NUST-School of Electrical Engineering and Computer Science(SEECS),H-12,Islaabad,Pakistan

course:E-891 Stochastic Systems Fall 2021

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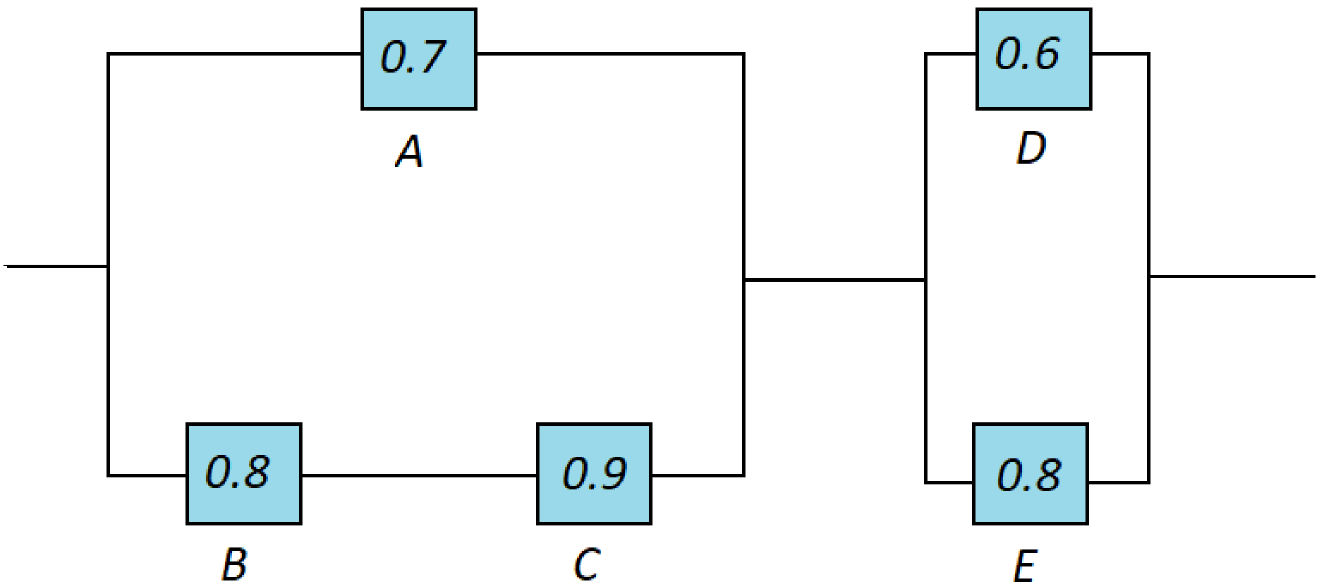
Assignment # 1

Problem 01:

A circuit system is given in the figure below. Assume the components fail independently. Probabilities shown are of each component working fine individually.



(a) What is the probability that the entire system works?
(b) Given that the system works, what is the probability that component B is not working?

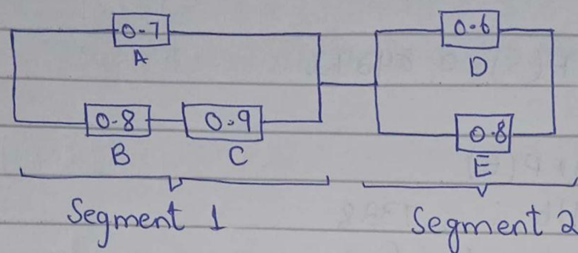
Solution:

Part(a)

Problem # 1

Solution:

(a) $P(S) = ?$



$P(A) = 0.7 \Rightarrow P(A') = 1 - P(A) = 1 - 0.7 = 0.3$

$P(B) = 0.8$

$P(C) = 0.9 \Rightarrow P(BC) = P(B) \times P(C) = 0.8 \times 0.9 = 0.72$

$P(BC)' = 1 - P(BC) = 1 - 0.72 = 0.28$

$P(\text{segment } 1) = 1 - \left(P(A') P(BC)' \right) = 1 - \left(0.28 \times 0.3 \right) = 0.916$

$P(D) = 0.6 \Longrightarrow P(D') = 1 - P(D) = 1 - 0.6 = 0.4$

$P(E) = 0.8 \Rightarrow P(E') = 1 - P(E) = 1 - 0.8 = 0.2.$

$P(\text{segment}2) = 1 - \left(P(D') \times P(E') \right) = 1 - \left(0.4 \times 0.2 \right) = 0.92.$

$P(\text{system}) = P(\text{segment}1) \times P(\text{segment } 2)$

$= 0.916 \times 0.92 = 0.84272.$

$P(\text{system}) = P(S) = 0.84272.$

Part(b)

Part (b)

Solution:

$P(D'|S) = ?$

using Baye's Rule

$$P(D'|S) = \frac{P(S|D') \times P(D')}{P(S)} \rightarrow (1)$$

$P(D') = 0.4$ $\quad$ $P(S) = 0.84272$

"D and E" are parallel.

$P(S|D') = P(\text{segment } 1) \times P(E)$

$= 0.8 \times 0.916 = 0.7328$

Put values in equation (1)

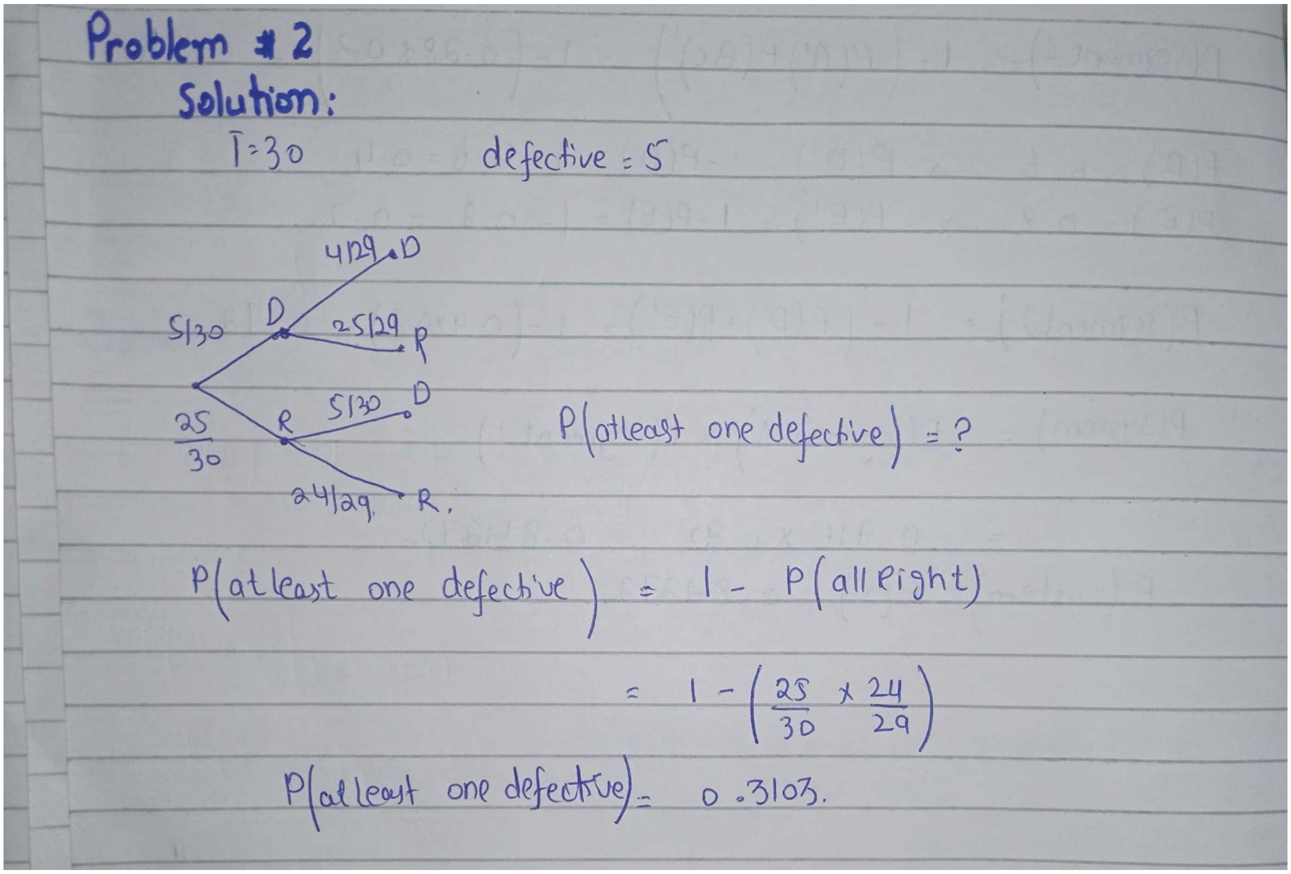
$$P(D'|S) = \frac{(0.7328 \times 0.4)}{(0.84272)} = 0.3478$$

$P(D'|S) = 0.3478$

Problem 2:

A box of 30 diodes is known to contain five defective ones. If two diodes are selected at random without replacement, what is the probability that at least one of these diodes is defective?

Solution:



Problem # 2

Solution:

T = 30 defective = 5

P(atleast one defective) = ?

$$P(\text{at least one defective}) = 1 - P(\text{all right})$$

$$= 1 - \left(\frac{25}{30} \times \frac{24}{29}\right)$$

$$P(\text{at least one defective}) = 0.3103.$$

Problem 3:

If there are three baskets i.e. Basket A, B and C. Basket A contains 2 red balls, Basket B contains 2 white balls and basket C contains 1 red ball and 1 white ball. A basket is selected at random, and one ball is taken at random from that basket.

(a) What is the probability of selecting a white ball?

(b) If the selected ball is white, what is the probability that the other ball in the basket is red?

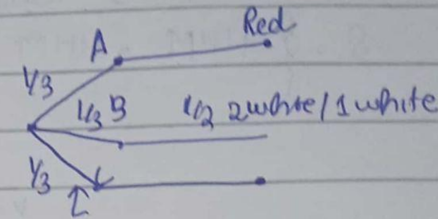
Solution:

Problem #3:

Solution:

(c) 3 baskets

A	B	C
2 Red balls	2 white balls	1 white, one Red



P(white ball) = ?

$P(box) = P(B_1) = 1/3 = P(B_2) = P(B_3)$ (Probability of a basket selection)

$P(white) = P(W|B_1)P(B_1) + P(W|B_2)P(B_2) + P(W|B_3)P(B_3)$

$= (0)(1/3) + (1/2)(1/3) + (1)(1/3) = \frac{1}{2}$

$P(white) = 1/2.$

(d) Only box/basket "C" has one white and Red ball.

$$P(C|W) = \frac{P(W|C)\,P(C)}{P(W)}$$

$$= \frac{(1/2)\times(1/3)}{(1/2)} = \frac{1}{3}.$$

$P(C|W) = 1/3.$

Problem 4:

We are given a number of darts. When we throw a dart at a target, we have a probability of ¼ of hitting the target. What is the probability of obtaining at least one hit if three darts are thrown? Calculate this probability two ways.

Solution:

Problem #4

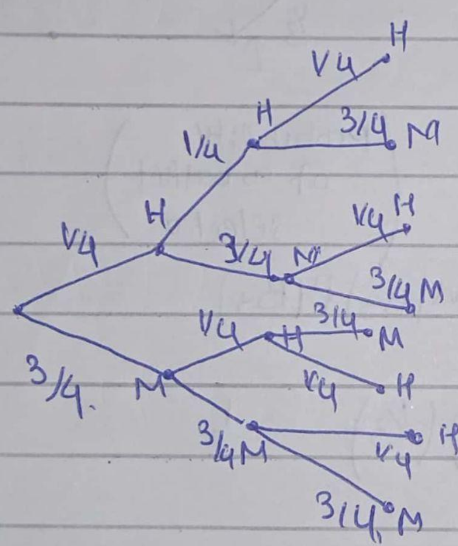
Solution:

P(hitting) = $1/4$.

P(missing) : $1 - 1/4 = 3/4$.

Total "3" chances"

S : { HHM , HHM , HMM , MHH , MHM , MMH , MMM }



P(atleast one hit) = $1 - P(\text{all misses})$

$$= 1 - \left(\frac{3}{4} + \frac{3}{4} \times \frac{3}{4}\right)$$

P(alleast one hit) = 0.578125.

Problem 5:

Consider a routine screening test for a disease. Suppose the frequency of the disease in the population (base rate) is 0.5%. The test is highly accurate with a 5% false positive rate and a 10% false negative rate. You take the test and it comes back positive. What is the probability that you have the disease?

Solution:

Problem #5

Solution:

frequency of disease $= 0.5\% = 0.005 = 5 \times 10^{-3}$

fake Positive $= 5\% = 0.05$

false negative $= 10\% = 0.1$.

$P(\text{disease} \mid \text{test +ive}) = ?$

$$P(\text{disease} \mid \text{test +ive}) = \frac{P(\text{+ive test} \mid \text{disease})\, P(\text{disease})}{P(\text{test +ive})} \rightarrow (1)$$

$$P(\text{test +ive}) = P(\text{test +ive} \mid \text{disease})\, P(\text{disease}) + P(\text{test +ive} \mid \text{no disease})\, P(\text{no disease}) \quad (2)$$

$$P(\text{test +ive} \mid \text{disease}) = 1 - P(\text{test -ive} \mid \text{disease})$$

$$= 1 - FP = 1 - 0.1 = 0.9$$

$$P(\text{no disease}) = 1 - P(\text{disease}) = 1 - (5\times10^{-3}) = 0.995$$

Put in equation (2)

$$P(\text{test +ive}) = (0.9 \times 5\times10^{-3}) + (0.05 \times 0.995)$$

$$P(\text{test +ive}) = 0.05425$$

Put values in equation (1)

$$P(\text{disease} \mid \text{test +ive}) = \frac{(5\times10^{-3}) \times (0.9)}{0.05425}$$

$$\boxed{P(\text{disease} \mid \text{test +ive}) = 0.0829}$$

Problem 6:(MATLAB)
Download the file “random_integers.csv” and read it in MATLAB using command X = csvread(). It will load data of random numbers. Now using your probability techniques find the following probabilities:

1) Plot the histogram of X.
2) P(X = -5) = ?, P(X = -2) = ?, P(X = 5) = ?
3) What is the probability that X is **EVEN?**
4) What is the probability that X is **ODD?**
5) What is the probability that X have values of -5 or +5? **HINT:** use or ‘|’ operator

Solution:

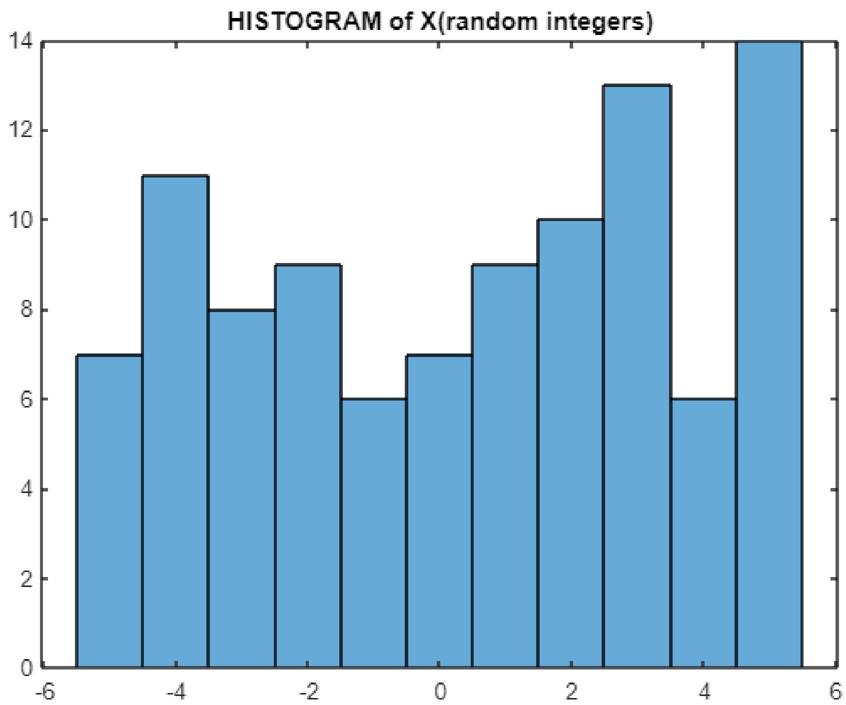
```
clc
%PART(1)
X = csvread('random_integers.csv'); %#ok<CSVRD>
histogram(X)
title("HISTOGRAM of X(random integers)");

%PART(2)
%(probability of x=-5)
% n = numel(A) returns the the number of elements, n , in array
p5=sum(X == -5)/numel(X);
fprintf('Probability of X=-5 is %f\n',p5);
%probability of (X=-2)
p2=sum(X == -2)/numel(X);
fprintf('Probability of X=-2 is %f\n',p2);
%probability of (X=+5)
p51=sum(X == 5)/numel(X);
fprintf('Probability of X=5 is %f\n',p51);
%PART(3)
%What is the probability that X is EVEN?
sum_even= sum(rem(X,2)==0);
p_even = sum_even /numel(X);
fprintf('Probability of EVEN X is %f\n',p_even);

%PART(4)
%What is the probability that X is ODD?
p_odd = (numel(X)-sum_even) /numel(X);
fprintf('Probability of ODD X is %f\n',p_odd);

%PART(5)
%probability of (X=+5 or X=-5)
p55=sum(X == 5 | X==-5 )/numel(X);
fprintf('Probability of X=5 OR X=-5 is %f\n',p55);
```

Outputs:

```
Probability of X=-5 is 0.070000
Probability of X=-2 is 0.090000
Probability of X=5 is 0.140000
Probability of EVEN X is 0.430000
Probability of ODD X is 0.570000
Probability of X=5 OR X=-5 is 0.210000
```

‘

Histogram:



Problem 7:

Now you need to perform following tasks:
1. Generate 100,000 random numbers as an array and plot histogram in all three cases and find their means.
2. Generate 100,000 random integers between 5 and 20 as an array and find their means, variance and standard deviation. Compute the PMF from this large observations of a random variable. It first computes the histogram and then normalizes the histogram to compute the PMF. Plot both histogram and pmf.

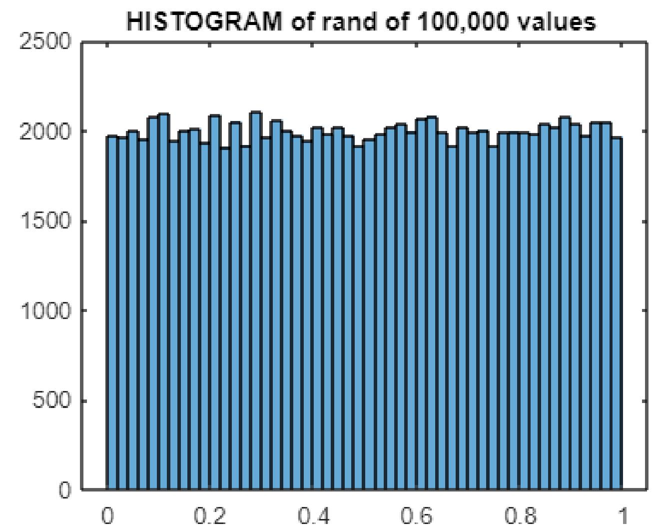
Solution:
rand:

```
%%PART(A.1)
X=rand([1,100000]);
histogram(X);
title("HISTOGRAM of rand of 100,000 values ");
M1=mean(X);
fprintf('MEAN of rand is %f\n',M1);
```

Output:

```
MEAN of rand is 0.500681
```

Histogram:



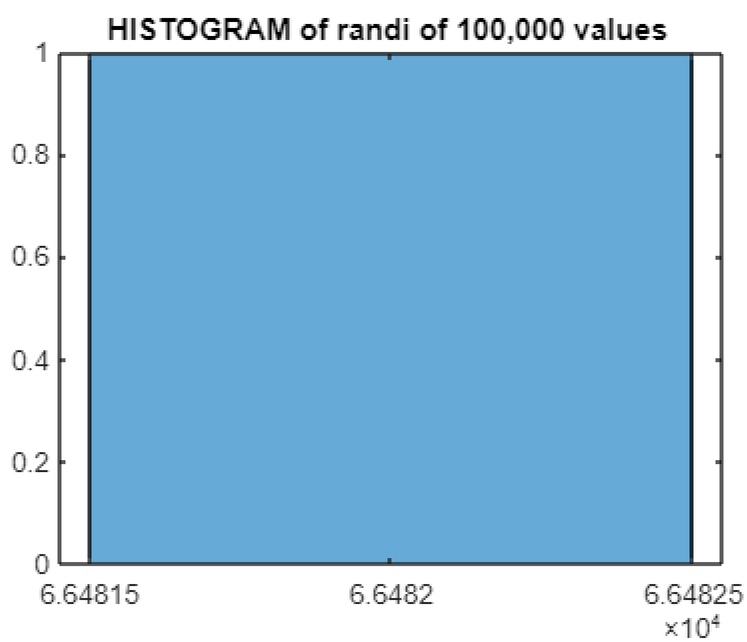
randi:

```
%%PART(A.2)
X1=randi([1,100000]);
histogram(X1);
title("HISTOGRAM of randi of 100,000 values ");
M1=mean(X1);
fprintf('MEAN of randi is %f\n',M1);
```

Output:

```
MEAN of randi is 66482.000000
```

Histogram:



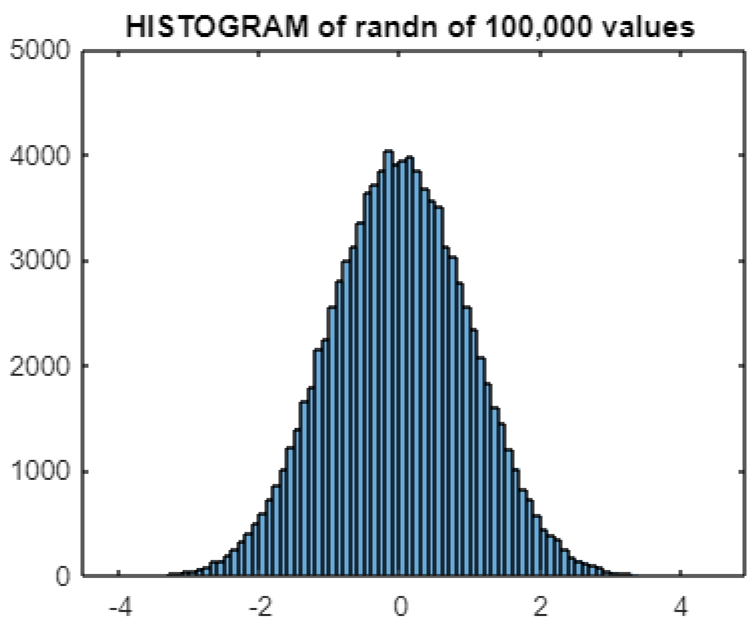
randn:

```
%%PART(A.3)
X2=randn([1,100000]);
histogram(X2);
title("HISTOGRAM of randn of 100,000 values ");
M2=mean(X2);
fprintf('MEAN of randn is %f\n',M2);
```

Output:

```
MEAN of randn is -0.001544
```

HistoGram:



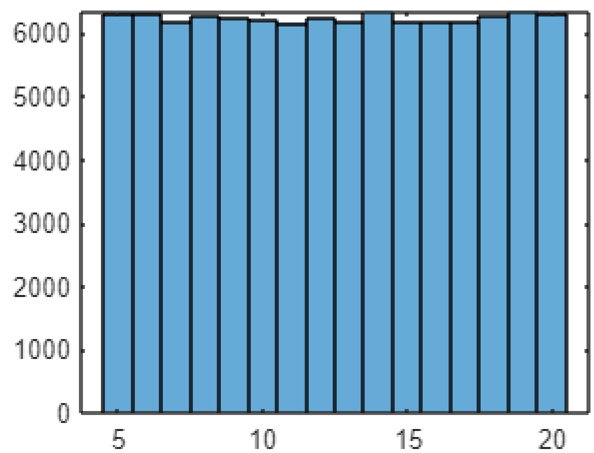
Part(b):
Code:

```
%PART(B)
A=randi([5,20],[1,100000]);
M_b=mean(A);
fprintf('MEAN of randI is %f\n',M_b);
V=var(A);
fprintf('VARIENCE of randi is %f\n',V);
S=std(A);
fprintf('Standarad Deviation of randi is %f\n',S);
histogram(A);
pmf = histcounts(A,[unique(A) Inf],'Normalization','probability');
bar(pmf);
```

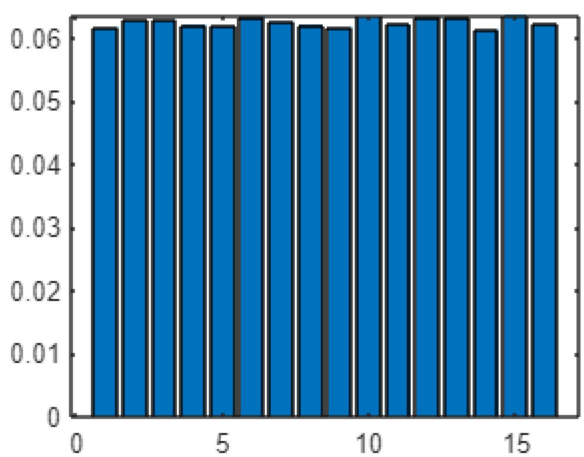
Output:

```
MEAN of randI is 12.503610
VARIENCE of randi is 21.353601
Standarad Deviation of randi is 4.620996
```

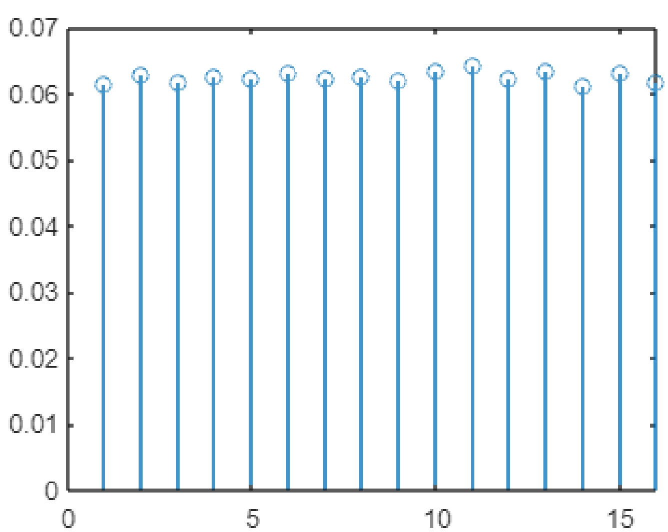
Histogram:



pmf(plot using bar):



pmf(plot using stem):



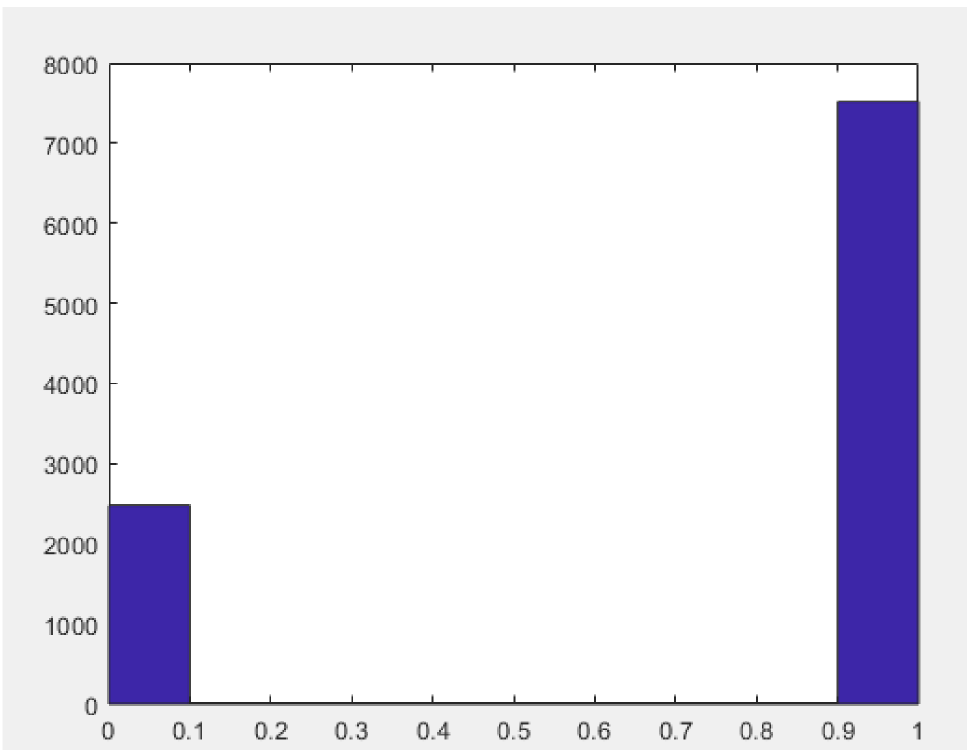
Problem 8:

Solution:

```
p_heads=0.25;
n=10000;
test_biased_coin( p_heads, n );
```

```
function test_biased_coin( p_heads, n )

    A = zeros(n,1);
    for i=1:n
        A(i) = biased_coin(p_heads)
    end
    hist(A);
    m=mean(A);
    fprintf('mean is %f',m);

end
```

Histogram:



Output:

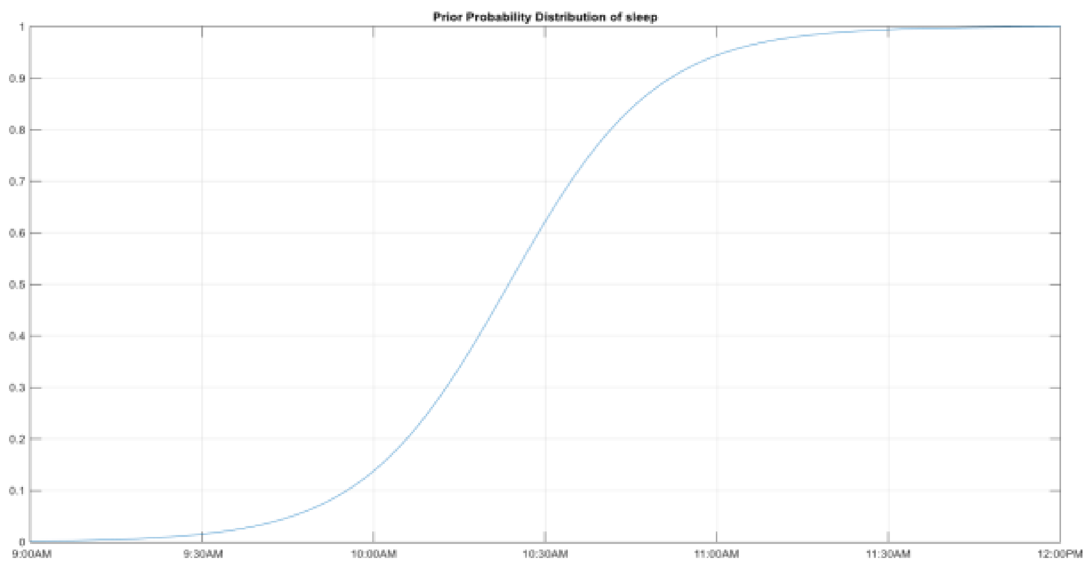
```
mean is 0.751900
```

Problem9:

With idea of Bayes' Rule, one problem that we can explore in our life is sleeping patterns. We have data a person showing when he falls asleep at night. We have figured out the probability using Markov Chain Monte Carlo (MCMC) methods and formula:

$$P(sleep) = \frac{1}{1 + e^{-0.0028 \times t + 6.5}}$$

The final model showing the most likely distribution of sleep as a function of time.



Based on person's habits, we know that he cannot sleep easily with lights ON and the probability of bedroom light is ON given that he is asleep is only about 1%. That is:

$$P(light|sleep) = 0.01$$

The probability that light is on given that he is not asleep, is around 80% probability that bedroom light is on if he is awake (which means there is a 20% chance that light is not on if he is awake).

a) Find the probability $P(-light|sleep) = ?$

(Where $-light$ indicates light OFF)

b) Find the probability that $light$ is ON at 09:30 PM i.e. $P(light)$.

c) What is the probability that person fall asleep with $light$ ON at 10:00 PM?

d) Suppose person fails to fall asleep at 11:00 PM, what is the probability that $light$ is ON?

NOTE: (solve ***a*** to ***d*** parts on paper and take estimated probabilities from graph at required times, but for **MATLAB** use formula to get probabilities)

Solution:(Paper calculations)

(a)

Problem#9

Solution:

(a) $P(-light \mid sleep) = ?$

$P(light \mid sleep) = 0.01$.

$P(light \mid nosleep) = 0.8$

$P(no\,light \mid nosleep) = 1 - P(light \mid nosleep)$

$= 1 - 0.8 = 0.2$

$P(-light \mid sleep) = 1 - P(light \mid sleep)$

$= 1 - 0.01$

$P(-light \mid sleep) = 0.99$.

(b)

(b) $P(light) = ?$ at 9:030PM

$P(light) = P(Light \mid sleep)\,P(sleep) + P(Light \mid -sleep)\,P(-sleep)$ — (1)

$P(sleep) = 0.02$ (from graph)

$P(-sleep) = 1 - P(sleep) = 1 - 0.02 = 0.98$.

Put in (1)

$P(light) = (0.01 \times 0.02) + (0.98 \times 0.8)$

$P(Light) = 0.7842$

©

(c) $P(sleep|Light) = ?$ at 10:00PM.

$$P(sleep|Light) = \frac{P(light|sleep)\,P(sleep)}{P(Light)} \rightarrow ①$$

$$P(Light) = P(Light|sleep)\,P(sleep) + P(light|-sleep)\,P(-sleep) \rightarrow ②$$

$P(sleep) = 0.15$ (approximate from graph)

$P(-sleep) = 1 - 0.15 = 0.98$

Put in ②

$$P(Light) = (0.01 \times 0.15) + (0.98 \times 0.8) = 0.7855$$

Put in ①

$$P(Sleep|Light) = \frac{(0.01)(0.15)}{(0.7855)} = 2 \times 10^{-3}.$$

$$P(sleep|light) = 2 \times 10^{-3}.$$

(d)

(d) $P(-sleep|light) = ?$ at 11:00PM

$$P(Light) = P(Light|sleep)\,P(sleep) + P(Light|-sleep)\,P(-sleep)$$

$P(sleep) = 0.95$ (approximate from graph) → ①

$P(-sleep) = 1 - 0.95 = 0.05.$

Put in ①

$$P(Light) = (0.01 \times 0.95) + (0.8 \times 0.05) = 0.0495.$$

$$P(-sleep|light) = \frac{P(light|-sleep)\,P(-sleep)}{P(Light)}$$

$$P(-sleep|light) = \frac{(0.8 \times 0.05)}{0.0495} = 0.8081.$$

Solution:(MATLAB)
part(a)

```
clc
clear
close all
% This program demonstrates probabilistic sleeping pattern
% of a person at night w.r.t time.
%
% Author: Asad khan
% Created: Sep-15-2021
%% DO NOT edit or disturb this portion only use required variables below.
t = 1:5000;
prob_sleep_time = 1./(1+exp(-0.0028.*t+6.5));
n = length(t);
sdate = linspace(datenum('01-Aug-2013 09:00:00'),datenum('01-Aug-2013 12:00:00'),n);
%% You cant edit or write your code line below here
plot(sdate,prob_sleep_time)
datetick('x','HH:MMPM')
title('Prior Probability Distribution of sleep');
grid ON
%my code here
p_light_sleep = 0.01;
%part(a)
p_nlight_sleep =1 - p_light_sleep;
fprintf('probability of no light given sleep is %f',p_nlight_sleep);
```

Output:

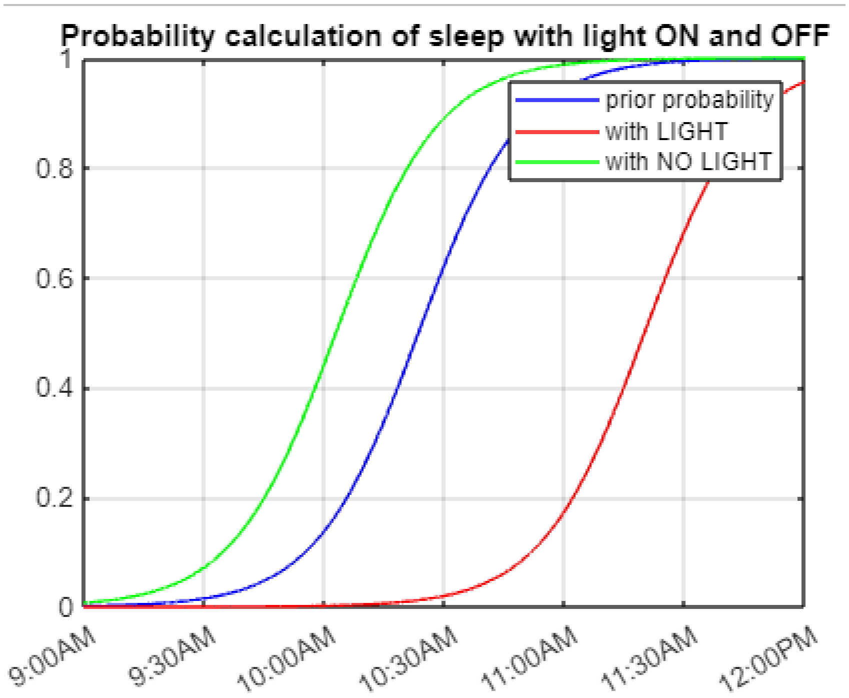
```
probability of no light given sleep is 0.990000
>>
```

Part(e):

```
%part(e)
p_light_nsleep = 0.8;
p_nlight_nsleep =1 - p_light_nsleep;
for i = 1:n
    p_light =p_light_sleep * prob_sleep_time(i)+p_light_nsleep *(1-prob_sleep_time(i));
    p_sleep_light(i)=p_light_sleep * prob_sleep_time(i) / p_light;
    %probbaility of sleep given no light%
    p_nlight=p_nlight_sleep * prob_sleep_time(i) + p_nlight_nsleep * (1-prob_sleep_time(i));
    p_sleep_nlight(i) = p_nlight_sleep * prob_sleep_time(i) / p_nlight;
end
plot(sdate,prob_sleep_time,color='blue');
hold on
plot(sdate,p_sleep_light,color='red');
plot(sdate,p_sleep_nlight,color='green');
legend('prior probability','with LIGHT','with NO LIGHT')
```

```
datetick('x','HH:MMPM')
title("Probability calculation of sleep with light ON and OFF")
grid on
```

Output:



Problem(9.2)

Applying Bayes' Rule to real life problems is a fun so why stop with only bedroom light? We can use more information in the model as we like and it will continue to get more precise results. For example, if I know the likelihood that person's mobile phone is charging given that he asleep is 95%, we can incorporate that knowledge into the model.

The probability that phone is charging given that he is not asleep, 50% probability that phone is on charging if he is awake.

> ***NOTE***: probability that phone is charging is conditionally independent of the probability that light is ON.

Bayes' equation using the extra information is expressed:

$$P(sleep|light, mobile) = \frac{P(light|sleep) \times P(mobile|sleep) \times P(sleep)}{P(light, mobile)}$$

a) Find the probability $P(-mobile|sleep) = ?$

(Where $mobile$ indicates Mobile charging)

b) Find the probability that $light$ is ON and $mobile$ is charging at 09:30PM i.e. $P(light, mobile)$.

c) What is the probability that person fall asleep with $light$ ON and $mobile$ **NOT** charging at 10:00 PM?

d) Use same **MATLAB** script file, find and plot the following probabilities from 09:00 PM to 12:00PM.

a. $P(sleep|light, mobile) = ?$

b. $P(sleep| -light, mobile) = ?$

c. $P(sleep|light, -mobile) = ?$

d. $P(sleep| -light, -mobile) = ?$

(Where $-mobile$ indicates Mobile **NOT** charging)

Plot all probabilities on same given figure i.e. prior sleep probability. Give *legends, title* and use different colors for each plot line.

Solution:(Paper Calculation)

Part(a,b)

9.2

Part(b)

(a) $P(-mobile|sleep) = ?$

$P(mobile|Sleep) = 0.95$

$P(mobile|-sleep) = 0.5$

$P(-mobile|sleep) = 1 - P(mobile|sleep)$

$= 1 - (0.95) = 0.05$

(b)

$P(light, mobile) = ?$ at 09:30PM

$P(light, mobile) = P(light|sleep)P(mobile|sleep)P(sleep) + P(light|-sleep) \times P(mobile|-sleep)P(-sleep)$

$P(sleep) = 0.02$ (from graph)

$P(-sleep) = 1 - 0.02 = 0.98$

$P(light, mobile) = (0.01 \times 0.95 \times 0.02) + (0.8 \times 0.5 \times 0.98)$

$P(light, mobile) = 0.39219$

Part(c):

(c) $P(sleep) = 0.15$ at 10:00

$P(sleep / light, -mobile) = ?$

$P(light, -mobile) = P(light|sleep)P(-mobile|sleep)P(sleep) + P(light|-sleep)P(-mobile|-sleep)P(-sleep)$

$= (0.01 \times 0.05 \times 0.15) + (0.8 \times 0.5 \times (1-0.15))$

$P(light, -mobile) = 0.340075$

$$P(sleep|light, -mobile) = \frac{P(light, -moble|sleep)P(sleep)}{P(light, -mobile)}$$

$$P(sleep|light, -mobile) = \frac{P(light|sleep)P(-mobile|sleep)P(sleep)}{P(light, -mobile)}$$

$$= \frac{(0.01 \times 0.05 \times 0.15)}{0.340075}$$

$P(sleep|light, -mobile) = 2.20539 \times 10^{-4}$.

Solution:(MATLAB)

Part(a)

Code:

```
%part(a)
p_light_nsleep = 0.8;
p_nlight_nsleep =1 - p_light_nsleep;
p_light_sleep = 0.01;
p_nlight_sleep =1 - p_light_sleep;
p_mobile_sleep=0.95;
p_mobile_nsleep=0.5;
p_nmobile_nsleep=1-p_mobile_nsleep;
p_nmobile_sleep=1-p_mobile_sleep;
fprintf("probability of not charging given sleep is %f",p_nmobile_sleep);
```

Output:

```
probability of not charging given sleep is 0.050000
```

Code:

```
for i=1:n
    %(a)
    p_light_mobile=p_light_sleep*p_mobile_sleep*prob_sleep_time(i)+p_light_nsleep*p_mobile_nsleep*(1-prob_sleep_time(i));
    p_sleep_light_mobile(i)=p_light_sleep*p_mobile_sleep*prob_sleep_time(i)/p_light_mobile;
    %(b)
    p_nlight_mobile=p_nlight_sleep*p_mobile_sleep*prob_sleep_time(i)+p_nlight_nsleep*p_mobile_nsleep*(1-prob_sleep_time(i));
    p_sleep_nlight_mobile(i)=p_nlight_sleep*p_mobile_sleep*prob_sleep_time(i)/p_nlight_mobile;
    %part(c)
    p_light_nmobile=p_light_sleep*p_nmobile_sleep*prob_sleep_time(i)+p_light_nsleep*p_nmobile_nsleep*(1-prob_sleep_time(i));
    p_sleep_light_nmobile(i)=p_light_sleep*p_nmobile_sleep*prob_sleep_time(i)/p_light_nmobile;
    %part(d)
    p_nlight_nmobile=p_nlight_sleep*p_nmobile_sleep*prob_sleep_time(i)+p_nlight_nsleep*p_nmobile_nsleep*(1-prob_sleep_time(i));
    p_sleep_nlight_nmobile(i)=p_nlight_sleep*p_nmobile_sleep*prob_sleep_time(i)/p_nlight_nmobile;
end
```

```
plot(sdate,prob_sleep_time,color='blue');
hold on
plot(sdate,p_sleep_light_mobile,color='red');
plot(sdate,p_sleep_nlight_mobile,color='green');
plot(sdate,p_sleep_light_nmobile,color='cyan');
plot(sdate,p_sleep_nlight_nmobile,color='magenta');
legend('prior probability','with LIGHT,MOBILE','with light OFF ,MOBILE ','with LIGHT ,mobile OFF','with NO light,NO mobile')
datetick('x','HH:MMPM')
title("Probability calculation of sleep with different conditions");
grid on
```

Solution: