Classifying Human Driving Behavior via Deep Neural Networks

A Thesis
Submitted to the Faculty
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of
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Abstract

Classifying Human Driving Behavior via Deep Neural Networks Jae Hoon Kim Santiago Ontañón, Ph.D.

The average person spends several hours a day behind the wheel of their vehicles, which are usually equipped with on-board computers capable of collecting real-time data concerning driving behavior. However, this data source has rarely been tapped for healthcare and behavioral research purposes. This MS thesis is done in the context of the *Diagnostic Driving* project, an NSF funded collaborative project between Drexel, Children Hospital of Philadelphia (CHOP) and the University of Central Florida that aims at studying the possibility of using driving behavior data to diagnose medical conditions. Specifically, this paper introduces focuses on the classification of driving behavior data collected in a driving simulator using deep neural networks. The target classification task is to differentiate novice versus expert drivers. The paper presents a comparative study on using different variants of LSTM (Long-Short Term Memory networks) and Auto-encoder networks to deal with the fact that we have a small amount of labels (16 examples of people driving in the simulator, each labeled with an "expert" or "inexpert" label), but each simulator drive is high dimensional and too densely sampled (each drive consists of 100 variables sampled at 60Hz). Our results show that using an intermediate number of neurons in the LSTM networks and using data filtering (only considering one out of each 10 samples) obtains better results, and that using Auto-encoders works worse than using manual feature selection.

Chapter 1: Introduction

Introduction Section

1. Problem Statement

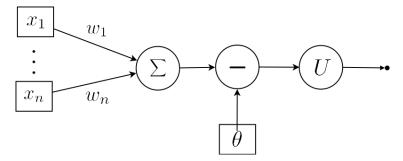


Figure 2.1: Basic Artificial Neuron

Chapter 2: Background

This chapter presents the necessary background to understand the experiments presented later in the paper. This chapter is dived into three sections. First Section 2.1 covers basic deep learning including recurrent neural network and auto-encoders. Second Section 2.2 compares deep learning libraries: DL4J, Theano, and Tensorflow. The last Section 2.3 introduces current trends in machine learning for driving data.

2.1 Deep Learning

This section covers basic concepts of deep learning. First Subsection 2.1.1 introduces deep learning, and next Section 2.1.2 deals with recurrent neural networks and long short term memory neural networks (LSTMs). The last Subsection 2.1.3 covers auto-encoders.

2.1.1 Basic Concepts

Artificial Neuron

Artificial neurons were introduced by Warren McCullock and Walter Pitts [1]. Initial artificial neurons are described as binary output logic gate from inputs. If sum of weighted inputs is greater than threshold, the output is 1. Otherwise, the output is 0.

Figure 2.1 shows a basic artificial neuron. The input is $\vec{x} = \{x_1, ..., x_n\} \in \mathbb{R}^n$ and the weights for each input are $\vec{w} = \{w_1, ..., w_n\} \in \mathbb{R}^n$. Let θ be threshold and define step function as below:

$$U(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{otherwise} \end{cases}$$

The function deciding output is called activation function. In this case, the step function U(z) is activation function.

Figure 2.1 also can be described by following equation:

$$o = U(\vec{x} \cdot \vec{w} - \theta)$$

Figure 2.2 shows an general artificial neuron. Threshold is replaced by bias and subtract node is replaced by add node. In general artificial neuron, activation function ϕ can be many different functions such as step, linear, or sigmoid function. An general artificial neuron also can be described by equation as follow:

$$o = \phi(\vec{x} \cdot \vec{w} + b)$$

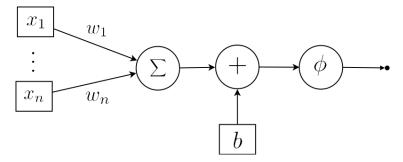


Figure 2.2: General Artificial Neuron

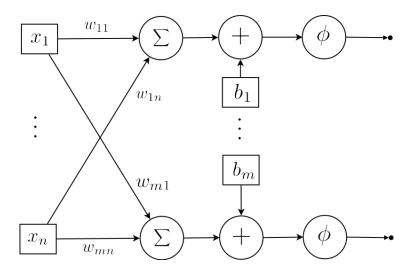


Figure 2.3: Single Layer in a Neural Network

Neural Network

Using single neuron has limitation because it can solve only few problems. For example, single neuron cannot solve xor problem. Instead of using single neuron, neural network has bundle of neurons. Figure 2.3 illustrates it with n input and m neurons. The input is $\vec{x} = \{x_1, ..., x_n\} \in \mathbb{R}^n$, weights for ith neuron are $\vec{w_i} = \{w_{i1}, ..., w_{in}\} \in \mathbb{R}^n$. The bias term for the ith neuron is b_i . The output \vec{o} of the network is calculated as follows:

$$\vec{o} = \phi(\{(\vec{x} \cdot \vec{w_1} + b_1), (\vec{x} \cdot \vec{w_2} + b_2), ..., (\vec{x} \cdot \vec{w_m} + b_m)\})$$

Where activation function is applied on each element of its input and output is m dimension vector $\vec{o} = \{o_1, ..., o_m\} \in \mathbb{R}^m$.

To simplify the equation, let a transform matrix $\mathbf{w} : \mathbb{R}^n \to \mathbb{R}^m$ be $\mathbf{w} = \begin{bmatrix} \vec{w_1} & \vec{w_2} & \cdots & \vec{w_m} \end{bmatrix}$ and bias vector $\vec{b} = \{b_1, ..., b_m\} \in \mathbb{R}^m$.

The equation is

$$\vec{o} = \phi((\vec{x}^T \mathbf{w})^T + \vec{b})$$

The bias term can be skipped by changing input vector and weight transform matrix. Let $\vec{X} \in \mathbb{R}^{n+1}$ be $\{x_1, x_2, ..., x_n, 1\}$ which is added one more dimension from \vec{x} with value 1 and $\vec{W_i} \in \mathbb{R}^{n+1}$ be $\{w_{i1}, ..., w_{in}, b_i\}$ which is added by bias term into weights and $\mathbf{W} : \mathbb{R}^{n+1} \to \mathbb{R}^m$ be $\mathbf{W} = \begin{bmatrix} \vec{W_1} & \vec{W_2} & \cdots & \vec{W_m} \end{bmatrix}$

Figure 2.4 shows same neural network as Figure 2.3 with \vec{X} and \mathbf{W} . Notice that the figure does not have add node anymore.

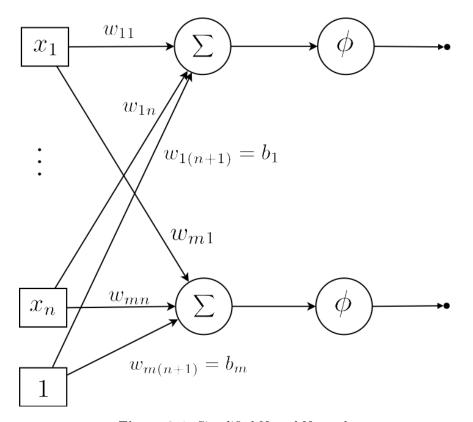


Figure 2.4: Simplified Neural Network

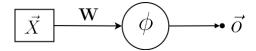


Figure 2.5: Simplified Neural Network

From this point on the paper, vector in large case is with bias term and vector in small case is without bias term. For example, \vec{X} is with bias and \vec{x} is without bias. Also all vectors is row vector. Transform matrix is bold character and if it includes bias, transform matrix uses large case, otherwise small case. For example, \mathbf{W} is transform matrix with bias and \mathbf{w} is transform matrix without bias. By applying this, the equation for neural network is

$$\vec{o} = \phi(\vec{X}\mathbf{W})$$

For all figures under this point, Σ node is skipped, square shape node represents input node, circle shape node represents compute node, edge with transform matrix represents weight matrix, edge without transform matrix represents identity matrix for weight matrix and arrow with small filled circle on end represents output vector. Figure 2.3, Figure 2.4, and 2.5 are equal.

Multilayer Neural Network

Single layer neural network has input layer and output layer but multilayer neural network has several hidden layers between input layer and output layer. This part introduces to multilayer neural network by explaining neural network with one hidden layer. Let $\vec{x} \in \mathbb{R}^n$ be input, $\vec{h} \in \mathbb{R}^m$ be output from hidden layer, $\vec{c} \in \mathbb{R}^m$ be bias for hidden layer, $\vec{o} \in \mathbb{R}^l$ be bias for output layer, $\mathbf{u} : \mathbb{R}^n \to \mathbb{R}^m$ be transform matrix for hidden layer and $\mathbf{w} : \mathbb{R}^m \to \mathbb{R}^l$ be transform matrix for output layer. The activation function \mathbf{h} is for hidden layer and the activation

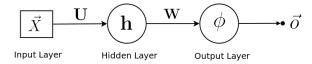


Figure 2.6: Two Layer Neural Network

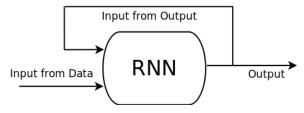


Figure 2.7: RNN

function ϕ is for output layer. Figure 2.6 shows two layer neural network. The output from hidden layer can be computed

$$\vec{h} = \mathbf{h}(\vec{x}\mathbf{u} + \vec{c}) = \mathbf{h}(\vec{X}\mathbf{U})$$

The output of the neural network can be computed

$$\vec{o} = \phi(\vec{h}\mathbf{w} + \vec{b}) = \phi(\vec{H}\mathbf{W})$$

Therefore, final equation for two layer neural network is

$$\vec{o} = \phi(\mathbf{h}(\vec{x}\mathbf{u} + \vec{c})\mathbf{w} + \vec{b})$$

Backpropagation

add backpropagation

2.1.2 Recurrent Neural Network

Basic concepts of recurrent neural networks:

A recurrent neural network (RNN) is a neural network that is specialized for processing a sequence of input values [2]. Figure 2.7 illustrates abstract structure of RNN. RNN has two input. One input is from data such as normal neural network input but another input is from previous output. The property gives benefit for sequential input data. This is because past outputs affect to current output. With sequential data, previous data can affect to current data and RNN considers previous outputs for current output. It means that even input from data are equal if previous outputs are different, current output are also different.

For example, human language sentences have series of words and meaning of words is different in different context. RNN can be used for that. Another example is driving data which is used later on the paper. Driving data are multi-dimension data with time domain. RNN can be used for the data because it is sequential data with time domain.

To describe RNN in math equation, let $\vec{x^t} = \{x_1^t, ..., x_n^t\} \in \mathbb{R}^n$ be a vector that represents the input data at time t, $\vec{h^t} = \{h_1^t, ..., h_m^t\} \in \mathbb{R}^m$ be result from hidden layer on time t, and $\vec{o^t} = \{o_1^t, ..., o_l^t\} \in \mathbb{R}^l$ be output on time t. For transform matrix, let $\mathbf{u} : \mathbb{R}^n \to \mathbb{R}^m$ be a transform matrix for input data, $\mathbf{v} : \mathbb{R}^m \to \mathbb{R}^l$ be a transform matrix for data from hidden layer, and $\mathbf{w} : \mathbb{R}^l \to \mathbb{R}^m$ be a transform matrix for previous output. Figure 2.8 illustrates unfolded RNN with defined symbols.

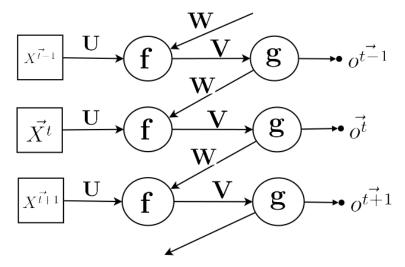


Figure 2.8: Unfold RNN

The result from hidden layer can be calculated by

$$\vec{h^t} = \mathbf{f}(\vec{X^t}\mathbf{U} + \vec{O^{t-1}}\mathbf{W})$$

where \mathbf{f} is activation function for hidden layer. Then the output can be calculated by

$$\vec{o^t} = \mathbf{g}(\vec{H^t}\mathbf{V})$$

where g is activation function for output layer. Therefore, the RNN with bias is

$$\vec{o^t} = \mathbf{g}(\mathbf{f}(\vec{x^t}\mathbf{u} + \vec{b_x} + \vec{o^{t-1}}\mathbf{w} + \vec{b_o})\mathbf{v} + \vec{b_h})$$

where $\vec{b_x} = \{b_{x1}, ..., b_{xm}\} \in \mathbb{R}^m$ is bias for input data, $\vec{b_o} = \{b_{o1}, ..., b_{om}\} \in \mathbb{R}^m$ is bias for previous output data, and $\vec{b_h} = \{b_{h1}, ..., b_{hl}\} \in \mathbb{R}^l$ is bias for data from hidden layer.

RNN is not always like Figure 2.8. Input from previous output can be replaced by result of previous hidden layer. The main idea of RNN is when neural network decides current output it considers previous state.

Long Short Term Memory (LSTM) networks:

Basic RNN has the problems of long term dependency. When RNN passes previous output for current output, some information in the input might not need or might need for future. RNN does not have ability to filter unnecessary information or to store necessary information. Long Short Term Memory (LSTM) neural network can handle these problems.

LSTM is a type of RNN and introduced by Hochreiter and Schmidhuber [3]. LSTM solves long term dependency problems in RNN by memory cells. LSTM manages memory cells as storage of knowledge. LSTM filters unnecessary information from memory cells and records necessary information on memory cells.

The structure of LSTM consists of four gates: forget gate, input gate, input modulation gate, and output gate. Each gates have different purpose and Christopher Olah's blog ¹ develops an intuition for concept of LSTM and explains purpose of each gates. The paper [4] describes LSTM in mathematical term. Let us first provide an intuitive description of how LSTM work by following Christopher Olah's blog, before providing a more in-depth formalization.

1. An intuitive explanation of LSTMs:

¹http://colah.github.io/posts/2015-08-Understanding-LSTMs

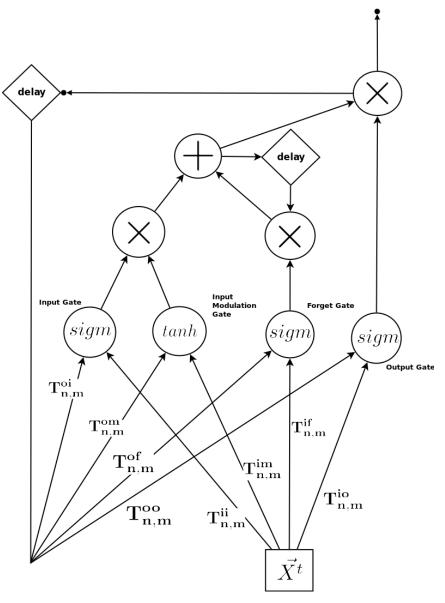


Figure 2.9: LSTM

The easiest way to understand LSTM is to understand memory cell and purpose of four different gates. Memory cell makes LSTM different from RNN. Memory cell stores important information and filters unimportant information for future. Three of four gates are involved in the memory cells to store and filter information.

The first gate affecting to memory cells is forget gate. The forget gate decides unnecessary data from input data and previous output then applies it to memory cells. Other two gates are input gate and input modulation gate and these also affect to memory cells. The two gates decide what information remember then apply it to memory cells. The last gate is output gate and it does not directly affect to memory cells. Output gate also has two inputs from input data and previous output, and output from output gate is multiplied by memory cells to make final output. Figure 2.9 illustrates LSTM neural network with four gates and next part describes more detail of LSTM and the figure in mathematical terms.

2. Modeling LSTMs in mathematical terms:

To describe LSTM in mathematical terms, let $\vec{x^t} = \{x_1^t, ..., x_n^t\} \in \mathbb{R}^n$ be a vector that represents the input data at time $t, \ \vec{i^t} = \{i_1^t, ..., i_m^t\} \in \mathbb{R}^m$ be result from input gate on time $t, \ \vec{m^t} = \{m_1^t, ..., m_m^t\} \in \mathbb{R}^m$ be result from input modulation gate on time $t, \ \vec{f^t} = \{f_1^t, ..., f_m^t\} \in \mathbb{R}^m$ be result from output gate on time $t, \ \vec{c^t} = \{c_1^t, ..., c_m^t\} \in \mathbb{R}^m$ be memory cells on time $t, \ and \ \vec{h^t} = \{h_1^t, ..., h_m^t\} \in \mathbb{R}^m$ be final result on time t. Each gates have transform matrices for inputs. Let $\mathbf{t_{n,m}} : \mathbb{R}^n \to \mathbb{R}^m$ be a transform matrix from n dimension to n dimension. Let $\mathbf{t_{n,m}^{ii}} = \mathbf{t_{n,m}^{ii}} = \mathbf{t_{n,m}^{ii}}$

The result of each gates are computed as following:

$$i^{\vec{t}} = \mathbf{sigm}(\vec{X}^t \mathbf{T}_{\mathbf{n} \mathbf{m}}^{\mathbf{i} \mathbf{i}} + H^{\vec{t}-1} \mathbf{T}_{\mathbf{m} \mathbf{m}}^{\mathbf{o} \mathbf{i}})$$

Input gate uses sigmoid function **sigm**() for activation function

$$\vec{m^t} = \mathbf{tanh}(\vec{X^t}\mathbf{T_{n,m}^{im}} + \vec{H^{t-1}}\mathbf{T_{m,m}^{om}})$$

Input modulation gate uses tanh function tanh() for activation function.

$$\vec{f}^t = \mathbf{sigm}(\vec{X^t} \mathbf{T_{n,m}^{if}} + \vec{H^{t-1}} \mathbf{T_{m,m}^{of}})$$

Forget gate uses sigmoid function sigm() for activation function.

$$\vec{o^t} = \mathbf{sigm}(\vec{X^t}\mathbf{T_{nm}^{io}} + H^{\vec{t}-1}\mathbf{T_{mm}^{oo}})$$

Output gate uses sigmoid function **sigm**() for activation function.

Memory cells are computed as

$$\vec{c^t} = \vec{i^t} * \vec{m^t} + \vec{f^t} * \vec{c^{t-1}}$$

And final result is computed as

$$\vec{h^t} = \vec{c^t} * \vec{o^t}$$

Where * is component-wise multiplication of two vectors.

LSTM neural network described above is one example of LSTMs. There are many other LSTMs but all LSTMs has memory cells to store information for long term memory and has four gates: input, input modulation, forget, and output gate.

2.1.3 Auto-encoder

This subsection introduces Auto-encoder and different kind of Auto-encoder: undercomplete Auto-encoder, overcomplete Auto-encoder, and multilayer Auto-encoder.

Basic Auto-encoder

An Auto-encoder (AE) is defined as a neural network that is trained to attempt to copy its input to its output [2]. It means that AE takes input and sends it as output. However, AE does not directly send input to output. AE has two layers: encode layer and decode layer. Figure 2.10 illustrates abstract structure of AE. Purpose of AE is to make output from decode same as input.

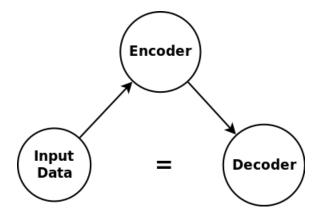


Figure 2.10: Abstract structure of Auto-encoder

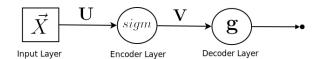


Figure 2.11: Basic Auto-encoder

To describe AE, let $\vec{x} = \{x_1, ..., x_n\} \in \mathbb{R}^n$ be input, $\vec{e} = \{e_1, ..., e_m\} \in \mathbb{R}^m$ be output from encoder layer, and $\vec{d} = \{d_1, ..., d_n\} \in \mathbb{R}^n$ be output from decoder layer. For transform matrix, let $\mathbf{u} : \mathbb{R}^n \to \mathbb{R}^m$ be a transform matrix for encode layer, $\mathbf{v} : \mathbb{R}^m \to \mathbb{R}^n$ be a transform matrix for decode layer. Usually AE uses sigmoid for activation function. Figure 2.11 describes AE.

The output of encoder is

$$\vec{e} = \mathbf{f}(\vec{X}\mathbf{U})$$

Where f is the activation function for encoder layer. It is usually sigmoid function.

The output of decoder is

$$\vec{d} = \mathbf{g}(\vec{E}\mathbf{V})$$

Where **g** is the activation function for decoder layer. It is usually sigmoid or linear function.

The purpose of AE copies input as output. Therefore, cost or loss function is calculated by

$$L(\vec{x}, \vec{d}) = L(\vec{x}, \mathbf{g}(\vec{E}\mathbf{V})) = L(\vec{x}, \mathbf{g}(\mathbf{f}(\vec{x}\mathbf{u} + \vec{b_e})\mathbf{v} + \vec{b_d}))$$

Where $\vec{b_e} = \{b_{e1}, ..., b_{em}\} \in \mathbb{R}^m$ is bias for encoder layer and $\vec{b_d} = \{b_{d1}, ..., b_{dm}\} \in \mathbb{R}^n$ is bias for decoder layer.

AE seems not useful because it only tries to copy input. However, AE is usually used with other neural network and other neural network uses data from encoder layer of AE not from decoder layer. Making similar values of input and decoder output guarantees that result from encode layer contains all information of input. It means that AE can represent same information of input data in different dimension. When dimension of encoder is higher than dimension of input, it is called overcomplete Auto-encoder. When dimension of encoder is lower than dimension of input, it is called undercomplete Auto-encoder.

On this paper and experiment, undercomplete AE is used for reducing input dimension. Undercomplete AE is often compared with PCA because both methods reduce dimension. The paper [5] compares undercomplete AE and PCA. The result on the paper is that undercomplete AE could keep more information than PCA. It means that reducing data to low dimension by undercomplete AE gives better performance. However, training AE takes long time than computing information gain for PCA.

Multilayer Auto-encoder

need to add tutorial

2.2 Deep Learning Library

2.2.1 DL4J

 $\mathrm{DL4J}$ ² is open source, distributed deep-learning library for Java and Scala. It also integrates with Hadoop and Spark.

2.2.2 Theano

The ano 3 is a Python library for machine learning research. It integrates with Numpy and uses GPU power. The library is written in C code and optimizes user functions.

2.2.3 Tensorflow

Tensorflow ⁴ is an open source software library for numerical computation using data flow graphs. The biggest difference from other libraries is that Tensorflow treats all operator as node. Another strong point is that Tesorflow allows developer to deploy computation to one or more CPUs or GPUs.

2.3 Machine Learning with Driving Data

 $^{^2 {\}tt https://deeplearning4j.org/}$

³http://deeplearning.net/software/theano/

⁴https://www.tensorflow.org/

Chapter 3: Data Set

The drive simulation data is from *Children's Hospital of Philadelphia* (CHOP). The simulator for the data can record 100 features with 60 samples per second from driver. The simulator has four different tracks. On the paper, the data has 16 set: 8 from expert and 8 from inexpert. Each expert and inexpert data set consist of two set of four different tracks.

Chapter 4: Technical Approach

4.1 Cross Validation

There are only 16 data set. It is not enough to train neural network for general case. Whether the neural network works well on the problem or not, I used cross validation. I split 16 data set to 4 groups. Each groups have 2 expert and 2 inexpert data. The neural network is trained four times with different train (3 groups) and test (1 group) set.

4.2 LSTM

The LSTM neural network is used to classify data because the data has time domain and LSTM neural network can handle serial data. On the paper, LSTM neural network is built with 16, 32, 64, 128, and 256 hidden neurons. The output from LSTM is sent to output layer which has two neurons. By using softmax, output from two neurons is classified. If it has [0, 1], it is classified to expert. Otherwise, it is classified to inexpert.

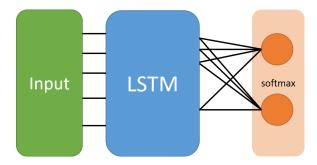


Figure 4.1: LSTM without Auto-Encoder

4.3 Dimensionality Reduction

The data has 100 features so dimensionality reduction is necessary. First way is selecting meaningful features and 23 features are selected. Another way to reduce dimension is auto-encoder. Before giving data to LSTM neural network, data is sent to auto-encoder layer and less dimension data from auto-encoder is sent to LSTM.

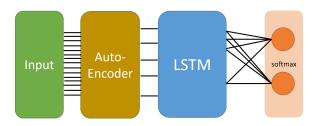


Figure 4.2: LSTM with Auto-Encoder

4.4 Sampling

The simulator records 60 samples per second. It is too often recorded. I reduce data by two way. First way is sampling one per ten and sampling one per twenty. The sampled data might not

represent ten or twenty data, so average data from every ten or twenty data is used. The second way can reduce noise because there is case in first way that sampled data has more noise than other data.

Chapter 5: Experiment Result

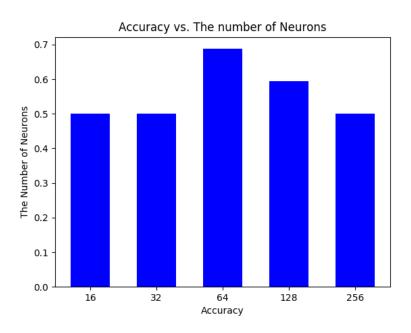


Figure 5.1: Accuracy

Table 5.1: Raw 1/10 Result Summary

		The number of neurans								
Test	n	16	32	64	128	256				
	1	0.5	0.25	1.0	0.5	0.5				
1	2	0.75	0.75	0.5	0.5	0.5				
1	3	1.0	0.5	0.5	0.75	0.5				
	4	0.5	0.5	0.75	0.5	0.5				
	1	0.5	0.75	0.75	0.75	0.75				
2	2	0.25	0.25	0.75	0.5	0.25				
	3	0.5	0.5	0.25	0.75	0.25				
	4	0.25	0.25	0.5	0.25	0.75				
	1	0.75	0.25	1.0	0.75	0.25				
3	2	0.25	0.5	0.75	0.5	1.0				
'	3	0.5	0.75	0.75	0.5	0.5				
	4	0.5	0.25	0.75	0.75	0.5				
	1	0.75	0.5	0.5	0.5	0.5				
4	2	0.25	0.5	0.5	1.0	0.25				
'	3	0.0	0.75	1.0	0.75	0.25				
	4	0.75	0.75	0.75	0.25	0.75				
Avera	ıge	0.5	0.5	0.6875	0.59375	0.5				

Chapter 6: Conclusion and Future work

Conclusion and Future work

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