

18

ELECTRO-OPTICS

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*18.4 PHOTOREFRACTIVE MATERIALS



Friedrich Pockels (1865–1913) was first to describe the linear electro-optic effect in 1893.



John Kerr (1824–1907) discovered the quadratic electro-optic effect in 1875.

Certain materials change their optical properties when subjected to an electric field. This is caused by forces that distort the positions, orientations, or shapes of the molecules constituting the material. The **electro-optic effect** is the change in the refractive index resulting from the application of a dc or low-frequency electric field (Fig. 18.0-1). A field applied to an *anisotropic* electro-optic material modifies its refractive indices and thereby its effect on polarized light.

The dependence of the refractive index on the applied electric field takes one of two forms:

- The refractive index changes in proportion to the applied electric field, in which case the effect is known as the **linear electro-optic effect** or the **Pockels effect**.
- The refractive index changes in proportion to the square of the applied electric field, in which case the effect is known as the **quadratic electro-optic effect** or the **Kerr effect**.

The change in the refractive index is typically very small. Nevertheless, its effect on an optical wave propagating a distance much greater than a wavelength of light in the medium can be significant. If the refractive index increases by 10^{-5} , for example, an optical wave propagating a distance of 10^5 wavelengths will experience an additional phase shift of 2π .

Materials whose refractive index can be modified by means of an applied electric field are useful for producing electrically controllable optical devices, as indicated by the following examples:

- A lens made of a material whose refractive index can be varied is a lens of controllable focal length.
- A prism whose beam bending ability is controllable can be used as an optical scanning device.
- Light transmitted through a transparent plate of controllable refractive index undergoes a controllable phase shift. The plate can be used as an optical phase modulator.

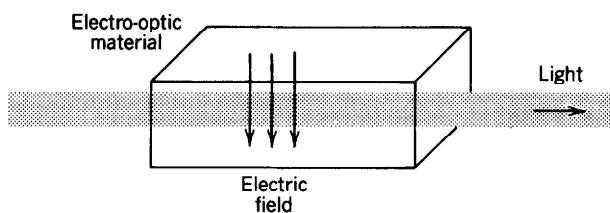


Figure 18.0-1 A steady electric field applied to an electro-optic material changes its refractive index. This, in turn, changes the effect of the material on light traveling through it. The electric field therefore controls the light.

- An anisotropic crystal whose refractive indices can be varied serves as a wave retarder of controllable retardation; it may be used to change the polarization properties of light.
- A wave retarder placed between two crossed polarizers gives rise to transmitted light whose intensity is dependent on the phase retardation (see Sec. 6.6B). The transmittance of the device is therefore electrically controllable, so that it can be used as an optical intensity modulator or an optical switch.

These are useful components for optical communication and optical signal-processing applications.

We begin with a simple description of the electro-optic effect and the principles of electro-optic modulation and scanning (Sec. 18.1). The initial presentation is simplified by deferring the detailed consideration of anisotropic effects to Sec. 18.2.

Section 18.3 is devoted to the electro-optic properties of liquid crystals. An electric field applied to the molecules of a liquid crystal causes them to alter their orientations. This leads to changes in the optical properties of the medium, i.e., it exhibits an electro-optic effect. The molecules of a twisted nematic liquid crystal are organized in a helical pattern so that they normally act as polarization rotators. An applied electric field removes the helical pattern, thereby deactivating the polarization rotatory power of the material. Removal of the electric field results in the material regaining its helical structure and therefore its rotatory power. The device therefore acts as a dynamic polarization rotator. The use of additional fixed polarizers permits such a polarization rotator to serve as an intensity modulator or a switch. This behavior is the basis of most liquid-crystal display devices.

The electro-optic properties of photorefractive media are considered in Sec. 18.4. These are materials in which the absorption of light creates an internal electric field which, in turn, initiates an electro-optic effect that alters the optical properties of the medium. Thus the optical properties of the medium are indirectly controlled by the light incident on it. Photorefractive devices therefore permit *light to control light*.

18.1 PRINCIPLES OF ELECTRO-OPTICS

A. Pockels and Kerr Effects

The refractive index of an electro-optic medium is a function $n(E)$ of the applied electric field E . This function varies only slightly with E so that it can be expanded in a Taylor's series about $E = 0$,

$$n(E) = n + a_1 E + \frac{1}{2} a_2 E^2 + \dots, \quad (18.1-1)$$

where the coefficients of expansion are $n = n(0)$, $a_1 = (dn/dE)|_{E=0}$, and $a_2 = (d^2n/dE^2)|_{E=0}$. For reasons that will become apparent subsequently, it is conventional to write (18.1-1) in terms of two new coefficients $r = -2a_1/n^3$ and $s = -a_2/n^3$, known as the electro-optic coefficients, so that

$$n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} s n^3 E^2 + \dots. \quad (18.1-2)$$

The second- and higher-order terms of this series are typically many orders of magnitude smaller than n . Terms higher than the third can safely be neglected.

For future use it is convenient to derive an expression for the electric impermeability, $\eta = \epsilon_o/\epsilon = 1/n^2$, of the electro-optic medium as a function of E . The parameter η is useful in describing the optical properties of anisotropic media (see Sec. 6.3A). The incremental change $\Delta\eta = (d\eta/dn)\Delta n = (-2/n^3)(-\frac{1}{2}rn^3E - \frac{1}{2}sn^3E^2) = rE +$

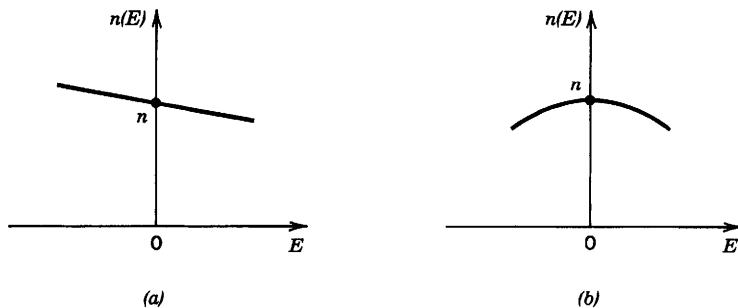


Figure 18.1-1 Dependence of the refractive index on the electric field: (a) Pockels medium; (b) Kerr medium.

$\propto E^2$, so that

$$\eta(E) \approx \eta + rE + sE^2, \quad (18.1-3)$$

where $\eta = \eta(0)$. The electro-optic coefficients r and s are therefore simply the coefficients of proportionality of the two terms of $\Delta\eta$ with E and E^2 , respectively. This explains the seemingly odd definitions of r and s in (18.1-2).

The values of the coefficients r and s depend on the direction of the applied electric field and the polarization of the light, as will be discussed in Sec. 18.2.

Pockels Effect

In many materials the third term of (18.1-2) is negligible in comparison with the second, whereupon

$$n(E) \approx n - \frac{1}{2}r n^3 E, \quad (18.1-4)$$

Pockels Effect

as illustrated in Fig. 18.1-1(a). The medium is then known as a Pockels medium (or a Pockels cell). The coefficient r is called the **Pockels coefficient** or the linear electro-optic coefficient. Typical values of r lie in the range 10^{-12} to 10^{-10} m/V (1 to 100 pm/V). For $E = 10^6$ V/m (10 kV applied across a cell of thickness 1 cm), for example, the term $\frac{1}{2}r n^3 E$ in (18.1-4) is on the order of 10^{-6} to 10^{-4} . Changes in the refractive index induced by electric fields are indeed very small. The most common crystals used as Pockels cells include $\text{NH}_4\text{H}_2\text{PO}_4$ (ADP), KH_2PO_4 (KDP), LiNbO_3 , LiTaO_3 , and CdTe .

Kerr Effect

If the material is centrosymmetric, as is the case for gases, liquids, and certain crystals, $n(E)$ must be an even symmetric function [see Fig. 18.1-1(b)] since it must be invariant to the reversal of E . Its first derivative then vanishes, so that the coefficient r must be zero, whereupon

$$n(E) \approx n - \frac{1}{2}s n^3 E^2. \quad (18.1-5)$$

Kerr Effect

The material is then known as a Kerr medium (or a Kerr cell). The parameter s is

called the **Kerr coefficient** or the quadratic electro-optic coefficient. Typical values of s are 10^{-18} to $10^{-14} \text{ m}^2/\text{V}^2$ in crystals and 10^{-22} to $10^{-19} \text{ m}^2/\text{V}^2$ in liquids. For $E = 10^6 \text{ V/m}$ the term $\frac{1}{2}s n^3 E^2$ in (18.1-5) is on the order of 10^{-6} to 10^{-2} in crystals and 10^{-10} to 10^{-7} in liquids.

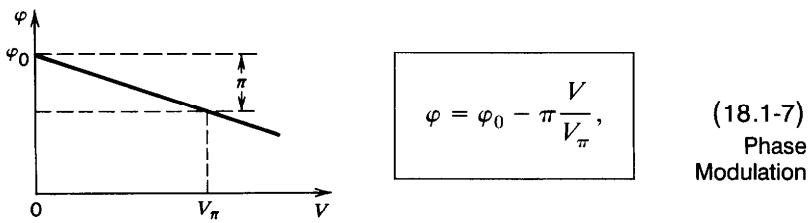
B. Electro-Optic Modulators and Switches

Phase Modulators

When a beam of light traverses a Pockels cell of length L to which an electric field E is applied, it undergoes a phase shift $\varphi = n(E)k_o L = 2\pi n(E)L/\lambda_o$, where λ_o is the free-space wavelength. Using (18.1-4), we have

$$\varphi \approx \varphi_0 - \pi \frac{r n^3 E L}{\lambda_o}, \quad (18.1-6)$$

where $\varphi_0 = 2\pi nL/\lambda_o$. If the electric field is obtained by applying a voltage V across two faces of the cell separated by distance d , then $E = V/d$, and (18.1-6) gives



where

$$V_\pi = \frac{d}{L} \frac{\lambda_o}{r n^3}. \quad (18.1-8)$$

Half-Wave Voltage

The parameter V_π , known as the **half-wave voltage**, is the applied voltage at which the phase shift changes by π . Equation (18.1-7) expresses a linear relation between the optical phase shift and the voltage. One can therefore modulate the phase of an optical wave by varying the voltage V that is applied across a material through which the light passes. The parameter V_π is an important characteristic of the modulator. It depends on the material properties (n and r), on the wavelength λ_o , and on the aspect ratio d/L .

The electric field may be applied in a direction perpendicular to the direction of light propagation (transverse modulators) or parallel to that direction (longitudinal modulators), in which case $d = L$ (Fig. 18.1-2). The value of the electro-optic coefficient r depends on the directions of propagation and the applied field since the crystal is anisotropic (as explained in Sec. 18.2). Typical values of the half-wave voltage are in the vicinity of 1 to a few kilovolts for longitudinal modulators, and hundreds of volts for transverse modulators.

The speed at which an electro-optic modulator operates is limited by electrical capacitive effects and by the transit time of the light through the material. If the

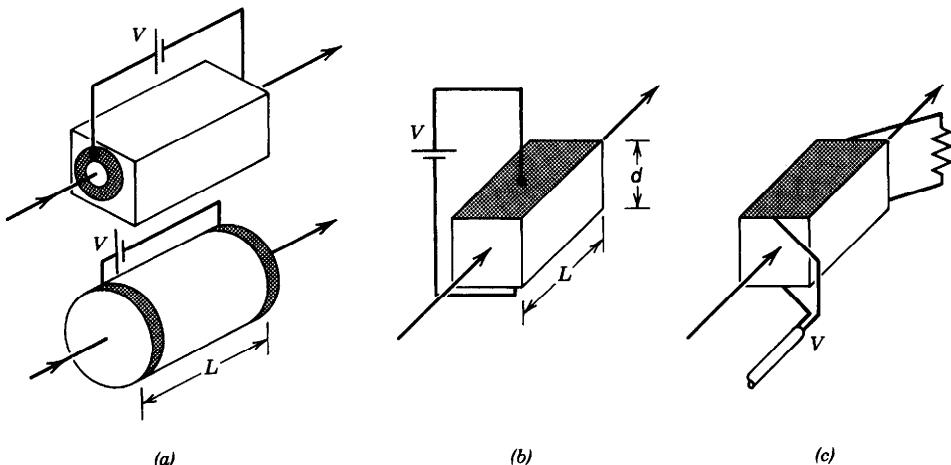


Figure 18.1-2 (a) Longitudinal modulator. The electrodes may take the shape of washers or bands, or may be transparent conductors. (b) Transverse modulator. (c) Traveling-wave transverse modulator.

electric field $E(t)$ varies significantly within the light transit time T , the traveling optical wave will be subjected to different electric fields as it traverses the crystal. The modulated phase at a given time t will then be proportional to the average electric field $E(t)$ at times from $t - T$ to t . As a result, the transit-time-limited modulation bandwidth is $\approx 1/T$. One method of reducing this time is to apply the voltage V at one end of the crystal while the electrodes serve as a transmission line, as illustrated in Fig. 18.1-2(c). If the velocity of the traveling electrical wave matches that of the optical wave, transit time effects can, in principle, be eliminated. Commercial modulators in the forms shown in Fig. 18.1-2 generally operate at several hundred MHz, but modulation speeds of several GHz are possible.

Electro-optic modulators can also be constructed as integrated-optical devices. These devices operate at higher speeds and lower voltages than do bulk devices. An optical waveguide is fabricated in an electro-optic substrate (often LiNbO_3) by indiffusing a material such as titanium to increase the refractive index. The electric field is applied to the waveguide using electrodes, as shown in Fig. 18.1-3. Because the configuration is transverse and the width of the waveguide is much smaller than its length ($d \ll L$), the half-wave voltage can be as small as a few volts. These modulators have been operated at speeds in excess of 100 GHz. Light can be conveniently coupled into, and out of, the modulator by the use of optical fibers.

Dynamic Wave Retarders

An anisotropic medium has two linearly polarized normal modes that propagate with different velocities, say c_o/n_1 and c_o/n_2 (see Sec. 6.3B). If the medium exhibits the Pockels effect, then in the presence of a steady electric field E the two refractive indices are modified in accordance with (18.1-4), i.e.,

$$n_1(E) \approx n_1 - \frac{1}{2}r_1 n_1^3 E$$

$$n_2(E) \approx n_2 - \frac{1}{2}r_2 n_2^3 E,$$

where r_1 and r_2 are the appropriate Pockels coefficients (anisotropic effects are examined in detail in Sec. 18.2). After propagation a distance L , the two modes

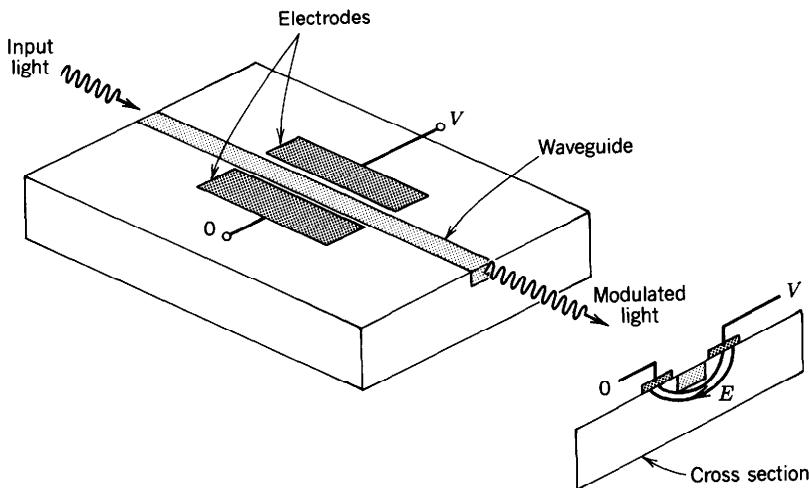
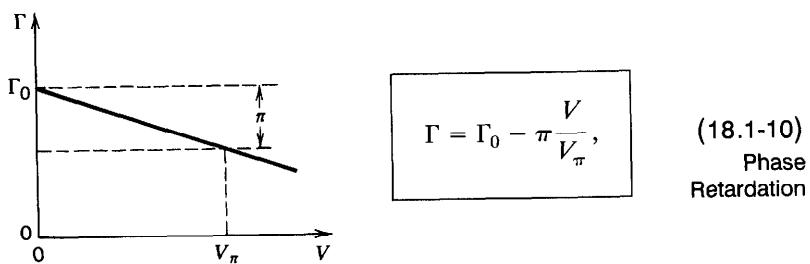


Figure 18.1-3 An integrated-optical phase modulator using the electro-optic effect.

undergo a phase retardation (with respect to each other) given by

$$\Gamma = k_o[n_1(E) - n_2(E)]L = k_o(n_1 - n_2)L - \frac{1}{2}k_o(r_1n_1^3 - r_2n_2^3)EL. \quad (18.1-9)$$

If E is obtained by applying a voltage V between two surfaces of the medium separated by a distance d , (18.1-9) can be written in the compact form



where $\Gamma_0 = k_o(n_1 - n_2)L$ is the phase retardation in the absence of the electric field and

$$V_\pi = \frac{d}{L} \frac{\lambda_o}{r_1n_1^3 - r_2n_2^3} \quad (18.1-11)$$

Retardation Half-Wave Voltage

is the applied voltage necessary to obtain a phase retardation π . Equation (18.1-10) indicates that the phase retardation is linearly related to the applied voltage. The medium serves as an electrically controllable dynamic wave retarder.

Intensity Modulators: Use of a Phase Modulator in an Interferometer

Phase delay (or retardation) alone does not affect the intensity of a light beam. However, a phase modulator placed in one branch of an interferometer can function as an intensity modulator. Consider, for example, the Mach-Zehnder interferometer

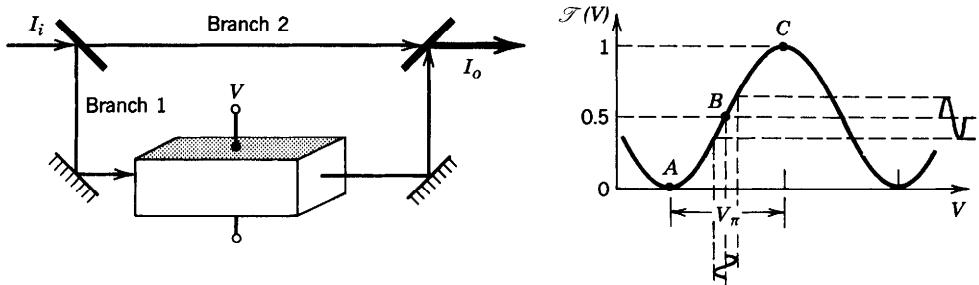


Figure 18.1-4 A phase modulator placed in one branch of a Mach-Zehnder interferometer can serve as an intensity modulator. The transmittance of the interferometer $\mathcal{T}(V) = I_o/I_i$ varies periodically with the applied voltage V . By operating in a limited region near point B , the device acts as a linear intensity modulator. If V is switched between points A and C , the device serves as an optical switch.

illustrated in Fig. 18.1-4. If the beamsplitters divide the optical power equally, the transmitted intensity I_o is related to the incident intensity I_i by

$$I_o = \frac{1}{2}I_i + \frac{1}{2}I_i \cos \varphi = I_i \cos^2 \frac{\varphi}{2}$$

where $\varphi = \varphi_1 - \varphi_2$ is the difference between the phase shifts encountered by light as it travels through the two branches (see Sec. 2.5A). The transmittance of the interferometer is $\mathcal{T} = I_o/I_i = \cos^2(\varphi/2)$.

Because of the presence of the phase modulator in branch 1, according to (18.1-7) we have $\varphi_1 = \varphi_{10} - \pi V/V_\pi$, so that φ is controlled by the applied voltage V in accordance with the linear relation $\varphi = \varphi_1 - \varphi_2 = \varphi_0 - \pi V/V_\pi$, where the constant $\varphi_0 = \varphi_{10} - \varphi_2$ depends on the optical path difference. The transmittance of the device is therefore a function of the applied voltage V ,

$$\mathcal{T}(V) = \cos^2\left(\frac{\varphi_0}{2} - \frac{\pi}{2} \frac{V}{V_\pi}\right). \quad (18.1-12)$$

Transmittance

This function is plotted in Fig. 18.1-4 for an arbitrary value of φ_0 . The device may be operated as a linear intensity modulator by adjusting the optical path difference so that $\varphi_0 = \pi/2$ and operating in the nearly linear region around $\mathcal{T} = 0.5$. Alternatively, the optical path difference may be adjusted so that φ_0 is a multiple of 2π . In this case $\mathcal{T}(0) = 1$ and $\mathcal{T}(V_\pi) = 0$, so that the modulator switches the light on and off as V is switched between 0 and V_π .

A Mach-Zehnder intensity modulator may also be constructed in the form of an integrated-optical device. Waveguides are placed on a substrate in the geometry shown in Fig. 18.1-5. The beamsplitters are implemented by the use of waveguide Y's. The optical input and output may be carried by optical fibers. Commercially available integrated-optical modulators generally operate at speeds of a few GHz but modulation speeds exceeding 25 GHz have been achieved.

Intensity Modulators: Use of a Retarder Between Crossed Polarizers

As described in Sec. 6.6B, a wave retarder (retardation Γ) sandwiched between two crossed polarizers, placed at 45° with respect to the retarder's axes (see Fig. 6.6-4), has

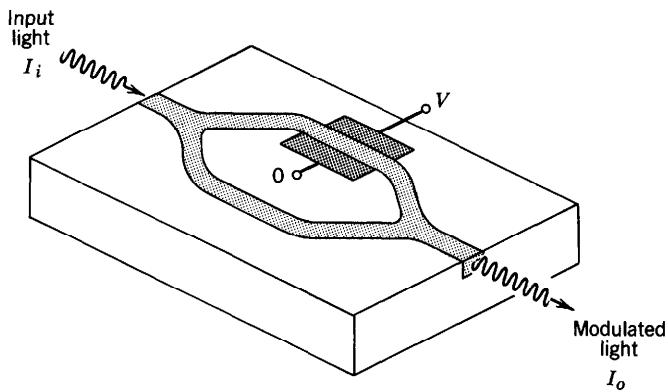


Figure 18.1-5 An integrated-optical intensity modulator (or optical switch). A Mach-Zehnder interferometer and an electro-optic phase modulator are implemented using optical waveguides fabricated from a material such as LiNbO_3 .

an intensity transmittance $\mathcal{T} = \sin^2(\Gamma/2)$. If the retarder is a Pockels cell, then Γ is linearly dependent on the applied voltage V as provided in (18.1-10). The transmittance of the device is then a periodic function of V ,

$$\mathcal{T}(V) = \sin^2\left(\frac{\Gamma_0}{2} - \frac{\pi}{2} \frac{V}{V_\pi}\right), \quad (18.1-13)$$

Transmittance

as shown in Fig. 18.1-6. By changing V , the transmittance can be varied between 0 (shutter closed) and 1 (shutter open). The device can also be used as a linear modulator if the system is operated in the region near $\mathcal{T}(V) = 0.5$. By selecting

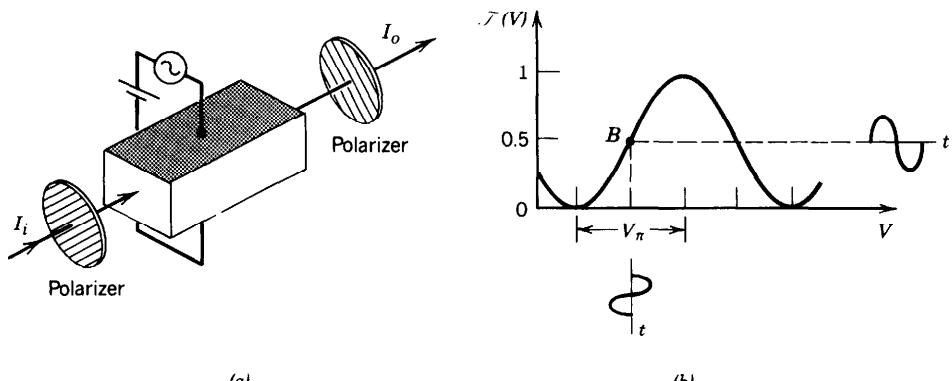


Figure 18.1-6 (a) An optical intensity modulator using a Pockels cell placed between two crossed polarizers. (b) Optical transmittance versus applied voltage for an arbitrary value of Γ_0 ; for linear operation the cell is biased near the point B .

$\Gamma_0 = \pi/2$ and $V \ll V_\pi$,

$$\begin{aligned}\mathcal{T}(V) &= \sin^2\left(\frac{\pi}{4} - \frac{\pi}{2} \frac{V}{V_\pi}\right) \\ &\approx \mathcal{T}(0) + \frac{d\mathcal{T}}{dV} \Big|_{V=0} V = \frac{1}{2} - \frac{\pi}{2} \frac{V}{V_\pi},\end{aligned}\quad (18.1-14)$$

so that $\mathcal{T}(V)$ is a linear function with slope $\pi/2V_\pi$ representing the sensitivity of the modulator. The phase retardation Γ_0 can be adjusted either optically (by assisting the modulator with an additional phase retarder, a compensator) or electrically by adding a constant bias voltage to V .

In practice, the maximum transmittance of the modulator is smaller than unity because of losses caused by reflection, absorption, and scattering. Furthermore, the minimum transmittance is greater than 0 because of misalignments of the direction of propagation and the directions of polarizations relative to the crystal axes and the polarizers. The ratio between the maximum and minimum transmittances is called the extinction ratio. Ratios higher than 1000:1 are possible.

C. Scanners

An optical beam can be deflected dynamically by using a prism with an electrically controlled refractive index. The angle of deflection introduced by a prism of small apex angle α and refractive index n is $\theta \approx (n - 1)\alpha$ [see (1.2-7)]. An incremental change of the refractive index Δn caused by an applied electric field E corresponds to an incremental change of the deflection angle,

$$\Delta\theta = \alpha \Delta n = -\frac{1}{2}\alpha \nabla n^3 E = -\frac{1}{2}\alpha \nabla n^3 \frac{V}{d}, \quad (18.1-15)$$

where V is the applied voltage and d is the prism width [Fig. 18.1-7(a)]. By varying the applied voltage V , the angle $\Delta\theta$ varies proportionally, so that the incident light is scanned.

It is often more convenient to place triangularly shaped electrodes defining a prism on a rectangular crystal. Two, or several, prisms can be cascaded by alternating the direction of the electric field, as illustrated in Fig. 18.1-7(b).

An important parameter that characterizes a scanner is its resolution, i.e., the number of independent spots it can scan. An optical beam of width D and wavelength λ_o has an angular divergence $\delta\theta \approx \lambda_o/D$ [see (4.3-6)]. To minimize that angle, the beam should be as wide as possible, ideally covering the entire width of the prism itself. For a given maximum voltage V corresponding to a scanned angle $\Delta\theta$, the number of independent spots is given by

$$N \approx \frac{|\Delta\theta|}{\delta\theta} = \frac{\frac{1}{2}\alpha \nabla n^3 V/d}{(\lambda_o/D)}. \quad (18.1-16)$$

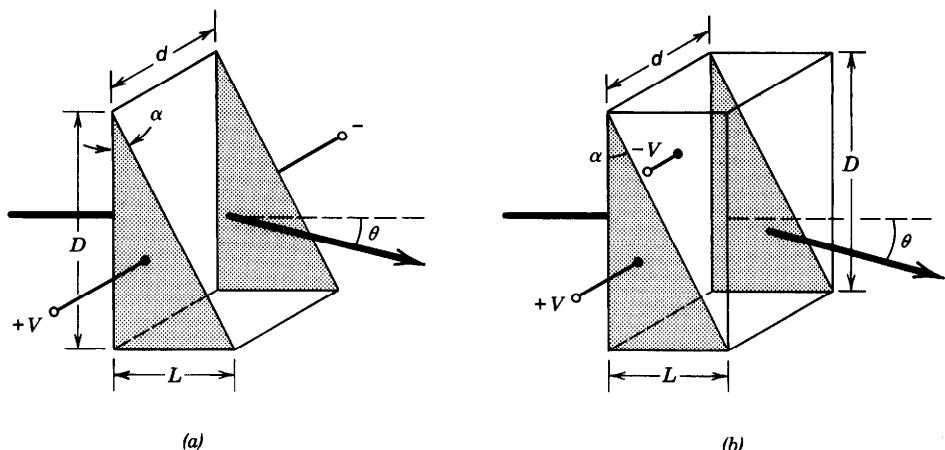


Figure 18.1-7 (a) An electro-optic prism. The deflection angle θ is controlled by the applied voltage. (b) An electro-optic double prism.

Substituting $\alpha \approx L/D$ and $V_\pi = (d/L)(\lambda_o/rn^3)$, we obtain

$$N \approx \frac{V}{2V_\pi}, \quad (18.1-17)$$

from which $V \approx 2NV_\pi$. This is a discouraging result. To scan N independent spots, a voltage $2N$ times greater than the half-wave voltage is necessary. Since V_π is usually large, making a useful scanner with $N \gg 1$ requires unacceptably high voltages. More popular scanners therefore include mechanical and acousto-optic scanners (see Secs. 20.2B and 21.1B).

The process of double refraction in anisotropic crystals (see Sec. 6.3E) introduces a lateral shift of an incident beam parallel to itself for one polarization and no shift for the other polarization. This effect can be used for switching a beam between two parallel positions by switching the polarization. A linearly polarized optical beam is

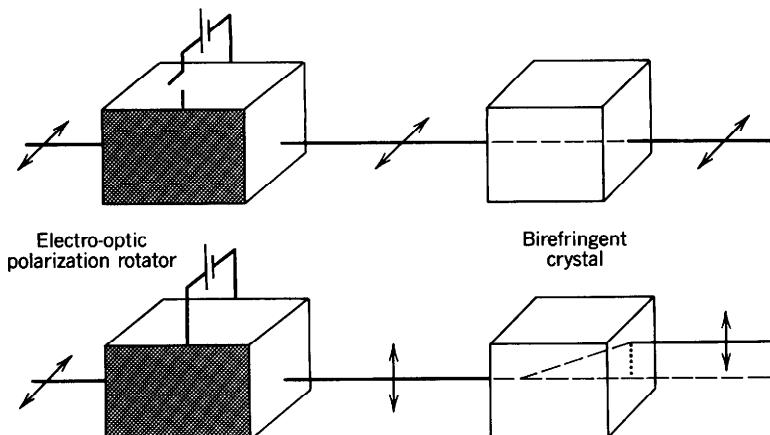


Figure 18.1-8 A position switch based on electro-optic phase retardation and double refraction.

transmitted first through an electro-optic wave retarder acting as a polarization rotator and then through the crystal. The rotator controls the polarization electrically, which determines whether the beam is shifted laterally or not, as illustrated in Fig. 18.1-8.

D. Directional Couplers

An important application of the electro-optic effect is in controlling the coupling between two parallel waveguides in an integrated-optical device. This can be used to transfer the light from one waveguide to the other, so that the device serves as an electrically controlled directional coupler.

The coupling of light between two parallel single-mode planar waveguides [Fig. 18.1-9(a)] was examined in Sec. 7.4B. It was shown that the optical powers carried by the two waveguides, $P_1(z)$ and $P_2(z)$, are exchanged periodically along the direction of propagation z . Two parameters govern the strength of this coupling process: the coupling coefficient C (which depends on the dimensions, wavelength, and refractive indices), and the mismatch of the propagation constants $\Delta\beta = \beta_1 - \beta_2 = 2\pi\Delta n/\lambda_0$, where Δn is the difference between the refractive indices of the waveguides. If the waveguides are identical, with $\Delta\beta = 0$ and $P_2(0) = 0$, then at a distance $z = L_0 = \pi/2C$, called the transfer distance or coupling length, the power is transferred completely from waveguide 1 into waveguide 2, i.e., $P_1(L_0) = 0$ and $P_2(L_0) = P_1(0)$, as illustrated in Fig. 18.1-9(a).

For a waveguide of length L_0 and $\Delta\beta \neq 0$, the power-transfer ratio $\mathcal{T} = P_2(L_0)/P_1(0)$ is a function of the phase mismatch [see (7.4-12)],

$$\mathcal{T} = \left(\frac{\pi}{2}\right)^2 \operatorname{sinc}^2\left\{\frac{1}{2}\left[1 + \left(\frac{\Delta\beta L_0}{\pi}\right)^2\right]^{1/2}\right\}, \quad (18.1-18)$$

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$. Figure 18.1-9(b) illustrates this dependence. The ratio has its maximum value of unity at $\Delta\beta L_0 = 0$, decreases with increasing $\Delta\beta L_0$, and vanishes when $\Delta\beta L_0 = \sqrt{3}\pi$, at which point the optical power is *not* transferred to waveguide 2.

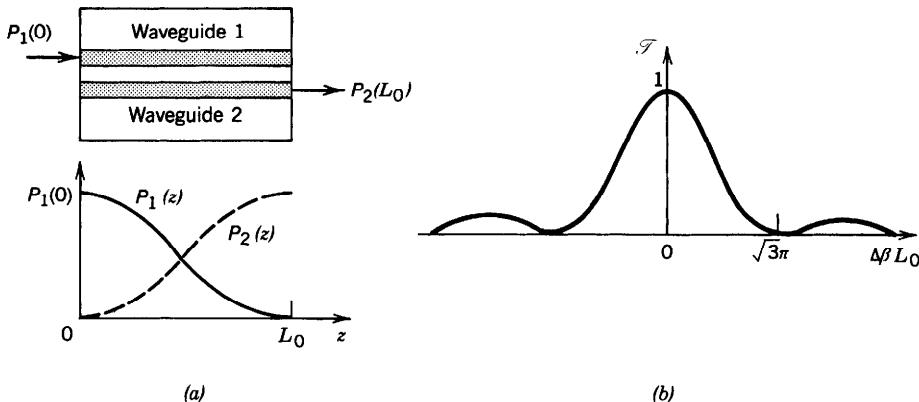


Figure 18.1-9 (a) Exchange of power between two parallel weakly coupled waveguides that are identical, with the same propagation constant β . At $z = 0$ all of the power is in waveguide 1. At $z = L_0$ all of the power is transferred into waveguide 2. (b) Dependence of the power-transfer ratio $\mathcal{T} = P_2(L_0)/P_1(0)$ on the phase mismatch parameter $\Delta\beta L_0$.

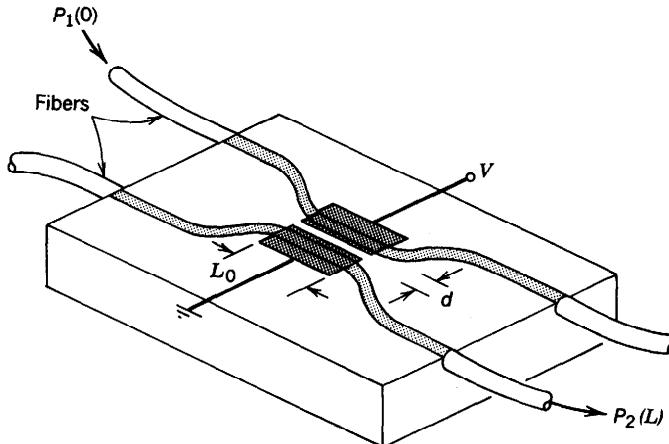


Figure 18.1-10 An integrated electro-optic directional coupler.

A dependence of the coupled power on the phase mismatch is the key to making electrically activated directional couplers. If the mismatch $\Delta\beta L_0$ is switched from 0 to $\sqrt{3}\pi$, the light remains in waveguide 1. Electrical control of $\Delta\beta$ is achieved by use of the electro-optic effect. An electric field E applied to one of two, otherwise identical, waveguides alters the refractive index by $\Delta n = -\frac{1}{2}n^3rE$, where r is the Pockels coefficient. This results in a phase shift $\Delta\beta L_0 = \Delta n(2\pi L_0/\lambda_o) = -(\pi/\lambda_o)n^3rL_0E$.

A typical electro-optic directional coupler has the geometry shown in Fig. 18.1-10. The electrodes are laid over two waveguides separated by a distance d . An applied voltage V creates an electric field $E \approx V/d$ in one waveguide and $-V/d$ in the other, where d is an effective distance determined by solving the electrostatics problem (the electric-field lines go downward at one waveguide and upward at the other). The refractive index is incremented in one guide and decremented in the other. The result is a net refractive index difference $2\Delta n = -n^3r(V/d)$, corresponding to a phase mismatch factor $\Delta\beta L_0 = -(2\pi/\lambda_o)n^3r(L_0/d)V$, which is proportional to the applied voltage V .

The voltage V_0 necessary to switch the optical power is that for which $|\Delta\beta L_0| = \sqrt{3}\pi$, i.e.,

$$V_0 = \sqrt{3} \frac{d}{L_0} \frac{\lambda_o}{2n^3r} = \frac{\sqrt{3}}{\pi} \frac{C\lambda_o d}{n^3r}, \quad (18.1-19)$$

where $L_0 = \pi/2C$ and C is the coupling coefficient. This is called the **switching voltage**. Since $|\Delta\beta L_0| = \sqrt{3}\pi V/V_0$, (18.1-18) gives

$$\mathcal{T} = \left(\frac{\pi}{2}\right)^2 \operatorname{sinc}^2 \left\{ \frac{1}{2} \left[1 + 3 \left(\frac{V}{V_0} \right)^2 \right]^{1/2} \right\}. \quad (18.1-20)$$

Coupling Efficiency

This equation (plotted in Fig. 18.1-11) governs the coupling of power as a function of the applied voltage V .

An electro-optic directional coupler is characterized by its coupling length L_0 , which is inversely proportional to the coupling coefficient C , and its switching voltage V_0 , which is directly proportional to C . The key parameter is therefore C , which is governed by the geometry and the refractive indices.

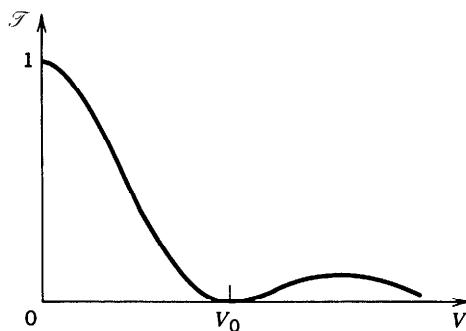


Figure 18.1-11 Dependence of the coupling efficiency on the applied voltage V . When $V = 0$, all of the optical power is coupled from waveguide 1 into waveguide 2; when $V = V_0$, all of the optical power remains in waveguide 1.

Integrated-optic directional couplers may be fabricated by diffusing titanium into high-purity LiNbO_3 substrates. The switching voltage V_0 is typically under 10 V, and the operating speeds can exceed 10 GHz. The light beams are focused to spot sizes of a few μm . The ends of the waveguide may be permanently attached to single-mode polarization-maintaining optical fibers (see Sec. 8.1C). Increased bandwidths can be obtained by using a traveling-wave version of this device.

EXERCISE 18.1-1

Spectral Response. Equation (18.1-19) indicates that the switching voltage V_0 is proportional to the wavelength. Assume that the applied voltage $V = V_0$ for a wavelength $\bar{\lambda}_o$; i.e., the coupling efficiency $\mathcal{T} = 0$ at $\bar{\lambda}_o$. If, instead, the incident wave has wavelength λ_o , plot the coupling efficiency \mathcal{T} as a function of $\lambda_o - \bar{\lambda}_o$. Assume that the coupling coefficient C and the material parameters n and r are approximately independent of wavelength.

E. Spatial Light Modulators

A spatial light modulator is a device that modulates the intensity of light at different positions by prescribed factors (Fig. 18.1-12). It is a planar optical element of control-

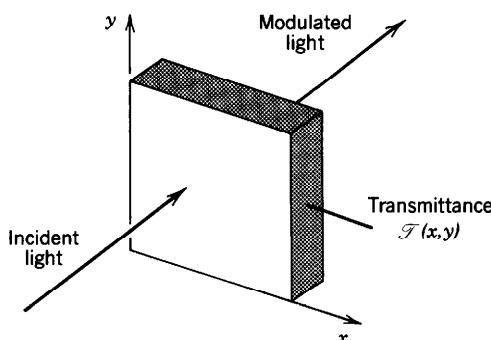


Figure 18.1-12 The spatial light modulator.

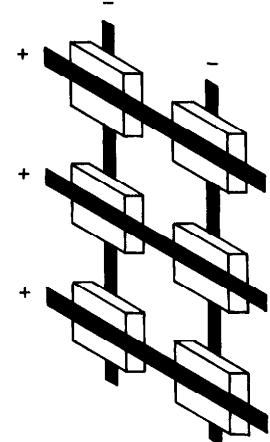


Figure 18.1-13 An electrically addressable array of longitudinal electro-optic modulators.

lable intensity transmittance $\mathcal{T}(x, y)$. The transmitted light intensity $I_o(x, y)$ is related to the incident light intensity $I_i(x, y)$ by the product $I_o(x, y) = I_i(x, y)\mathcal{T}(x, y)$. If the incident light is uniform [i.e., $I_i(x, y)$ is constant], the transmitted light intensity is proportional to $\mathcal{T}(x, y)$. The “image” $\mathcal{T}(x, y)$ is then imparted to the transmitted light, much like “reading” the image stored in a transparency by uniformly illuminating it in a slide projector. In a spatial light modulator, however, $\mathcal{T}(x, y)$ is controllable. In an electro-optic modulator the control is electrical.

To construct a spatial light modulator using the electro-optic effect, some mechanism must be devised for creating an electric field $E(x, y)$ proportional to the desired transmittance $\mathcal{T}(x, y)$ at each position. This is not easy. One approach is to place an array of transparent electrodes on small plates of electro-optic material placed between crossed polarizers and to apply on each electrode an appropriate voltage (Fig. 18.1-13). The voltage applied to the electrode centered at the position (x_i, y_i) , $i = 1, 2, \dots$ is made proportional to the desired value of $\mathcal{T}(x_i, y_i)$ (see, e.g., Fig. 18.1-6). If the number of electrodes is sufficiently large, the transmittance approximates $\mathcal{T}(x, y)$. The system is in effect a parallel array of longitudinal electro-optic modulators operated as intensity modulators. However, it is not practical to address a large number of these electrodes independently; nevertheless we will see that this scheme is practical in the liquid-crystal spatial light modulators used for display, since the required voltages are low (see Sec. 18.3B).

Optically Addressed Electro-Optic Spatial Light Modulators

One method of optically addressing an electro-optic spatial light modulator is based on the use of a thin layer of photoconductive material to create the electric field required to operate the modulator (Fig. 18.1-14). The conductivity of a photoconductive material is proportional to the intensity of light to which it is exposed (see Sec. 17.2). When illuminated by light of intensity distribution $I_W(x, y)$, a spatial pattern of conductance $G(x, y) \propto I_W(x, y)$ is created. The photoconductive layer is placed between two electrodes that act as a capacitor. The capacitor is initially charged and the electrical charge leakage at the position (x, y) is proportional to the local conductance $G(x, y)$. As a result, the charge on the capacitor is reduced in those regions where the conductance is high. The local voltage is therefore proportional to $1/G(x, y)$ and the corresponding electric field $E(x, y) \propto 1/G(x, y) \propto 1/I_W(x, y)$. If the transmittance $\mathcal{T}(x, y)$ [or the reflectance $R(x, y)$] of the modulator is proportional to the applied field, it must be inversely proportional to the initial light intensity $I_W(x, y)$.

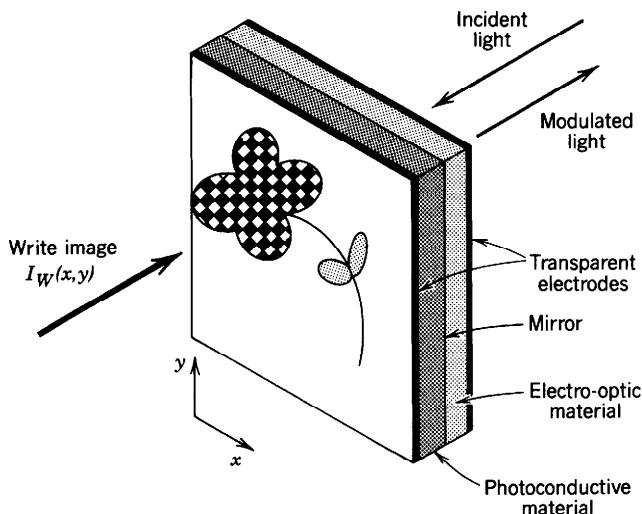


Figure 18.1-14 The electro-optic spatial light modulator uses a photoconductive material to create a spatial distribution of electric field which is used to control an electro-optic material.

The Pockels Readout Optical Modulator

An ingenious implementation of this principle is the Pockels readout optical modulator (PROM). The device uses a crystal of bismuth silicon oxide, $\text{Bi}_{12}\text{SiO}_{20}$ (BSO), which has an unusual combination of optical and electrical properties: (1) it exhibits the electro-optic (Pockels) effect; (2) it is photoconductive for blue light, but not for red light; and (3) it is a good insulator in the dark. The PROM (Fig. 18.1-15) is made of a thin wafer of BSO sandwiched between two transparent electrodes. The light that is to be modulated (read light) is transmitted through a polarizer, enters the BSO layer, and is reflected by a dichroic reflector, whereupon it crosses a second polarizer. The reflector reflects red light but is transparent to blue light. The PROM is operated as

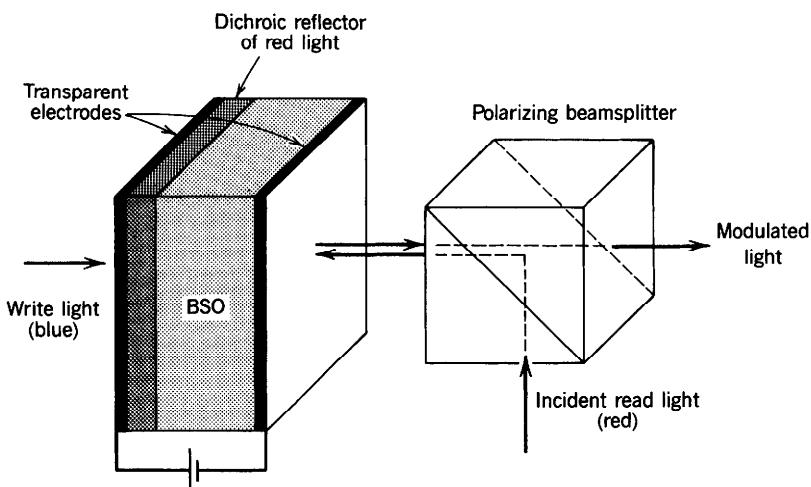


Figure 18.1-15 The Pockels readout optical modulator (PROM).

follows:

- **Priming:** A large potential difference (≈ 4 kV) is applied to the electrodes and the capacitor is charged (with no leakage since the crystal is a good insulator in the dark).
- **Writing:** Intense blue light of intensity distribution $I_W(x, y)$ illuminates the crystal. As a result, a spatial pattern of conductance $G(x, y) \propto I_W(x, y)$ is created, the voltage across the crystal is selectively lowered, and the electric field decreases proportionally at each position, so that $E(x, y) \propto 1/G(x, y) \propto 1/I_W(x, y)$. As a result of the electro-optic effect, the refractive indices of the BSO are altered, and a spatial pattern of refractive-index change $\Delta n(x, y) \propto 1/I_W(x, y)$ is created and stored in the crystal.
- **Reading:** Uniform red light is used to read $\Delta n(x, y)$ as with usual electro-optic intensity modulators [see Fig. 18.1-6(a)] with the polarizing beamsplitter playing the role of the crossed polarizers.
- **Erasing:** The refractive-index pattern is erased by the use of a uniform flash of blue light. The crystal is again primed by applying 4 kV, and a new cycle is started.

Incoherent-to-Coherent Optical Converters

In an optically addressed spatial light modulator, such as the PROM, the light used to write a spatial pattern into the modulator need not be coherent since photoconductive materials are sensitive to optical intensity. A spatial optical pattern (an image) may be written using incoherent light, and read using coherent light. This process of real-time conversion of a spatial distribution of natural incoherent light into a proportional spatial distribution of coherent light is useful in many optical data- and image-processing applications (see Sec. 21.5B).

***18.2 ELECTRO-OPTICS OF ANISOTROPIC MEDIA**

The basic principles and applications of electro-optics have been presented in Sec. 18.1. For simplicity, polarization and anisotropic effects have been either ignored or introduced only generically. In this section a more complete analysis of the electro-optics of anisotropic media is presented. The following is a brief reminder of the important properties of anisotropic media, but the reader is expected to be familiar with the material in Sec. 6.3 on propagation of light in anisotropic media.

Crystal Optics: A Brief Refresher

The optical properties of an anisotropic medium are characterized by a geometric construction called the index ellipsoid,

$$\sum_{ij} \eta_{ij} x_i x_j = 1, \quad i, j = 1, 2, 3, \quad (18.2-1)$$

where $\eta_{ij} = \eta_{ji}$ are elements of the impermeability tensor $\eta = \epsilon_0 \epsilon^{-1}$. The ellipsoid's principal axes are the optical principal axes of the medium; its principal dimensions along these axes are the principal refractive indices n_1 , n_2 , and n_3 (Fig. 18.2-1). The index ellipsoid may be used to determine the polarizations and refractive indices n_a and n_b of the two normal modes of a wave traveling in an arbitrary direction. This is accomplished by drawing a plane

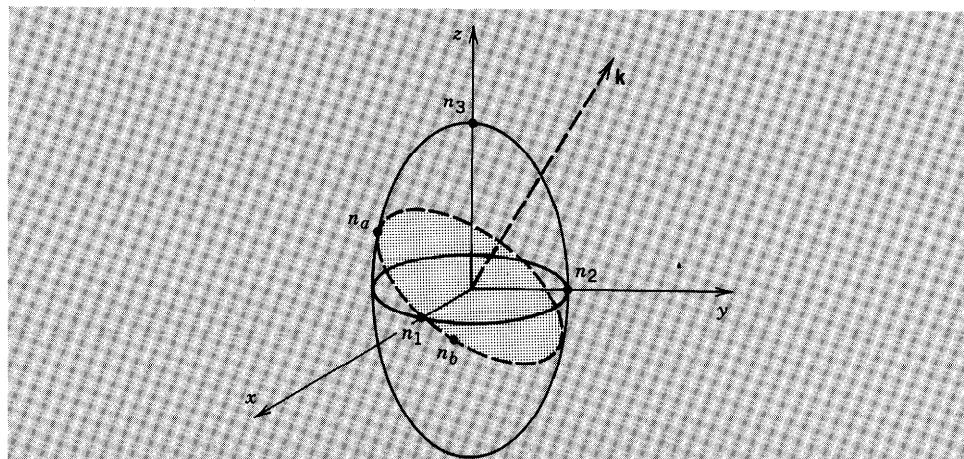


Figure 18.2-1 The index ellipsoid. The coordinates (x, y, z) are the principal axes and n_1, n_2, n_3 are the principal refractive indices. The refractive indices of the normal modes of a wave traveling in the direction \mathbf{k} are n_a and n_b .

perpendicular to the direction of propagation and passing through the center of the ellipsoid. Its intersection with the ellipsoid is an ellipse whose major and minor axes have half-lengths equal to n_a and n_b (see Sec. 6.3C).

A. Pockels and Kerr Effects

When a steady electric field \mathbf{E} with components (E_1, E_2, E_3) is applied to a crystal, elements of the tensor η are altered, so that each of the nine elements η_{ij} becomes a function of E_1, E_2 , and E_3 , i.e., $\eta_{ij} = \eta_{ij}(\mathbf{E})$. As a result, the index ellipsoid is modified (Fig. 18.2-2). Once we know the function $\eta_{ij}(\mathbf{E})$, we can determine the index ellipsoid and the optical properties at any applied electric field \mathbf{E} . The problem is simple in principle, but the implementation is often lengthy.

Each of the elements $\eta_{ij}(\mathbf{E})$ is a function of the three variables $\mathbf{E} = (E_1, E_2, E_3)$, which may be expanded in a Taylor's series about $\mathbf{E} = \mathbf{0}$,

$$\eta_{ij}(\mathbf{E}) = \eta_{ij} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} E_k E_l, \quad i, j, k, l = 1, 2, 3, \quad (18.2-2)$$

where $\eta_{ij} = \eta_{ij}(\mathbf{0})$, $r_{ijk} = \partial \eta_{ij} / \partial E_k$, $s_{ijkl} = \frac{1}{2} \partial^2 \eta_{ij} / \partial E_k \partial E_l$, and the derivatives are evaluated at $\mathbf{E} = \mathbf{0}$. Equation (18.2-2) is a generalization of (18.1-3), in which r is replaced by $3^3 = 27$ coefficients $\{r_{ijk}\}$, and s is replaced by $3^4 = 81$ coefficients $\{s_{ijkl}\}$. The coefficients $\{r_{ijk}\}$ are known as the linear electro-optic (Pockels) coefficients. They

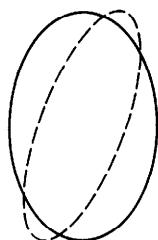


Figure 18.2-2 The index ellipsoid is modified as a result of applying a steady electric field.

TABLE 18.2-1 Lookup Table for the Index I That Represents the Pair of Indices $(i, j)^a$

j	$i : 1$	2	3
1	1	6	5
2	6	2	4
3	5	4	3

^aThe pair $(i, j) = (3, 2)$, for example, is labeled $I = 4$.

form a tensor of third rank. The coefficients $\{s_{ijkl}\}$ are the quadratic electro-optic (Kerr) coefficients. They form a fourth-rank tensor.

Symmetry

Because η is symmetric ($\eta_{ij} = \eta_{ji}$), r and s are invariant under permutations of the indices i and j , i.e., $r_{ijk} = r_{jik}$ and $s_{ijkl} = s_{jikl}$. Also, the coefficients $s_{ijkl} = \frac{1}{2}\partial^2\eta_{ij}/\partial E_k \partial E_l$ are invariant to permutations of k and l (because of the invariance to the order of differentiation), so that $s_{ijkl} = s_{ijlk}$. Because of this permutation symmetry the nine combinations of the indices i, j generate six instead of nine independent elements. The same reduction applies to the indices k, l . Consequently, r_{ijk} has 6×3 independent elements, whereas s_{ijkl} has 6×6 independent elements.

It is conventional to rename the pair of indices (i, j) , $i, j = 1, 2, 3$ as a single index $I = 1, 2, \dots, 6$ in accordance with Table 18.2-1. The pair (k, l) is similarly replaced by an index $K = 1, 2, \dots, 6$, in accordance with the same rule. Thus the coefficients r_{ijk} and s_{ijkl} are replaced by r_{Ik} and s_{IK} , respectively. For example, r_{12k} is denoted as r_{6k} , s_{1231} is renamed s_{65} , and so on. The third-rank tensor r is therefore replaced by a 6×3 matrix and the fourth-rank tensor s by a 6×6 matrix.

Crystal Symmetry

The symmetry of the crystal adds more constraints to the entries of the r and s matrices. Some entries must be zero and others must be equal, or equal in magnitude and opposite in sign, or related by some other rule. For centrosymmetric crystals r vanishes and only the Kerr effect is exhibited. Lists of the coefficients of r and s and their symmetry relations for the 32 crystal point groups may be found in several of the books referenced in the reading list. Representative examples are provided in Tables 18.2-2 and 18.2-3.

Pockels Effect

To determine the optical properties of an anisotropic material exhibiting the Pockels effect in the presence of an electric field $\mathbf{E} = (E_1, E_2, E_3)$, the following sequence is

TABLE 18.2-2 Pockels Coefficients r_{Ik} for Some Representative Crystal Groups

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$	$\begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$
---	---	--

Cubic $\bar{4}3m$
[e.g., GaAs, CdTe, InAs]

Tetragonal $\bar{4}2m$
[e.g., KDP, ADP]

Trigonal $3m$
[e.g., LiNbO₃, LiTaO₃]

TABLE 18.2-3 Kerr Coefficients \mathfrak{s}_{IK} for an Isotropic Medium

$$\begin{bmatrix} \mathfrak{s}_{11} & \mathfrak{s}_{12} & \mathfrak{s}_{12} & 0 & 0 & 0 \\ \mathfrak{s}_{12} & \mathfrak{s}_{11} & \mathfrak{s}_{12} & 0 & 0 & 0 \\ \mathfrak{s}_{12} & \mathfrak{s}_{12} & \mathfrak{s}_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{s}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathfrak{s}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathfrak{s}_{44} \end{bmatrix}, \quad \mathfrak{s}_{44} = \frac{\mathfrak{s}_{11} - \mathfrak{s}_{12}}{2}$$

followed:

- Find the principal axes and principal refractive indices n_1 , n_2 , and n_3 in the absence of \mathbf{E} .
- Find the coefficients $\{r_{ijk}\}$ by using the appropriate matrix for r_{IK} (e.g., Table 18.2-2) together with the contraction rule relating i, j to I (Table 18.2-1);
- Determine the elements of the impermeability tensor using

$$\eta_{ij}(\mathbf{E}) = \eta_{ij}(\mathbf{0}) + \sum_k r_{ijk} E_k,$$

where $\eta_{ij}(\mathbf{0})$ is a diagonal matrix with elements $1/n_1^2$, $1/n_2^2$, and $1/n_3^2$.

- Write the equation for the modified index ellipsoid

$$\sum_{ij} \eta_{ij}(\mathbf{E}) x_i x_j = 1.$$

- Determine the principal axes of the modified index ellipsoid by diagonalizing the matrix $\eta_{ij}(\mathbf{E})$, and find the corresponding principal refractive indices $n_1(\mathbf{E})$, $n_2(\mathbf{E})$, and $n_3(\mathbf{E})$.
- Given the direction of light propagation, find the normal modes and their associated refractive indices by using the index ellipsoid.

EXAMPLE 18.2-1. Trigonal 3m Crystals (e.g., LiNbO_3 and LiTaO_3). Trigonal 3m crystals are uniaxial ($n_1 = n_2 = n_o$, $n_3 = n_e$) with the matrix \mathbf{r} provided in Table 18.2-2. Assuming that $\mathbf{E} = (0, 0, E)$, i.e., that the electric field points along the optic axis (see Fig. 18.2-3), we find that the modified index ellipsoid is

$$\left(\frac{1}{n_o^2} + r_{13}E \right) (x_1^2 + x_2^2) + \left(\frac{1}{n_e^2} + r_{33}E \right) x_3^2 = 1. \quad (18.2-3)$$

This is an ellipsoid of revolution whose principal axes are independent of E . The ordinary and extraordinary indices $n_o(E)$ and $n_e(E)$ are given by

$$\frac{1}{n_o^2(E)} = \frac{1}{n_o^2} + r_{13}E \quad (18.2-4)$$

$$\frac{1}{n_e^2(E)} = \frac{1}{n_e^2} + r_{33}E. \quad (18.2-5)$$

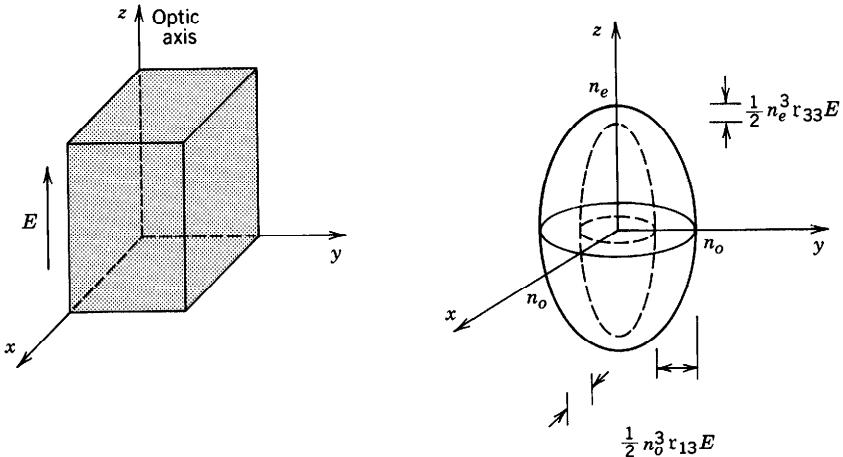


Figure 18.2-3 Modification of the index ellipsoid of a trigonal 3m crystal caused by an electric field in the direction of the optic axis.

Because the terms $r_{13}E$ and $r_{33}E$ in (18.2-4) and (18.2-5) are usually small, we use the approximation $(1 + \Delta)^{-1/2} \approx 1 - \frac{1}{2}\Delta$, when $|\Delta|$ is small, to obtain

$$n_o(E) \approx n_o - \frac{1}{2}n_o^3 r_{13}E \quad (18.2-6)$$

$$n_e(E) \approx n_e - \frac{1}{2}n_e^3 r_{33}E. \quad (18.2-7)$$

We conclude that when an electric field is applied along the optic axis of this uniaxial crystal it remains uniaxial with the same principal axes, but its refractive indices are modified in accordance with (18.2-6) and (18.2-7) (Fig. 18.2-3). Note the similarity between these equations and the generic equation (18.1-4).

EXAMPLE 18.2-2. Tetragonal 42m Crystals (e.g., KDP and ADP). Repeating the same steps for these uniaxial crystals and assuming that the electric field also points along the optic axis (Fig. 18.2-4), we obtain the equation of the index ellipsoid

$$\frac{x_1^2 + x_2^2}{n_o^2} + \frac{x_3^2}{n_e^2} + 2r_{63}Ex_1x_2 = 1. \quad (18.2-8)$$

The modified principal axes are obtained by rotating the coordinate system 45° about the z axis. Substituting $u_1 = (x_1 - x_2)/\sqrt{2}$, $u_2 = (x_1 + x_2)/\sqrt{2}$, $u_3 = x_3$ in (18.2-8), we obtain

$$\frac{u_1^2}{n_1^2(E)} + \frac{u_2^2}{n_2^2(E)} + \frac{u_3^2}{n_3^2(E)} = 1,$$

where

$$\frac{1}{n_1^2(E)} = \frac{1}{n_o^2} + r_{63}E$$

$$\frac{1}{n_2^2(E)} = \frac{1}{n_e^2} - r_{63}E$$

$$n_3(E) = n_e.$$

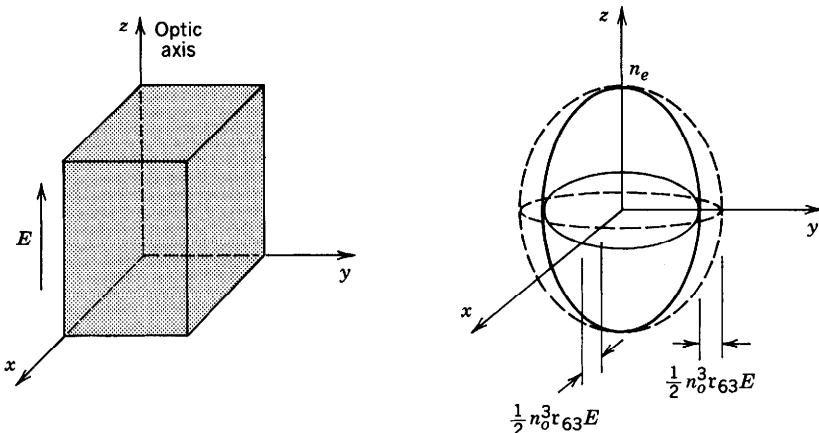


Figure 18.2-4 Modification of the index ellipsoid resulting from an electric field E along the direction of the optic axis of a uniaxial tetragonal $\bar{4}2m$ crystal.

Using the approximation $(1 + \Delta)^{-1/2} \approx 1 - \frac{1}{2}\Delta$ yields

$$n_1(E) \approx n_o - \frac{1}{2}n_o^3 r_{63}E \quad (18.2-9)$$

$$n_2(E) \approx n_o + \frac{1}{2}n_o^3 r_{63}E \quad (18.2-10)$$

$$n_3(E) = n_e. \quad (18.2-11)$$

Thus the originally uniaxial crystal becomes biaxial when subjected to an electric field in the direction of its optic axis (Fig. 18.2-4).

EXAMPLE 18.2-3. Cubic $\bar{4}3m$ Crystals (e.g., GaAs, CdTe, and InAs). These crystals are isotropic ($n_1 = n_2 = n_3 = n$). Without loss of generality, the coordinate system may be selected such that the applied electric field points in the z direction (Fig. 18.2-5). Following the same steps, an equation for the index ellipsoid is obtained,

$$\frac{x_1^2 + x_2^2 + x_3^2}{n^2} + 2r_{41}Ex_1x_2 = 1. \quad (18.2-12)$$

As in Example 18.2-2, the new principal axes are rotated 45° about the z axis and the principal refractive indices are

$$n_1(E) \approx n - \frac{1}{2}n^3 r_{41}E \quad (18.2-13)$$

$$n_2(E) \approx n + \frac{1}{2}n^3 r_{41}E \quad (18.2-14)$$

$$n_3(E) = n. \quad (18.2-15)$$

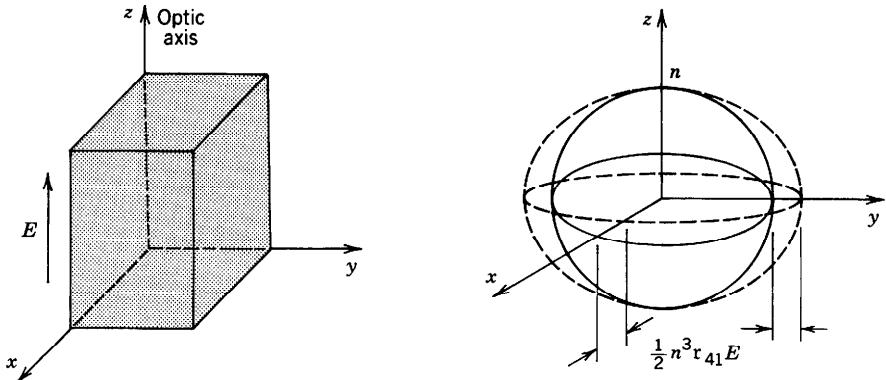


Figure 18.2-5 Modification of the index ellipsoid as a result of application of an electric field E to a cubic $\bar{4}3m$ crystal.

Thus the applied field changes the originally isotropic crystal into a biaxial crystal (Fig. 18.2-5).

The three examples above share the property that the principal axes do not change as the applied steady electric field E increases. The directions of polarization of the normal modes therefore remain the same, but their associated refractive indices are functions of E . The medium can then be used as a phase modulator, wave retarder, or intensity modulator in accordance with the generic theory provided in Sec. 18.1B. This principle is described further in Sec. 18.2B.

Kerr Effect

The optical properties of a Kerr medium can be determined by using the same procedure used for the Pockels medium, except that the coefficients $\eta_{ij}(E)$ are now given by $\eta_{ij}(E) = \eta_{ij}(0) + \sum_{kl} s_{ijkl} E_k E_l$.

EXAMPLE 18.2-4. Isotropic Medium. Using the Kerr coefficients s_{IK} in Table 18.2-3 for an isotropic medium, and taking the z axis to point along the applied electric field E , we easily find the equation of the index ellipsoid,

$$\left(\frac{1}{n^2} + s_{12} E^2 \right) (x_1^2 + x_2^2) + \left(\frac{1}{n^2} + s_{11} E^2 \right) x_3^2 = 1. \quad (18.2-16)$$

This is the equation of an ellipsoid of revolution whose axis is the z axis. The principal refractive indices $n_o(E)$ and $n_e(E)$ are determined from

$$\frac{1}{n_o^2(E)} = \frac{1}{n^2} + s_{12} E^2 \quad (18.2-17)$$

$$\frac{1}{n_e^2(E)} = \frac{1}{n^2} + s_{11} E^2. \quad (18.2-18)$$

Noting that the second terms in (18.2-17) and (18.2-18) are small, and using the approximation $(1 + \Delta)^{-1/2} \approx 1 - \frac{1}{2}\Delta$ when $|\Delta| \ll 1$, we obtain

$$n_o(E) \approx n - \frac{1}{2}n^3\mathfrak{s}_{12}E^2 \quad (18.2-19)$$

$$n_e(E) \approx n - \frac{1}{2}n^3\mathfrak{s}_{11}E^2. \quad (18.2-20)$$

Thus a steady electric field E applied to an originally isotropic medium converts it into a uniaxial crystal with the optic axis pointing in the direction of the electric field. The ordinary and extraordinary indices are *quadratic* decreasing functions of E .

B. Modulators

The principles of phase and intensity modulation using the electro-optic effect were outlined in Sec. 18.1B. Anisotropic effects were introduced only generically. Using the anisotropic theory presented in this section, the generic parameters r and \mathfrak{s} , which were used in Sec. 18.1, can now be determined for any given crystal and directions of the applied electric field and light propagation. Only Pockels modulators will be discussed, but the same approach can be applied to Kerr modulators. For simplicity, we assume that the direction of the electric field is such that the principal axes of the crystal are not altered as a result of modulation. We shall also assume that the direction of the wave relative to these axes is such that the planes of polarization of the normal modes are also not altered by the electric field.

Phase Modulators

A normal mode is characterized by a refractive index $n(E) \approx n - \frac{1}{2}r n^3 E$, where n and r are the appropriate refractive index and Pockels coefficient, respectively, and $E = V/d$ is the electric field obtained by applying a voltage V across a distance d . A wave traveling a distance L undergoes a phase shift

$$\varphi = \varphi_o - \pi \frac{V}{V_\pi} \quad (18.2-21)$$

where $\varphi_o = 2\pi nL/\lambda_o$ and

$$V_\pi = \frac{d}{L} \frac{\lambda_o}{rn^3} \quad (18.2-22)$$

is the half-wave voltage. The appropriate coefficients generically called n and r can be easily determined as demonstrated in the following example.

EXAMPLE 18.2-5. Trigonal 3m Crystal (LiNbO_3 and LiTaO_3). When an electric field is directed along the optic axis of this type of uniaxial crystal, the crystal remains uniaxial with the same principal axes (see Fig. 18.2-3). The principal refractive indices are given by (18.2-6) and (18.2-7). The crystal can be used as a phase modulator in either of

two configurations:

Longitudinal Modulator: If a linearly polarized optical wave travels along the direction of the optic axis (parallel to the electric field), the appropriate parameters for the phase modulator are $n = n_o$, $r = r_{13}$, and $d = L$. For LiNbO₃, $r_{13} = 9.6 \text{ pm/V}$, and $n_o = 2.3$ at $\lambda_o = 633 \text{ nm}$. Equation (18.2-22) then gives $V_\pi = 5.41 \text{ kV}$, so that 5.41 kV is required to change the phase by π .

Transverse Modulator: If the wave travels in the x direction and is polarized in the z direction, the appropriate parameters are $n = n_e$ and $r = r_{33}$. The width d is generally not equal to the length L . For LiNbO₃ at $\lambda_o = 633 \text{ nm}$, $r_{33} = 30.9 \text{ pm/V}$, and $n_e = 2.2$, giving a half-wave voltage $V_\pi = 1.9(d/L) \text{ kV}$. If $d/L = 0.1$, we obtain $V_\pi \approx 190 \text{ V}$, which is significantly lower than the half-wave voltage for the longitudinal modulator.

Intensity Modulators

The difference in the dependence on the applied field of the refractive indices of the two normal modes of a Pockels cell provides a voltage-dependent retardation,

$$\Gamma = \Gamma_0 - \pi \frac{V}{V_\pi}, \quad (18.2-23)$$

where

$$\Gamma_0 = \frac{2\pi(n_1 - n_2)L}{\lambda_o} \quad (18.2-24)$$

$$V_\pi = \frac{(d/L)\lambda_o}{r_1 n_1^3 - r_2 n_2^3}. \quad (18.2-25)$$

If the cell is placed between crossed polarizers, the system serves as an intensity modulator (see Sec. 18.1B). It is not difficult to determine the appropriate indices n_1 and n_2 , and coefficients r_1 and r_2 , as illustrated by the following example.

EXAMPLE 18.2-6. Tetragonal 42m Crystal (e.g., KDP and ADP). As described in Example 18.2-2, when an electric field is applied along the optic axis of this uniaxial crystal, it changes into a biaxial crystal. The new principal axes are the original axes rotated by 45° about the optic axis. Assume a longitudinal modulator configuration ($d/L = 1$) in which the wave travels along the optic axis. The two normal modes have refractive indices given by (18.2-9) and (18.2-10). The appropriate coefficients to be used in (18.2-25) are therefore $n_1 = n_2 = n_o$, $r_1 = r_{63}$, $r_2 = -r_{63}$, and $d = L$, so that $\Gamma_0 = 0$ and

$$V_\pi = \frac{\lambda_o}{2r_{63}n_o^3}. \quad (18.2-26)$$

For KDP at $\lambda_o = 633 \text{ nm}$, $V_\pi = 8.4 \text{ kV}$.

EXERCISE 18.2-1

Intensity Modulation Using the Kerr Effect. Use (18.2-19) and (18.2-20) to determine an expression for the phase shift φ , and the phase retardation Γ , in a longitudinal Kerr modulator made of an isotropic material, as functions of the applied voltage V . Derive expressions for the half-wave voltages V_π in each case.

18.3 ELECTRO-OPTICS OF LIQUID CRYSTALS

As described in Sec. 6.5, the elongated molecules of nematic liquid crystals tend to have ordered orientations that are altered when the material is subjected to mechanical or electric forces. Because of their anisotropic nature, liquid crystals can be arranged to serve as wave retarders or polarization rotators. In the presence of an electric field, their molecular orientation is modified, so that their effect on polarized light is altered. Liquid crystals can therefore be used as electrically controlled optical wave retarders, modulators, and switches. These devices are particularly useful in display technology.

A. Wave Retarders and Modulators

Electrical Properties of Nematic Liquid Crystals

The liquid crystals used to make electro-optic devices are usually of sufficiently high resistivity that they can be regarded as ideal dielectric materials. Because of the elongated shape of the constituent molecules, and their ordered orientation, liquid crystals have anisotropic dielectric properties with uniaxial symmetry (see Sec. 6.3A). The electric permittivity is ϵ_{\parallel} for electric fields pointing in the direction of the molecules and ϵ_{\perp} in the perpendicular direction. Liquid crystals for which $\epsilon_{\parallel} > \epsilon_{\perp}$, (positive uniaxial) are usually selected for electro-optic applications.

When a steady (or low frequency) electric field is applied, electric dipoles are induced and the resultant electric forces exert torques on the molecules. The molecules rotate in a direction such that the free electrostatic energy, $-\frac{1}{2}\mathbf{E} \cdot \mathbf{D} = -\frac{1}{2}[\epsilon_{\perp}E_1^2 + \epsilon_{\perp}E_2^2 + \epsilon_{\parallel}E_3^2]$, is minimized (here, E_1 , E_2 , and E_3 are components of \mathbf{E} in the directions of the principal axes). Since $\epsilon_{\parallel} > \epsilon_{\perp}$, for a given direction of the electric field, minimum energy is achieved when the molecules are aligned with the field, so that $E_1 = E_2 = 0$, $\mathbf{E} = (0, 0, E)$, and the energy is then $-\frac{1}{2}\epsilon_{\parallel}E^2$. When the alignment is complete the molecular axis points in the direction of the electric field (Fig. 18.3-1). Evidently, a reversal of the electric field effects the same molecular rotation. An alternating field generated by an ac voltage also has the same effect.

Nematic Liquid-Crystal Retarders and Modulators

A nematic liquid-crystal cell is a thin layer of nematic liquid crystal placed between two parallel glass plates and rubbed so that the molecules are parallel to each other. The

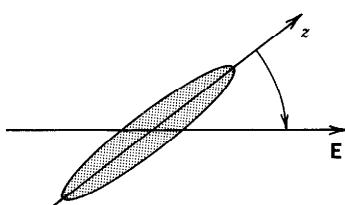


Figure 18.3-1 The molecules of a positive uniaxial liquid crystal rotate and align with the applied electric field.

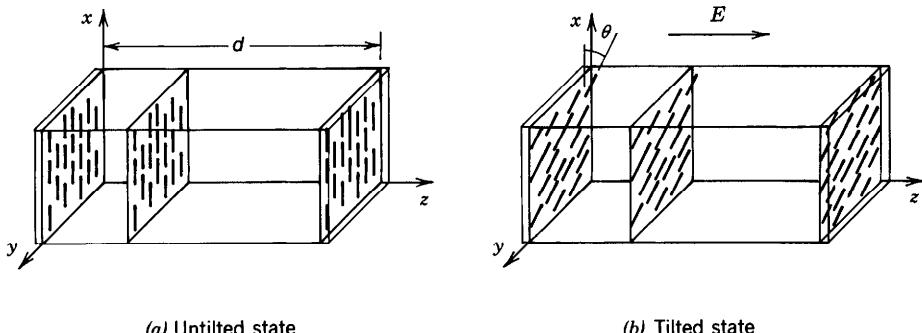


Figure 18.3-2 Molecular orientation of a liquid-crystal cell (a) in the absence of a steady electric field and (b) when a steady electric field is applied. The optic axis lies along the direction of the molecules.

material then acts as a uniaxial crystal with the optic axis parallel to the molecular orientation. For waves traveling in the z direction (perpendicular to the glass plates), the normal modes are linearly polarized in the x and y directions, (parallel and perpendicular to the molecular directions), as illustrated in Fig. 18.3-2(a). The refractive indices are the extraordinary and ordinary indices n_e and n_o . A cell of thickness d provides a wave retardation $\Gamma = 2\pi(n_e - n_o)d/\lambda_o$.

If an electric field is applied in the z direction (by applying a voltage V across transparent conductive electrodes coated on the inside of the glass plates), the resultant electric forces tend to tilt the molecules toward alignment with the field, but the elastic forces at the surfaces of the glass plates resist this motion. When the applied electric field is sufficiently large, most of the molecules tilt, except those adjacent to the glass surfaces. The equilibrium tilt angle θ for most molecules is a monotonically increasing function of V , which can be described by[†]

$$\theta = \begin{cases} 0, & V \leq V_c \\ \frac{\pi}{2} - 2 \tan^{-1} \exp\left(-\frac{V - V_c}{V_0}\right), & V > V_c, \end{cases} \quad (18.3-1)$$

where V is the applied rms voltage, V_c a critical voltage at which the tilting process begins, and V_0 a constant. When $V - V_c = V_0$, $\theta \approx 50^\circ$; as $V - V_c$ increases beyond V_0 , θ approaches 90° , as indicated in Fig. 18.3-3(a).

When the electric field is removed, the orientations of the molecules near the glass surfaces are reasserted and all of the molecules tilt back to their original orientation (in planes parallel to the plates). In a sense, the liquid-crystal material may be viewed as a *liquid with memory*.

For a tilt angle θ , the normal modes of an optical wave traveling in the z direction are polarized in the x and y directions and have refractive indices $n(\theta)$ and n_o , where

$$\frac{1}{n^2(\theta)} = \frac{\cos^2 \theta}{n_e^2} + \frac{\sin^2 \theta}{n_o^2}, \quad (18.3-2)$$

so that the retardation becomes $\Gamma = 2\pi[n(\theta) - n_o]d/\lambda_o$ (see Sec. 6.3C). The retardation achieves its maximum value $\Gamma_{\max} = 2\pi(n_e - n_o)d/\lambda_o$ when the molecules are not

[†]See, e.g., P.-G. de Gennes, *The Physics of Liquid Crystals*, Clarendon Press, Oxford, 1974, Chap. 3.

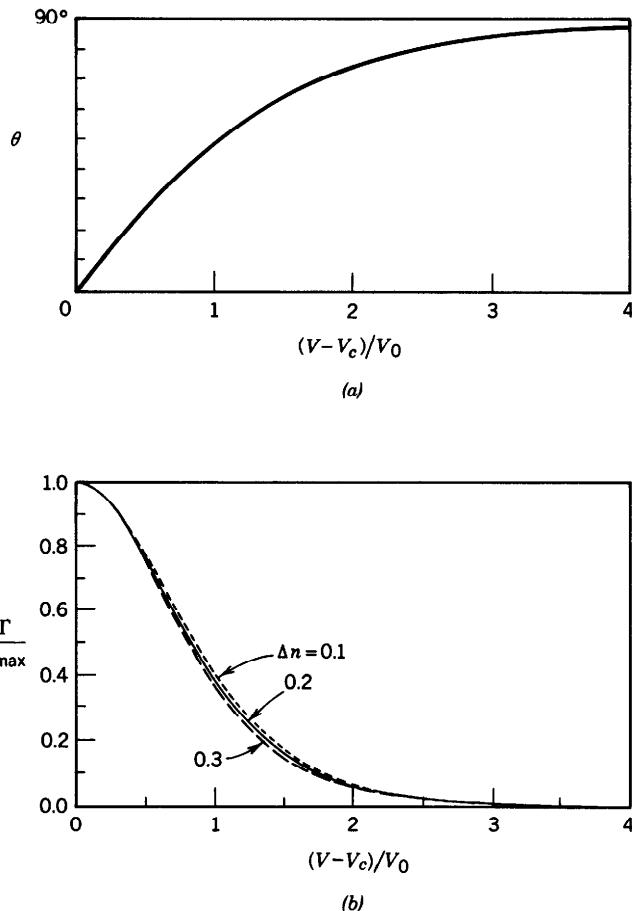


Figure 18.3-3 (a) Dependence of the tilt angle θ on the normalized rms voltage. (b) Dependence of the normalized retardation $\Gamma/\Gamma_{\max} = [n(\theta) - n_o]/(n_e - n_o)$ on the normalized rms voltage when $n_o = 1.5$, for the values of $\Delta n = n_e - n_o$ indicated. This plot is obtained from (18.3-1) and (18.3-2).

tilted ($\theta = 0$), and decreases monotonically toward 0 when the tilt angle reaches 90° , as illustrated in Fig. 18.3-3(b).

The cell can readily be used as a voltage-controlled *phase modulator*. For an optical wave traveling in the z direction and linearly polarized in the x direction (parallel to the unilted molecular orientation), the phase shift is $\varphi = 2\pi n(\theta)d/\lambda_o$. For waves polarized at 45° to the x axis in the x - y plane, the cell serves as a voltage-controlled *wave retarder*. When placed between two crossed polarizers (at $\pm 45^\circ$), a half-wave retarder ($\Gamma = \pi$) becomes a voltage-controlled *intensity modulator*. Similarly, a quarter-wave retarder ($\Gamma = \pi/2$) placed between a mirror and a polarizer at 45° with the x axis serves as an intensity modulator, as illustrated in Fig. 18.3-4.

The liquid-crystal cell is sealed between optically flat glass windows with antireflection coatings. A typical thickness of the liquid crystal layer is $d = 10 \mu\text{m}$ and typical values of $\Delta n = n_e - n_o = 0.1$ to 0.3. The retardation Γ is typically given in terms of the retardance $\varrho = (n_e - n_o)d$, so that the retardation $\Gamma = 2\pi\varrho/\lambda_o$. Retardances of several hundred nanometers are typical (e.g., a retardance of 300 nm corresponds to a retardation of π at $\lambda_o = 600 \text{ nm}$).

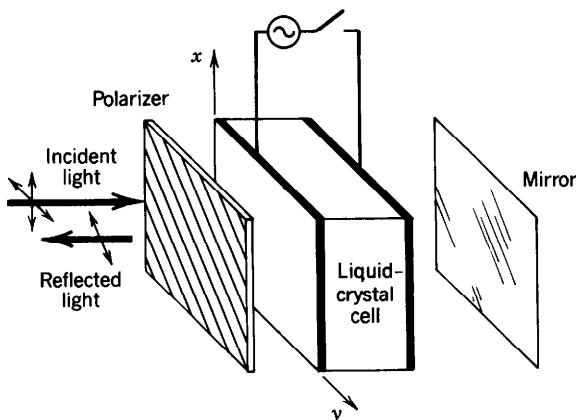


Figure 18.3-4 A liquid-crystal cell provides a retardation $\Gamma = \pi/2$ in the absence of the field (“off” state), and $\Gamma = 0$ in the presence of the field (“on” state). After reflection from the mirror and a round trip through the crystal, the plane of polarization rotates 90° in the “off” state, so that the light is blocked. In the “on” state, there is no rotation, and the reflected light is not blocked.

The applied voltage usually has a square waveform with a frequency in the range between tens of Hz and a few kHz. Operation at lower frequencies tends to cause electromechanical effects that disrupt the molecular alignment and reduce the lifetime of the device. Frequencies higher than 100 Hz result in greater power consumption because of the increased conductivity. The critical voltage V_c is typically a few volts rms.

Liquid crystals are slow. Their response time depends on the thickness of the liquid-crystal layer, the viscosity of the material, temperature, and the nature of the applied drive voltage. The rise time is of the order of tens of milliseconds if the operating voltage is near the critical voltage V_c , but decreases to a few milliseconds at higher voltages. The decay time is insensitive to the operating voltage but can be reduced by using cells of smaller thickness.

Twisted Nematic Liquid-Crystal Modulators

A *twisted nematic* liquid-crystal cell is a thin layer of nematic liquid crystal placed between two parallel glass plates and rubbed so that the molecular orientation rotates helically about an axis normal to the plates (the axis of twist). If the angle of twist is 90°, for example, the molecules point in the x direction at one plate and in the y direction at the other [Fig. 18.3-5(a)]. Transverse layers of the material act as uniaxial crystals, with the optic axes rotating helically about the axis of twist. It was shown in Sec. 6.5 that the polarization plane of linearly polarized light traveling in the direction of the axis of twist rotates with the molecules, so that the cell acts as a polarization rotator.

When an electric field is applied in the direction of the axis of twist (the z direction) the molecules tilt toward the field [Fig. 18.3-5(b)]. When the tilt is 90°, the molecules lose their twisted character (except for those adjacent to the glass surfaces), so that the polarization rotatory power is deactivated. If the electric field is removed, the orientations of the layers near the glass surfaces dominate, thereby causing the molecules to return to their original twisted state, and the polarization rotatory power to be regained.

Since the polarization rotatory power may be turned off and on by switching the electric field on and off, a shutter can be designed by placing a cell with 90° twist

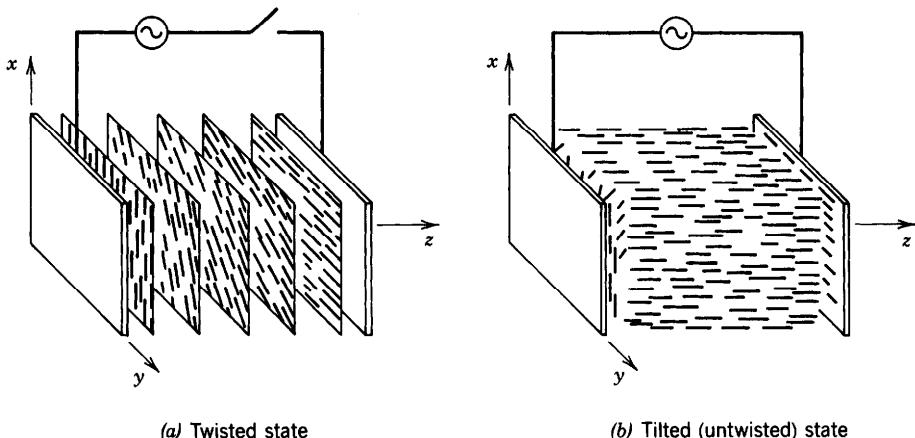


Figure 18.3-5 In the presence of a sufficiently large electric field, the molecules of a twisted nematic liquid crystal tilt and lose their twisted character.

between two crossed polarizers. The system transmits the light in the absence of an electric field and blocks it when the electric field is applied, as illustrated in Fig. 18.3-6.

Operation in the reflective mode is also possible, as illustrated in Fig. 18.3-7. Here, the twist angle is 45° ; a mirror is placed on one side of the cell and a polarizer on the other side. When the electric field is absent the polarization plane rotates a total of 90° upon propagation a round trip through the cell; the reflected light is therefore blocked by the polarizer. When the electric field is present, the polarization rotatory power is suspended and the reflected light is transmitted through the polarizer. Other reflective and transmissive modes of operation with different angles of twist are also possible.

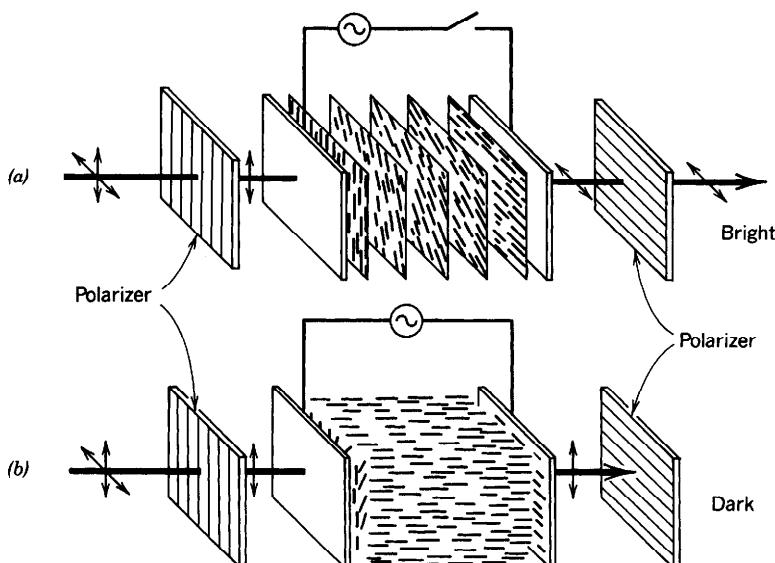


Figure 18.3-6 A twisted nematic liquid-crystal switch. (a) When the electric field is absent, the liquid-crystal cell acts as a polarization rotator; the light is transmitted. (b) When the electric field is present, the cell's rotatory power is suspended and the light is blocked.

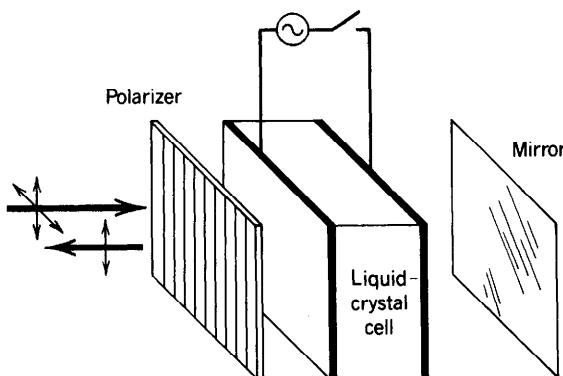


Figure 18.3-7 A twisted nematic liquid-crystal cell with 45° twist angle provides a round-trip polarization rotation of 90° in the absence of the electric field (blocked state) and no rotation when the field is applied (unblocked state). The device serves as a switch.

The twisted liquid-crystal cell placed between crossed polarizers may also be operated as an analog modulator. At intermediate tilt angles, there is a combination of polarization rotation and wave retardation. Analysis of the transmission of polarized light through tilted and twisted molecules is rather complex, but the overall effect is a partial intensity transmittance. There is an approximately linear range of transition between the total transmission of the fully twisted (untilted) state and zero transmission in the fully tilted (untwisted) state. However, the dynamic range is rather limited.

Ferroelectric Liquid Crystals

Smectic liquid crystals are organized in layers, as illustrated in Fig. 6.5-1(b). In the smectic-C phase, the molecular orientation is tilted by an angle θ with respect to the normal to the layers (the x axis), as illustrated in Fig. 18.3-8. The material has ferroelectric properties. When placed between two close glass plates the surface interactions permit only two stable states of molecular orientation at the angles $\pm\theta$, as shown in Fig. 18.3-8. When an electric field $+E$ is applied in the z direction, a torque is produced that switches the molecular orientation into the stable state $+\theta$ [Fig. 18.3-8(a)]. The molecules can be switched into the state $-\theta$ by use of an electric field of opposite polarity $-E$ [Fig. 18.3-8(b)]. Thus the cell acts as a uniaxial crystal whose optic axis may be switched between two orientations.

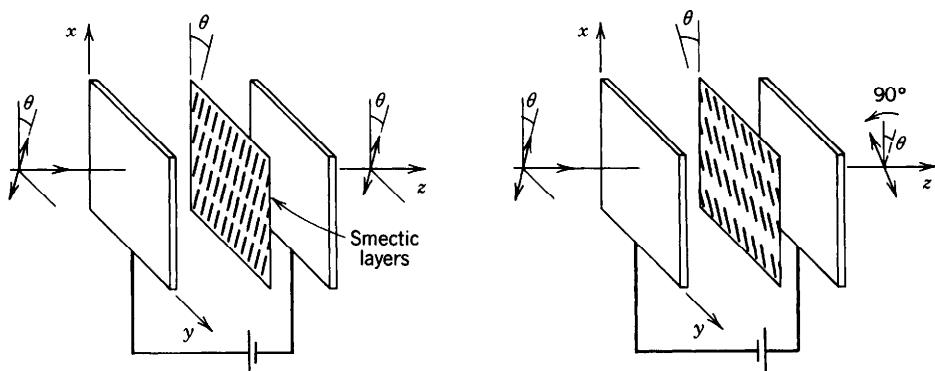


Figure 18.3-8 The two states of a ferroelectric liquid-crystal cell.

In the geometry of Fig. 18.3-8, the incident light is linearly polarized at an angle θ with the x axis in the x - y plane. In the $+θ$ state, the polarization is parallel to the optic axis and the wave travels with the extraordinary refractive index n_e without retardation. In the $-θ$ state, the polarization plane makes an angle 2θ with the optic axis. If $2\theta = 45^\circ$, the wave undergoes a retardation $\Gamma = 2\pi(n_e - n_o)d/\lambda_0$, where d is the thickness of the cell and n_o is the ordinary refractive index. If d is selected such that $\Gamma = \pi$, the plane of polarization rotates 90° . Thus, reversing the applied electric field has the effect of rotating the plane of polarization by 90° .

An intensity modulator can be made by placing the cell between two crossed polarizers. The response time of ferroelectric liquid-crystal switches is typically $< 20 \mu\text{s}$ at room temperature, which is far faster than that of nematic liquid crystals. The switching voltage is typically $\pm 10 \text{ V}$.

B. Spatial Light Modulators

Liquid-Crystal Displays

A liquid-crystal display (LCD) is constructed by placing transparent electrodes of different patterns on the glass plates of a reflective liquid-crystal (nematic, twisted-nematic, or ferroelectric) cell. By applying voltages to selected electrodes, patterns of reflective and nonreflective areas are created. Figure 18.3-9 illustrates a pattern for a seven-bar display of the numbers 0 to 9. Larger numbers of electrodes may be addressed sequentially. Indeed, charge-coupled devices (CCDs) can be used for addressing liquid-crystal displays. The resolution of the device depends on the number of segments per unit area. LCDs are used in consumer items such as digital watches, pocket calculators, computer monitors, and televisions.

Compared to light-emitting diode (LED) displays, the principal advantage of LCDs is their low electrical power consumption. However, LCDs have a number of disadvantages:

- They are passive devices that modulate light that is already present, rather than emitting their own light; thus they are not useful in the dark.
- Nematic liquid crystals are relatively slow.
- The optical efficiency is limited as a result of the use of polarizers that absorb at least 50% of unpolarized incident light.
- The angle of view is limited; the contrast of the modulated light is reduced as the angle of incidence/reflectance increases.

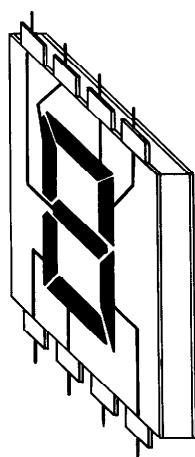


Figure 18.3-9 Electrodes of a seven-bar-segment LCD.

Optically Addressed Spatial Light Modulators

Most LCDs are addressed electrically. However, optically addressed spatial light modulators are attractive for applications involving image and optical data processing. Light with an intensity distribution $I_W(x, y)$, the “write” image, is converted by an optoelectronic sensor into a distribution of electric field $E(x, y)$, which controls the reflectance $\mathcal{R}(x, y)$ of a liquid-crystal cell operated in the reflective mode. Another optical wave of uniform intensity is reflected from the device and creates the “read” image $I(x, y) \propto \mathcal{R}(x, y)$. Thus the “read” image is controlled by the “write” image (see Fig. 18.1-14).

If the “write” image is carried by incoherent light, and the “read” image is formed by coherent light, the device serves as a spatial incoherent-to-coherent light converter, much like the PROM device discussed in Sec. 18.1E. Furthermore, the wavelengths of the “write” and “read” beams need not be the same. The “read” light may also be more intense than the “write” light, so that the device may serve as an image intensifier.

There are several means for converting the “write” image $I_W(x, y)$ into a pattern of electric field $E(x, y)$ for application to the liquid-crystal cell. A layer of photoconductive material placed between the electrodes of a capacitor may be used. When illuminated by the distribution $I_W(x, y)$, the conductance $G(x, y)$ is altered proportionally. The capacitor is discharged at each position in accordance with the local conductance, so that the resultant electric field $E(x, y) \propto 1/I_W(x, y)$ is a negative of the original image (much as in Fig. 18.1-14). An alternative is the use of a sheet photodiode [a *p-i-n* photodiode of hydrogenated amorphous silicon (a-Si : H), for example]. The reverse-biased photodiode conducts in the presence of light, thereby creating a potential difference proportional to the local light intensity.

An example of a liquid-crystal spatial light modulator is the **Hughes liquid-crystal light valve**. This device is essentially a capacitor with two low-reflectance transparent electrodes (indium–tin oxide) with a number of thin layers of materials between (Fig. 18.3-10). There are two principal layers: the liquid crystal, which is responsible for the modulation of the “read” light; and the photoconductor layer [cadmium sulfide (CdS)], which is responsible for sensing the “write” light distribution and converting it into an electric-field distribution. These two layers are separated by a dielectric mirror, which reflects the “read” light, and a light blocking dielectric material [cadmium telluride (CdTe)], which prevents the “write” light from reaching the “read” side of the device. The polarizers are placed externally (by use of a polarizing beamsplitter, for example).

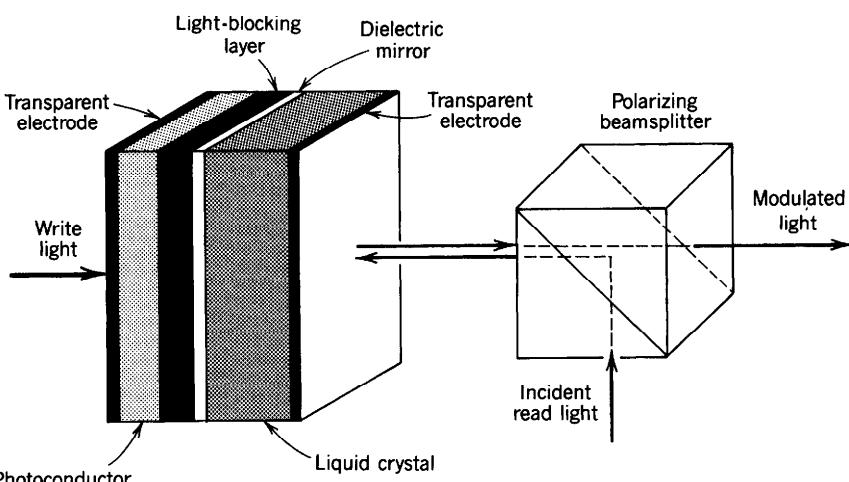


Figure 18.3-10 A liquid-crystal light valve is an optically addressed spatial light modulator.

*18.4 PHOTOREFRACTIVE MATERIALS

Photorefractive materials exhibit photoconductive and electro-optic behavior, and have the ability to detect and store spatial distributions of optical intensity in the form of spatial patterns of altered refractive index. Photoinduced charges create a space-charge distribution that produces an internal electric field, which, in turn, alters the refractive index by means of the electro-optic effect.

Ordinary *photoconductive* materials are often good insulators in the dark. Upon illumination, photons are absorbed, free charge carriers (electron–hole pairs) are generated, and the conductivity of the material increases. When the light is removed, the process of charge photogeneration ceases, and the conductivity returns to its dark value as the excess electrons and holes recombine. Photoconductors are used as photon detectors (see Sec. 17.2).

When a *photorefractive* material is exposed to light, free charge carriers (electrons or holes) are generated by excitation from impurity energy levels to an energy band, at a rate proportional to the optical power. This process is much like that in an extrinsic photoconductor (see Sec. 17.2). These carriers then diffuse away from the positions of high intensity where they were generated, leaving behind fixed charges of the opposite sign (associated with the impurity ions). The free carriers can be trapped by ionized impurities at other locations, depositing their charge there as they recombine. The result is the creation of an inhomogeneous space-charge distribution that can remain in place for a period of time after the light is removed. This charge distribution creates an *internal* electric field pattern that modulates the local refractive index of the material by virtue of the (Pockels) electro-optic effect. The image may be accessed optically by monitoring the spatial pattern of the refractive index using a probe optical wave. The material can be brought back to its original state (erased) by illumination with uniform light, or by heating. Thus the material can be used to record and store images, much like a photographic emulsion stores an image. The process is illustrated in Fig. 18.4-1 for doped lithium niobate (LiNbO_3).

Important photorefractive materials include barium titanate (BaTiO_3), bismuth silicon oxide ($\text{Bi}_{12}\text{SiO}_{20}$), lithium niobate (LiNbO_3), potassium niobate (KNbO_3), gallium arsenide (GaAs), and strontium barium niobate (SBN).

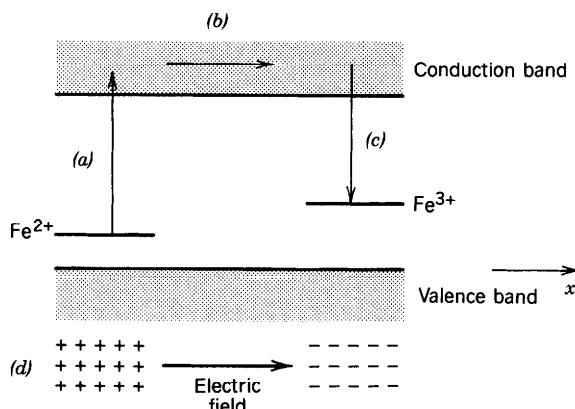


Figure 18.4-1 Energy-level diagram of LiNbO_3 illustrating the processes of (a) photoionization, (b) diffusion, (c) recombination, and (d) space-charge formation and electric-field generation. Fe^{2+} impurity centers act as donors, becoming Fe^{3+} when ionized, while Fe^{3+} centers act as traps, becoming Fe^{2+} after recombination.

Simplified Theory of Photorefactivity

When a photorefractive material is illuminated by light of intensity $I(x)$ that varies in the x direction, the refractive index changes by $\Delta n(x)$. The following is a step-by-step description of the processes that mediate this effect (illustrated in Fig. 18.4-1) and a simplified set of equations that govern them:

- *Photogeneration.* The absorption of a photon at position x raises an electron from the donor level to the conduction band. The rate of photoionization $G(x)$ is proportional both to the optical intensity and to the number density of nonionized donors. Thus

$$G(x) = s(N_D - N_D^+)I(x), \quad (18.4-1)$$

where N_D is the number density of donors, N_D^+ is the number density of ionized donors, and s is a constant known as the photoionization cross section.

- *Diffusion.* Since $I(x)$ is nonuniform, the number density of excited electrons $n(x)$ is also nonuniform. As a result, electrons diffuse from locations of high concentration to locations of low concentration.
- *Recombination.* The electrons recombine at a rate $R(x)$ proportional to their number density $n(x)$, and to the number density of ionized donors (traps) N_D^+ , so that

$$R(x) = \gamma_R n(x)N_D^+, \quad (18.4-2)$$

where γ_R is a constant. In equilibrium, the rate of recombination equals the rate of photoionization, $R(x) = G(x)$, so that

$$sI(x)(N_D - N_D^+) = \gamma_R n(x)N_D^+, \quad (18.4-3)$$

from which

$$n(x) = \frac{s}{\gamma_R} \frac{N_D - N_D^+}{N_D^+} I(x). \quad (18.4-4)$$

- *Space Charge.* Each photogenerated electron leaves behind a positive ionic charge. When the electron is trapped (recombines), its negative charge is deposited at a different site. As a result, a nonuniform space-charge distribution is formed.
- *Electric Field.* This nonuniform space charge generates a position-dependent electric field $E(x)$, which may be determined by observing that in steady state the drift and diffusion electric-current densities must be of equal magnitude and opposite sign, so that the total current density vanishes, i.e.,

$$J = e\mu_e n(x)E(x) - k_B T \mu_e \frac{dn}{dx} = 0, \quad (18.4-5)$$

where μ_e is the electron mobility, k_B is Boltzmann's constant, and T is the temperature. Thus

$$E(x) = \frac{k_B T}{e} \frac{1}{n(x)} \frac{dn}{dx}. \quad (18.4-6)$$

- **Refractive Index.** Since the material is electro-optic, the internal electric field $E(x)$ locally modifies the refractive index in accordance with

$$\Delta n(x) = -\frac{1}{2}n^3rE(x), \quad (18.4-7)$$

where n and r are the appropriate values of refractive index and electro-optic coefficient for the material [see (18.1-4)].

The relation between the incident light intensity $I(x)$ and the resultant refractive index change $\Delta n(x)$ may readily be obtained if we assume that the ratio $(N_D/N_D^+ - 1)$ in (18.4-4) is approximately constant, independent of x . In that case $n(x)$ is proportional to $I(x)$, so that (18.4-6) gives

$$E(x) = \frac{k_B T}{e} \frac{1}{I(x)} \frac{dI}{dx}. \quad (18.4-8)$$

Finally, substituting this into (18.4-7), provides an expression for the position-dependent refractive-index change as a function of intensity,

$$\boxed{\Delta n(x) = -\frac{1}{2}n^3r \frac{k_B T}{e} \frac{1}{I(x)} \frac{dI}{dx}}. \quad (18.4-9)$$

Refractive-Index
Change

This equation is readily generalized to two dimensions, whereupon it governs the operation of a photorefractive material as an image storage device.

Many assumptions have been made to keep the foregoing theory simple: In deriving (18.4-8) from (18.4-6) it was assumed that the ratio of number densities of unionized to ionized donors is approximately uniform, despite the spatial variation of the photoionization process. This assumption is approximately applicable when the ionization is caused by other more effective processes that are position independent in addition to the light pattern $I(x)$. Dark conductivity and volume photovoltaic effects were neglected. Holes were ignored. It was assumed that no external electric field was applied, when in fact this can be useful in certain applications. The theory is valid only in the steady state although the time dynamics of the photorefractive process are clearly important since they determine the speed with which the photorefractive material responds to the applied light. Yet in spite of all these assumptions, the simplified theory carries the essence of the behavior of photorefractive materials.

EXAMPLE 18.4-1. Detection of a Sinusoidal Spatial Intensity Pattern. Consider an intensity distribution in the form of a sinusoidal grating of period Λ , contrast m , and mean intensity I_0

$$I(x) = I_0 \left(1 + m \cos \frac{2\pi x}{\Lambda} \right), \quad (18.4-10)$$

as shown in Fig. 18.4-2. Substituting this into (18.4-8) and (18.4-9), we obtain the internal

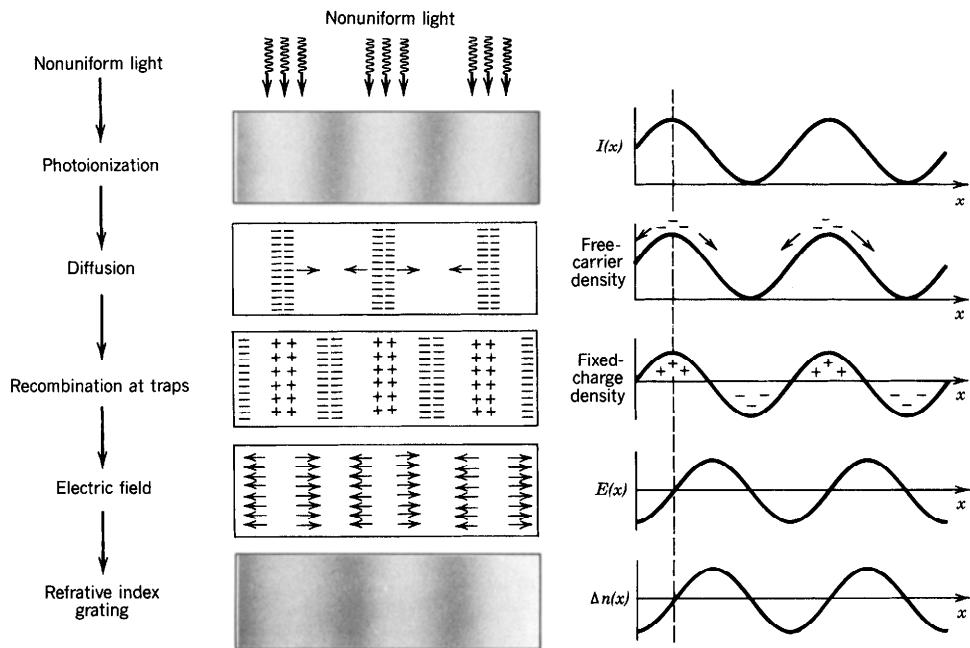


Figure 18.4-2 Response of a photorefractive material to a sinusoidal spatial light pattern.

electric field and refractive index distributions

$$E(x) = E_{\max} \frac{-\sin(2\pi x/\Lambda)}{1 + m \cos(2\pi x/\Lambda)}, \quad \Delta n(x) = \Delta n_{\max} \frac{\sin(2\pi x/\Lambda)}{1 + m \cos(2\pi x/\Lambda)}, \quad (18.4-11)$$

where $E_{\max} = 2\pi(k_B T/e\Lambda)m$ and $\Delta n_{\max} = \frac{1}{2}n^3 r E_{\max}$ are the maximum values of $E(x)$ and $\Delta n(x)$, respectively.

If $\Lambda = 1 \mu\text{m}$, $m = 1$, and $T = 300 \text{ K}$, for example, $E_{\max} = 1.6 \times 10^5 \text{ V/m}$. This internal field is equivalent to applying 1.6 kV across a crystal of 1-cm width. The maximum refractive index change Δn_{\max} is directly proportional to the contrast m and the electro-optic coefficient r , and inversely proportional to the spatial period Λ . The grating pattern $\Delta n(x)$ is totally insensitive to the uniform level of the illumination I_0 .

When the image contrast m is small, the second term of the denominators in (18.4-11) may be neglected. The internal electric field and refractive index change are then sinusoidal patterns shifted by 90° relative to the incident light pattern,

$$\Delta n(x) \approx \Delta n_{\max} \sin \frac{2\pi x}{\Lambda}. \quad (18.4-12)$$

These patterns are illustrated in Fig. 18.4-2.

Applications of the Photorefractive Effect

An image $I(x, y)$ may be stored in a photorefractive crystal in the form of a refractive-index distribution $\Delta n(x, y)$. The image can be read by using the crystal as a spatial-phase modulator to encode the information on a uniform optical plane wave

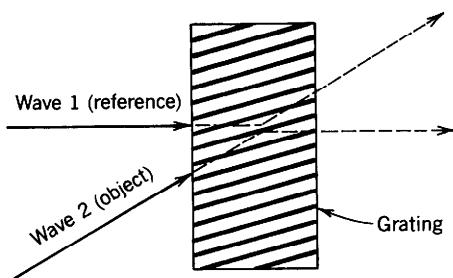


Figure 18.4-3 Two-wave mixing is a form of dynamic holography.

acting as a probe. Phase modulation may be converted into intensity modulation by placing the cell in an interferometer, for example.

Because of their capability to record images, photorefractive materials are attractive for use in real-time holography (see Sec. 4.5 for a discussion of holography). An object wave is holographically recorded by mixing it with a reference wave, as illustrated for two plane waves in Fig. 18.4-3. The intensity of the sum of two such waves forms a sinusoidal interference pattern, which is recorded in the photorefractive crystal in the form of a refractive-index variation. The crystal then serves as a volume phase hologram (see Sec. 4.5, Fig. 4.5-10). To reconstruct the stored object wave, the crystal is illuminated with the reference wave. Acting as a volume diffraction grating, the crystal reflects the reference wave and reproduces the object wave.

Since the recording process is relatively fast, the processes of recording and reconstruction can be carried out simultaneously. The object and reference waves travel together in the medium and exchange energy via reflection from the created grating. This process is called **two-wave mixing**. As shown in Fig. 18.4-3 (see also Fig. 4.5-8), waves 1 and 2 interfere and form a volume grating. Wave 1 reflects from the grating and adds to wave 2; wave 2 reflects from the grating and adds to wave 1. Thus the two waves are coupled together by the grating they create in the medium. Consequently, the transmission of wave 1 through the medium is controlled by the presence of wave 2, and *vice versa*. For example, wave 1 may be amplified at the expense of wave 2.

The mixing of two (or more) waves also occurs in other nonlinear optical materials with light-dependent optical properties, as discussed in Chap. 19. Wave mixing has numerous applications in optical data processing (see Chaps. 19 and 21), including image amplification, the removal of image aberrations, cross correlation of images, and optical interconnections.

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PROBLEMS

- 18.1-1 Response Time of a Phase Modulator.** A GaAs crystal with refractive index $n = 3.6$ and electro-optic coefficient $r = 1.6 \text{ pm/V}$ is used as an electro-optic phase modulator operating at $\lambda_o = 1.3 \mu\text{m}$ in the longitudinal configuration. The crystal is 3 cm long and has a 1-cm² cross-sectional area. Determine the half-wave voltage V_π , the transit time of light through the crystal, and the electric capacitance of the device (the dielectric constant of GaAs is $\epsilon/\epsilon_0 = 13.5$). The voltage is applied using a source with 50Ω resistance. Which factor limits the speed of the device, the transit time of the light through the crystal or the response time of the electric circuit?
- 18.1-2 Sensitivity of an Interferometric Electro-Optic Intensity Modulator.** An integrated-optic intensity modulator using the Mach-Zehnder configuration, illustrated in Fig. 18.1-5, is used as a linear analog modulator. If the half-wave voltage is $V_\pi = 10 \text{ V}$, what is the sensitivity of the device (the incremental change of the intensity transmittance per unit incremental change of the applied voltage)?
- 18.1-3 An Elasto-Optic Strain Sensor.** An elasto-optic material exhibits a change of the refractive index proportional to the strain. Design a strain sensor based on this effect. Consider an integrated-optical implementation. If the material is also electro-optic, consider a design based on compensating the elasto-optic and the electro-optic refractive index change, and measuring the electric field that nulls the reading of the photodetector in a Mach-Zehnder interferometer.
- 18.1-4 Magneto-Optic Modulators.** Describe how a Faraday rotator (see Sec. 6.4B) may be used as an optical intensity modulator.
- ***18.2-1 Cascaded Phase Modulators.** (a) A KDP crystal ($r_{41} = 8 \text{ pm/V}$; $r_{63} = 11 \text{ pm/V}$; $n_o = 1.507$, $n_e = 1.467$ at $\lambda_o = 633 \text{ nm}$) is used as a longitudinal phase modulator. The orientation of the crystal axes and the applied electric field are as shown in Examples 18.2-2 and 18.2-6. Determine the half-wave voltage V_π at $\lambda_o = 633 \text{ nm}$. (b) An electro-optic phase modulator consists of 9 KDP crystals separated by electrodes that are biased as shown in Fig. P18.2-1. How should the plates be oriented relative to each other so that the total phase modulation is maximized? Calculate V_π for the composite modulator.

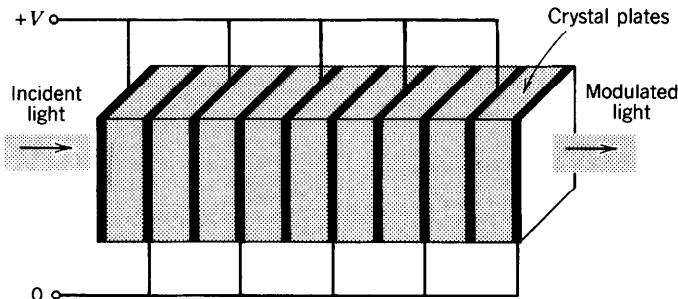


Figure P18.2-1

- *18.2-2 **The “Push-Pull” Intensity Modulator.** An optical intensity modulator uses two integrated electro-optic phase modulators and a 3-dB directional coupler, as shown in Fig. P18.2-2. The input wave is split into two waves of equal amplitudes, each of which is phase modulated, reflected from a mirror, phase modulated once more, and the two returning waves are added by the directional coupler to form the output wave. Derive an expression for the intensity transmittance of the device in terms of the applied voltage, the wavelength, the dimensions, and the physical parameters of the phase modulator.

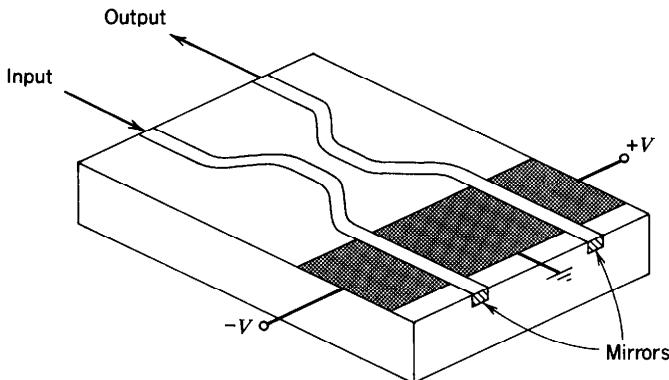


Figure P18.2-2

- *18.2-3 **A LiNbO₃ Integrated-Optic Intensity Modulator.** Design a LiNbO₃ integrated-optic intensity modulator using the Mach-Zehnder interferometer shown in Fig. 18.1-5. Select the orientation of the crystal and the polarization of the guided wave for the smallest half-wave voltage V_{π} . Assume that the active region has length $L = 1$ mm and width $d = 5 \mu\text{m}$, the wavelength is $\lambda_o = 0.85 \mu\text{m}$, the refractive indices are $n_o = 2.29$, $n_e = 2.17$; and the electro-optic coefficients are $r_{33} = 30.9$, $r_{13} = 8.6$, $r_{22} = 3.4$, and $r_{42} = 28 \text{ pm/V}$.
- *18.2-4 **Double Refraction in an Electro-Optic Crystal.** (a) An unpolarized He-Ne laser beam ($\lambda_o = 633 \text{ nm}$) is transmitted through a 1-cm-thick LiNbO₃ plate ($n_e = 2.17$, $n_o = 2.29$, $r_{33} = 30.9 \text{ pm/V}$, $r_{13} = 8.6 \text{ pm/V}$). The beam is orthogonal to the plate and the optic axis lies in the plane of incidence of the light at 45° with the beam. The beam is double refracted (see Sec. 6.3E). Determine the lateral displacement and the retardation between the ordinary and extraordinary beams. (b) If an electric field $E = 30 \text{ V/m}$ is applied in a direction parallel to the optic axis, what is the effect on the transmitted beams? What are possible applications of this device?