NUST
NATIONAL UNIVERSITY
OF SCIENCES & TECHNOLOGY

NUST-SEECS School of Electrical Engineering and Computer Science H-12, Islamabad, Pakistan

E-891: Stochastic Systems Fall 2021

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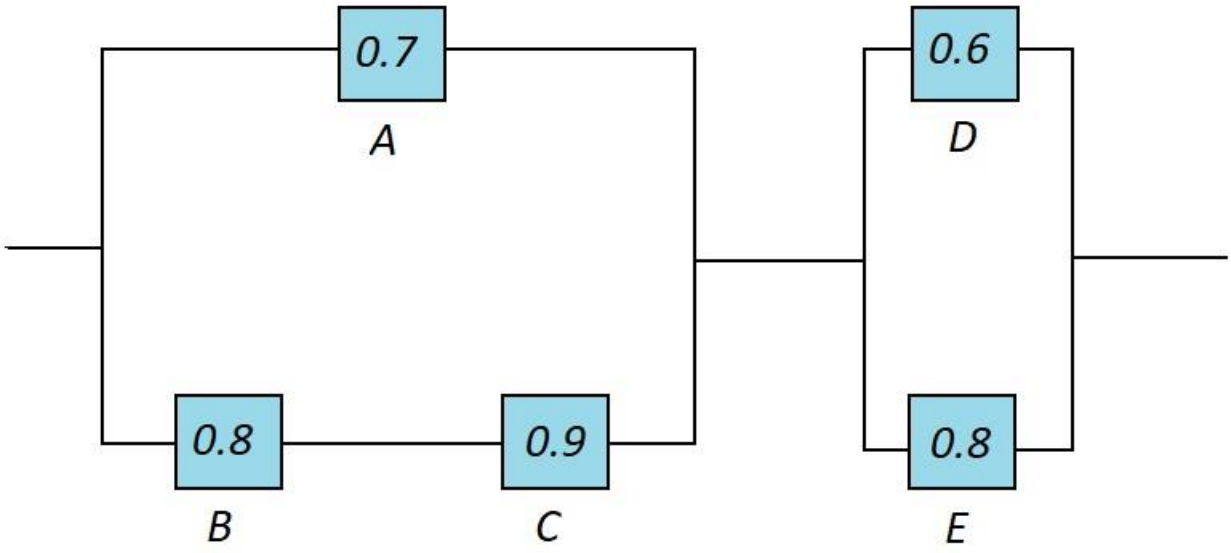
Note: For any issues regarding the assignment, drop an email at akhan.phdee21seecs@seecs.edu.pk

Assignment # 1

Problem 01:

A circuit system is given in the figure below. Assume the components fail independently. Probabilities shown are of each component working fine individually.



(a) What is the probability that the entire system works?
(b) Given that the system works, what is the probability that the component D is not working?

Problem 02:

A box of 30 diodes is known to contain five defective ones. If two diodes are selected at random without replacement, what is the probability that at least one of these diodes is defective?

Problem 03:

If there are three baskets i.e. Basket A, B and C. Basket A contains 2 red balls, Basket B contains 2 white balls and basket C contains 1 red ball and 1 white ball. A basket is selected at random, and one ball is taken at random from that basket.

(c) What is the probability of selecting a white ball?
(d) If the selected ball is white, what is the probability that the other ball in the basket is red?

Problem 04:

We are given a number of darts. When we throw a dart at a target, we have a probability of ¼ of hitting the target. What is the probability of obtaining at least one hit if three darts are thrown? Calculate this probability two ways.

Hint: Construct the sample space. How many outcomes are in the sample space? Are all outcomes in the sample space equally likely?

Problem 05:

Consider a routine screening test for a disease. Suppose the frequency of the disease in the population (base rate) is 0.5%. The test is highly accurate with a 5% false positive rate and a 10% false negative rate. You take the test and it comes back positive. What is the probability that you have the disease?

MATLAB Problem 06:

Download the file "random_integers.csv" and read it in MATLAB using command `X = csvread()`. It will load a data of random numbers. Now using your probability techniques find the following probabitlities:

1) Plot the histogram of X.
2) P(X = -5) = ?, P(X = -2) = ?, P(X = 5) = ?
3) What is the probability that X is **EVEN?**
4) What is the probability that X is **ODD?**
5) What is the probability that X have values of -5 or +5**?** **HINT:** use or '|' operator

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MATLAB Problem 07:

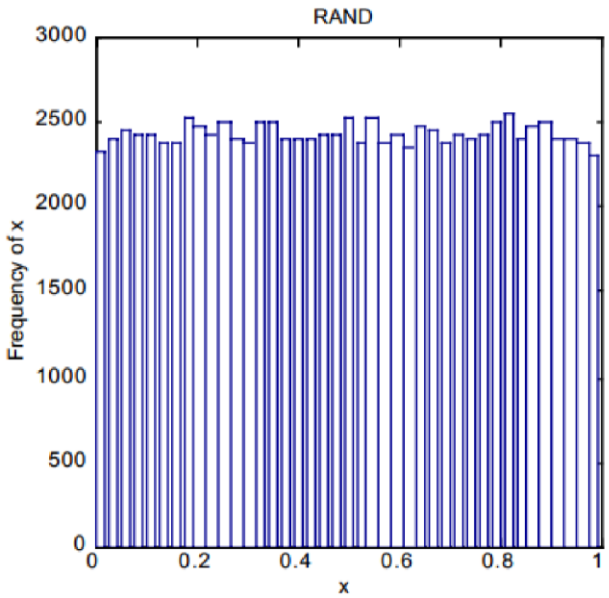
In MATLAB we can Generate Random variable from certain distribution. e.g.

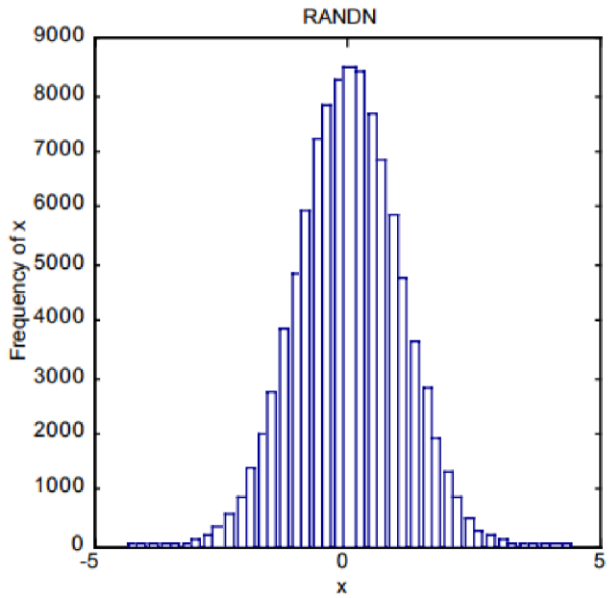
`rand()` used to generate uniformly distributed random numbers.

`randn()` used to generate normally distributed random numbers.

`randi()` used to generate uniformly distributed random integers.

Shown below are histograms of the distributions of 100,000 random numbers produced by these first two generators.





The numbers produced by `rand()` have a flat histogram, indicating that all values are equally likely. They are distributed between a minimum value of 0 and a maximum value of 1.0. The ones produced by `randn()`, on the other hand, are distributed along a classic bell-shaped curve.

`randi()` gives you uniformly distributed random Integers as output. e.g. to generate integers between 6 and 15 of array of size 10:

```
>> randi([6,15],[1,10]) ans

=

    15    11     6    10     7    15    11    14    12    10
```

Now you need to perform following tasks:

1. Generate 100,000 random numbers as an array and plot histogram in all three cases and find their means.
2. Generate 100,000 random integers between 5 and 20 as an array and find their means, variance and standard deviation. Compute the PMF from this large observations of a random variable. It first computes the histogram and then normalizes the histogram to compute the PMF. Plot both histogram and pmf.

You can follow these Steps for second task or **you can think of your own way**:

I. Generate integers.

II. Make image array which will include the range of generated numbers, e.g. in above example image array will be: '`ans = [6 7 8 9 10 11 12 13 14 15]`' and can be generated as '`image = 6:15`'

III. Now pick each element one by one from image array, compare and find the occurrences of that number in the generated random integers. Following loop will help you.

```
for i = 1:nX      cX
= imageX(i);
    cntX(i) = sum(observX == cX);  end
```

where: `nX` is the length if image array `cX` will take value from image array one by one at each iteration. `cntX` initially an empty array and will store the number of occurrences against each element `cX` of image.

IV. To get normalized pmf simply divide the cntX (histogram) by their sum. And plot against image array.

V. Find statistics parameters (means, variances etc)

MATLAB Problem 08:

We know that how we can generate random variables from different distribution. now we need to conduct experiment of a biased coin. Which have distribution:

$$P(\text{heads}) = 0.25 \quad \text{and} \quad P(\text{Tails}) = 0.75$$

How can we get random values from this distribution instead of uniform or normal distribution? Attached are the function files named "biased_coin.m" and "test_biased_coin.m", download them and use it to get random values from this distribution.

e.g. `test_biased_coin(p_heads, n)`

where: `p_heads` is the probability of head i.e. 0.25 and `n` is the number of random numbers to generate Generate 100,000 random numbers as an array and plot histogram find their means.

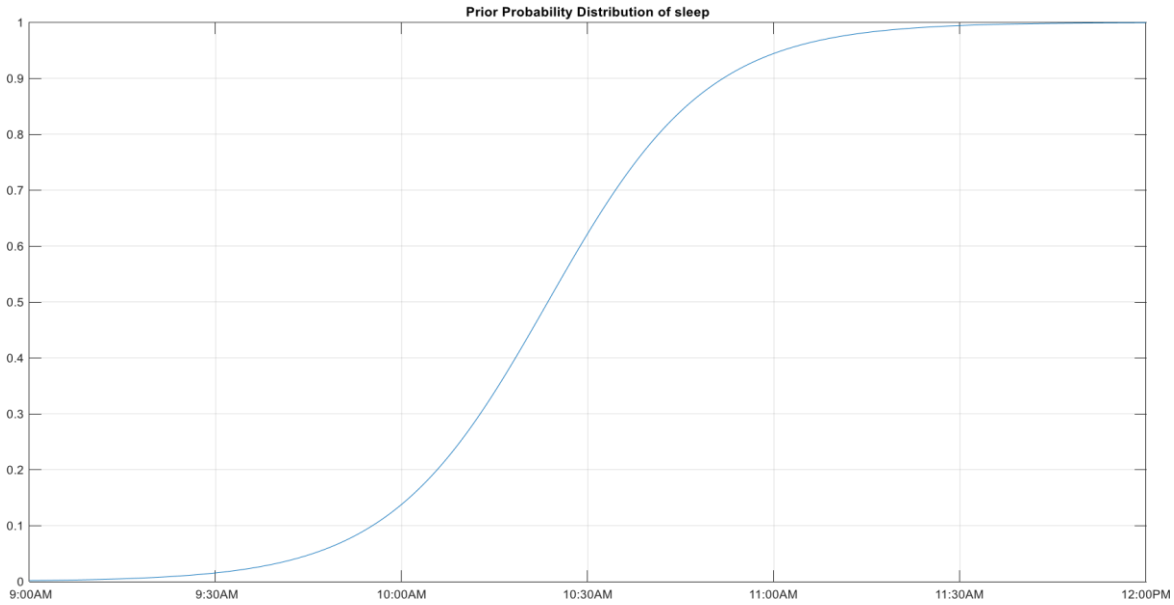
NOTE: understand the code used in the function this can help in future assignments

MATLAB Problem 09:

With idea of Bayes' Rule, one problem that we can explore in our life is sleeping patterns. We have data a person showing when he falls asleep at night. We have figured out the probability using Markov Chain Monte Carlo (MCMC) methods and formula:

$$P(sleep) = \frac{1}{1 + e^{-0.0028 \times t + 6.5}}$$

The final model showing the most likely distribution of sleep as a function of time.



Bedroom light ON can change the probability of person's sleep. This is where we use Bayes' Rule to update our estimate. For a specific time, if we know information about bedroom light, we can use the probability from the distribution above as the *prior* and then apply Bayes' equation:

$$P(sleep|light) = \frac{P(light|sleep) \times P(sleep)}{P(light)}$$

Where $light$ indicates light ON

Based on person's habits, we know that he cannot sleep easily with lights ON and the probability of bedroom light is ON given that he is asleep is only about 1%. That is:

$$P(light|sleep) = 0.01$$

The probability that light is on given that he is not asleep, is around 80% probability that bedroom light is on if he is awake (which means there is a 20% chance that light is not on if he is awake).

a) Find the probability $P(-light|sleep) = ?$
   (Where $-light$ indicates light OFF)
b) Find the probability that $light$ is ON at 09:30 PM i.e. $P(light)$.
c) What is the probability that person fall asleep with $light$ ON at 10:00 PM?
d) Suppose person fails to fall asleep at 11:00 PM, what is the probability that $light$ is ON? ***NOTE***: (solve ***a*** to ***d*** parts on paper and take estimated probabilities from graph at required times, but for **MATLAB** use formula to get probabilities)

e) You are given with a **MATLAB** script file "Sleep_bayes.m" that plots the above graph using formula. Use that script file, find and plot the following probabilities from 09:00 PM to 12:00PM.

   a. $P(sleep|light) = ?$
   b. $P(sleep|-light) = ?$

   Plot both probabilities on same given figure of prior sleep probability. Give *legends, title* and use different colors for each plot line.

Applying Bayes' Rule to real life problems is a fun so why stop with only bedroom light? We can use more information in the model as we like and it will continue to get more precise results. For example, if I know the likelihood that person's mobile phone is charging given that he asleep is 95%, we can incorporate that knowledge into the model.

The probability that phone is charging given that he is not asleep, 50% probability that phone is on charging if he is awake.

NOTE: probability that phone is charging is conditionally independent of the probability that light is ON.

Bayes' equation using the extra information is expressed:

$$P(sleep|light, mobile) = \frac{P(light|sleep) \times P(mobile|sleep) \times P(sleep)}{P(light, mobile)}$$

a) Find the probability $P(-mobile|sleep) = ?$

(Where $mobile$ indicates Mobile charging)

b) Find the probability that $light$ is ON and $mobile$ is charging at 09:30PM i.e. $P(light, mobile)$.

c) What is the probability that person fall asleep with $light$ ON and $mobile$ **NOT** charging at 10:00 PM?

NOTE: (solve ***a*** to ***c*** parts on paper and take estimated probabilities from graph at required times, but for **MATLAB** use formula to get probabilities)

d) Use same **MATLAB** script file, find and plot the following probabilities from 09:00 PM to 12:00PM.

   a. $P(sleep|light, mobile) = ?$
   b. $P(sleep|-light, mobile) = ?$
   c. $P(sleep|light, -mobile) = ?$

d. $P(sleep| - light, -mobile) = ?$

(Where $-mobile$ indicates Mobile **NOT** charging)

Plot all probabilities on same given figure i.e. prior sleep probability. Give *legends*, *title* and use different colors for each plot line.

In case of more events to find the total probability just multiply the conditional probability of every event with prior probability e.g. in this case:

$$P(light, mobile) = P(light|sleep) \times P(mobile|sleep) \times P(sleep)$$
$$+ P(light| - sleep) \times P(mobile| - sleep) \times P(-sleep)$$

Requirements:

Make a pdf file including your details (name, registration number etc.), snaps of Solved Questions, codes, snaps of graphs, values of means, variances and standard deviation.

Graphs should be well self-explained. Use 'graph titles', 'x and y axis titles' and "legends" in each plots.