Série 2 – EC 260 – Misael Levi Costa Cavalcanti

01) a) $P_i = \oint_\Omega U(\theta,\phi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta \sin^2\phi \sin\theta \, d\theta\, d\phi = \int_0^{2\pi} \sin^2\phi \left(\int_0^{\pi/2} \cos^4\theta \sin\theta\, d\theta \right) d\phi$

tomando $u = \cos\theta \Rightarrow du = -\sin\theta\, d\theta \Rightarrow \int_0^{\pi/2} \cos^4\theta \sin\theta\, d\theta = \int_0^1 u^4\, du = \frac{1}{5}$

$\Rightarrow P_i = \frac{1}{5}\int_0^{2\pi} \sin^2\phi\, d\phi = \frac{1}{5} \cdot \int_0^{2\pi} \frac{1}{2}(1 - \cos 2\phi)\, d\phi = \frac{\pi}{5}$

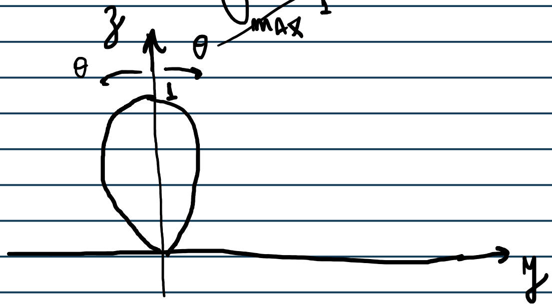
b) $D_{dB} = 10 \log[D]$, $D = \frac{4\pi \cdot U(\theta,\phi)_{máx}}{P_i} = \frac{4\cancel{\pi} \cdot 1}{\cancel{\pi}/5} = 20$

$\Rightarrow \boxed{D_{dB} = 10 \log 20 = 10(1 + 0,3) \cong 13}$ //

c) No plano yz, $\phi = \pm \pi/2 \Rightarrow \sin^2\phi = 1$.

$\Rightarrow 0 \leq \theta \leq \pi/2 \Rightarrow u = \frac{U(\theta,\phi)}{\cancel{U_{máx}}^{\,1}} = \cos^4\theta$

$\Rightarrow$



d) $|u| = 0,5 \Rightarrow \cos^4(\alpha_{3dB}) = \frac{1}{2} \Rightarrow \alpha_{3dB} = \arccos\left(1/\sqrt[4]{2}\right) = 0,572$ rad

$\Rightarrow HPBW = 2 \cdot \alpha_{3dB} = 1,144$ rad ou $65,53°$

02) $P_r = \underbrace{P_{IN}\left(1-|\Gamma_t|^2\right)}_{P_i \text{ da transmissora}}\left(1-|\Gamma_r|^2\right) p \cdot \frac{\lambda^2}{(4\pi R)^2} \cdot G_t \cdot \underbrace{G_r}_{K \cdot D_r}$

$$p = \frac{\left|(\hat{x}+i\hat{y})\cdot(2\hat{x}+i\hat{y})\right|^2}{|\hat{x}+i\hat{y}|^2 \cdot |2\hat{x}+i\hat{y}|^2} = \frac{|2-1|^2}{2 \cdot 5} = \frac{1}{10}$$

$VSWR = 2 \Rightarrow 1+|\Gamma_r| = 2-2|\Gamma_r| \Rightarrow |\Gamma_r| = 1/3$. Assim, pela fórmula de Friis:

$$10^{-14} = 5 \cdot \frac{8}{9} \cdot \frac{1}{10} \cdot \frac{(3 \cdot 10^8 / 2 \cdot 10^9)^2}{(4\pi \cdot 4 \cdot 10^6)^2} \cdot 10^3 \cdot 0{,}95 \cdot D_r$$

$$\Rightarrow \boxed{D_r = 2{,}66}$$

$$A_{em} = K \frac{\lambda^2}{4\pi} D_r = 0{,}95 \cdot \left(\frac{3 \cdot 10^8}{2 \cdot 10^9}\right)^2 \cdot \frac{1}{4\pi} \cdot 2{,}66 \cong 45{,}23 \text{ cm}^2$$

03) a) $\vec{E}^{i}_{w} = (i\hat{y} + 3\hat{z})E_0 \cdot e^{iK_0 x}$ (já está no referencial da receptora)

⇒ elipticamente polarizada à esquerda

b) $\vec{E}_a = (\hat{y}+2\hat{z})E_a\, e^{-iK_0 x}$

⇒ polarização linear.

c) $PLF = |\vec{P}_w \cdot \vec{P}_a|^2 = \left|(i\hat{y}+3\hat{z})\frac{1}{\sqrt{10}} \cdot (\hat{y}+2\hat{z})\frac{1}{\sqrt{5}}\right|^2$

$\Rightarrow PLF = \left|\frac{i}{3\sqrt{2}} + \frac{6}{5\sqrt{2}}\right|^2 = \frac{1}{50}\cdot 37$

$\Rightarrow PLF_{dB} = 10\log\left(\frac{37}{50}\right) \cong -1{,}31\ dB$

04) a) $\vec{E}_w = (4\hat{a}_z + j\cdot 2\hat{a}_x)E_w\, \underset{y}{\underline{e}}^{jky}$ ⇒ Elipticamente polarizada para a direita

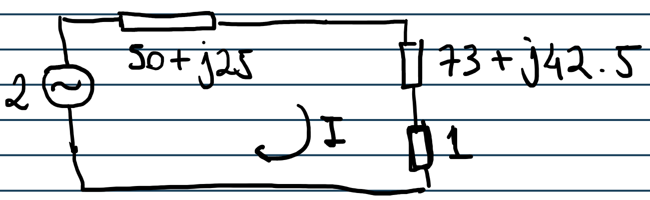
b) A componente adiantada é em x. Logo $\hat{z}$ gira na direção de $\hat{x}$, estabelecendo sentido para a direita.

c) $AR = \frac{4}{2} = 2$

d) $PLF = |\vec{P}_w \cdot \vec{P}_a|^2 = \left|(4\hat{z}+2j\hat{x})\frac{1}{2\sqrt{5}}(\hat{z})\right|^2 = \frac{4}{5} = 0{,}8$

$\Rightarrow PLF_{dB} = 10\log 0{,}8 = -0{,}97\ dB$

05) $\boxed{2.53}$



a) $P_s = \frac{1}{2}Re\{2\cdot I^*\}$

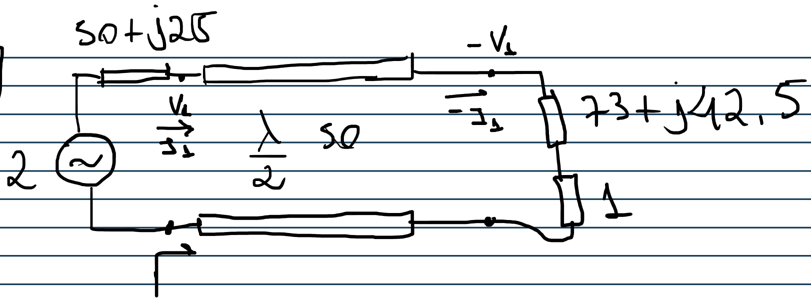
$\rightarrow \boxed{P_s = 0{,}0125\ W}$

$I = \frac{2}{(124+j67{,}5)}$

b) $P_i = \frac{1}{2}Re\{(73+j42.5)\cdot|I_g|^2\} = 0{,}00732\ W$

c) $P_d = \frac{1}{2}Re\{1\cdot|I_g|^2\} = 0{,}0001\ W$

05) $\boxed{2.54}$



$Z_{IN} = Z_L = 74 + j42,5$

→ a) Potência fornecida pela fonte é igual a do caso anterior.

b) Como $l = \lambda/2$, a tensão na carga e a corrente I são as mesmas do caso anterior, mas opostas. Assim a potência irradiada continua sendo a mesma do caso anterior.

c) Mesma do caso anterior.

06) $D = \dfrac{4\pi\, U(\theta,\phi)_{MAX}}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \operatorname{sen}\theta\, d\theta\, d\phi}$. Como máximo de $\operatorname{sinc}(x)$ é 1:

$$\Rightarrow D = \frac{2}{\int_0^{\pi} \frac{\operatorname{sen}^2(\pi \operatorname{sen}\theta)}{\pi^2 \operatorname{sen}\theta}\, d\theta}$$

Por integração numérica → $D = \dfrac{2}{0{,}284332}$

$\Rightarrow \boxed{D = 7{,}034}$

→ $\boxed{D_{dB} = 8{,}66\ dB}$