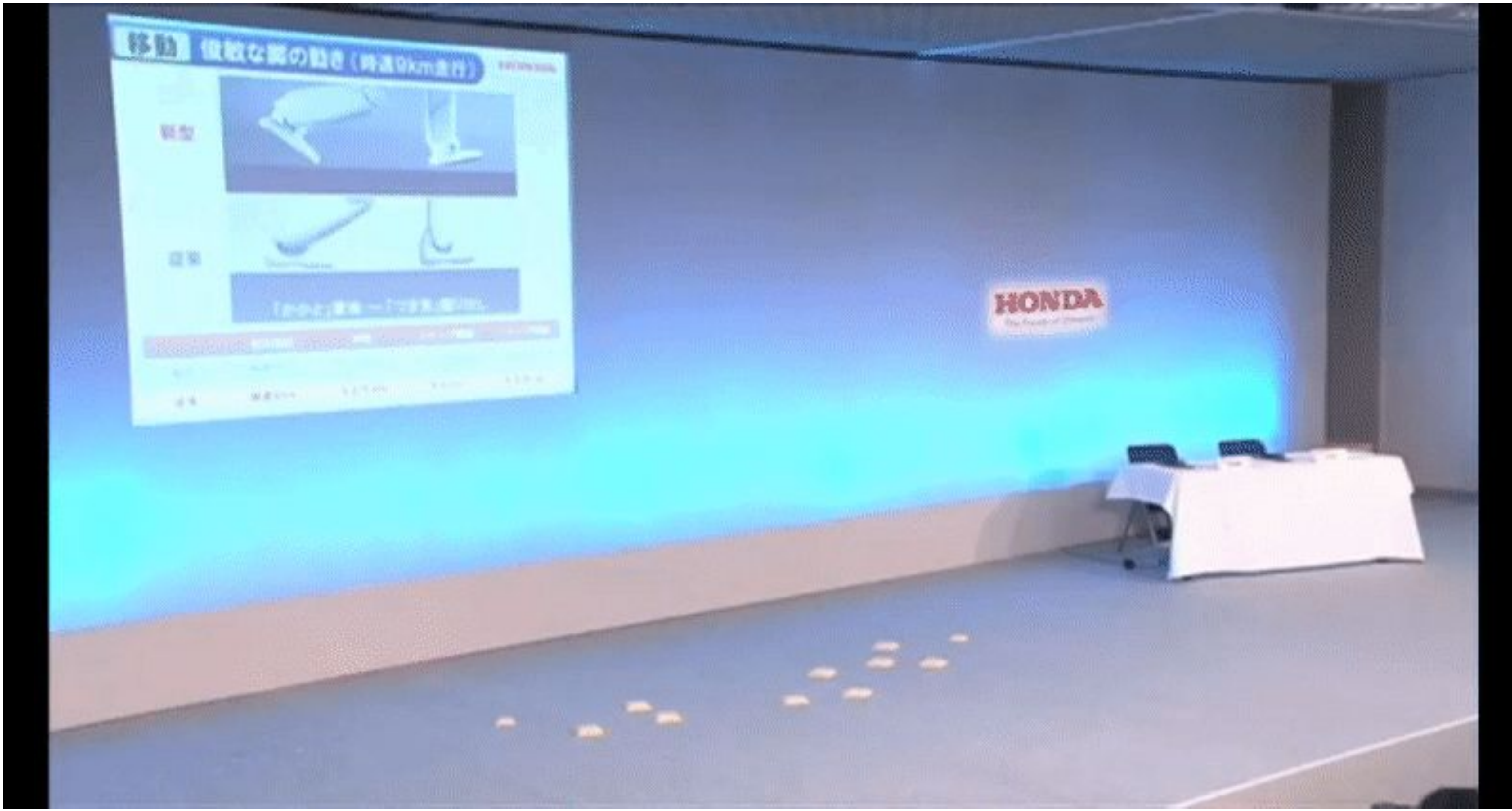
Bipedal Locomotion Control Based on Divergent Component of Motion(DCM)

Legged Robots Course

First Mini-project

Professor: Auke Ijspeert

Asimo runs at 9 Km/h by using the DCM concept for locomotion planning!



Takenaka, Toru, Takashi Matsumoto, and Takahide Yoshiike. "Real time motion generation and control for biped robot-1 st report: Walking gait pattern generation." 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2009.

IHMC came in 2nd place in Darpa Robotics Challenge(DRC) by having the DCM planner that you will implement in the project!

Englsberger, Johannes, Christian Ott, and Alin Albu-Schäffer. "Three-dimensional bipedal walking control based on divergent component of motion." Ieee transactions on robotics 31.2 (2015): 355-368.(254)

Koolen, Twan, et al. "Design of a momentum-based control framework and application to the humanoid robot atlas." International Journal of Humanoid Robotics 13.01 (2016): 1650007.(160)

Linear Inverted Pendulum (LIP) model (2D

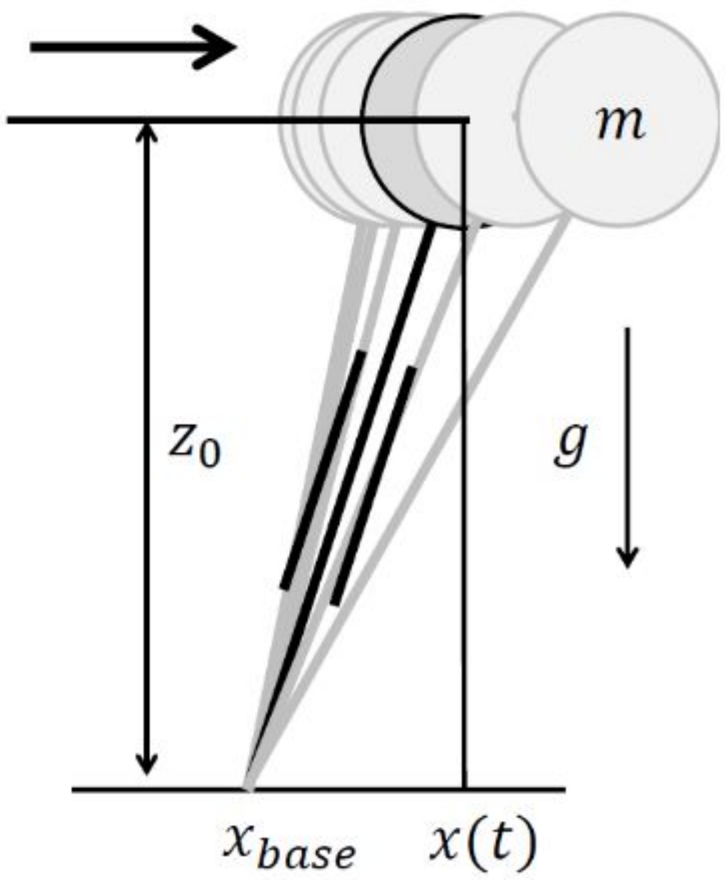
- Solving the equation of motion:

$$\ddot{x}(t) = \frac{g}{z_0}(x(t) - x_{base}),\ \omega = \sqrt{g/z_0}$$

$$x(t) = Ae^{-\omega t} + \boxed{Be^{\omega t}} + x_{base}$$

$$A = (-\dot{x}(0)/\omega + x(0) - x_{base})/2$$

$$B = \boxed{(\boxed{\dot{x}(0)/\omega + x(0)} - x_{base})/2}$$

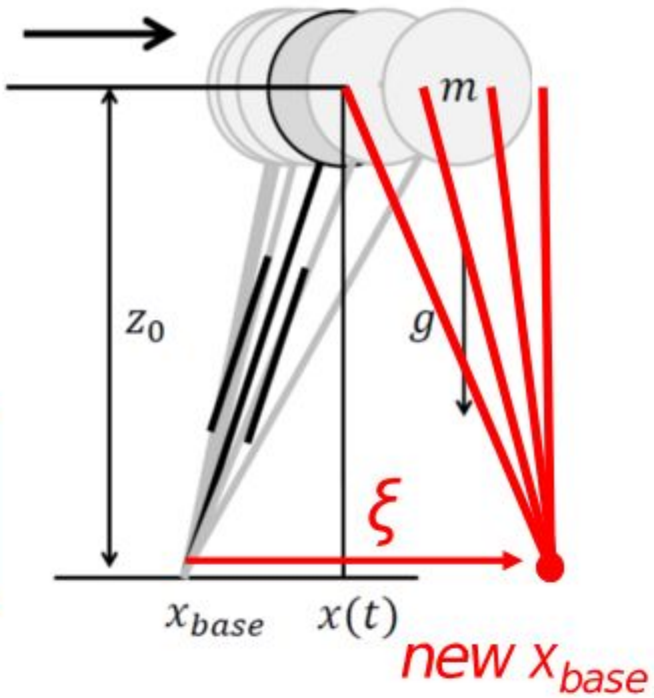


- We thus have a closed form (i.e. analytical solution) that allows us to predict the forward movement of the center of mass

Divergent component of motion (DCM)

- It is therefore important to monitor the DCM, and to plan the foot steps (i.e. x_{base}) to bring the robot (the CoM) where you want and to prevent falling.

$$\boxed{\xi = x + \frac{\dot{x}}{\omega}}$$



- The DCM corresponds to the ***instantaneous capture point.***
- Indeed by setting the next foot step (i.e. x_{base}) to ξ at the beginning of a step, the time evolution of x goes to zero (i.e. the pendulum stops).

$$\dot{\xi} = \omega(\xi - x_{base})$$

$$\dot{x} = -\omega(x - \xi)$$

- Takenaka, Toru, Takashi Matsumoto, and Takahide Yoshiike. 2009. "Real Time Motion Generation and Control for Biped Robot: 1st Report: Walking Gait Pattern Generation." In *Proceedings of* IROS 2009.
- Englsberger et al (2014). Trajectory generation for continuous leg forces during double support and heel-to-toe shift based on divergent component of motion. In *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems* (pp. 4022-4029). IEEE.

This will be tested in the mini project with the Atlas simulation!

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Linear Inverted Pendulum Model(LIPM)-Divergent Component of Motion(DCM)

$$\ddot{x}_c = \omega^2(x_c - CoP)$$

x_c Center of mass position

CoP Center of Pressure

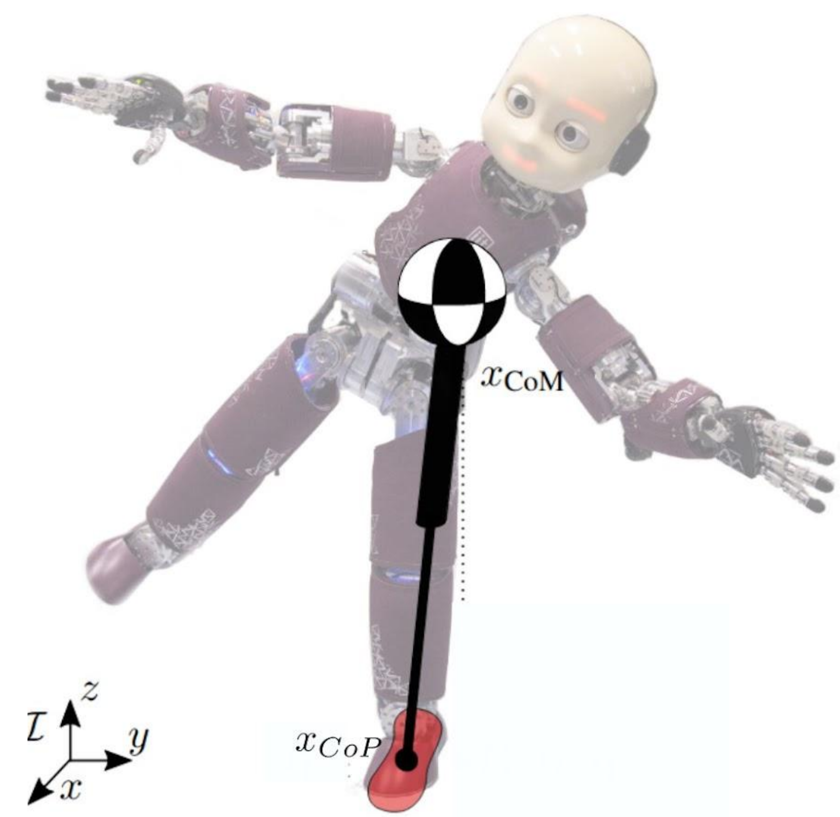
g Gravity Acceleration

z Center of mass height(constant)

$$\omega = \sqrt{\frac{g}{z}}$$

$$\xi_x = x_c + \frac{\dot{x}_c}{\omega}$$

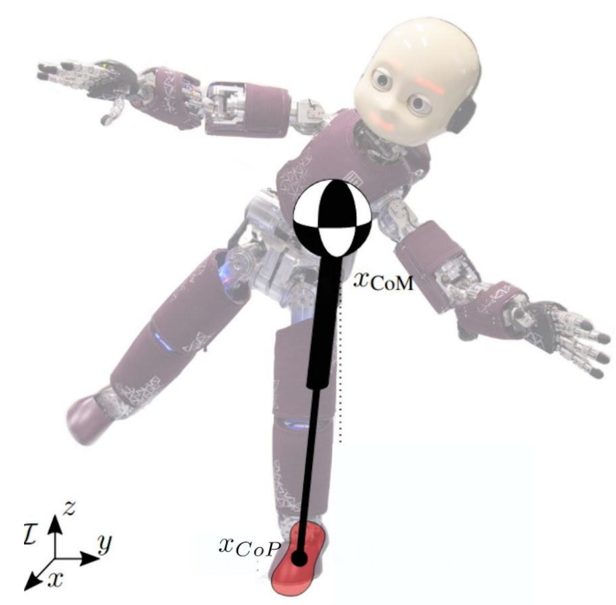
ξ DCM



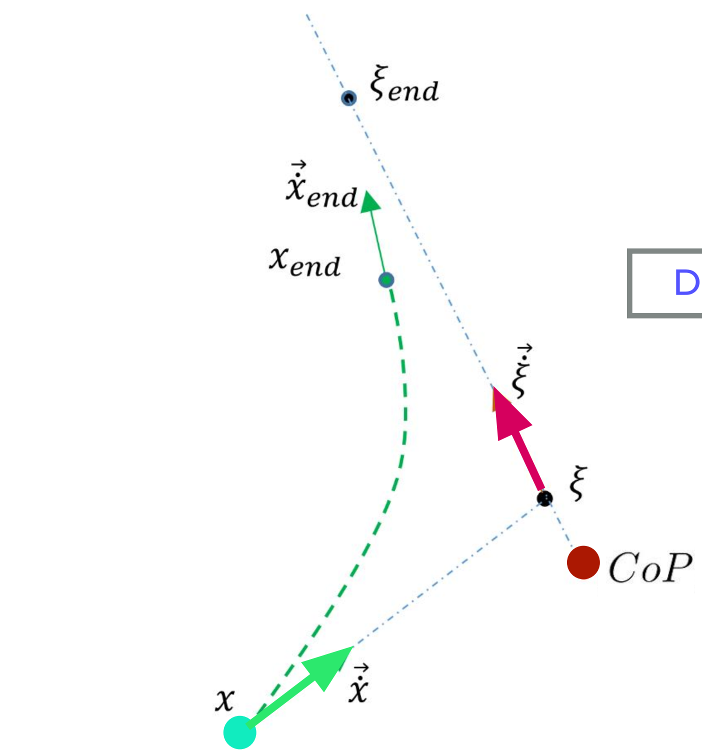
Pratt, Jerry, et al. "Capture point: A step toward humanoid push recovery." 2006 6th IEEE-RAS international conference on humanoid robots. IEEE, 2006.(Citation:1013)

DCM Dynamics

$$\ddot{x}_c = \omega^2 (x_c - CoP)$$



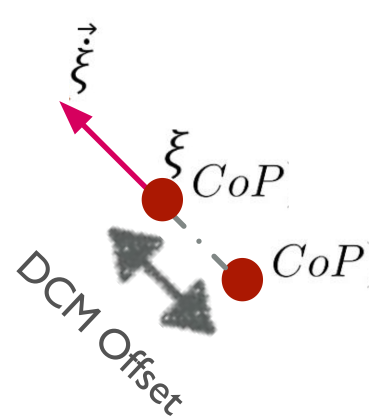
$$\xi_x = x_c + \frac{\dot{x}_c}{\omega}$$



$$\dot{\xi} = \omega(\xi - CoP) \qquad \dot{x} = -\omega(x - \xi)$$

Divergent Component of Motion(DCM) | Convergent Component of Motion

Why is capture point another name for DCM?



Koolen, Twan, et al. "Capturability-based analysis and control of legged locomotion, Part 1: Theory and application to three simple gait models." The international journal of robotics research 31.9 (2012): 1094-1113.(Citation:433)

Solution of DCM ode

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \mathbf{cop})$$

$$\downarrow$$

$$\dot{\boldsymbol{\xi}} - \omega\boldsymbol{\xi} = -\omega\,\mathbf{cop}_0$$

$$\downarrow$$

We can solve this ode by considering that CoP is a constant value (During single support phase)

$$\downarrow$$

$$\boldsymbol{\xi}(t) = e^{\int \omega dt}\left[\int (-\mathbf{cop}_0\,\omega)e^{\int -\omega dt}dt + \mathbf{C}\right],$$

Solution of DCM ode

$$\boldsymbol{\xi}(t) = e^{\int \omega dt}\left[\int (-\mathbf{cop}_0\,\omega) e^{\int -\omega dt} dt + \mathbf{C}\right],$$

$$\downarrow$$

$$\boldsymbol{\xi}(0) = \boldsymbol{\xi}_0 = \mathbf{cop}_0 + \mathbf{C}_0, \qquad \boldsymbol{\xi}(T) = \boldsymbol{\xi}_T = \mathbf{cop_0} + \mathbf{C_f}\, e^{\omega T}.$$

$$\downarrow$$

$$\boldsymbol{\xi}_0 - \mathbf{cop}_0 = (\boldsymbol{\xi}_T - \mathbf{cop}_0) e^{-\omega T}.$$

$$\downarrow$$

$$\sigma = e^{\omega T} \qquad \boldsymbol{\gamma}_\mathbf{T} = \boldsymbol{\xi}_\mathbf{T} - \mathbf{cop}_\mathbf{T}$$

$$\downarrow$$

$$\boldsymbol{\gamma}_T + \mathbf{cop}_T + (\mathbf{cop}_0 - \boldsymbol{\xi}_0)\sigma = \mathbf{cop}_0$$

Optimization based on QP

$$J = \alpha_1 |cop_{xT} - cop_{xT,nom}|^2 + \alpha_2 |cop_{yT} - cop_{yT,nom}|^2 + \alpha_3 |\gamma_{T,x} - \gamma_{nom,x}|^2 + \alpha_4 |\gamma_{T,y} - \gamma_{nom,y}|^2 + \alpha_5 |\sigma - e^{\omega T_{nom}}|^2,$$

$$\begin{bmatrix} I_2 & 0_{2\times1} & 0_2 \\ -I_2 & 0_{2\times1} & 0_2 \\ 0_{1\times2} & I_1 & 0_{1\times2} \\ 0_{1\times2} & -I_1 & 0_{1\times2} \end{bmatrix} \begin{bmatrix} \mathbf{cop}_T \\ \sigma \\ \gamma_T \end{bmatrix} \leq \begin{bmatrix} \mathbf{cop}_{T,max} \\ -\mathbf{cop}_{T,min} \\ \sigma_{max} \\ -\sigma_{min} \end{bmatrix},$$

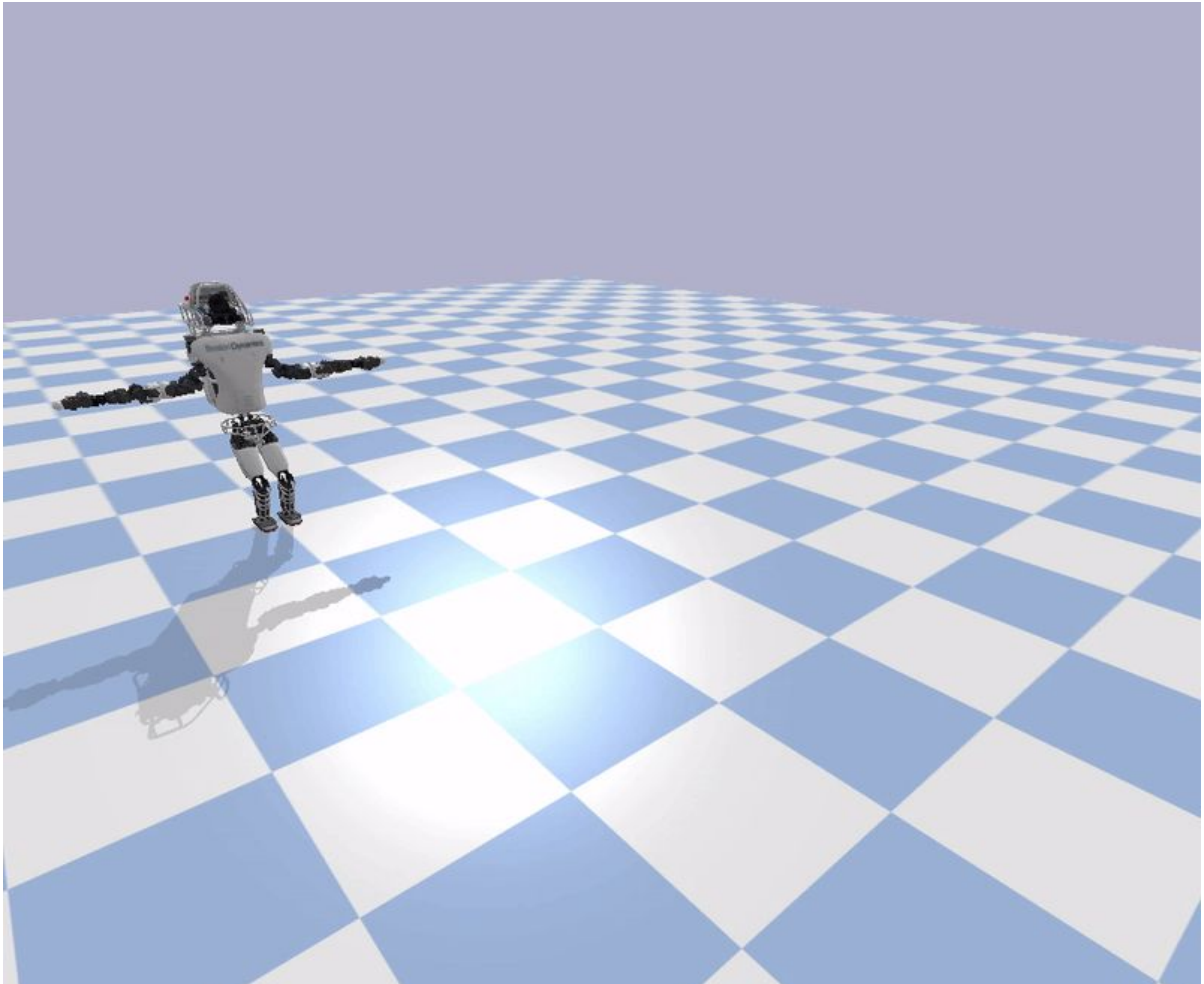
$$\boxed{\boldsymbol{\gamma}_T} + \boxed{\mathbf{cop}_T} + (\mathbf{cop}_0 - \boldsymbol{\xi}_0)\boxed{\sigma} = \mathbf{cop}_0$$

Quadratic Programming

$$\begin{array}{ll} \underset{x}{\text{minimize}} & \frac{1}{2}x^T P x + q^T x \\ \text{subject to} & Gx \leq h \\ & Ax = b \end{array}$$

Push Recovery During walking

QP Optimization based on DCM



- Please clone this repo: https://github.com/MiladShafiee/lr-biped-start
- This assignment: 15% of course grade.
- Upload your code and report.

Thank you!