**Latent-space variational inversion for geologic models from multi-source dynamic data**

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1. **Introduction**

Subsurface modeling plays a crucial role in the characterization and forecasting of energy resources. Subsurface modeling is the integration of all available subsurface data and knowledge for the construction of accurate geological feature spatial distribution uncertainty models to support accurate flow response forecasts for optimum development decision making. However, it is difficult to describe subsurface formations and the distribution of geologic features due to their inherent sparse sampling, non-stationarity, and heterogeneity. Furthermore, the integration of dynamic response data measurements in subsurface modeling to reconstruct spatial features is an ill-posed and extremely underdetermined inverse problem. Most geologic feature inversion techniques thus suffer from issues such as high computational complexity due to the frequency, variability, and variety of measurements, massive parameterization to generate implicit feature spaces, and imprecise conditioning techniques for dynamic data integration and inversion.

The general problem of inversion is widely studied and applied in subsurface energy resource engineering. Fundamentally, any system can be described in three steps: the parameterization of the system, the forward simulation, and the inverse model (Tarantola, 2005). Any system can be described numerically through a set of parameters whose values characterize the system, in this case the subsurface geologic feature distribution. The forward simulation is a physical representation of the dynamic behavior given a set of parameters and represented as a set of observations, namely the numerical reservoir simulation dynamic responses. The inverse problem is the inference of the inherent numerical model by minimizing the difference between the forward simulation and the true measurements. In subsurface engineering, geologic inversion is the reconstruction of a realistic feature distribution that generates the given dynamic states of a numerical reservoir simulation.

Geologic feature inversion workflows are introduced by Backus and Gilbert (1967) to efficiently reconstruct Earth models from high-dimensional geophysical measurements through physics-based computational algorithms. Given the inherent uncertainty and randomness of the subsurface, geostatistics-based workflows are widely used for the conditional and unconditional reconstruction of subsurface geologic feature distributions. Zimmermann et al. (1998) compare different geostatistical-based approaches for transmissibility reconstruction in groundwater flow. Nonetheless, the principal assumption is that prior models can be described through two-point geostatistical models assuming a multi-Gaussian, which is rarely the case (Gomez-Hernandez and Wen, 1998). Thus, multiple-point statistics (MPS) and conditional model simulations show better estimation of geologic feature spatial distributions (Guardiano and Srivastava, 1993; Mariethoz et al., 2010). Despite significant improvement in the inverse geologic estimation problem, these methods still require significant computational efforts and do not properly honor conditional data for accurate spatial reconstruction of geologic features.

More recently, deep learning-based methods have expanded geologic inversion into a new paradigm. Reduced-dimensionality models provide a data-driven efficient parameterization of the inherent geologic model for the inversion framework and are useful for applications such as noise suppression (Canchumuni et al., 2019) and dynamic data integration (Razak et al. 2020, 2022). Different parametrization schemes for latent-space representations of geologic data have aided in the reconstruction (Chan & Elsheikh, 2019; Misra et al., 2022) and forecasting (Alsulaimani & Wheeler, 2021) of subsurface energy resources. These techniques are expanded into applications such as geologic carbon sequestration (Rezaee & Ekundayo, 2022; Yan et al., 2022), or fractured systems (S. Jiang et al., 2021). Deep learning-based dimensionality reduction techniques are used for parameterization of dynamic data and subsequent data assimilation and closed-loop optimization workflows. These frameworks are used for dynamic data integration such as geophysical data (Brankovic et al., 2021) or dynamic well response data (A. Jiang & Jafarpour, 2019, 2021; S. Jiang & Durlofsky, 2023).

Two popular deep learning methods for reduced-dimensionality modeling are Variational AutoEncoders (VAE) (Kingma et al., 2019) and Generative Adversarial Networks (GAN) (Goodfellow et al., 2020). These techniques exploit a latent representation of the geologic feature distributions, obtaining an optimal parameterization of the models that best represents the entire set. General AutoEncoders (AE) aim to compress the given models into a latent-space representation through an encoder network, and to reconstruct the original data through a decoder network. VAEs follow a similar procedure but constrain the latent space to follow a certain distribution, most commonly a multi-Gaussian, to ensure that the latent space is regularized. VAEs also provide the advantage of new (also known as fake) model generation by providing a set of random parameters from the latent distribution and reconstructing new (or fake) geologic feature distributions. GANs on the other hand follow a generator-discriminator approach, where a network is trained to reconstruct models from a latent-space distribution and the discriminator is trained to differentiate realistic from imprecise models, which over time allows the model to create new realistic geologic feature distributions. The main disadvantage of GANs, despite relatively better performance, is their extreme complexity in terms of trainable parameters and their computational requirements in terms of training time and number of examples required.

Multiple efforts are made to enhance MPS-based geologic inversion by incorporating a latent-space distribution models in deep learning methods (Laloy et al. 2017a, 2017b). Abdellatif et al. (2022) show the application of a GAN for non-stationary random fields of binary geologic distributions. Feng et al. (2022) expand GANs by implementing a Bayesian framework for probabilistic binary geologic model reconstruction. The previous methods focus on 2D geologic feature distribution reconstruction, but VAEs and GANs can also be used for 3D geologic model reconstruction (Yang et al., 2022; T. Zhang et al., 2022; T.F. Zhang et al., 2019). However, most examples of these methods are limited to the reconstruction of binary geologic models or Gaussian-distributed geologic models only, not both.

Furthermore, deep learning-based methods can also be conditioned to dynamic observation data for improved reconstruction of subsurface geologic feature distributions. Chan and Elsheikh (2020), demonstrate the potential GANs to generate realistic geologic feature distributions by conditioning the networks to data from observation wells. Further examples of geologic inversion from different sources of conditioning data are shown for geophysical data (Laloy et al., 2019; Pan et al., 2020), and well log data (Bressan et al., 2020; Qi and Carr, 2006). Methods for the initialization of reservoir models from production data are also shown to improve history matching and uncertainty quantification (Emami Niri & Lumley, 2017).

We propose a geological model inversion method from multi-source dynamic response data by training three independent autoencoders and combining them into an inversion network model that reconstructs the corresponding geologic feature realization based on the producer well response data and the dynamic reservoir response data. It follows a transfer learning protocol, where three independent autoencoders are trained offline prior to the inversion step, to find the best latent-space representation of the different data sources. The latent distributions are then used to predict the latent geologic representation with a dense regression network, which is finally passed through the pre-trained geologic decoder network for the geologic inversion step. Our proposed method is directly applicable to subsurface energy resource operations, where we can obtain well measurements and observation well data and utilize these to reconstruct the underlying geological feature distributions. Our method is capable of reconstructing both binary geologic features, such as channel facies, as well as Gaussian-distributed spatial distributions, such as porosity and permeability, for 2D and 3D geologic models – a task that has not been solved through a single model before – through the use of separable depth-wise convolutional layers. We are also able to assimilate multiple data sources through a combined variational latent space, regardless of resolution or dimensionality of the observations. Our method accounts for the geologic uncertainty and the ill-posed nature of the geologic inversion problem and can accurately and efficiently reconstruct realistic geologic feature distributions from discrete dynamic observation data.

This article is structured as follows: **Section 2** discusses the theory and formulation of geologic inversion, **Section 3** describes our proposed methodology, **Section 4** provides a discussion of the results and key findings, and **Section 5** delineates the conclusions.

1. **Theory**

The classical problem of parameter estimation or inverse modeling is based on the approximation of a numerical model from which the response data originates. This requires a numerical description of the original model, a set of physics-based forward simulation equations, and the minimization of a pre-defined loss for the difference between true and estimated models. The typical workflow of an inverse problem involves an original model, , a physics-based forward simulation function, , and the forward simulation data, . The forward modeling follows that . The inverse model, on the other hand, follows that , the inversion function used to reconstruct the model from the data, is used to obtain the estimated model, , such that , and we try to minimize the difference to obtain identical estimated and true models.

In our workflow, our model is the set of geologic prior spatial distributions, and the forward simulator, , is a physics-based finite-difference numerical reservoir simulator. The simulated data in this case is two-fold, with producer well response and dynamic reservoir response, namely and , respectively. The inversion function in this case, , is a deep neural network model optimized to generate geologic models that minimize a loss compared to the true original geologic prior spatial distributions. Thus, it is formulated as .

Furthermore, it is possible to exploit the concept of dimensionality reduction, which attempts to parameterize the input and output data into a latent vector of reduced order but maintaining all the critical information for lossless compression. In general, for any data , we can encode the data into a latent space, , and decode back into an estimated data that attempts to minimize the difference between the true and estimated, . This is formulated as , where is the autoencoder loss, and . Combined, this is interpreted as minimizing .

Moreover, by constructing the latent variables and to follow a variational latent space, it is ensured that the parameterization of the dynamic data in latent space is properly regularized. This means that the latent vector is sampled randomly to generate a latent multi-Gaussian distribution of the latent space by computing and for the case . It is assumed that this latent distribution is an ideal representation of the latent space and can be used to reconstruct the original data through a decoder. This not only allows for a more compressed yet accurate latent representations, but also provides the power to generate new or fake models by sampling the latent distribution at new locations and creating unseen reconstructions.

For the multi-source dynamic data inversion, we propose three different latent spaces: one for the producer response data, , one for the dynamic reservoir response data from observation wells, , and one for the prior geologic spatial feature distributions, . All of them follow an encoder-decoder structure, such that we have the following: , , and . Each one of these three independent autoencoders is trained separately and then used as static estimators in a transfer learning protocol through a latent inversion network model to predict from and . Let be the regressor between and , then we have that . The full workflow then follows that the estimated geologic models is therefore given by the following formulation: .

The minimization of the differences between the true and predicted geologic models, , is optimized by adjusting the neural network parameters, namely weights and biases in each layer. Multiple epochs, or iterations, of a modified gradient descent program are done to adjust the weights for the optimal compression and reconstruction of the data. Each independent autoencoder is first trained separately, with losses of , , and , respectively, for the producer response data, dynamic reservoir response from observation wells, and geologic feature distributions. These are expertly-designed using a combination of metrics for the best reconstruction of the static and dynamic data, as explained in Section 3. The pretrained autoencoders are then used within the inversion network architecture in a transfer learning protocol, without further training them, but only optimizing the latent regressor parameters in with loss . The total loss in the system is then given by .

1. **Methodology**

The proposed inversion framework is designed for the reconstruction of three-layer geologic model from time-lapse production data and time-lapse observation wells dynamic data. The purpose is to reconstruct the facies, porosity, and permeability maps from the dynamic production response of oil production rate, water production rate, and water cut, as well as dynamic observations in monitoring wells of bottomhole pressure and fluid saturation. Further details on the generation of the geologic models, numerical forward simulation for the dynamic data, data processing, and the model architecture are explained in the following subsections, respectively.

Notes to self:

Free form hypothesis and conclusion (*draft*) + outline:

**Hypothesis:** We train three independent autoencoders and combine them into an inversion network model that reconstructs the corresponding geologic feature realization based on the producer well response data and the dynamic reservoir response data. […] Our method is capable of reconstructing both binary geologic features, such as channel facies, as well as Gaussian-distributed spatial distributions, such as porosity and permeability – a task that has not been solved through a single model before […]. We are also able to assimilate multiple data sources through a combined variational latent space, regardless of resolution or dimensionality of the observations.

**Conclusions:** We propose a novel method for the problem of geologic inversion from dynamic response data in subsurface energy resource engineering applications. We present the workflow for generating realistic prior geologic models, running forward reservoir simulation using a finite-difference approach, and training a decoupled triple-autoencoder network to reconstruct geologic maps from producer well response data and dynamic observation well data, which is then used in a transfer learning protocol to estimate geologic maps from the dynamic data. While this model was designed as a fit-for-purpose tool, it can be easily generalized to any subsurface energy application. For demonstration purposes, we observe how the inversion network can reconstruct the geologic models from dynamic data of a two-dimensional waterflooding simulation. The entire process is computationally efficient and fast, with minimal training and prediction time compared to previous deep learning-based techniques and is capable of reconstructing binary and Gaussian-distributed geologic spatial distribution in a single workflow by inversion of multiple data sources.

**Outline:**

Section 2: Theory and Formulation

* Classical formulation of inverse problems 🡪 ; ;
* AutoEncoder formulation 🡪 ;
* Triple AutoEncoder formulation (and losses) 🡪 , ,
* Latent inversion formulation 🡪 and

Section 3: Methodology

* Geomodel generation 🡪 permeability & porosity (Gaussian), facies (binary)
* Numerical forward simulation 🡪 waterflood line-drive
* Data processing 🡪 normalization, sampling observation wells from dynamic response
* Model architecture 🡪 describe the triple-AE and inversion regression network layers.
* Model training 🡪 describe training process, losses, etc.

Section 4: Results and Discussion

* Numeric accuracy (mse, ssim)
* Training time and prediction time
* Discussion of results

Section 5: Conclusions