Exercises 7
07.04.2014

Rules: The document contains a set of 4 exercises: exercises 1 to 2 worth 2 points each, exercises 3 and 4 worth 3 points each. You need to provide a single PDF file for the solutions of all exercises. The file must be named FirstName_LastName_ex*n*.zip, where n is the number of the exercise session (see ex_set_1.pdf). The PDF file must be uploaded on ILIAS until the specified deadline.
Good luck!

Exercise 1. Use the lecture slides for Markov chains (05 Markov Models.pdf). By following the weather example (slide 12), compute the probability of the sequences:

a) {cloudy, rainy, sunny, sunny}
b) {sunny, snowy, sunny, rainy}

Exercise 2. A particle moves on a circle through points which have been marked 0, 1, 2, 3, 4 (in a clockwise order). At each step, it has a probability (p) of moving to the right and (1 - p) to the left. Let Xn denote its location on the circle after the nth step.
The process {Xn, where n equals or more than 0} is a Markov chain.

a) Find the transition probability matrix.
b) Is this Markov chain irreducible? Why?

Exercise 3. Having a transition matrix:

	A	B	C	D
A	1	0	0	0
B	.9	0	.1	0
C	0	.9	0	.1
D	0	0	0	1

a) Draw the graph of this Markov chain. Vertices are states, edges should be labeled with transition probabilities.
b) If we start in state B, what is the probability of being in state A after 8 transitions?
c) If we start in state C, what is the probability of being in state D after 6 transitions?

Exercise 4. Use the definition of the Crazy Machine in the course (06 Hidden Markov Model.pdf, slides 15-16):

a) what is the probability of the sequence O = {coffee, coffee, lemonade}? Apply the Forward Algorithm.
b) For the same sequence as before, apply the Backward Algorithm.
c) Find the state sequence that best explains the observations (argmax Prob[X|O]).