Sensorless Control of Interior Permanent Magnet Synchronous Motor by Estimation of an Extended Electromotive Force

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Abstract— **Recently, many methods have been proposed for Surface Permanent Magnet Synchronous Motors (SPMSMs)' sensorless control based on estimation of electromotive force (EMF), in which motor's position information is included. However, these methods can not be applied to Internal Permanent Magnet Synchronous Motors (IPMSMs) directly, because the position information is included in not only the EMF but also the inductance of stator. In this paper, a new mathematical model for IPMSM is proposed and an extended EMF is defined, which includes both position information from the EMF and the stator inductance. As a result sensorless controls for SPMSMs can easily be applied to IPMSMs. As an example, a disturbance observer is studied and the experimental results show that the proposed method is very effective.**

I. Introduction

Permanent Magnet Synchronous Motors (PMSMs) have been widely used as servo motors for their high performances. To control the PMSM, rotor position and velocity information are necessary. However, sensors for detection of these signals are expensive and mechanically bulky. Therefore, the position and velocity sensorless control has been desired.

To estimate the position and velocity of the PMSM, the motor's mathematical model can be used. During recent years, many sensorless control methods for Surface Permanent Magnet Synchronous Motors (SPMSMs) based on the motor's mathematical model have been proposed [1]–[3]. The basic idea of these methods is estimating the permanent magnet's flux or Electromotive Force (EMF), because rotor's position information is included in these physical variables. The authors proposed a disturbance observer for EMF estimation of which stability is clarified by using a linear state equation [4].

However, these methods for the SPMSM can not be used directly to Interior Permanent Magnet Synchronous Motors (IPMSMs). The mathematical model of the SPMSM is a special symmetrical case of the IPMSM, which is relatively easy for mathematical procession. In the model of the IPMSM, position information is included not only in the flux or EMF term but also in the changing inductance because of its saliency. To apply the methods for the SPMSM to a wide class of motors, i.e. the IPMSM, some extension should be made.

To solve this problem, some attempts have been made [5]–[7]. In [5] and [6], observers based on rotating frames have been proposed. This is an easy approach as the IPMSM's mathematical model on rotating frame has the similar structure to the SPMSM's. However, because the real rotating frame is unknown, some approximations are made and this may destroy the stability of the proposed sensorless control systems under certain conditions.

In [7], two unique voltages are defined and by calculating these voltages, it is possible to obtain the motor's position and velocity. However, the authors did not give a clear physical meaning of the voltages. Also, there were derivation errors in defining the voltages. Besides, current's differentiation is used in calculating the voltages, which makes the estimation sensitive to measurement noises.

In this paper, a new mathematical model of IPMSMs on fixed frame is proposed and an Extended EMF (EEMF) is defined. The EEMF includes position information from not only the traditionally defined EMF but also the stator inductance. This makes it possible to obtain motor's position and velocity by just estimating the EEMF. And because the new model of the IPMSM on fixed frame has the similar structure to that of the SPMSM, methods for the SPMSM can be easily extended to the IPMSM. As an example, the disturbance observer for the SPMSM in [4] is applied to the IPMSM. Experimental results show that the proposed method is very effective.

II. Mathematical Model of SPMSM

The circuit equation of the SPMSM on $d-q$ rotating coordinate is given by (1).

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R+pL & -\omega_{re}L \\ \omega_{re}L & R+pL \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{re}K_E \end{bmatrix}, \tag{1}$$

where

$[v_d \;\; v_q]^T$	:	voltage on rotating frame,
$[i_d \;\; i_q]^T$	:	current on rotating frame,
R	:	stator resistance,
L	:	stator inductance,
p	:	differential operator,
K_E	:	EMF constant,
ω_{re}	:	angular velocity at electrical angle,
θ_{re}	:	rotor position at electrical angle.

The first term on the right side of (1) is a voltage drop on motor's impedance, and the second term is an EMF term.

Transforming (1) into $\alpha-\beta$ fixed coordinate, (2) is derived.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} R+pL & 0 \\ 0 & R+pL \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega_{re}K_E \begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix}, \tag{2}$$

where

$[v_\alpha \;\; v_\beta]^T = v$	:	voltage on fixed frame,
$[i_\alpha \;\; i_\beta]^T = i$	:	current on fixed frame.

From (2), it is known that rotor's position information is included in the EMF term. By estimating the EMF, it is possible to get θ_{re} from its phase.

The physical meaning of the sensorless estimation can be interpreted by Fig.1(a). The stator voltage v is sum of three voltage vectors A, B and C which correspond to three terms on the right side of (3) derived from (2), respectively. Here, the stator voltage v and current i can be detected by sensors. And voltage drop A on resistance and B on inductance can be calculated from current i, nominal values of resistance R and inductance L. Therefore, the only unknown variable is the EMF term C. So it is possible to solve (3) to get term C.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + pL\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega_{re}K_E\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix} \quad (3)$$

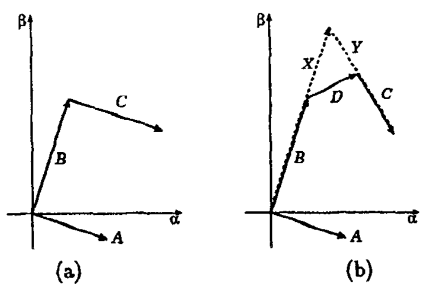

Fig. 1. Vector Diagram of (a) SPMSM, (b) IPMSM

III. Mathematical Model of IPMSM

The circuit equation of the IPMSM on $d-q$ rotating coordinate is given by (4).

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R+pL_d & -\omega_{re}L_q \\ \omega_{re}L_d & R+pL_q \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{re}K_E \end{bmatrix}, \quad (4)$$

where

$$\begin{aligned} L_d &: \text{ inductance of d axis,} \\ L_q &: \text{ inductance of q axis.} \end{aligned}$$

Transforming (4) into $\alpha-\beta$ fixed coordinate, (5) is derived.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} R+pL_\alpha & pL_{\alpha\beta} \\ pL_{\alpha\beta} & R+pL_\beta \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega_{re}K_E\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix}, \quad (5)$$

where

$$\begin{aligned} L_\alpha &= L_0 + L_1\cos 2\theta, \quad L_{\alpha\beta} = L_1 \sin 2\theta, \\ L_\beta &= L_0 - L_1\cos 2\theta, \\ L_0 &= (L_d+L_q)/2, \quad L_1 = (L_d-L_q)/2. \end{aligned}$$

Equation (5) is obviously different from (2) in that both $2\theta_{re}$ and θ_{re} terms are included, which is not easy for mathematical processing. This can be interpreted by Fig.1(b). The vectors A, B, C and D correspond to terms on the right side of (6) derived from (5), respectively. But at this time, there are two unknown vectors C and D, having functions of rotor's position θ_{re}, which makes the equation difficult to solve.

$$\begin{aligned} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} &= R\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + pL_0\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega_{re}K_E\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix} \\ &+ pL_1\begin{bmatrix} \cos 2\theta_{re} & \sin 2\theta_{re} \\ \sin 2\theta_{re} & -\cos 2\theta_{re} \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \end{aligned} \quad (6)$$

An easy way to solve this problem is to use the estimated position $\hat{\theta}_{re}$ in place of θ_{re} to calculate vector D. This is possible if the amplitude of D is smaller enough than that of C, i.e. $|L_1 i| \ll K_E$. The approximation made in [5] and [6] is based on the assumption that this condition is valid.

This is true for those motors with relatively small reluctance torque. But if the motor's reluctance torque can not be neglected comparing with the permanent magnet torque, sensorless estimation may be unstable.

However, the $2\theta_{re}$ term in (5) is avoidable. It is known that there is no secondary harmonic in motor's current if higher harmonic voltages generated by an inverter is neglected. Therefore, it is natural to think about eliminating the $2\theta_{re}$ terms by some pure mathematical method.

Comparing (4) with (1), the reason why $2\theta_{re}$ terms appear can be concluded as that the impedance matrix in (4) is asymmetrical. If the impedance matrix is rewritten symmetrically as

$$\begin{aligned} \begin{bmatrix} v_d \\ v_q \end{bmatrix} &= \begin{bmatrix} R+pL_d & -\omega_{re}L_q \\ \omega_{re}L_q & R+pL_d \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ (L_d-L_q)(\omega_{re}i_d - \dot{i}_q) + \omega_{re}K_E \end{bmatrix}, \end{aligned} \quad (7)$$

the circuit equation on $\alpha-\beta$ coordinate can be derived as (8), in which there is no $2\theta_{re}$ term.

$$\begin{aligned} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} &= \begin{bmatrix} R+pL_d & \omega_{re}(L_d-L_q) \\ -\omega_{re}(L_d-L_q) & R+pL_d \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \\ &+ \left\{(L_d-L_q)(\omega_{re}i_d - \dot{i}_q) + \omega_{re}K_E\right\}\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix} \end{aligned} \quad (8)$$

It should be noted that the differential operator " $\cdot$ " in the second term on the right side of (8) is only effective to i_q, and that it is not effective to the following $\sin\theta_{re}$ and $\cos\theta_{re}$. This usage is different from the differential operator p used in (5).

The second term and third term on the right side of (8) are depicted in Fig.1(b) as vectors X and Y in dashed lines, respectively. They can be explained as extensions of vectors B, C with decomposition of vector D in their directions. Here, X has no relation with rotor's position θ_{re} so that Y might be the only vector having functions of θ_{re}. This makes the sensorless estimation problem of IPMSMs as easy to be solved as that of SPMSMs.

The third term on the right side of (8) is defined as an Extended EMF (EEMF) by (9). In this term, besides the traditionally defined EMF generated by permanent magnet, there is a kind of voltage related to saliency of IPMSM. It includes position information from both the EMF and the stator inductance. If the EEMF can be estimated, the position of magnet can be obtained from its phase just like EMF in SPMSMs.

$$e = \begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = \left\{(L_d-L_q)(\omega_{re}i_d - \dot{i}_q) + \omega K_E\right\}\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix} \quad (9)$$

Equation (8) is a transformation of (5) without any approximation. It is a general form of mathematical model for all synchronous motors. When $L_d = L_q$, it devolutes into the equation of SPMSMs. And when $K_E = 0$, it becomes the equation of Synchronous Reluctance Motors (SYNRMs). Based on this model, it is easy to apply methods for SPMSMs to IPMSMs. Also it is reasonable to think that those methods are valid for SYNRMs.

There is a differential term of i_q in the EEMF. This means that even when motor's velocity is near to zero, if the q-axis current i_q is changing, the EEMF e is not zero. This property will be useful for zero and low speed drives.

IV. Disturbance Observer

Regarding the EEMF in (8) as a disturbance, the EEMF can be estimated by a disturbance observer for middle and high speed drives. This method was at first proposed for SPMSMs [4]. With the new model of (8), it becomes applicable for IPMSMs.

A. Linear State Equation

From the new model of (8), IPMSMs can be described by a linear state equation as (10) [8]. Here, the state variables are stator current i and EEMF e. The system's input is stator voltage v and the output is stator current i. Assuming that the electrical system's time constants are shorter enough than the mechanical ones, the velocity ω_{re} is regarded as a constant parameter.

$$\begin{aligned} p\begin{bmatrix} i \\ e \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}\begin{bmatrix} i \\ e \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ W \end{bmatrix}, \\ i &= C \cdot \begin{bmatrix} i \\ e \end{bmatrix}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} A_{11} &= -(R/L_d)I + \{\omega_{re}(L_d - L_q)/L_d\}J, \\ A_{12} &= (-1/L_d)J, \quad A_{22} = \omega_{re}J, \\ B_1 &= (1/L_d)I, \quad C = [I \quad 0], \\ W &= (L_d - L_q)(\omega_{re} i_d - \dot{i}_q)\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix}, \\ I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \end{aligned}$$

The W term in (10) is a linearization error. This term appears only when i_d or i_q is changing. However, under motor's velocity control, this happens in a very short time because of high response of current control loop. Besides, the proposed disturbance observer has an embedded low-pass-filter which can cut the effect of W.

B. Configuration of Disturbance Observer

To estimate the state variable e in a system described by (10), a reduced order observer is constructed as (11).

$$\begin{aligned} \dot{\hat{i}} &= \tilde{A}_{11} i + \tilde{A}_{12}\hat{e} + \tilde{B}_1 v \\ \dot{\hat{e}} &= \tilde{A}_{22}\hat{e} + G(\dot{\hat{i}} - \dot{i}) \\ &= \tilde{A}_{11}Gi + (\tilde{A}_{12}G + \tilde{A}_{22})\hat{e} + \tilde{B}_1 G v - G\dot{i}, \end{aligned} \tag{11}$$

where

$\hat{}$: estimated state variable,

$\tilde{}$: parameter's nominal value,

$G = g_1 I + g_2 J$: feedback gain.

To avoid differentiation of current i, an intermediate variable ξ is introduced as (12).

$$\begin{aligned} \xi &= \hat{e} + Gi \\ \dot{\xi} &= \dot{\hat{e}} + G\dot{i} \end{aligned} \tag{12}$$

Substituting (12) into (11) yields (13).

$$\begin{aligned} \dot{\xi} &= (\tilde{A}_{12}G + \tilde{A}_{22})\xi + \tilde{B}_1 G v + G(\tilde{A}_{11}I - \tilde{A}_{12}G - \tilde{A}_{22})i \\ \hat{e} &= \xi - Gi \end{aligned} \tag{13}$$

This is an equivalent disturbance observer as shown in Fig.2, where the filter's transfer function $H(s)$ is shown as (14).

$$H(s) = \frac{\alpha}{(s+\alpha)^2 + \beta^2}\left\{(s+\alpha)I + \beta J\right\} \tag{14}$$

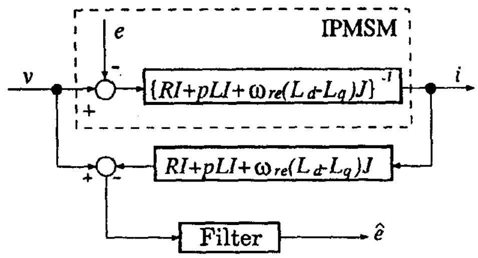

Fig. 2. Configuration of Disturbance Observer

C. Pole Assignment of Disturbance Observer

The poles of the disturbance observer is assigned for two considerations. One is for robust EEMF estimation against velocity estimation error. The other is for suppression of the twelfth harmonic in EMF.

In the proposed disturbance observer, velocity ω_{re} is used as a parameter. However, the real value of it is unknown so that the estimated one $\hat{\omega}_{re}$ is used in place. At acceleration and deceleration, transient velocity estimation error is inevitable. This may cause EEMF estimation error, and EEMF estimation error may lead to more velocity estimation error. To stop this vicious spiral, the EEMF estimation should be robust against velocity estimation error. In this section, a pole assignment to make the disturbance observer robust is described at first.

From (10) and (11), considering velocity estimation error $\Delta\omega_{re} = \hat{\omega}_{re} - \omega_{re}$, the observer's error equation is derived as (15).

$$\begin{aligned} \dot{\varepsilon} &= (\tilde{A}_{22} + G A_{12})\varepsilon + \begin{bmatrix} G(\tilde{A}_{11} - A_{11}) & \tilde{A}_{22} - A_{22} \end{bmatrix}\begin{bmatrix} i \\ e \end{bmatrix} \\ &= (-\alpha I + \beta J)\varepsilon + FD \end{aligned} \tag{15}$$

where $\varepsilon = \hat{e} - e$ is the EEMF estimation error. The normalized disturbance vector D and its input matrix F are defined as follows.

$$\begin{aligned} D &= \begin{bmatrix} \frac{i}{|i|} & \frac{e}{\omega_{re}K_E} \end{bmatrix}^T \Delta\omega_{re} \\ F &= [(L_d - L_q)\{-\alpha I + (\beta - \hat{\omega}_{re})J\}|i| \quad \omega_{re}K_E]\, J \end{aligned}$$

The error equation's block diagram is shown in Fig.3. From the figure, the feedback gain G, or the closed loop poles $(-\alpha, \beta)$, decide not only the response of EEMF estimation, but also the input matrix of disturbance.

The transfer function $T(s)$ from the disturbance vector D to EEMF estimation error ε is defined as

$$T(s) = (sI - \overline{A})^{-1}F. \tag{16}$$

The influence of velocity estimation error $\Delta\omega_{re}$ can be evaluated by the H_∞ norm of $T(s)$ as

$$\|T\|_\infty = \sup_\omega \sigma_{\max}[T(j\omega)]$$

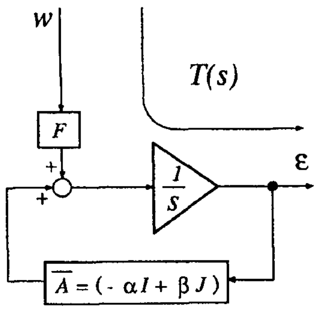

Fig. 3. Block diagram of error equation.

$$
\begin{aligned}
&= \sup_{\omega} \sqrt{\lambda_{\max}[T^H(j\omega) \cdot T(j\omega)]} \\
&= \sup_{\omega} \frac{\|F_{\omega}\|_2}{\sqrt{(\omega - |\beta|)^2 + \alpha^2}} \\
&= \frac{\|F\|_2}{\alpha} \quad (\text{at } \omega = |\beta|),
\end{aligned} \tag{17}
$$

where

$\sigma_{\max}[\cdot]$: maximum singular value,
$\lambda_{\max}[\cdot]$: maximum eigenvalue,
$(\cdot)^H$: complex conjugate transposition,
$\|\cdot\|_2$: induced norm.

The induced norm of F is calculated as

$$
\begin{aligned}
&\|F\|_2 = \sigma_{\max}[F] \\
&= \sqrt{(L_d - L_q)^2 \left\{\alpha^2 + (\beta - \hat{\omega}_{re})^2\right\} |i|^2 + (\omega_{re} K_E)^2}.
\end{aligned} \tag{18}
$$

From (18), the best β assignment which makes $\|F\|_2$ minimum is

$$
\beta = \hat{\omega}_{re}. \tag{19}
$$

Then (17) becomes

$$
\begin{aligned}
\|T\|_{\infty} &= \sqrt{(L_d - L_q)^2 |i|^2 + \left(\frac{\omega_{re} K_E}{\alpha}\right)^2} \\
&\leq \sqrt{(L_d - L_q)^2 |i|^2_{\max} + \left(\frac{\omega_{re} K_E}{\alpha}\right)^2} \\
&= \nu_1',
\end{aligned} \tag{20}
$$

where ν_1' is a given parameter for robust estimation.

According to (20), the assignment of pole α which makes the EEMF estimation robust against velocity estimation error is derived as

$$
\alpha \geq |\omega_{re}| \nu_1, \tag{21}
$$

where the relation between ν_1 and ν_1' is given as

$$
\nu_1 = \frac{K_E}{\sqrt{\nu_1'^2 - (L_d - L_q)^2 |i|^2_{\max}}}.
$$

From (21), the lower limit of pole α for robust velocity estimation is decided. However, an unlimited high gain is not desired, either. Next, the decision of higher limit of pole α will be discussed in consideration of higher harmonic suppression.

Usually, for salient-pole motors such as IPMSMs and SYNRMs, there are higher harmonics in EMF because of interactions between stator teeth and permanent magnet or rotor teeth. The frequency of the harmonic is decided by the motor's structure. For an example, the motor in our experimental system has 24 slots and 2 pairs of poles, so twelfth harmonic is included in the EMF as Fig.4.

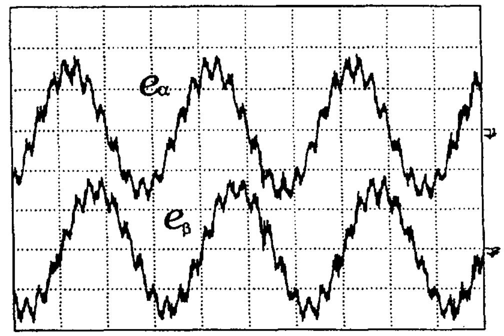

Fig. 4. Twelfth harmonic in EMF

This kind of higher order harmonics in EMF may spoil the accuracy of position estimation. Using the embedded filter in the disturbance observer, it is possible to get rid of them.

The diagonal term's amplitude $|H_d(j\omega)|$ of transfer function matrix $H(s)$ is calculated as (22)

$$
|H_d(j\omega)| = \alpha \sqrt{\frac{\alpha^2 + \omega^2}{(\omega_{re}^2 - \omega^2)^2 + 4\alpha\omega^2}} \tag{22}
$$

The relative influence of the twelfth harmonic is evaluated by the ratio of the twelfth harmonic's $|H_d(12j\omega_{re})|$ to the basic harmonic's $|H_d(j\omega_{re})|$.

$$
\frac{|H_d(12j\omega_{re})|}{|H_d(j\omega_{re})|} = \frac{\sqrt{\frac{\alpha^2 + 12^2 \cdot \omega_{re}^2}{(12^2 - 1)\alpha^2 \omega_{re}^2 - 4 \cdot 12^2 \cdot \alpha^2 \omega_{re}^2}}}{\sqrt{\frac{\alpha^2 + \omega_{re}^2}{4\alpha^2 \omega_{re}^2}}} \leq \nu_2' \tag{23}
$$

Then the assignment of pole α suppressing the influence of the twelfth harmonic is derived as

$$
\alpha \leq |\omega_{re}| \nu_2, \tag{24}
$$

where ν_2 is a constant deciding the relative amplitude of the twelfth harmonic, which can be calculated from (23).

From the above, the poles assignment of the disturbance observer is decided as

$$
\begin{aligned}
|\omega_{re}| \nu_1 \leq \quad \alpha = |\omega_{re}| \nu \quad &\leq |\omega_{re}| \nu_2, \\
\beta = \hat{\omega}_{re}.&
\end{aligned} \tag{25}
$$

V. Position and Velocity Estimation

The motor's position and velocity can be calculated from the estimated EEMF $\hat{e}$. With the estimated position and velocity, the IPMSM can be controlled sensorlessly.

A. Position Estimation

Using the estimated value of the EEMF, the position is calculated as follows.

$$
\hat{\theta}_{re} = \tan^{-1}\left(-\frac{\hat{e}_{\alpha}}{\hat{e}_{\beta}}\right) \tag{26}
$$

Note that the amplitude of EEMF is not used for position estimation.

B. Adaptive Velocity Estimation

The rotor's velocity can be estimated by an adaptive velocity estimator [4]. The concept of estimator is shown in Fig.5.

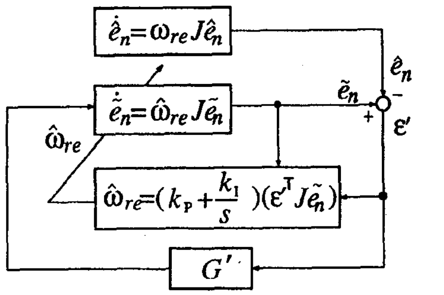

Fig. 5. Configuration of adaptive velocity estimator

First, the estimated EEMF is normalized as (27).

$$\hat{e}_n = \frac{1}{\sqrt{\hat{e}_\alpha^2 + \hat{e}_\beta^2}} \begin{bmatrix} \hat{e}_\alpha \\ \hat{e}_\beta \end{bmatrix} \tag{27}$$

For the normalized EEMF, the next equation is valid by assuming ω_{re} as a constant parameter.

$$\dot{\hat{e}}_n = \omega_{re} J \hat{e}_n \tag{28}$$

This equation is used as the reference model in the adaptive velocity estimator. Based on (28), the estimation model is defined as

$$\dot{\tilde{e}}_n = \hat{\omega}_{re} J \tilde{e}_n + G'(\tilde{e}_n - \hat{e}_n), \tag{29}$$

where $\tilde{e}_n$ is the output of the estimation model and $\hat{\omega}_{re}$ is the estimated velocity. For convergence, a feedback loop is introduced and the feedback gain is $G' = g'I$.

If the velocity estimation error exists, it will lead to the EEMF estimation error $\varepsilon' = \tilde{e}_n - \hat{e}_n$. Then this error together with the estimation model's output $\tilde{e}_n$ is used to update the estimated velocity $\hat{\omega}_{re}$ by the adaptive scheme as (30).

$$\hat{\omega}_{re} = (k_P + \frac{k_I}{s})(\varepsilon'^T J \tilde{e}_n) \tag{30}$$

The stability of the estimation is guaranteed by Popov's hyper stability theory so that the error ε' converges to zero, and the estimated velocity $\hat{\omega}_{re}$ converges to its real value ω_{re} at last.

VI. Experimental Results

Using the proposed method, experiments of position and velocity sensorless velocity control are carried out on an IPMSM with parameters listed in Table I. The parameters for the controllers are list in Table II. Configuration of the experimental system is shown in Fig.6.

The motor's stator voltage and current are detected by sensors and then sent into a digital signal processor (DSP TMS320C31) through 12 bit A/D converters. In DSP program, 3-phase voltage and current signals (u-v-w coordinate) are converted into 2-phase coordinate signals (α-β coordinate). Using these signals, the EEMF $\hat{e}$ are estimated by the proposed disturbance observer. Rotor's position $\hat{\theta}_{re}$ is obtained from the phase of estimated EEMF $\hat{e}$. Velocity $\hat{\omega}_{re}$ is estimated by the adaptive estimator. The difference between the velocity command

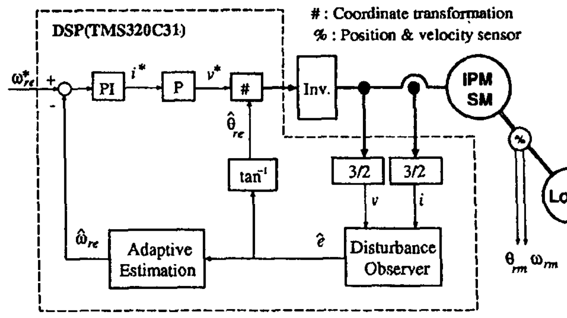

Fig. 6. Configuration of experimental system

ω^*_{re} and the estimated velocity $\hat{\omega}_{re}$ is used to generate current command i^* at rotor reference frame (d-q coordinate) by the PI controller, so that maximum torque might be generated. Using i^*, voltage command v^* is calculated by the current is controller and then converted into the stator reference 3-phase frame using the estimated rotor position $\hat{\theta}_{re}$. Next, the voltage command v^* is sent to the inverter to drive the motor. The PWM inverter's maximum switching frequency is 5kHz. A 1.5kW surface permanent magnet synchronous motor is used as a load. An absolute encoder is used to detect the rotor's real position and velocity for verification.

TABLE I
Parameters of test motor

Rated power		500	[W]
Rated current		5	[A]
Rated torque		22.5	[kgf·cm]
Stator resistance	R	0.45	[Ω]
Stator inductance	Ld	4.15	[mH]
	Lq	16.74	[mH]
Rotor inertia	J	0.06	[kg·cm·s^2]
Number of pole pairs	P	2	
EMF constant	K_E	0.107	[V·s/rad]
DC link voltage		220	[V]

In Fig.7, the estimated EEMF $\hat{e}_\alpha, \hat{e}_\beta$ at 1,000rpm are compared with their theoretical values e_α, e_β calculated from detected position, current, nominal inductance and EMF constant according to (9). From the figure, it is known that the observer estimates the EEMF and filtering the twelfth harmonic of it successfully.

TABLE II
Parameters for controller

Constant for observer's pole	ν	2	
Estimator feedback gain	g'	1000	
Adaptive scheme P gain	k_P	0.01	[rad/s/V^2]
Adaptive scheme I gain	k_I	50000	[rad/s^2/V^2]
Velocity controller P gain	k_{SP}	0.08	[A·s/rad]
Velocity controller I gain	k_{SI}	0.7	[A/rad]
Maximum current	i_{max}	14	[A]

In Fig.8, the estimated position $\hat{\theta}_{re}$ calculated from the phase of EEMF is compared with real position θ_{re} and position estimation error $\Delta\theta_{re} = \hat{\theta}_{re} - \theta_{re}$. The maximum position estimation error is 3°. This may lead to less than 1% of torque drop, which is an excellent estimation.

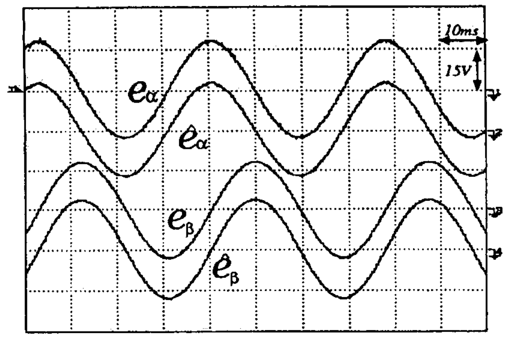

Fig. 7. Result of Extended EMF Estimation

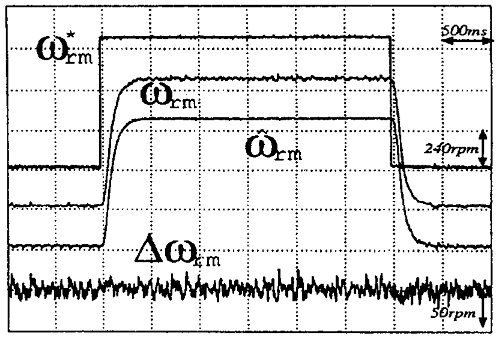

Fig. 9. Result of Velocity Control

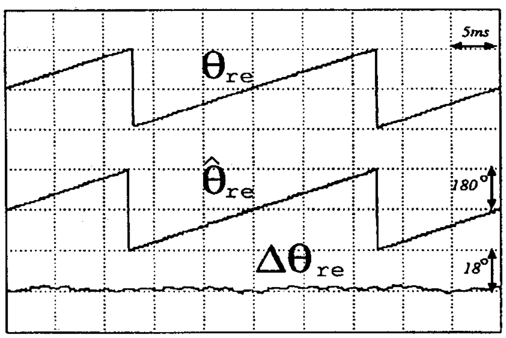

Fig. 8. Result of Position Estimation

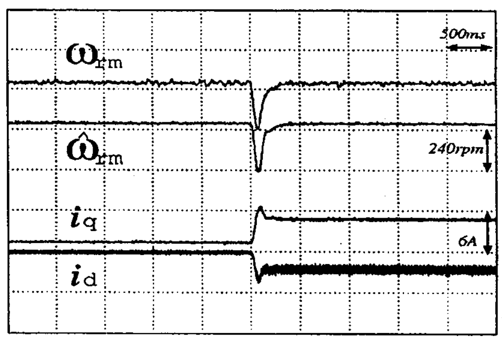

Fig. 10. Impact Drop Characteristic

In Fig.9, the estimated velocity $\hat{\omega}_{rm}$ at mechanical angle is compared with velocity command ω^*_{rm}, detected velocity ω_{rm} and velocity estimation error $\Delta\omega_{rm} = \hat{\omega}_{rm} - \omega_{rm}$ at acceleration and deceleration between 400rpm and 1,000rpm without load. From the figure, the maximum error of velocity estimation is within $\pm$20rpm and the average error is near to zero. At acceleration and deceleration, the velocity estimation error does not increase, which shows the effectiveness of the proposed pole assignment.

Fig.10 is an experimental result of torque impact drop. The detected velocity ω_{rm}, estimated velocity $\hat{\omega}_{rm}$ and currents i_d, i_q before and after a rated torque being applied to the motor are shown. From the result, a stable sensorless control is realized at both transient and steady state with full-load running.

VII. Conclusions

This paper presented a new mathematical model for IPMSMs with the concept of Extended EMF defined. Using the new model, a disturbance observer proposed for SPMSMs was applied to IPMSMs for position and velocity sensorless controls. The proposed method was shown feasible by experiments.

As it is mentioned in Section 3, further researches will be carried out on zero and low speed drives of IPMSMs using the EEMF, and the possibility of SYNRM's sensorless control using the proposed disturbance observer. The results will be reported later.

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