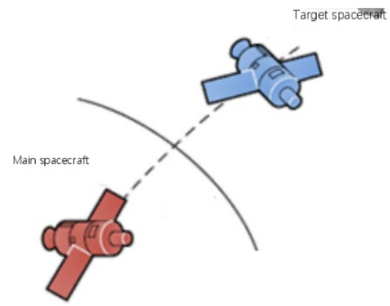

Figure 1. Spacecraft non cooperative target approach game scenario

3. ATTITUDE ORBIT INTEGRATED DYNAMIC MODEL OF SPACECRAFT

The orbit dynamics of the relative motion between the main spacecraft and the target spacecraft can be expressed as:

$$\begin{cases} \ddot{x}_i = \dfrac{\mu}{r_o^2} - \dfrac{\mu}{r_i^3}(x_i + r_o) + 2\omega \dot{y}_i + \dot{\omega} y_i + \omega^2 x_i \\ \ddot{y}_i = -\dfrac{\mu}{r_i^3} y_i - 2\omega \dot{x}_i - \dot{\omega} x_i + \omega^2 y_i \\ \ddot{z}_i = -\dfrac{\mu}{r_i^3} z_i \end{cases} \tag{1}$$

Where, $r_i = \sqrt{(r_o + x_i)^2 + y_i^2 + z_i^2}$, $i = P, E$ 。The relative motion dynamics equation of two spacecraft is expressed as follows:

$$\begin{aligned} \dot{\boldsymbol{x}}_P &= f(\boldsymbol{x}_P) + \boldsymbol{B}_P \boldsymbol{u}_P \\ \dot{\boldsymbol{x}}_E &= f(\boldsymbol{x}_E) + \boldsymbol{B}_E \boldsymbol{u}_E \end{aligned} \tag{2}$$

Among them, $\boldsymbol{B}_P = \boldsymbol{B}_E = \begin{bmatrix} \boldsymbol{0}_{3\times3} \\ I_{3\times3} \end{bmatrix}$ The relative state quantity is introduced to reduce the dimension of the model, and the nonlinear dynamic model is obtained as follows:

$$\dot{\boldsymbol{x}}_{PE} = f(x_P) - f(x_E) + \boldsymbol{B}_P \boldsymbol{u}_P - \boldsymbol{B}_E \boldsymbol{u}_E \tag{3}$$

Its differential equation is expanded as follows:

$$\begin{cases} \ddot{x} = \left[\frac{\mu}{r_E^3}(x_E + r_o) - \frac{\mu}{r_P^3}(x_P + r_o) \right] + 2\omega\dot{y} + \dot{\omega}y + \omega^2 x + u_{Px} - u_{Ex} \\ \ddot{y} = \left[\frac{\mu}{r_E^3} y_E - \frac{\mu}{r_P^3} y_P \right] - 2\omega\dot{x} - \dot{\omega}x + \omega^2 y + u_{Py} - u_{Ey} \\ \ddot{z} = \left[\frac{\mu}{r_E^3} z_E - \frac{\mu}{r_P^3} z_P \right] + u_{Pz} - u_{Ez} \end{cases} \tag{4}$$

hypothesis

$$\begin{gathered} b_1 \triangleq \frac{\mu}{r_E^3}(x_E + r_o) - \frac{\mu}{r_P^3}(x_P + r_o) \\ b_2 \triangleq \frac{\mu}{r_E^3} y_E - \frac{\mu}{r_P^3} y_P \\ b_3 \triangleq \frac{\mu}{r_E^3} z_E - \frac{\mu}{r_P^3} z_P \end{gathered} \tag{5}$$

At the same time b_1、b_2、b_3 Multiply and divide by $\boldsymbol{x}_{PE}$ The square of module, then the above formula can be written as:

$$\begin{cases} \ddot{x} = b_1 \frac{|\boldsymbol{x}_{PE}|^2}{|\boldsymbol{x}_{PE}|^2} + 2\omega\dot{y} + \dot{\omega}y + \omega^2 x + u_{Px} - u_{Ex} \\ \ddot{y} = b_2 \frac{|\boldsymbol{x}_{PE}|^2}{|\boldsymbol{x}_{PE}|^2} - 2\omega\dot{x} - \dot{\omega}x + \omega^2 y + u_{Py} - u_{Ey} \\ \ddot{z} = b_3 \frac{|\boldsymbol{x}_{PE}|^2}{|\boldsymbol{x}_{PE}|^2} + u_{Pz} - u_{Ez} \end{cases} \tag{6}$$

order $\boldsymbol{r}^2 = |\boldsymbol{x}_{PE}|^2$, the above formula can be written as:

$$\begin{cases} \ddot{x} = \left(b_1 \frac{x}{r^2}\right)x + \left(b_1 \frac{y}{r^2}\right)y + \left(b_1 \frac{z}{r^2}\right)z + 2\omega\dot{y} + \dot{\omega}y + \omega^2 x + u_{Px} - u_{Ex} \\ \ddot{y} = \left(b_2 \frac{x}{r^2}\right)x + \left(b_2 \frac{y}{r^2}\right)y + \left(b_2 \frac{z}{r^2}\right)z - 2\omega\dot{x} - \dot{\omega}x + \omega^2 y + u_{Py} - u_{Ey} \\ \ddot{z} = \left(b_3 \frac{x}{r^2}\right)x + \left(b_3 \frac{y}{r^2}\right)y + \left(b_3 \frac{z}{r^2}\right)z + u_{Pz} - u_{Ez} \end{cases} \tag{7}$$

Then the above equation is written in the form of state space:

$$\dot{\boldsymbol{x}}_{PE} = \boldsymbol{A}_{PE}(\boldsymbol{x}_{PE})\boldsymbol{x}_{PE} + \boldsymbol{B}_P\boldsymbol{u}_P - \boldsymbol{B}_E\boldsymbol{u}_E \tag{8}$$

Where

$$
\boldsymbol{A}_{PE}(\boldsymbol{x}_{PE}) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \omega^2 + b_1 \frac{x}{r^2} & \dot{\omega} + b_1 \frac{y}{r^2} & b_1 \frac{z}{r^2} & 0 & 2\omega & 0 \\ -\dot{\omega} + b_2 \frac{x}{r^2} & \omega^2 + b_2 \frac{y}{r^2} & b_2 \frac{z}{r^2} & -2\omega & 0 & 0 \\ b_3 \frac{x}{r^2} & b_3 \frac{y}{r^2} & b_3 \frac{z}{r^2} & 0 & 0 & 0 \end{bmatrix} \tag{9}
$$

The modified Rodrigues parameter methods (MRPs) are used to describe the spacecraft attitude, which is defined as follows

$$
\sigma = \frac{\hat{\boldsymbol{q}}}{1+q_0} = \boldsymbol{k}_e \tan\frac{\phi}{4} \tag{10}
$$

among $\boldsymbol{k}_e \in \mathbb{R}^3$ Is the attitude rotation Euler axis; $\phi \in (-2\pi, 2\pi)$ Is Euler angle; $\boldsymbol{q} = [q_0, \hat{\boldsymbol{q}}^T]^T$ Represents the attitude quaternion. Then the spacecraft attitude dynamics equation and kinematics equation are expressed as follows:

$$
\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{u}_\tau + \boldsymbol{d}_\tau \tag{11}
$$

$$
\dot{\boldsymbol{\sigma}} = G(\boldsymbol{\sigma})\boldsymbol{\omega}
$$

$$
G(\boldsymbol{\sigma}) = \frac{1}{4}\left[(1-\boldsymbol{\sigma}^T\boldsymbol{\sigma})\boldsymbol{I}_{3\times3} + 2\boldsymbol{\sigma}^\times + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^\mathrm{T}\right] \tag{12}
$$

Among them, $\boldsymbol{u}_\tau$ Is the control torque, $\boldsymbol{d}_\tau$ Is the disturbing torque, $\boldsymbol{I}$ Is the identity matrix.

hypothesis $\boldsymbol{\sigma}_E$ and $\boldsymbol{\omega}_E$ Are the attitude and angular velocity of the target spacecraft, $\boldsymbol{\sigma}_P$ and $\boldsymbol{\omega}_P$ Attitude and angular velocity of main spacecraft respectively.Then the two sides take a relative attitude $\boldsymbol{\sigma}_{PE}$ and $\boldsymbol{\omega}_{PE}$ They are respectively expressed as:

$$
\boldsymbol{\sigma}_{PE} = \frac{(1-\boldsymbol{\sigma}_E^\mathrm{T}\boldsymbol{\sigma}_E)\boldsymbol{\sigma}_{PE} - (1-\boldsymbol{\sigma}_P^\mathrm{T}\boldsymbol{\sigma}_P)\boldsymbol{\sigma}_E + 2\boldsymbol{\sigma}_P^\times\boldsymbol{\sigma}_E}{1+\boldsymbol{\sigma}_P^\mathrm{T}\boldsymbol{\sigma}_P\boldsymbol{\sigma}_E^\mathrm{T}\boldsymbol{\sigma}_E + 2\boldsymbol{\sigma}_E^\mathrm{T}\boldsymbol{\sigma}_P} \tag{13}
$$

$$
\boldsymbol{\omega}_{PE} = \boldsymbol{\omega}_P - \boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E = G^{-1}(\boldsymbol{\sigma}_{PE})\dot{\boldsymbol{\sigma}}_{PE} \tag{14}
$$

Among them, $\boldsymbol{R}(\boldsymbol{\sigma}_{PE})$ It is the coordinate transformation matrix of the two spacecraft system.

$$
\boldsymbol{R}(\boldsymbol{\sigma}_{PE}) = \boldsymbol{R}(\boldsymbol{\sigma}_P)[\boldsymbol{R}(\boldsymbol{\sigma}_E)]^\mathrm{T} \tag{15}
$$

$$
\boldsymbol{R}(\boldsymbol{\sigma}) = I_{3\times3} - \frac{4(1-\boldsymbol{\sigma}^T\boldsymbol{\sigma})}{(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^2}\boldsymbol{\sigma}^\times + \frac{8\boldsymbol{\sigma}^\times}{(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^2}\boldsymbol{\sigma}^\times \tag{16}
$$

The relative attitude kinematics equation of the spacecraft can be obtained from equation (14), and the relative attitude dynamics equation can be obtained from the derivation of equation (14) combined with equation (11):

$$
\begin{aligned}
\dot{\boldsymbol{\omega}}_{PE} &= -\boldsymbol{J}^{-1}\boldsymbol{\omega}_P{}^\times\boldsymbol{J}\boldsymbol{\omega}_P + \boldsymbol{J}^{-1}\boldsymbol{\tau} + \boldsymbol{J}^{-1}\boldsymbol{d} + \\
&\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{J}^{-1}\boldsymbol{\omega}_E{}^\times\boldsymbol{J}\boldsymbol{\omega}_E - \boldsymbol{\omega}_{PE}{}^\times\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E
\end{aligned} \tag{17}
$$

It is assumed that the moment of inertia matrix of the main spacecraft and the target spacecraft are the same, $\boldsymbol{d}$ and $\boldsymbol{\tau}$ They are disturbance torque and relative attitude control torque respectively, and equation (14) is substituted into equation (17) to get

$$\begin{aligned}
&\dot{\boldsymbol{\omega}}_{PE} = -\boldsymbol{J}^{-1}[\boldsymbol{\omega}_{PE} + \boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E]^{\times}\boldsymbol{J}[\boldsymbol{\omega}_{PE} + \boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E] + \\
&\boldsymbol{J}^{-1}\boldsymbol{\tau} + \boldsymbol{J}^{-1}\boldsymbol{d} + \boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{J}^{-1}\boldsymbol{\omega}_E{}^{\times}\boldsymbol{J}\boldsymbol{\omega}_E - \boldsymbol{\omega}_{PE}{}^{\times}\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E \\
&\quad = -\boldsymbol{J}^{-1}\boldsymbol{\omega}_{PE}^{\times}\boldsymbol{J}\boldsymbol{\omega}_{PE} - \boldsymbol{J}^{-1}[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E]^{\times}\boldsymbol{J}\boldsymbol{\omega}_{PE} - \\
&\boldsymbol{J}^{-1}\boldsymbol{\omega}_{PE}{}^{\times}\boldsymbol{J}[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E] - \boldsymbol{J}^{-1}[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E]^{\times}\boldsymbol{J}[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E] + \\
&\qquad \boldsymbol{J}^{-1}\boldsymbol{\tau} + \boldsymbol{J}^{-1}\boldsymbol{d} + \boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{J}^{-1}\boldsymbol{\omega}_E{}^{\times}\boldsymbol{J}\boldsymbol{\omega}_E - \boldsymbol{\omega}_{PE}{}^{\times}\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E
\end{aligned} \tag{18}$$

order

$$\begin{aligned}
&\boldsymbol{T} = \boldsymbol{\tau} + \boldsymbol{d} - [\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E]^{\times}\boldsymbol{J}\boldsymbol{\omega}_{PE} - \boldsymbol{\omega}_{PE}{}^{\times}\boldsymbol{J}[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E] - \\
&[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E]^{\times}\boldsymbol{J}[\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E] + \\
&\qquad \boldsymbol{J}\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{J}^{-1}\boldsymbol{\omega}_E{}^{\times}\boldsymbol{J}\boldsymbol{\omega}_E - \boldsymbol{J}\boldsymbol{\omega}_{PE}{}^{\times}\boldsymbol{R}(\boldsymbol{\sigma}_{PE})\boldsymbol{\omega}_E
\end{aligned} \tag{19}$$

From this, it can be concluded that the attitude dynamics equation is

$$\dot{\boldsymbol{\omega}}_{PE} = -\boldsymbol{J}^{-1}\boldsymbol{\omega}_{PE}^{\times}\boldsymbol{J}\boldsymbol{\omega}_{PE} + \boldsymbol{J}^{-1}\boldsymbol{T} \tag{20}$$

Design state quantity $\left[\boldsymbol{\sigma}_{PE}^{\mathrm{T}}, \boldsymbol{\omega}_{PE}^{\mathrm{T}}\right]^{\mathrm{T}}$ $\boldsymbol{\sigma}_{PE}^{\mathrm{T}} = [\sigma_1, \sigma_2, \sigma_3]^{\mathrm{T}}$， $\boldsymbol{\omega}_{PE}{}^{\mathrm{T}} = [\omega_1, \omega_2, \omega_3]^{\mathrm{T}}$ Then the state space is expressed as

$$\begin{bmatrix} \dot{\boldsymbol{\sigma}}_{PE} \\ \dot{\boldsymbol{\omega}}_{PE} \end{bmatrix} = \boldsymbol{M}\begin{bmatrix} \boldsymbol{\sigma}_{PE} \\ \boldsymbol{\omega}_{PE} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{J}^{-1} \end{bmatrix}\boldsymbol{T} \tag{21}$$

Among them,

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{0} & G(\boldsymbol{\sigma}_{PE}) \\ \boldsymbol{0} & -\boldsymbol{J}^{-1}\boldsymbol{\omega}_{PE}{}^{\times}\boldsymbol{J} \end{bmatrix} \tag{22}$$

Define a new state quantity $\boldsymbol{\xi} = \left[\boldsymbol{\sigma}_{PE}^{\mathrm{T}}, \boldsymbol{\omega}_{PE}^{\mathrm{T}}, \boldsymbol{x}_{PE}^{\mathrm{T}}, \dot{\boldsymbol{x}}_{PE}^{\mathrm{T}}\right]^{\mathrm{T}}$, define a new control quantity $\boldsymbol{\upsilon}_P = \left[T_P^{\mathrm{T}}, u_P^{\mathrm{T}}\right]^{\mathrm{T}}$, $\boldsymbol{\upsilon}_E = \left[T_E^{\mathrm{T}}, u_E^{\mathrm{T}}\right]^{\mathrm{T}}$, Formula (10) and Formula (21) are sorted out, and the relative dynamic model of nonlinear attitude orbit integrated spacecraft is obtained as follows:

$$\dot{\boldsymbol{\xi}} = \boldsymbol{A}_{cop}(\boldsymbol{\xi})\boldsymbol{\xi} + \boldsymbol{B}_{cop}\boldsymbol{\upsilon}_P + \boldsymbol{C}_{cop}\boldsymbol{\upsilon}_E \tag{23}$$

Where, $\boldsymbol{A}_{cop} = \begin{bmatrix} \boldsymbol{M} & 0_{6\times6} \\ 0_{6\times6} & \boldsymbol{A}_{PE} \end{bmatrix}$, $\boldsymbol{B}_{cop} = \begin{bmatrix} \boldsymbol{0}_{3\times3} \\ \boldsymbol{J}^{-1} \\ \boldsymbol{0}_{3\times3} \\ \boldsymbol{A}_{Hc} \end{bmatrix}$, $\boldsymbol{C}_{cop} = -\boldsymbol{B}_{cop}$。

4. DESIGN OF PAYMENT FUNCTION MODEL

Player P , that is, the quadratic payment function of the main spacecraft is written as:

$$J_P = \frac{1}{2}\int_{t_0}^{\infty}\left[\boldsymbol{x}^T(t)\boldsymbol{Q}_i\boldsymbol{x}(t) + \boldsymbol{u}_P^T(t)\boldsymbol{R}_P\boldsymbol{u}_P(t) - \boldsymbol{u}_E^T(t)\boldsymbol{R}_E\boldsymbol{u}_E(t)\right]\mathrm{d}t \tag{24}$$

According to the two-person zero sum differential game theory, we can get:

$$J_P + J_E = 0 \tag{25}$$

with $\boldsymbol{u}_P^*$ and $\boldsymbol{u}_E^*$ Represents the optimal control strategy, which can be obtained from the definition of the saddle point solution of the game problem. The function under the optimal control quantity should meet the following relationship:

$$\begin{aligned} J_P\left(\boldsymbol{u}_P^*, \boldsymbol{u}_E^*\right) \le J_P\left(\boldsymbol{u}_P, \boldsymbol{u}_E^*\right) \\ J_E\left(\boldsymbol{u}_P^*, \boldsymbol{u}_E^*\right) \le J_E\left(\boldsymbol{u}_P^*, \boldsymbol{u}_E\right) \end{aligned} \tag{26}$$

It can be seen from the payment function shown in equation (26) that in this problem, the payment function of both sides not only includes the distance between the two sides, but also includes the control amount of both sides. Among $\boldsymbol{u}_P^T(t)\boldsymbol{R}_P\boldsymbol{u}_P(t)$ Represents the energy consumed by the main spacecraft during the game, $\boldsymbol{u}_E^T(t)\boldsymbol{R}_E\boldsymbol{u}_E(t)$ Indicates the energy consumed by the target spacecraft in the game process. $\boldsymbol{R}_P$ and $\boldsymbol{R}_E$ Is the weight matrix of energy. The value of the matrix represents the proportion of fuel consumption in the total payment during the game. When $\boldsymbol{R}_P$ and $\boldsymbol{R}_E$ When the weight in is large, it indicates that more attention should be paid to saving energy in this process. Therefore, it can be seen that this type of payment function combines the distance and energy loss of both parties, which is more practical.

5. SOLUTION OF OPTIMAL CONTROL RATE

The Hamiltonian function is introduced as follows:

$$\begin{aligned} & H(t, \boldsymbol{\lambda}, \boldsymbol{x}, \boldsymbol{u}_P, \boldsymbol{u}_E) = \\ & \frac{1}{2}(\boldsymbol{x}^T(t)\boldsymbol{Q}\boldsymbol{x}(t) + \boldsymbol{u}_P^T(t)\boldsymbol{R}_P\boldsymbol{u}_P(t) - \boldsymbol{u}_E^T(t)\boldsymbol{R}_E\boldsymbol{u}_E(t)) \\ & +\boldsymbol{\lambda}^T(\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}_P\boldsymbol{u}_P(t) - \boldsymbol{B}_E\boldsymbol{u}_E(t)) \end{aligned} \tag{27}$$

If the control is not constrained, the optimal control quantity is expressed as:

$$\begin{cases} 0 = \dfrac{\partial H}{\partial \boldsymbol{u}_P} \\ 0 = \dfrac{\partial H}{\partial \boldsymbol{u}_E} \end{cases} \tag{28}$$

The control equation is obtained:

$$\begin{aligned} \boldsymbol{u}_P(t) = -\boldsymbol{R}_P^{-1}\boldsymbol{B}_P^T\boldsymbol{\lambda}(t) \\ \boldsymbol{u}_E(t) = \boldsymbol{R}_E^{-1}\boldsymbol{B}_E^T\boldsymbol{\lambda}(t) \end{aligned} \tag{29}$$

The co state vector satisfies the following differential equation:

$$\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H}{\partial \boldsymbol{x}(t)} = -\boldsymbol{Q}\boldsymbol{x}(t) - \boldsymbol{A}^T\boldsymbol{\lambda}(t) \tag{30}$$

Here it is assumed that the co state vector is a linear function of the state vector, namely:

$$\boldsymbol{\lambda}(t) = \boldsymbol{P}(t)\boldsymbol{x}(t) \tag{31}$$

Then the feedback control law is:

$$\begin{aligned} \boldsymbol{u}_P &= -\boldsymbol{R}_P^{-1}\boldsymbol{B}_P^T\boldsymbol{P}(t)\boldsymbol{x}(t) \\ \boldsymbol{u}_E &= \boldsymbol{R}_E^{-1}\boldsymbol{B}_E^T\boldsymbol{P}(t)\boldsymbol{x}(t) \end{aligned} \tag{32}$$

The differential of the co state vector can be expressed as:

$$\dot{\boldsymbol{\lambda}}(t)=\dot{\boldsymbol{P}}(t)\boldsymbol{x}(t)+\boldsymbol{P}(t)\dot{\boldsymbol{x}}(t) \tag{33}$$

Equation (33), combined with Equation (30), gives:

$$\begin{aligned} \dot{\boldsymbol{P}}(t) = &-\boldsymbol{P}(t)\boldsymbol{A}-\boldsymbol{A}^T\boldsymbol{P}(t) \\ &+\boldsymbol{P}(t)\left(\boldsymbol{B}_P\boldsymbol{R}_P^{-1}\boldsymbol{B}_P^T-\boldsymbol{B}_E\boldsymbol{R}_E^{-1}\boldsymbol{B}_E^T\right)\boldsymbol{P}(t) \\ &-\boldsymbol{Q} \end{aligned} \tag{34}$$

For the infinite time domain differential game problem, change equation (14) to:

$$\dot{\boldsymbol{P}}(t)=0 \tag{35}$$

The following Riccati algebraic equation is obtained:

$$-\boldsymbol{P}\boldsymbol{A}-\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\left(\boldsymbol{B}_P\boldsymbol{R}_P^{-1}\boldsymbol{B}_P^T-\boldsymbol{B}_E\boldsymbol{R}_E^{-1}\boldsymbol{B}_E^T\right)\boldsymbol{P}-\boldsymbol{Q}=0 \tag{36}$$

By solving equation (36), the optimal feedback control strategy of both sides can be obtained. The solution is based on solving the state dependent Riccati algebraic equation (SDRE).According to the nonlinear dynamic model shown in (23), the control laws of the two spacecraft are expressed as:

$$\begin{cases} \boldsymbol{\upsilon}_P = -\boldsymbol{R}^{-1}\boldsymbol{B}_{cop}{}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{\xi})\boldsymbol{\xi} \\ \boldsymbol{\upsilon}_E = -\gamma^{-2}\boldsymbol{R}^{-1}\boldsymbol{C}_{cop}{}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{\xi})\boldsymbol{\xi} \end{cases} \tag{37}$$

For the infinite time domain control problem $\boldsymbol{P}(\boldsymbol{\xi})$ Meet Riccati algebraic equation:

$$\begin{aligned} 0= &\boldsymbol{P}(\xi)\boldsymbol{A}_{cop}(\xi)+\boldsymbol{A}_{cop}^{\mathrm{T}}(\xi)\boldsymbol{P}(\xi)- \\ &\boldsymbol{P}(\xi)\left(\boldsymbol{B}_{cop}\boldsymbol{R}^{-1}\boldsymbol{B}_{cop}-\gamma^{-2}\boldsymbol{C}_{cop}\boldsymbol{R}^{-1}\boldsymbol{C}_{cop}^{\mathrm{T}}\right)\boldsymbol{P}(\xi)+\boldsymbol{Q} \end{aligned} \tag{38}$$

And $\boldsymbol{P}(\boldsymbol{\xi})\geq 0$, $\boldsymbol{R}>0$ 。Because matrix $\boldsymbol{A}_{cop}(\boldsymbol{\xi})$ It contains state quantity information, so its value will change with the change of state. The nonlinear dynamic system can be decomposed or parameterized into the product of state vector and state dependent matrix function, which can contain the nonlinear characteristics of the system.

6. MATHEMATICAL SIMULATION AND ANALYSIS

The initial conditions shown in Table 1 are given:

Table 1. Initial orbit elements of reference orbit.

essential factor	a	e	i	ω	Ω	tp
numerical value	6900km	0	0	0	0	0

The difference between two spacecraft status quantities in LVLH coordinate system is:

Table 2. State Quantity Difference between Two Spacecrafts

name	Three axis initial position state value difference (km)			Three axis initial speed state value difference (km/s)		
Symbol	Δx	Δy	Δz	$\Delta \dot{x}$	$\Delta \dot{y}$	$\Delta \dot{z}$
numerical value	10	10	10	0.005	0.005	0.005

The rotational inertia matrix of the two spacecraft is:

$$I_{P,E} = \begin{bmatrix} I_x & & \\ & I_y & \\ & & I_z \end{bmatrix}$$

among $I_x = 25kg \cdot m^2$, $I_y = 20kg \cdot m^2$, $I_z = 18kg \cdot m^2$.

Orbital control quantity of main spacecraft $\|u_{Pi}\| \le 2m/s$, orbital control quantity of target spacecraft $\|u_{Ei}\| \le 1m/s$, attitude control quantity of main spacecraft $\|T_{Pi}\| \le 0.02N \cdot m$, attitude control quantity of target spacecraft $\|T_{Ei}\| \le 0.01N \cdot m$。The game simulation of spacecraft under the attitude orbit integration model is carried out.

Example (1): Assume the coefficient of control capability difference between two spacecraft α by $\frac{1}{\sqrt{2}}$ The simulation results are as follows:

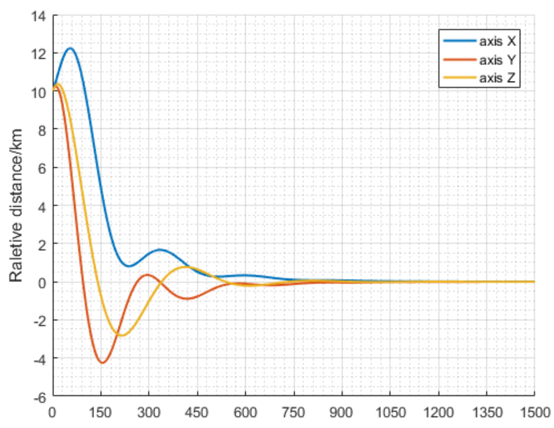

Figure 2. Three axis relative position change of two spacecraft

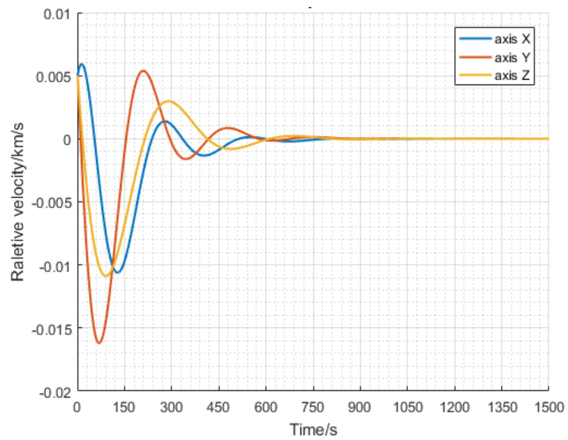

Figure 3. Three axis relative velocity change of two spacecraft

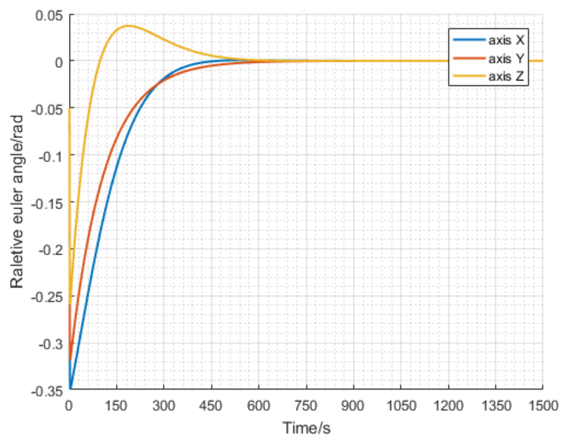

Figure 4. Variation of Three axis Relative Attitude Angle Difference

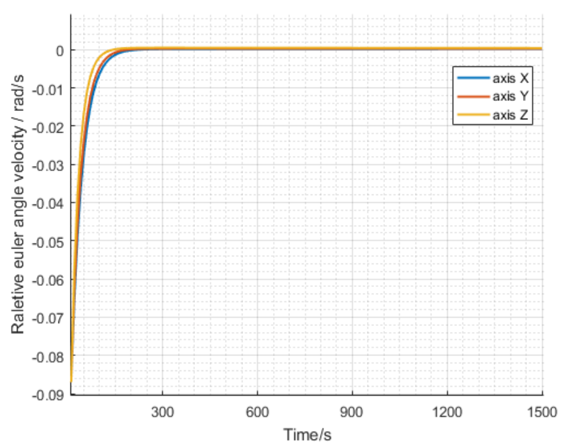

Figure 5. Variation of Relative Attitude Angular Velocity Difference

Example (2): Change the control capability difference coefficient of two spacecraft α , set α 0.5, the initial conditions are the same, and the simulation results are as follows:

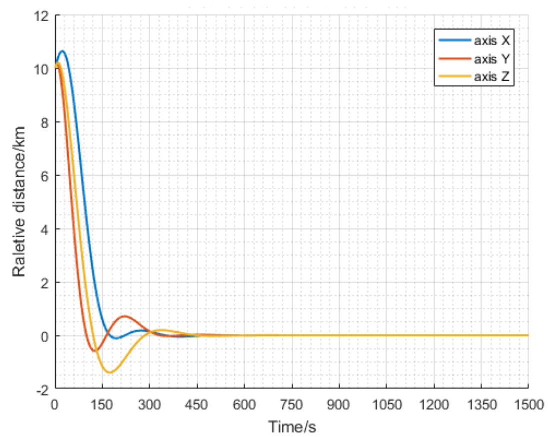

Figure 6. Change of Three Axis Relative Position of Two Spacecrafts

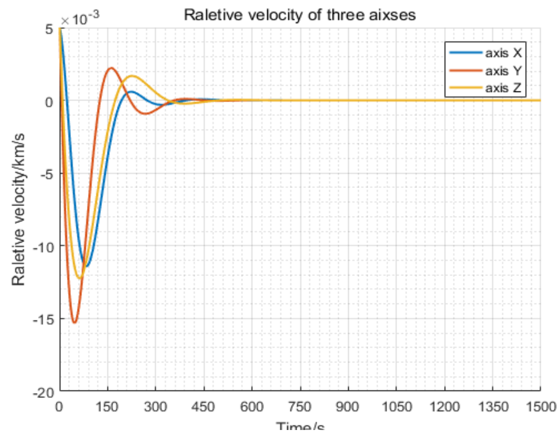

Figure 7. Three axis relative velocity change of two spacecraft

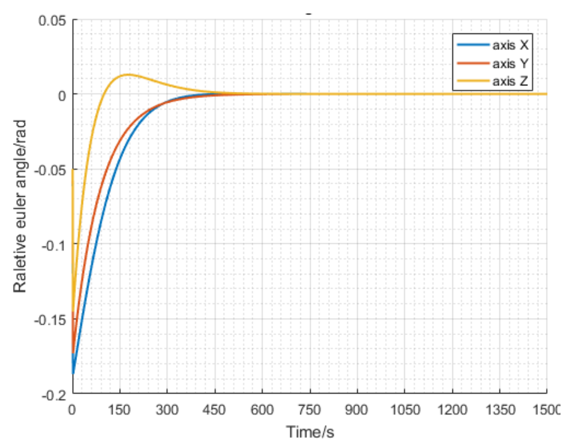

Figure 8. Variation of Three axis Relative Attitude Angle Difference

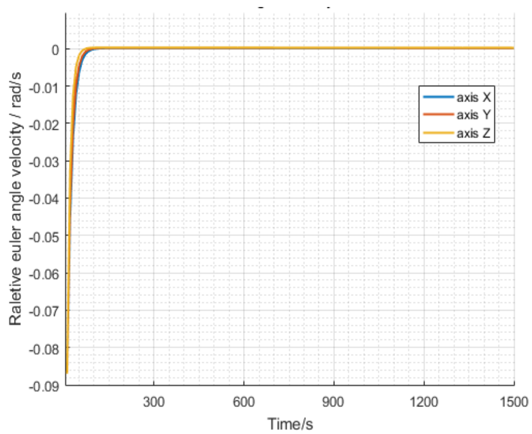

Figure 9. Change of Relative Attitude Angular Velocity Difference

By changing α And change the level of mobility difference between the main spacecraft and the target spacecraft, so different game simulation results can be obtained. By comparing example (1) and example (2), it can be found that the system can be stable near zero. That is to say, when the mobility of the target spacecraft is less than that of the main spacecraft, the main spacecraft will finally successfully approach the target spacecraft after the game between the two parties, and the stronger the mobility of the main spacecraft, the higher the efficiency of approach (the faster the system convergence speed).The optimal trajectories of both sides can be obtained by using game strategies, and the control strategy solution at each time is based on the state quantity at the previous time, which has good dynamic performance.

7. CONCLUSION

Based on spacecraft relative motion dynamics and dynamic game theory, this paper studies the scenario construction and game strategy of spacecraft close approach for space targets with autonomous mobility. Considering the energy consumption of the non-cooperative target approaching the mission, the relative distance of the spacecraft, the relative attitude and other indicators, a relatively complete evaluation criterion of mission income is formulated, and the expression of the spacecraft payment function is designed; The necessary conditions for the optimal control strategies of both sides are derived by establishing the integrated dynamic model of attitude and orbit for short distance.

Combining the above theories and set scenarios, the nonlinear model is simulated, and the simulation results of the non-cooperative target approach game of the close-range attitude orbit integrated spacecraft are obtained, which verifies the effectiveness of the approach strategy, and shows that the mobility has an important impact on the approach results.

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