NUST-SEECS School of Electrical Engineering and Computer Science H-12, Islamabad, Pakistan

E-891: Stochastic Systems Fall 2019

Instructor: Dr. Shahzad Younis **TA:** Rafi Ul Zaman
Salsabeel Ahmad

Note: For any issues regarding the assignment, drop an email at rzaman.msee16seecs@seecs.edu.pk or sahmed.msee17seecs@seecs.edu.pk

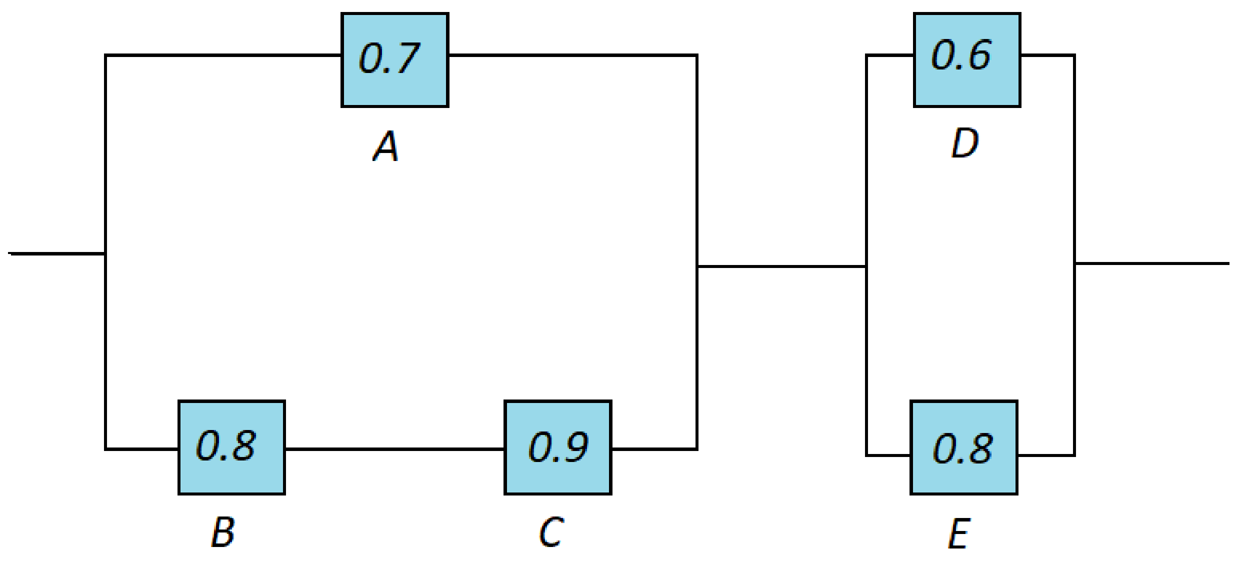
Assignment # 1

Name:Malik Awais (DSSP)

Reg:00000319169

Problem 01:

A circuit system is given in the figure below. Assume the components fail independently. Probabilities shown are of each component working fine individually.



(a) What is the probability that the entire system works?

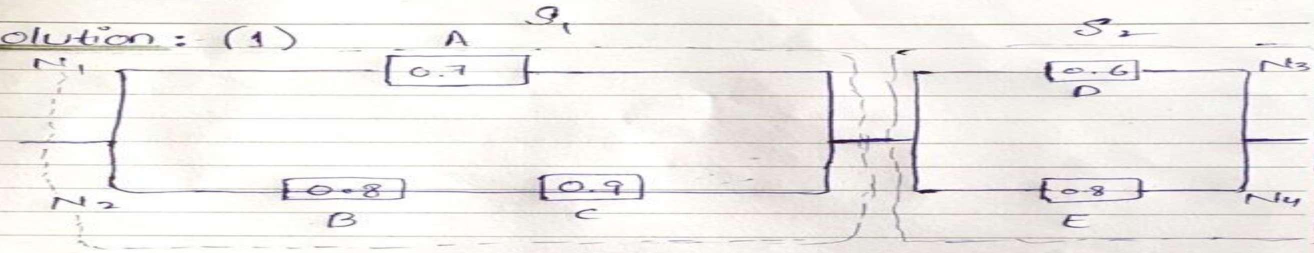
(b) Given that the system works, what is the probability that the component B is not working?

Solution:

P ①

Date:

Solution: (1)



Events:

N_1	:	Node - 1	works	properly
N_2	:	Node - 2	"	"
N_3	:	Node - 3	"	"
N_4	:	Node - 4	"	"
S_1	:	System 1	"	"
S_2	:	System 2	"	"
S	:	Entire System	"	"

A, B, C, D are components

(a) $P(S) = ?$

P(2)

Date:

$P(N_1) = P(A) = 0.7$

$P(\bar{N}_1) = 1 - 0.7 = 0.3$

$P(N_2) = P(B) \cdot P(C)$

$= 0.8 \times 0.9$

$= 0.72$

$P(\bar{N}_2) = 1 - 0.72$

$= 0.28$

$P(N_3) = P(D) = 0.6$

$P(\bar{N}_3) = 1 - 0.6 = 0.4$

$P(N_4) = P(E) = 0.8$

$P(\bar{N}_4) = 1 - 0.8 = 0.2$

$P(\bar{S}_1) = P(\bar{N}_1 \cap \bar{N}_2) = P(\bar{N}_1) P(\bar{N}_2)$

$= 0.3 \times 0.28$

$= 0.084$

Date:

$P(S_1) = 1 - 0.084$
$= 0.916$

$P(\bar{S_2}) = P(\bar{N_3} \cap \bar{N_4})$
$= P(\bar{N_3}) \cdot P(\bar{N_4})$
$= 0.4 \times 0.2 = 0.08$

$P(S_2) = 1 - P(\bar{S_2})$
$= 1 - 0.08$
$= 0.92$

$P(S) = P(S_1) \cdot P(S_2)$
$= 0.916 \times 0.92$

$\boxed{P(S) = 0.84}$

(b) $P(\bar{B}/S) = \dfrac{P(S/\bar{B})\,P(\bar{B})}{P(S)}$

$P(S/\bar{B}) = P(S_1 \cap N_4)$
$= P(S_1) \cdot P(N_4)$
$= 0.916 \times 0.8$
$P(S/\bar{B}) = 0.7328$

Date:

$$P(\bar{B}/S) = \boxed{\frac{(0.7328)(0.4)}{0.84} = 0.34}$$

Solution: (2)

Alive $P(T/25) = 0.8$

Alive $P(N/25) = 0.95$

Not alive $P(\overline{Tom}/25) = 1 - 0.8 = 0.2$

Not alive $P(\bar{N}/25) = 1 - 0.95 = 0.05$

Now both alive

$= P(T/25) \cdot P(N/25)$

$= 0.8 \times 0.95$

$= 0.76$

Now both not alive

$P(\bar{T}/25) \cdot P(\bar{N}/25)$

0.2×0.05

$= 0.01$

Problem 02:

The probability that Tom will be alive in 25 years is 0.8, and the probability that Nancy will be alive in 25 years is 0.95. If we assume independence for both, what is the probability that neither will be alive in 25 years?

Date:______________

$$\boxed{P(\bar{B}/S) = \frac{(0.7328)(0.4)}{0.84} = 0.34}$$

Solution: (2)

Alive $P(T/25) = 0.8$

Alive $P(N/25) = 0.95$

Not alive $P(\bar{Tom}/25) = 1 - 0.8 = 0.2$

Not alive $P(\bar{N}/25) = 1 - 0.95 = 0.05$

Now both alive

$= P(T/25) \cdot P(N/25)$

$= 0.8 \times 0.95$

$= 0.76$

Now both not alive

$P(\bar{T}/25) \cdot P(\bar{N}/25)$

0.2×0.05

$= 0.01$

Problem 03:

In a certain region of the country having equal number of men and women, it is known from past experience that the probability of selecting a man over 40 years of age with cancer from total population is 0.05 and woman over 40 years of age with cancer is 0.03. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06.

1. What is the probability that an adult is a man if a selected adult having cancer?
2. What is the probability that an adult over 40 years of age is diagnosed as having cancer?
3. What is the probability that a person diagnosed as having cancer actually has the disease?

Sol:-

$P(\text{Cancer}/\text{man}) = 0.05$

$P(\text{Cancer}/\text{women}) = 0.03$

$P(D/C) = 0.78$

$P(D/\bar{C}) = 0.06$

$P(M) = 0.5$, $P(W) = 0.5$ ∵ half

a) Prob of man if selected having cancer,

$$P(\text{Man}/\text{Cancer}) = \frac{P(C/M)\,P(M)}{P(C)}$$

Now we have

$$P(C) = P(C/M)\,P(M) + P(C/W)\,P(W)$$
$$= 0.05 \times 0.5 + (0.03)(0.5)$$
$$= 0.025 + 0.15$$
$$P(C) = 0.04$$

$$P(\text{Man}/\text{cancer}) = \frac{0.05 \times 0.5}{0.04}$$

$$P(\text{Man}/\text{cancer}) = \boxed{0.625}$$

b) As we have to find to of the probability of man and woman having Cancer age greater than 40.

$$P(\text{cancer}/\text{adult}) = P(C/M)\,P(M) + P(C/W)\,P(W)$$
$$= 0.05 \times 0.5 + 0.03 \times 0.5$$
$$P(\text{cancer}/\text{adult}) = \boxed{0.04}$$

3) $P(C/D) = ?$

$$P(C/D) = \frac{P(D/C)P(C)}{P(D/C)P(C) + P(D/\bar{C})P(\bar{C})}$$

$$P(\bar{C}) = 1 - 0.04 = 0.96$$

$$P(C/D) = \frac{0.78 \times 0.04}{0.78 \times 0.04 + 0.06 \times 0.96}$$

$$= \frac{0.0312}{0.0888}$$

$$P(C/D) = 0.3514$$

Problem 04:

If there are three baskets i.e. Basket A, B and C. Basket A contains 2 red balls, Basket B contains 2 white balls and basket C contains 1 red ball and 1 white ball. A basket is selected at random, and one ball is taken at random from that basket.

(a) What is the probability of selecting a white ball?

(b) If the selected ball is white, what is the probability that the other ball in the basket is red?

P(5)

Date:

Solution : (4)

(a) $P(W) = ?$ $\quad P(W) = P[(W \cap A) \cup (W \cap B) \cup (W \cap C)]$

$P(W) = P(W \cap A) + P(W \cap B) + P(W \cap C)$

$P(W) = P(W/A) . P(A) + P(W/B) . P(B) + P(W/C) . P(C)$

$P(W) = (0 \times 1/3) + (1 \times 1/3) + (1/2 + 1/3) = 1/2$

(b) $P(C/W) = ?$ $\quad$ * Because C contain 1 white & 1 red ball

$P(C/W) = P(C \cap W) / P(W)$

$= P(W/C) \; P(C) / P(W)$

$= (1/2 \times 1/3) / 1/2 = (1/6)/(1/2) = 2/6 = 1/3$

Problem 05:

A deck of cards has 52 cards in it. Event A is drawing a King first, Event B is drawing a King second and Event C is drawing a red card.

1. What is the probability of Event A?
2. What is the probability of Event B given Event A?
3. What is the probability of Event C?
4. What is the probability of getting red king?
5. What is the probability of getting two kings?

Date:

Solution: (5)

Events:

A = { drawing king 1st }
B = { drawing king 2nd }
C = { drawing red card }

part (1) Prob of event A = ?

$$P(A) = \frac{4}{52} = \frac{1}{13}$$

part (2) $B/A = \frac{P(A \cap B)}{P(A)} = \frac{P(A/B)\,P(B)}{P(A)}$

part (3) $P(C) = \frac{26}{52} = \frac{1}{2}$

part (4) Number of red card are 2 out of 52, therefore

$$P(R') = \frac{2}{52} = \frac{1}{26}$$

(17)

Date: ____________

part (5) Assume that we have 4 kings and after 1st draw king is not replaced so

$P_1 = \frac{4}{52} \rightarrow$ for 1st draw

$P_2 = \frac{3}{51} \longrightarrow$ for 2nd draw

Now

$P = P_1 \cdot P_2$

$= \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$

Assume cards are replaced

$P = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

MATLAB Problem 06:

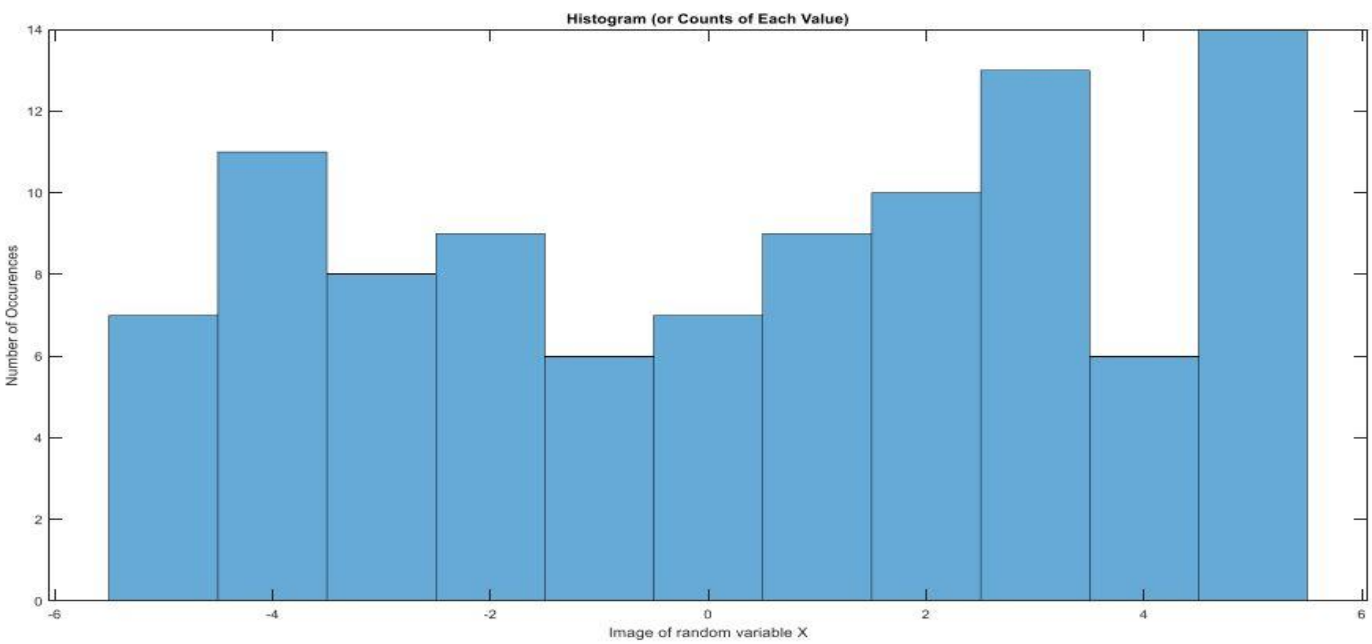
Download the file "random_integers.csv" and read it in MATLAB using command `X = csvread()`. It will load a data of random numbers. Now using your probability techniques find the following probabitlities:

1) Plot the histogram of X.
2) P(X = -5) = ?, P(X = -2) = ?, P(X = 5) = ?
3) What is the probability that X is **EVEN?**
4) What is the probability that X is **ODD?**
5) What is the probability that X have values of -5 or +5**?** **HINT:** use or '|' operator

```
clc
clear
close all
X = csvread('random_integers.csv');
%% 1)   Plot the histogram of X.
histogram(X);
%% 2)   P(X = -5) = ?, P(X = -2) = ?, P(X = 5) = ?
prob_m5 = sum(X == -5)/numel(X);

fprintf('P(X = -5) = %f\n',prob_m5);
prob_m2 = sum(X == -2)/numel(X);
fprintf('P(X = -2) = %f\n',prob_m2);
prob_5 = sum(X == 5)/numel(X);
fprintf('P(X = 5) = %f\n',prob_5);

%% 3)   What is the probability that X is EVEN?
even_X = sum(~rem(X,2));
prob_even = even_X/numel(X);
fprintf('P(X = EVEN) = %f\n',prob_even);

%% 4)   What is the probability that X is ODD?
prob_odd = (numel(X)-even_X)/numel(X);
fprintf('P(X = ODD) = %f\n',prob_odd);

%% 5)   What is the probability that X have values of -5 or +5?
prob_5m5 = sum(X == 5|X == -5)/numel(X);
fprintf('P(X = -5|X = 5) = %f\n',prob_5m5);
```

New to MATLAB? See reso

```
P(X = -5) = 0.070000
P(X = -2) = 0.090000
P(X = 5) = 0.140000
P(X = EVEN) = 0.430000
P(X = ODD) = 0.570000
P(X = -5|X = 5) = 0.210000
fx >>
```



Problem 07:

Suppose the airforce designs a new radar system. If an aircraft is present in the range of the radar system, then the aircraft is detected with probability 0.99. If the aircraft is not present, then the radar reports that an aircraft is present with probability 0.1. Suppose the probability that an aircraft is present is 0.05.

1. What is the probability that the system gives a false alarm, meaning that an aircraft is not presented but is detected?
2. What is the probability that an aircraft is present and detected?
3. What is the probability that an aircraft is present given that the radar reports an aircraft?

P(8)

Date:

Solution : (7)

Let A is the event in which air craft is present.
Let R is the event that radar detects an aircraft.

$P(R/A) = 0.99$ (air craft is present & radar detects)

$P(R/A') = 0.01$ (if radar detects and no aircraft).

$P(A) = 0.05$

probability of false alarm means that

(1) $A' \cap R$

$P(R/A') = \frac{P A' \cap R}{P(A')}$

$P(R/A')\, P(A) = P(A' \cap R)$

So

$A' \cap R = P(R/A')\, P(A')$

$= 0.01 \times (1-0.05)$

$= 0.01 \times 0.95$

$= 0.095$

(2) $A \cap R \longrightarrow$ Aircraft is present and radar detects.

$P(A)$

Date:

$$(A \cap R) = P(R/A) \cdot P(A)$$
$$= 0.99 \times 0.05$$
$$= 0.0495$$

(3) $P(A/R) \rightarrow$ Aircraft is present given that radar reports

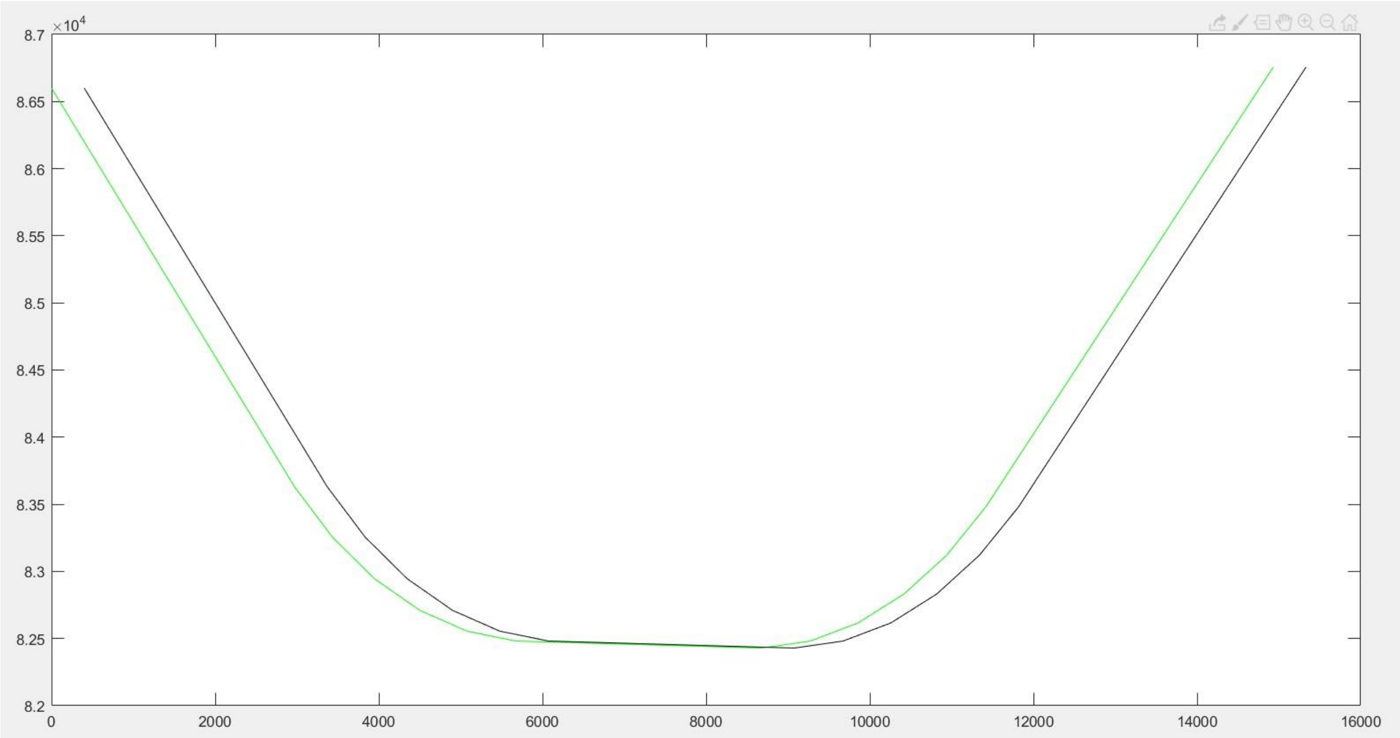
$$P(A/R) = \frac{P(R/A)\,P(A)}{P(R)} = \frac{0.99 \times 0.05}{P(A)}$$

$$P(R) = P(A \cap R) + P(R \cap A')$$

$$= 0.095 + 0.0495$$
$$= 0.1445$$

$$P(A/R) = \frac{0.99 \times 0.05}{0.1445} = 0.34256$$

MATLAB Problem 08:



We consider a ground target tracking scenario where two sensors are located at (−50, 0)km and (50, 0)km, respectively. The sampling interval of all sensors T = 1s. The two targets are initially at (0, 86.6)km and (0.4, 86.6)km, respectively. Both targets move in parallel with a speed of 300m/s. Both targets initially move toward south-east on a course of approximately 135◦ North. Then at t = 15s both targets make a course change with a constant turn rate of 4◦/s and heads toward east. Both targets make a second course change at t = 35s with a constant turn rate of 4◦/s and heads toward north-east. The ground truth of trajectories of the two targets are shown in Fig.

We know noise is the most apparent part of sensors, so radars also give us noisy measurements as given in the file “RadarTrack.xlsx”. first two columns are the x and y coordinates of target 01 and second two columns are the x and y coordinates of target 02 respectively.

Suppose we only have that data from the radars and we are supposed to find that they are actually two targets or just one target with shared noisy blips of other sensor. We make decision on the base of their distances at each interval. If distance is greater than threshold i.e. 30m system will consider it as two different targets and vice versa. Then on the basis of maximum out of all reading it finally make decision about that. Read the “RadarTrack.xlsx” file and:

1. Draw the noisy tracks in Matlab graph.
2. Find that is it actually a single target or two targets?
3. Probabilities of these being single or two targets?
4. Plot histogram and probability graphs.

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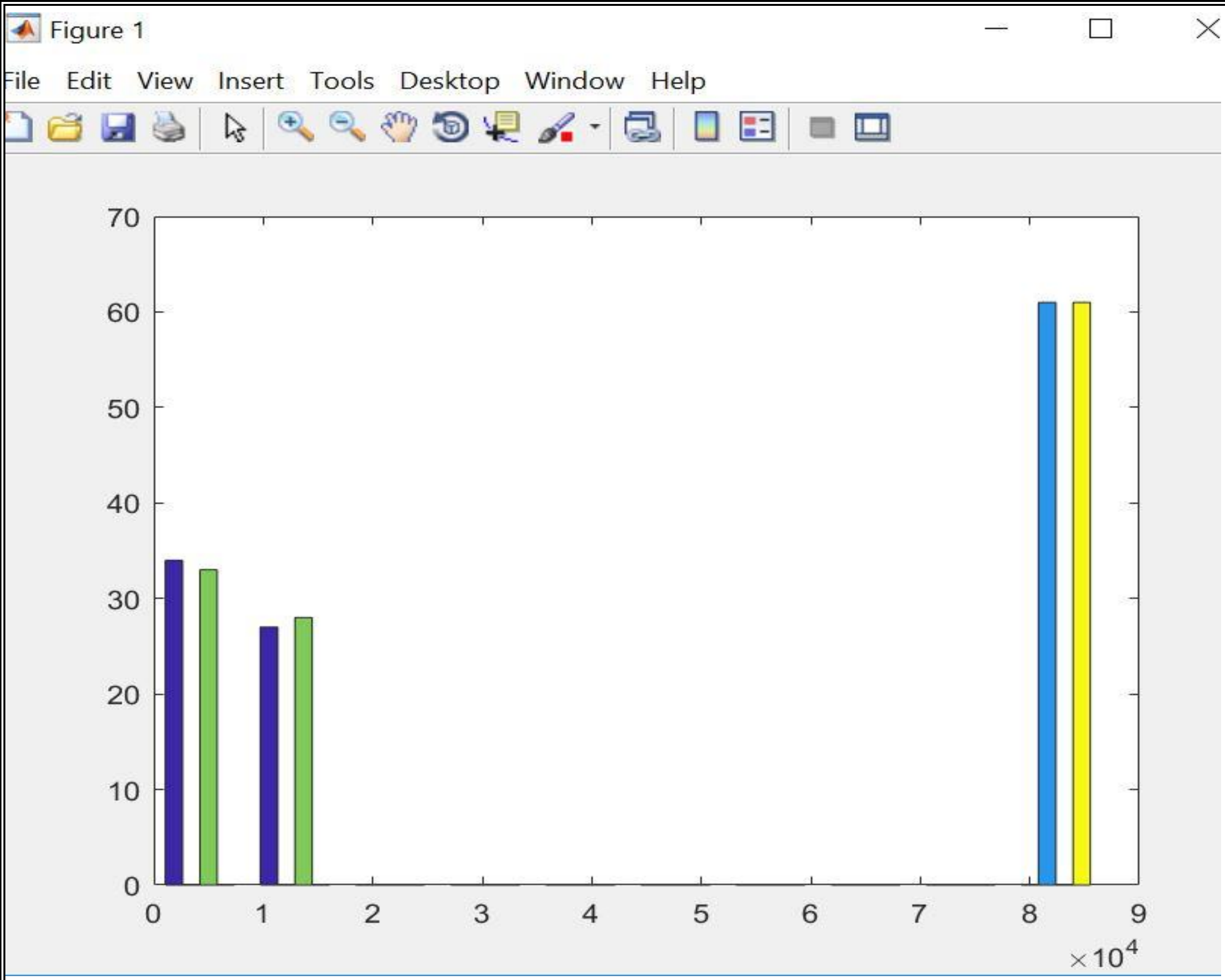
Q6.m | Radar1.m | +

```
close all
[X]=xlsread('RadarTrack.xlsx');
data_x1=X(:,1);
data_y1=X(:,2);
data_x2=X(:,3);
data_y2=X(:,4);


a=0;
hist(X)

j=length(data_y1);
for i=1:j
    if data_y2(i)-data_y1(i)>=30
      a=a+1 ;
    end
    if a>=1

    end
end
 disp(a)
 disp('its is actully two target')
 probability=a/length(X)
```

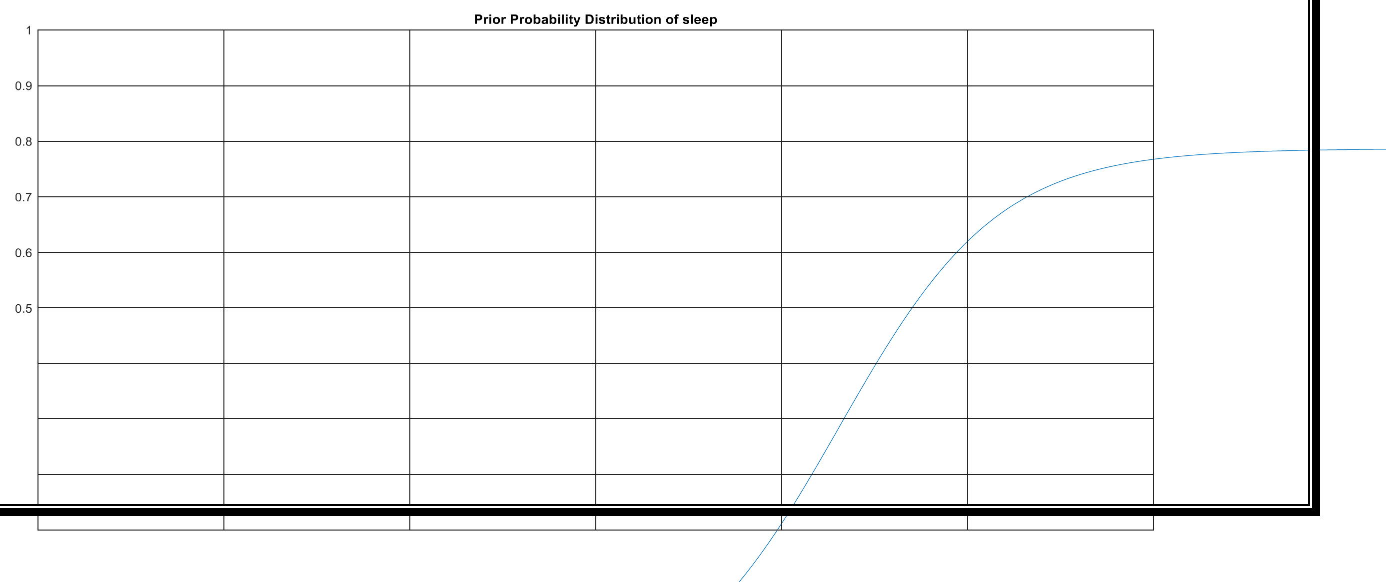


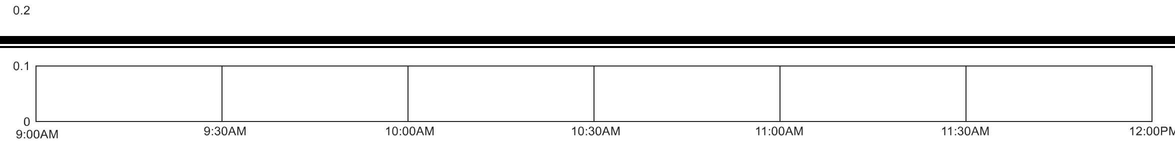
MATLAB Problem 09:

With idea of Bayes' Rule, one problem that we can explore in our life is sleeping patterns. We have data a person showing when he falls asleep at night. We have figured out the probability using Markov Chain Monte Carlo (MCMC) methods and formula:

$$P(sleep) = \frac{1}{1 + e^{-0.0028 \times t + 6.5}}$$

The final model showing the most likely distribution of sleep as a function of time.





Bedroom light ON can change the probability of person's sleep. This is where we use Bayes' Rule to update our estimate. For a specific time, if we know information about bedroom light, we can use the probability from the distribution above as the *prior* and then apply Bayes' equation:

$$P(sleep|light) = \frac{P(light|sleep) \times P(sleep)}{P(light)}$$

Where $light$ indicates light ON

Based on person's habits, we know that he cannot sleep easily with lights ON and the probability of bedroom light is ON given that he is asleep is only about 1%. That is:

$$(light|sleep) = 0.01$$

a) Find the probability $(-light|sleep) = ?$

(Where $-light$ indicates light OFF)

b) Find the probability that $light$ is ON at 09:30 PM i.e. $(light)$.

c) What is the probability that person fall asleep with $light$ ON at 10:00 PM?

d) Suppose person fails to fall asleep at 11:00 PM, what is the probability that $light$ is ON?

NOTE: (solve ***a*** to ***d*** parts on paper and take estimated probabilities from graph at required times, but for **MATLAB** use formula to get probabilities)

e) You are given with a **MATLAB** script file "Sleep_bayes.m" that plots the above graph using formula. Use that script file, find and plot the following probabilities from 09:00 PM to 12:00PM.

   a. $(sleep|light) = ?$

   b. $(sleep| - light) =?$

   Plot both probabilities on same given figure of prior sleep probability. Give *legends*, *title* and use different colors for each plot line.

Solution:

Solution : (9)

a) $P(-\text{light}/\text{Sleep}) = 1 - P(\text{light}/\text{sleep})$

$= 1 - 0.01$

$= 0.99$

b) $P(\text{light}) = P(\text{light}/\text{sleep}) \times P(\text{sleep}) + P(\text{light}/-\text{sleep}) \times P(-\text{sleep})$

Est. $P(\text{sleep})$ at 09:30 PM $= 0.02$

$P(-\text{sleep}) = 1 - P(\text{sleep})$

$= 1 - 0.02$

$= 0.98$

$P(\text{light}) = 0.01 \times 0.02 + 0.80 \times 0.98 = 0.7842$

c) $P(\text{Sleep}/\text{light}) = \dfrac{P(\text{light}/\text{sleep}) \times P(\text{sleep})}{P(\text{light})}$

Est. $P(\text{Sleep})$ at 10:00 PM $= 0.14$

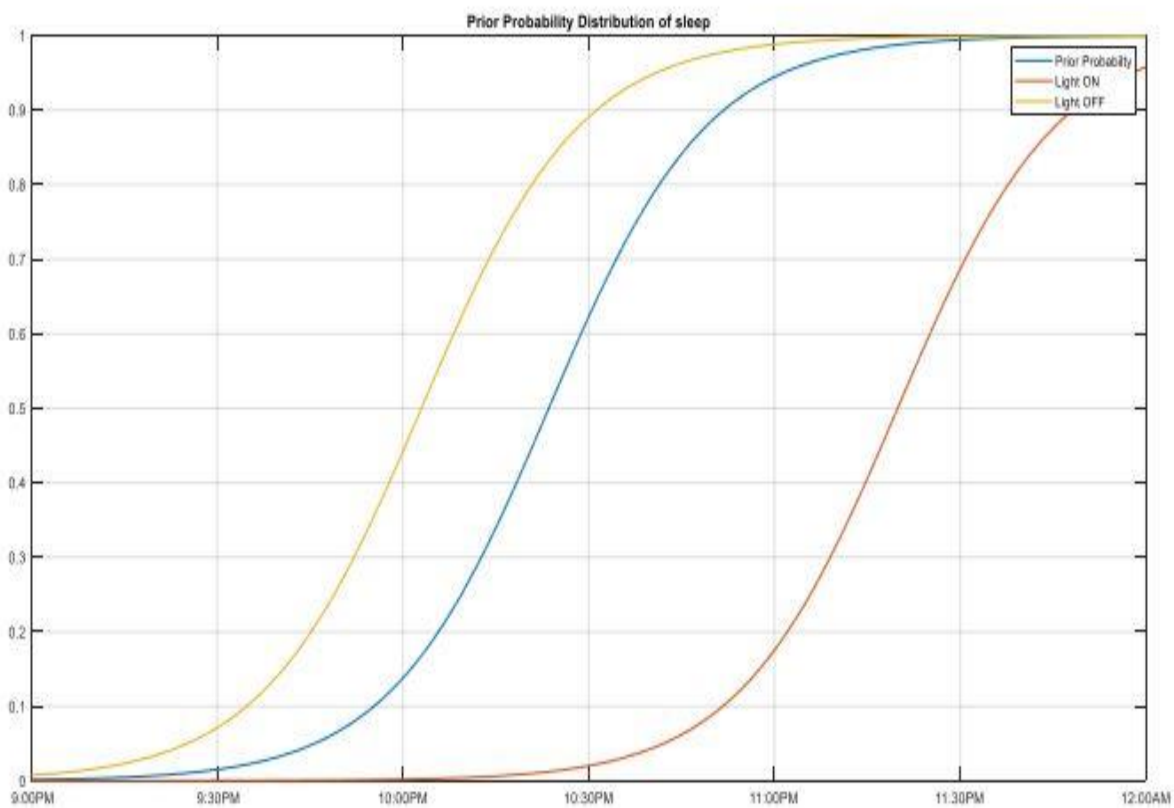
$P(\text{light}) = P(\text{light}/\text{Sleep}) \times P(\text{sleep}) + P(\text{light}/-\text{sleep}) \times P(-\text{sleep})$

$P(\text{light}) = 0.01 \times 0.14 + 0.80 \times 0.86 = 0.6894$

$P(\text{sleep}/\text{light}) = \dfrac{0.01 \times 0.14}{0.6894} = 0.0020$

d) $P(\text{light}/-\text{sleep})$ at any time $= 80\%$ (Given in Question)

```
clc
clear
close all

% This program demonstrates probabilistic sleeping pattern
% of a person at night w.r.t time.
%
% Author: Rafi Ul Zaman
% Created: Sep-15-2018

%% DO NOT edit or disturb this portion only use required variables
below.
t = 1:5000;
prob_sleep_time = 1./(1+exp(-0.0028.*t+6.5));
n = length(t);
sdate = linspace(datenum('01-Aug-2013 21:00:00'),datenum('01-Aug-
2013 24:00:00'),n);
%% You cant edit or write your code line below here
prob_lit_slp = 0.01;
prob_nlit_slp = 1 - prob_lit_slp;

prob_lit_nslp = 0.8;
prob_nlit_nslp = 1 - prob_lit_nslp;

for i = 1:n
    prob_lit = prob_lit_slp*prob_sleep_time(i) + prob_lit_nslp*(1-
prob_sleep_time(i));
    prob_slp_lit(i) = prob_lit_slp*prob_sleep_time(i)/prob_lit;

    prob_nlit = prob_nlit_slp*prob_sleep_time(i) +
prob_nlit_nslp*(1-prob_sleep_time(i));
    prob_slp_nlit(i) = prob_nlit_slp*prob_sleep_time(i)/prob_nlit;
end

plot(sdate,prob_sleep_time);
hold on
plot(sdate,prob_slp_lit);
plot(sdate,prob_slp_nlit);
legend('Prior Probabilty', ...
    'Light ON', ...
    'Light OFF');
datetick('x','HH:MMPM');
title('Prior Probability Distribution of sleep');
grid on;
```



Applying Bayes' Rule to real life problems is a fun so why stop with only bedroom light? We can use more information in the model as we like and it will continue to get more precise results. For example, if I know the likelihood that person's mobile phone is charging given that he asleep is 95%, we can incorporate that knowledge into the model.

> ***NOTE***: probability that phone is charging is conditionally independent of the probability that light is ON.

Bayes' equation using the extra information is expressed:

$$P(sleep|light, mobile) = \frac{P(light|sleep) \times P(mobile|sleep) \times P(sleep)}{P(light, mobile)}$$

a) Find the probability $(-mobile|sleep) = ?$

(Where $mobile$ indicates Mobile charging)

b) Find the probability that $light$ is ON and $mobile$ is charging at night i.e. $P(light, mobile)$.

c) What is the probability that person fall asleep with $light$ ON and $mobile$ **NOT** charging at 10:00 PM?

NOTE: (solve ***a*** to ***c*** parts on paper and take estimated probabilities from graph at required times, but for **MATLAB** use formula to get probabilities)

d) Use same **MATLAB** script file, find and plot the following probabilities from 09:00 PM to 12:00PM.
   a. $(sleep|light, mobile) = ?$
   b. $(sleep| - light, mobile) = ?$
   c. $(sleep|light, -mobile) = ?$
   d. $(sleep| - light, -mobile) = ?$

(Where $-mobile$ indicates Mobile **NOT** charging)

Plot all probabilities on same given figure i.e. prior sleep probability. Give *legends*, *title* and use different colors for each plot line.

P(11)

Solution :

a) $P(-mobile|sleep) = 1 - P(mobile|sleep)$
$= 1 - 0.95$
$= 0.05$

b) $P(light, mobile) = P(light|sleep) \times P(mobile|sleep) \times P(sleep) + P(light|-sleep) \times P(mobile|-sleep) \times P(-sleep)$

Est. $P(sleep)$ at 9:30 PM $= 0.02$
$P(-sleep) = 1 - P(sleep)$
$= 1 - 0.02$
$= 0.98$

$P(light, mobile) = 0.01 \times 0.95 \times 0.02 + 0.80 \times 0.5 \times 0.98 = 0.3922$

c) $P(sleep|light, -mobile) = \dfrac{P(light|sleep) \times P(-mobile|sleep) \times p(sleep)}{p(light, -mobile)}$

Est. $P(sleep)$ at 10:00 PM $= 0.14$

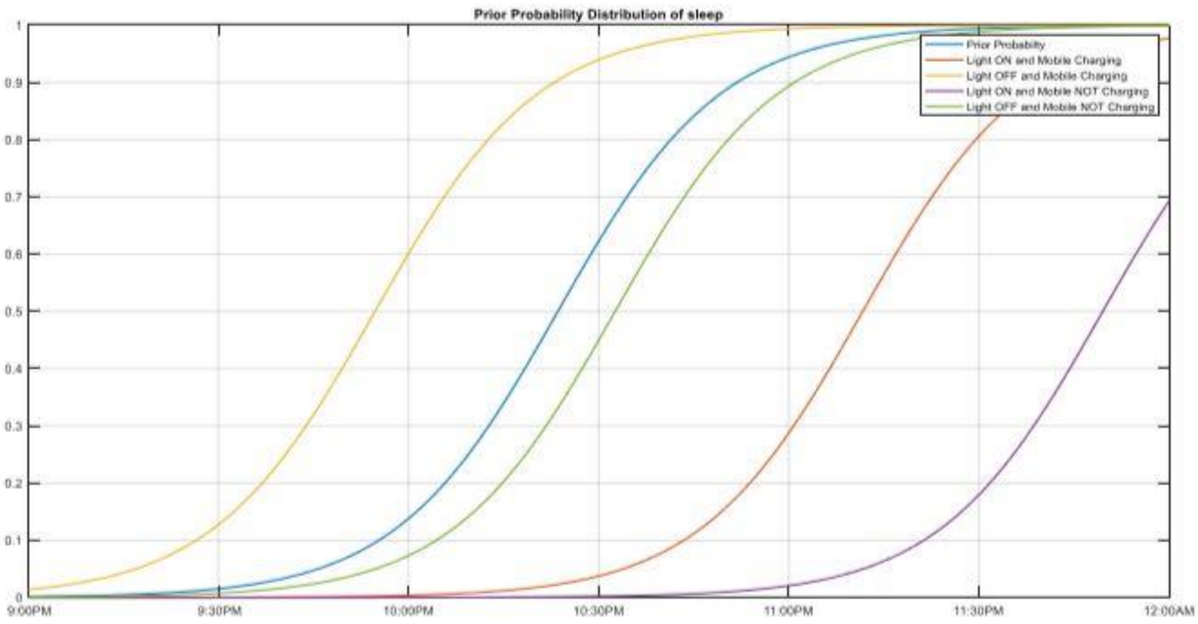
$P(light, -mobile) = P(light|sleep) \times P(-mobile|sleep) \times P(sleep) + P(light|-sleep) \times P(-mobile|-sleep) \times P(-sleep)$
$= 0.01 \times 0.05 \times 0.14 + 0.80 \times 0.5 \times 0.86 = 0.3441$

$P(sleep|light, -mobile) = \dfrac{0.01 \times 0.05 \times 0.14}{0.3441} = 0.0002$

d)

```
clc
clear
close all

% This program demonstrates probabilistic sleeping pattern
% of a person at night w.r.t time.
%
% Author: Rafi Ul Zaman
% Created: Sep-15-2018

%% DO NOT edit or disturb this portion only use required variables
below.
t = 1:5000;
prob_sleep_time = 1./(1+exp(-0.0028.*t+6.5));
n = length(t);
sdate = linspace(datenum('01-Aug-2013 21:00:00'),datenum('01-Aug-
2013 24:00:00'),n);
%% You cant edit or write your code line below here
prob_lit_slp = 0.01;
prob_nlit_slp = 1 - prob_lit_slp;
prob_lit_nslp = 0.8;
prob_nlit_nslp = 1 - prob_lit_nslp;
prob_mob_slp = 0.95;
prob_nmob_slp = 1 - prob_mob_slp;
prob_mob_nslp = 0.5;
prob_nmob_nslp = 1 - prob_mob_nslp;
for i = 1:n
    prob_lit_mob = prob_lit_slp*prob_mob_slp*prob_sleep_time(i) +
prob_lit_nslp*prob_mob_nslp*(1-prob_sleep_time(i));
    prob_slp_lit_mob(i) =
prob_lit_slp*prob_mob_slp*prob_sleep_time(i)/prob_lit_mob;

    prob_nlit_mob = prob_nlit_slp*prob_mob_slp*prob_sleep_time(i) +
prob_nlit_nslp*prob_mob_nslp*(1-prob_sleep_time(i));
    prob_slp_nlit_mob(i) =
prob_nlit_slp*prob_mob_slp*prob_sleep_time(i)/prob_nlit_mob;

    prob_lit_nmob = prob_lit_slp*prob_nmob_slp*prob_sleep_time(i) +
prob_lit_nslp*prob_nmob_nslp*(1-prob_sleep_time(i));
    prob_slp_lit_nmob(i) =
prob_lit_slp*prob_nmob_slp*prob_sleep_time(i)/prob_lit_nmob;

    prob_nlit_nmob = prob_nlit_slp*prob_nmob_slp*prob_sleep_time(i)
+ prob_nlit_nslp*prob_nmob_nslp*(1-prob_sleep_time(i));
    prob_slp_nlit_nmob(i) =
prob_nlit_slp*prob_nmob_slp*prob_sleep_time(i)/prob_nlit_nmob;
```

```
end

plot(sdate,prob_sleep_time);
hold on;
plot(sdate,prob_slp_lit_mob);
plot(sdate,prob_slp_nlit_mob);
plot(sdate,prob_slp_lit_nmob);
plot(sdate,prob_slp_nlit_nmob);
legend('Prior Probabilty', ...
    'Light ON and Mobile Charging', ...
    'Light OFF and Mobile Charging', ...
    'Light ON and Mobile NOT Charging', ...
    'Light OFF and Mobile NOT Charging');
datetick('x','HH:MMPM');
title('Prior Probability Distribution of sleep');
grid on;
```



End