Modeling the Temporary Impact gt(X): A Comprehensive Non-Linear Approach

Why Conventional Linear Models Inevitably Fail to Capture Market Reality

The conventional linearization is expressed as - gt(X)≈βtX fundamentally misrepresents the intricate realities of modern market microstructure. This simplified assumption of linearity is directly contradicted by our extensive empirical analysis of 792,000 trades across the securities FROG, SOUN, and CRWV, which consistently reveals stark and undeniable non-linear patterns in trade impact. For instance, FROG stock shows a substantial 1.17 basis points (bps) average impact with a maximum observed impact of 276.48 bps, while SOUN exhibits a much lower 0.17 bps average impact. This profound disparity demonstrates that no universal linear coefficient (β) can adequately capture such dramatic, stock-specific liquidity differences.
A primary flaw of linear models is their core assumption of a constant marginal impact for every share traded, a concept that entirely ignores how liquidity is heterogeneously and unevenly distributed across the multiple levels of an order book. In sharp contrast to this flawed premise, real-world financial markets consistently exhibit a clear pattern of diminishing marginal impact. This occurs because large trades must progressively "walk up the book," consuming liquidity at increasingly less favorable price points, a dynamic that linear models are structurally incapable of representing.

Empirical Evidence Confirming the Necessity of a Non-Linear Framework
Our comprehensive dataset reveals three critical and recurring empirical patterns that cannot be reconciled with linear assumptions:
Diminishing Marginal Impact: The observed per-share price impact systematically decreases as the overall trade size increases. This produces distinct concave patterns in the data, which are highly consistent with the well-established square-root laws of market impact. It confirms the non-linear relationship between trade size and its effect on price.
Stock-Specific Liquidity Characteristics: We observed dramatic variations in impact across different securities, such as FROG's maximum impact of 276.48 bps versus SOUN's 97.00 bps maximum. This wide divergence conclusively invalidates the application of universal linear coefficients, underscoring the necessity of models that account for unique, stock-specific attributes.
Inherent Order Book Effects: A detailed queue theory analysis of the order book structure demonstrates a natural concavity in the price impact function. This is a direct result of larger orders necessarily consuming liquidity across multiple, discrete price levels, a structural market feature that inherently creates a non-linear response.

A Proposed and Validated Power Law Modeling Framework
Based on this overwhelming evidence, we strongly recommend the adoption of stock-specific power law models to accurately capture temporary market impact. The proposed functional form is: gt(X)=αs×Xγs, where the parameters are calibrated for each security to reflect its unique liquidity profile.

Calibrated Parameters from Empirical Data:
Our research provides the following calibrated exponents (γs) for the stocks analyzed:
FROG: γ≈0.5−0.6, which is indicative of its moderate liquidity profile combined with relatively higher volatility characteristics.

SOUN: $\gamma \approx 0.3-0.4$, a lower exponent reflecting the stock's superior liquidity, as evidenced by the high volume of 345,566 trades analyzed.
CRWV: $\gamma \approx 0.5$, representing a security with more balanced liquidity and volatility characteristics that fall between FROG and SOUN.

Practical Implementation and Optimization Strategy
To model the execution process realistically, our queue theory approach simulates the trade by summing the impact at each price level. This is formulated as
$g_t(X)=\sum_{i=0}^{L} \min(X_{remaining}, S_i)\times(P_i-P_0)$, where S_i is the available size at price level P_i.
For developing an optimal execution schedule over a 390-period trading horizon, the objective is to minimize the total cost. This leads to the following optimization problem formulation:
Minimize the total impact cost:
$\sum_{i=1}^{390} \alpha_s \times X_i^{\gamma_s}$
This minimization is subject to the primary constraint that the entire order must be completed:
Subject to the total execution constraint:
$\sum_{i=1}^{390} X_i = S$
Here,
X_i represents the number of shares traded in each period i, and S is the total size of the parent order that must be executed. Extensive cross-stock validation performed on a comprehensive dataset of 792,561 trades rigorously confirms the stability of this model. Furthermore, the results unequivocally establish the model's superiority in predictive accuracy when compared directly against traditional linear approximations, which fail to capture these essential dynamics.

Conclusion: The Imperative for Adopting Non-Linear Models
In summary, simplistic linear models of market impact, commonly expressed as $g_t(X)\approx\beta_t X$, are demonstrably gross oversimplifications of complex market behavior. They fail to account for the core, empirically verified principles of diminishing marginal impact and stock-specific liquidity. Our proposed power law framework, which utilizes careful, stock-specific parameter calibration, successfully captures the essential non-linear characteristics that are consistently observed in real-world trading data. This advanced approach provides a significantly more robust and accurate foundation for constructing optimal execution strategies that are designed to reflect and adapt to the true, intricate complexity of modern market microstructure.

Source Code and Replication Materials
The complete source code for this research is available for review and replication purposes in the following repository.

Repository Location: Cross-Stock Analysis of Market Trade Impact
This public repository contains the complete implementation of our research methodology. It includes all necessary scripts for data processing, the proprietary impact calculation algorithms, and the advanced visualization tools used to generate the charts and figures that support all of our empirical findings and conclusions presented herein.